Design and Behavior of Dapped-End Beams

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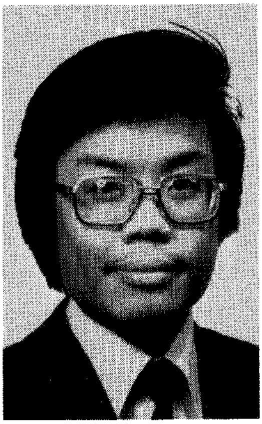

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The dapped-end beam is a useful concept. It enables the construction depth of a precast concrete floor or roof structure to be reduced, by recessing the supporting corbels into the depth of the beams supported.

In a "cantilever and suspended span" type of structure, the suspended span is a dapped-end beam, and the ends of the supporting cantilevers are similar to the ends of the dapped-end beams, but inverted.

The use of dapped-end beams facilitates the erection of a precast concrete structure, due to the greater lateral stability of an isolated dapped-end beam than that of an isolated beam supported at its bottom face.

Despite the fairly extensive use made of this form of construction, few studies[1,2] appear to have been made of its behavior. These were primarily analytical, utilizing the finite element method to analyze the stresses in and around the dapped end under service load conditions.

From these analyses, Werner and Dilger[2] were able to predict the shear at which diagonal tension cracking would occur at the re-entrant corner. They also proposed that the shear strength of the dapped end could be calculated using:

$$V_n = V_c + V_p + V_s$$

where

NOTE: This paper, which is based on a study supported by a PCI Research Fellowship, has been reviewed by the PCI Technical Activities Committee and is hereby endorsed for publication in the PCI JOURNAL.

Reports the results of an experimental investigation on reinforced concrete dapped-end beams subject to vertical load only and to combinations of vertical and horizontal load.
Based on these test results and a consideration of equilibrium conditions, a design procedure for reinforced or prestressed dapped-end beams is proposed and a design example is given.

V_c = shear at diagonal tension cracking
V_p = vertical component of prestress force for tendon anchored in the dapped end
V_s = shear force in web reinforcement near end face of beam

The present study was undertaken to provide an improved understanding of the behavior of dapped-end beams both at service load and at ultimate, with a view to developing a rational design procedure.

Preliminary Considerations

In many respects the nib (or reduced depth part) of a dapped end resembles an inverted corbel. However, in the case of the corbel, the inclined concrete compression force in the corbel is resisted by a compression force in the column (Fig. 1a); but in the case of the dapped end, the inclined compression force in the nib must be resisted by a tension force in the stirrup reinforcement placed close to the full-depth end face of the beam (Fig. 1b). For equilibrium of the nib, it appears that this stirrup reinforcement must provide a tension force equal to the shear force acting on the nib.

The strength of the full-depth part of the dapped end will be adversely affected by the formation of diagonal tension cracks. The principal diagonal tension cracks will be those originating at the re-entrant corner A and at the bottom corner B of the full-depth beam (see Fig. 2). Sufficient reinforcement must cross these cracks to prevent failure.

In the light of these considerations, the design of the test specimens was based on the following initial proposals:

1. Design the reduced depth part of the dapped end as if it were a corbel, using the design proposals of Mattock.[3]

2. Provide a group of closed stirrups close to the end face of the full depth beam, to resist the vertical component of the inclined compression force in the nib; that is, the yield strength of this stirrup group $A_{vh}f_y$, to be not less than V_u/ϕ.

3. Design the full depth part of the beam so as to satisfy moment and force equilibrium requirements across inclined cracks AY and BZ shown in

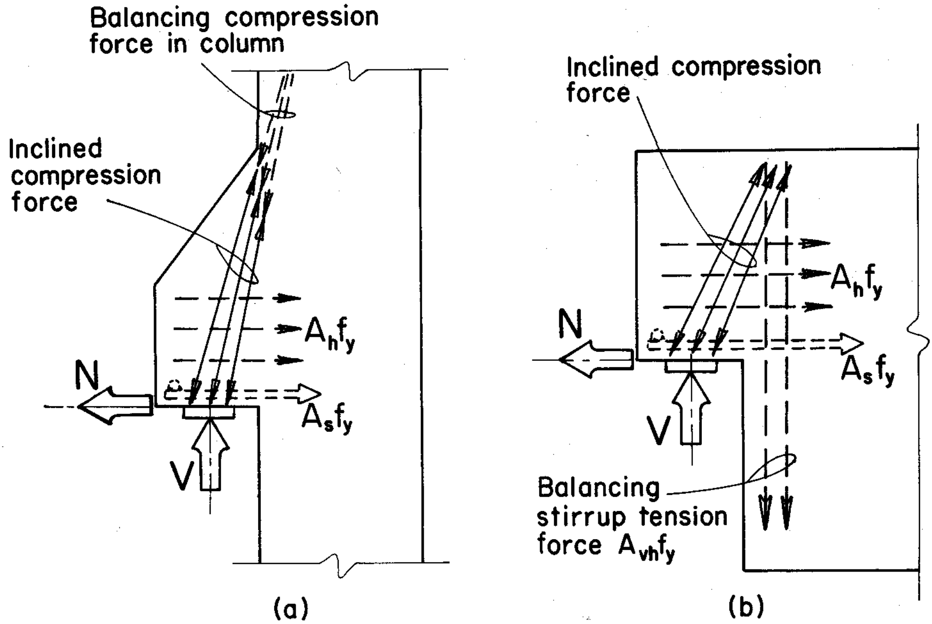

Fig. 1. Comparison of internal force systems: (a) in corbel on a column and (b) in a dapped-end beam.

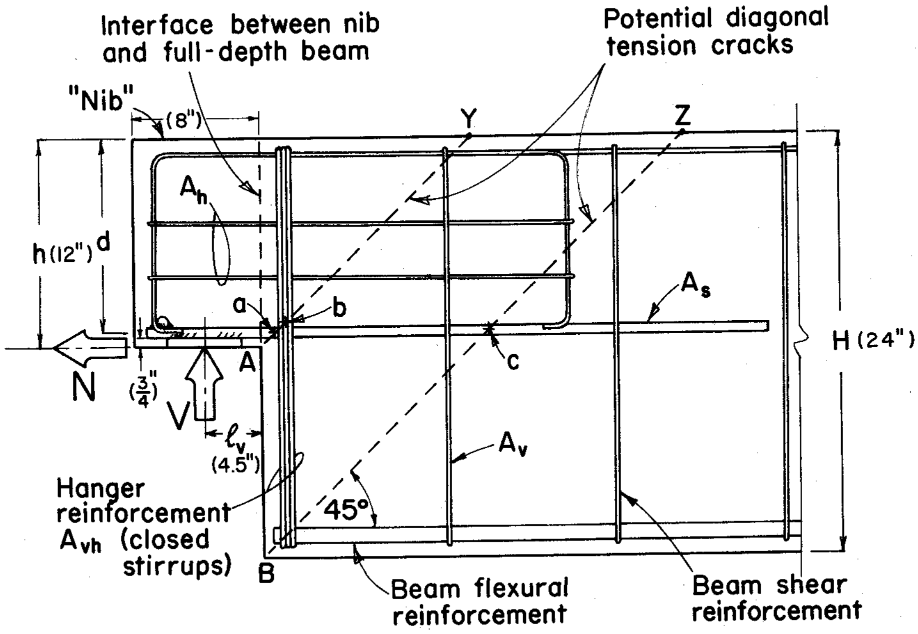

Fig. 2. Typical dapped-end reinforcement and location of potential diagonal tension cracks. Note that the dimensions in parentheses are those of the test specimens.

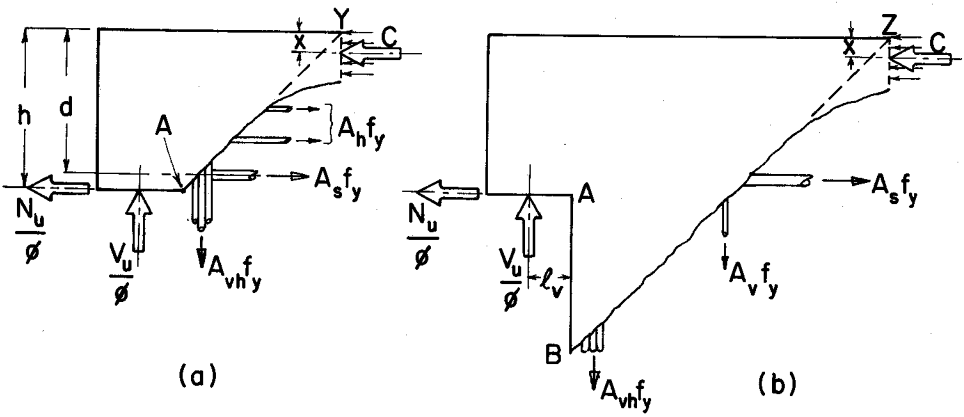

Fig. 3. Forces assumed acting on free bodies cut off by diagonal tension cracks in full-depth beam.

Fig. 2 in addition to carrying out the usual design of sections normal to the longitudinal axis of the beam for flexure and shear.

The horizontal stirrup reinforcement A_h was provided in the form of closed ties which extended 1.5 l_d [18 in. (or 457 mm)] beyond the nib-beam interface, as recommended in the *PCI Design Handbook*.[4] The main nib reinforcement A_s was extended a distance 1.4 l_d beyond the point at which the potential crack BZ crosses it, in order to develop its yield strength at that location.

When checking moment equilibrium about Point Y of that part of the dapped end to the left of AY in Fig. 2, it was assumed that A_s, A_h and A_{vh} all developed their yield strength, as indicated in Fig. 3a.

When checking moment equilibrium about Point Z of that part of the dapped end to the left of BZ in Fig. 2, it was assumed that A_s, A_{vh}, and the beam stirrup A_v all developed their yield strength, as indicated in Fig. 3b. In both cases, the depth of the compression zone and the location of the resultant concrete compression force C were calculated using the equivalent rectangular stress distribution of the ACI Building Code.[5]

Experimental Study

The test program was conducted at the structural engineering laboratories of the University of Washington.

Test Specimens

Eight dapped ends were tested, four being subjected to vertical load only, and four to a combination of vertical and horizontal loads. The dapped ends were formed on opposite ends of 5 x 24-in. (127 x 610 mm) cross section beams, 10 ft (3.05 m) long. The nibs all had a length of 8 in. (203 mm) and an overall depth of 12 in. (305 mm), i.e., one-half the overall depth of the beam.

Typical reinforcement details are shown in Fig. 2. The sizes and amounts of reinforcement in each specimen are listed in Table 1, together with the concrete strength at the time of testing. A ¾-in. (19 mm) cover was provided to the stirrups and the main dapped-end reinforcement.

The design of successive specimens differed. The design criteria were changed after study of the behavior of preceding specimens. The design of the specimens will therefore be discussed along with their behavior.

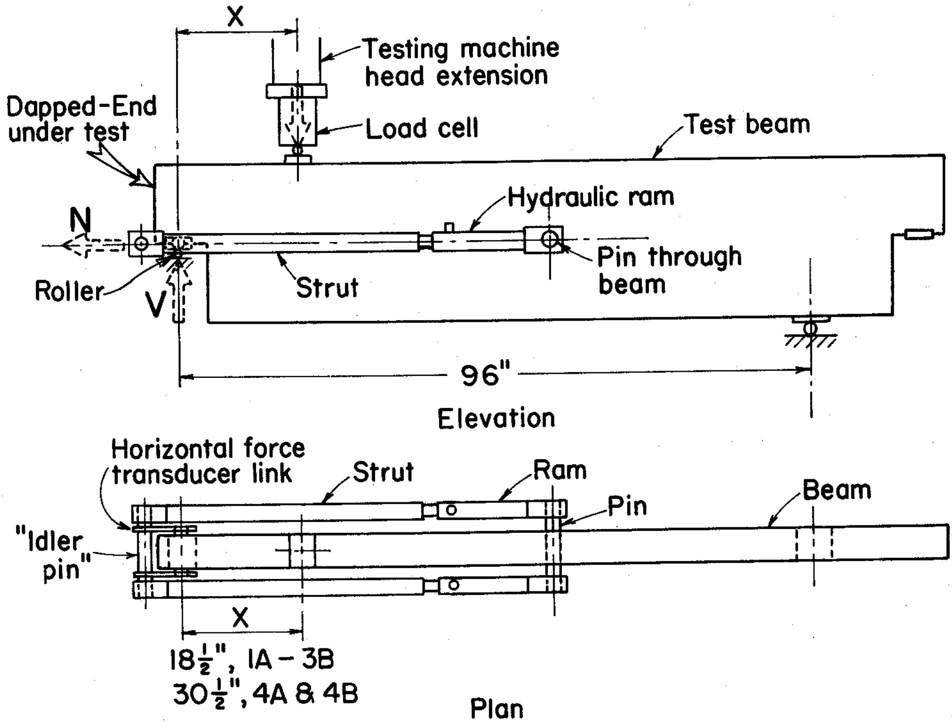

Fig. 4. Testing arrangements for dapped-end beam.

Materials and Fabrication

The concrete was made from Type III portland cement, sand and ¾-in. (19 mm) maximum size gravel, in the proportions 1.0:2.77:3.45 by weight. The freshly cast concrete was moist cured for 2 days, then cured in the air of the laboratory until test at about 5 days.

The deformed reinforcing bars of size #3 and larger conformed to ASTM Specification A 615. The #2 bars used for horizontal stirrup reinforcement had deformations similar to those on the larger bars conforming to ASTM Specification A 615.

The dapped-end bearing plates were welded to the main reinforcement A_s, so as to be able to transmit the horizontal force N directly to that reinforcement.

Testing Arrangements and Instrumentation

The dapped ends were tested independently by supporting the 10-ft (3.05 m) long beam through the dapped end at one end of the beam, and under the beam bottom face at the opposite end, using an 8-ft (2.4 m) span between centers of support.

Typical arrangements for test are shown in Fig. 4. A shot of the test setup in which both horizontal and vertical loads are acting on a dapped-end beam is shown in Fig. 5.

After test of one dapped end, damage was mostly confined to the region of that dapped end. It was therefore possible to turn the beam end-for-end, and test the other dapped end.

Specimens 1A, 1B, 2A, 2B, 3A, and 3B were loaded so that the outer edge

Table 1. Specimen reinforcement details and concrete strength.

Specimen No.	Main dapped-end reinforcement			Horizontal stirrups			Hanger reinforcement			Concrete strength
	Bars	A_s (in.^{2})	f_y (ksi)	Stirrups	A_h (in.^{2})	f_y (ksi)	Stirrups	A_{vh} (in.^{2})	f_y (ksi)	f'_c (psi)
1A	2#3	0.22	69.1	1#2	0.10	67.0	3#3	0.66	65.5	4875
1B	2#6	0.88	59.8	2#2	0.20	66.0	3#3	0.66	67.7	4425
2A	3#3	0.33	69.4	2#2	0.20	67.0	2#3	0.44	67.1	4785
2B	2#6	0.88	59.8	2#2	0.20	66.8	2#3	0.44	68.2	4475
3A	3#3	0.33	69.1	2#2	0.20	65.0	2#3 +1#2	0.44 +0.10	68.2 65.0	5370
3B	2#6	0.88	63.6	2#2	0.20	70.2	2#3 +1#2	0.44 +0.10	70.9 70.2	4590
4A	3#3	0.33	69.1	2#2	0.20	63.2	2#3 +1#2	0.44 +0.10	69.1 63.2	4590
4B	2#6	0.88	63.6	2#2	0.20	67.0	2#3 +1#2	0.44 +0.10	71.3 67.0	4260

*Measured on 6 x 12-in. cylinders.
Note: 1 in. = 25.4 mm; 1 in.^{2} = 645.16 mm^2, 1 psi = 6.895 kPa; 1 ksi = 6.895 MPa.

of the 4 x 5-in. (102 x 127 mm) loading plate was at Point Y in Fig. 2. This load location was chosen so that, for these specimens, the behavior should be controlled by the behavior of the nib and/or by the behavior of the reinforcement crossing the diagonal tension crack originating at the re-entrant corner A in Fig. 2.

In the case of Specimens 4A and 4B, the outer edge of the loading plate was at Point Z in Fig. 2, so that the influence on behavior of the diagonal tension crack originating at the bottom corner (Point B in Fig. 2) could be studied.

In tests with vertical load only, the 4 x 5-in. (102 x 127 mm) dapped-end bearing plate was supported on a free roller.

In tests with combined vertical and horizontal loads (Fig. 5), the face of the dapped-end bearing plate had a ½ x 2-in. (13 x 51 mm) transverse groove. This plate was supported by another plate with stub axles on each end, and a ½ x 2-in. (13 x 51 mm) transverse upstand which engaged with the groove in the bearing plate.

The second plate was supported on a free roller, and equal horizontal forces were applied to each stub axle by hydraulic rams acting through transducer links as shown in Fig. 4. In this way it was possible to apply to the dapped end, independently controlled vertical and horizontal forces. The hydraulic rams reacted against a 2-in. (51 mm) diameter steel pin which passed

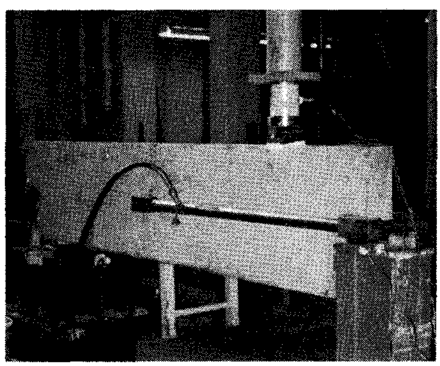

Fig. 5. Test setup in which both horizontal and vertical loads are acting on dapped-end beam.

Table 2. Test results of dapped-end beams.

Specimen No.	N_n (kips)	V_n(calc) (kips)	V_n(test) (kips)	V_y(test) (kips)	$\frac{V_n(\text{test})}{V_n(\text{calc})}$	$\frac{V_y(\text{test})}{V_n(\text{calc})}$	Maximum crack width at service loads (in.)	Distance of flexural failure plane from Corner A (in.)	Measured force in hanger reinforcement at V_y(test) (kips)
1A	0	36.12	32.40	23.49	0.90	0.65	0.017	2.42	26.7
1B	30	44.13	42.93	29.97	0.97	0.68	0.011	2.13	20.6
2A	0	36.98	40.10	31.25	1.08	0.85	0.019	3.19	27.9
2B	25	38.84	38.10	31.00	0.98	0.80	0.018	3.64	30.0*
3A	0	37.04	48.52	35.50	1.31	0.96	0.015	2.28	35.0
3B	28	38.91	39.70	36.50	1.02	0.94	0.016	2.43	38.2*
4A	0	36.74	42.43	32.00	1.15	0.87	0.009	2.96	30.2
4B	28	38.67	39.78	34.30	1.03	0.89	0.006	2.83	34.8

*Yielded.
Note: 1 in. = 25.4 mm; 1 kip = 4.448 kN.

through the center of the beam, which was locally reinforced.

The strain in the main dapped end reinforcement A_s was measured at locations a and c in Fig. 2, and that in the hanger reinforcement A_{vh} at Point b, using electrical resistance gages.

The load cell for vertical load, the transducer links for horizontal load, and the strain gages, were all monitored continuously during the test using a Sanborn strip chart recorder.

Test Procedure

It was considered desirable to obtain a measure of serviceability as well as strength, by measuring crack widths at service load. It was arbitrarily decided to consider that the dead load shear V_D and the live load shear V_L were equal. Then with $\phi = 0.85$, we have $V_D = V_L = 0.275V_n$ in the limit, since the ACI Code[5] requires that

$$\phi V_n \geqslant 1.4V_D + 1.7V_L.$$

Also, since the ACI Code requires that the horizontal force acting on a corbel be treated as a live load, the service horizontal force $N = N_u/1.7$, i.e., $N = \phi N_n/1.7 = 0.5N_n$.

The following loading sequence was therefore used:

1. $N = 0$, V increased incrementally to $V_D = 0.275V_n$.
2. $V = V_D$, N increased incrementally to $N = 0.5N_n$.
3. $N = 0.5N_n$, V increased incrementally to $(V_D + V_L) = 0.55V_n$.
4. $V = (V_D + V_L)$, N increased incrementally to N_n.
5. $N = N_n$, V increased incrementally until failure occurred.

The maximum crack widths at $V = (V_D + V_L)$ and $N = 0.5N_n$ were recorded, this being regarded as the service load. In all tests, the cracks were marked at each increment of loading.

Specimen Design and Behavior

All Specimens—The design of all the specimens followed the initial proposals, with modifications as described below. In the design of the

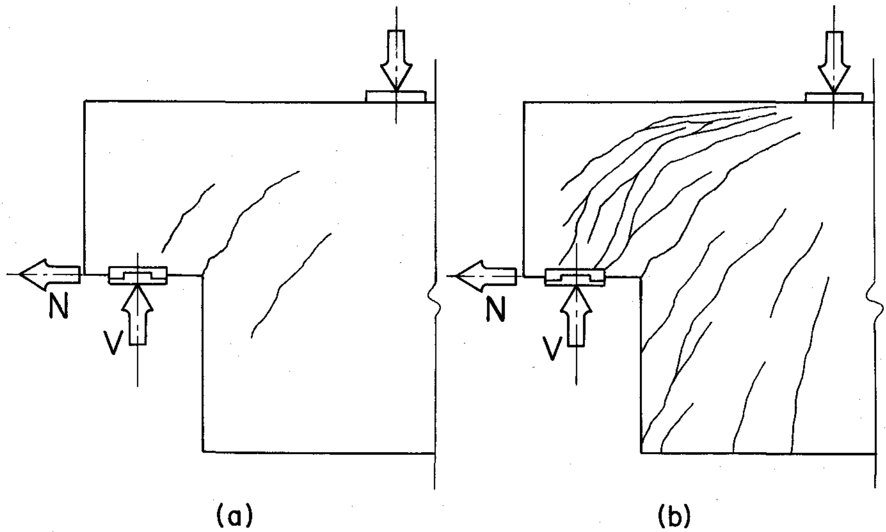

Fig. 6. Typical cracking patterns of dapped-end beams tested: (a) at service load and (b) just before failure.

"nib" as a corbel, the "Modified Shear Friction" method[3] was used for shear design rather than the "Shear Friction" method of the ACI Code, so as to be the least conservative.

The initial "target" design strengths for the specimens were $V_n = 40$ kips (178 kN) and $N_n = 30$ kips (133 kN). However, the strengths and sizes of reinforcing bars available for use in the tests were not exactly suitable. The design strengths were therefore varied as necessary in order to match the available reinforcing bars.

The design strengths V_n and N_n for each specimen, and the strengths obtained in the tests, are summarized in Table 2. In this table, V_n(test) is the shear strength measured in the test (i.e., the applied shear at failure), V_y is the shear acting when yield of the dapped-end main reinforcement A_s occurred. The design of the dapped-end main reinforcement was always controlled by flexure, so in all the specimens V_n(calc) is the calculated shear corresponding to a flexural failure of the dapped end.

Specimens 1A and 1B—In the design of these specimens, the shear span "*a*" used in the design of the "nib" as an inverted corbel, was taken to be the distance from the center of support to the interface between the nib and the full-depth beam, i.e., $a = l_v$. This was as indicated in the *PCI Design Handbook.*[3] The amount of hanger reinforcement A_{vh} was chosen to make $A_{vh}f_y = V_n$ as closely as possible.

The behavior of Specimens 1A and 1B was unsatisfactory, both at service load and at ultimate.

The general process of cracking was similar for all specimens. The first crack initiated at the re-entrant corner A at about 20 percent of ultimate. This crack propagated at approximately 45 deg to the horizontal, and at service load extended to about two-thirds the height of the nib.

At this time additional diagonal tension cracks occurred in the nib and in the full depth beam, as shown in Fig. 6a. The original crack had the greatest width at service load and this occurred close to the re-entrant corner A.

As the load was further increased, additional cracks formed and existing cracks lengthened. The diagonal tension cracks in the nib assumed a flatter trajectory on reaching the hanger reinforcement, propagating toward the loading plate. A typical cracking pattern just prior to failure is shown in Fig. 6b.

In all cases, the dapped-end main reinforcement yielded before maximum load was reached. At that time, the cracks crossing this reinforcement widened markedly. Shortly before failure, compression spalling and bulging of the top face of the beam occurred adjacent to the loading plate. At failure, inclined crushing of the concrete occurred in this same region. In some cases this crushing spread downwards and backwards into the nib as failure progressed.

In the case of Specimens 1A and 1B, the main dapped-end reinforcement yielded at about 70 percent of V_n(test) and about 66 percent of V_n(calc). This resulted in undesirable permanently wide cracks occurring at about 20 percent above service load.

Since the flexural strength of an under-reinforced section can be calculated with good accuracy, it was deduced that the interface between the nib and the full-depth beam was not the location of flexural failure, as assumed in the design calculations. Assuming that the flexural strength M_n could be calculated satisfactorily, the distance "a" from the load V to the vertical plane in which flexural failure was apparently occurring, was calculated using:

$$a = \frac{M_n - N_n(h - d)}{V_y(\text{test})}$$

The values of "a" so calculated for Specimens 1A and 1B were 6.92 and 6.63 in. (176 and 168 mm), respectively. This corresponds to the flexural failure plane being approximately at the center of gravity of the hanger reinforcement. This is in agreement with the concept of a truss-like force system developing in the dapped end, as shown in Fig. 1b.

It was therefore decided that, for all succeeding specimens, flexural design of the dapped end would be based on a shear span "a" measured from the center of support to the center of gravity of the hanger reinforcement.

Specimens 2A and 2B—These specimens were tested to determine whether the amount of hanger reinforcement could be reduced. The hanger reinforcement was designed to have a yield strength $A_{vh}f_y$ equal to $(V_n - V_c)$, where V_c is the shear that could be carried by the concrete in a beam having the same cross section as the nib, according to ACI 318-77. (In this case, V_c was taken to be $2\sqrt{f'_c}\,bd$.)

In designing the main dapped-end reinforcement for flexure, it was assumed that the center of gravity of the hanger reinforcement would be 2 in. (51 mm) from the interface between the nib and the full-depth beam, i.e., "a" = 6.5 in. (165 mm).

The behavior of Specimens 2A and 2B was unsatisfactory both at service load and at ultimate load. At service load, the cracks were excessively wide and the main dapped-end reinforcement yielded at about 80 percent of the ultimate shear. At yield of the main dapped-end reinforcement in Specimen 2B, the hanger reinforcement had yielded, and in Specimen 2A was very close to yield. It was therefore decided to check whether an increase in the hanger reinforcement would improve behavior.

Specimens 3A and 3B—In these specimens, the hanger reinforcement was designed to have a yield strength $A_{vh}f_y$ equal to V_n. In designing the main dapped-end reinforcement for flexure, it was again assumed that the center of gravity of the hanger reinforcement would be 2 in. (51 mm) from the interface between the nib and the full-depth beam, i.e., "a" = 6.5 in. (165 mm).

Specimens 3A and 3B behaved in a more satisfactory manner, the main dapped-end reinforcement not yielding until about 95 percent of V_n(calc.) Also, the maximum crack widths at service load were reduced.

Specimens 4A and 4B—The previous specimens were tested with the edge of the loading plate at Point Y in Fig. 2. This was done to examine behavior associated with failure modes involving the nib itself and the principal diagonal tension crack originating at the re-entrant corner "A".

Specimens 4A and 4B were designed in the same way as Specimens 3A and 3B, but were tested with the edge of the loading plate at Point Z in Fig. 2. This was done to check the influence on strength and behavior of a diagonal tension crack propagating freely from the bottom corner of the beam (Point B in Fig. 2).

The behavior of Specimens 4A and 4B was considered satisfactory. Although the main flexural reinforcement yielded at a slightly lower fraction of V_n(calc.) than in the case of Specimens 3A and 3B, the maximum crack widths at service load were less than in Specimens 3A and 3B.

It should be noted that, in Specimens 4A and 4B, the main dapped-end reinforcement yielded before failure at Point "c" in Fig. 2, i.e., where the bar intercepted the diagonal tension crack originating at the bottom corner of the beam. This was also the widest crack at high loads.

It is clearly necessary to extend the main dapped-end reinforcement sufficiently far into the beam so that it can develop its yield strength at the point where it is intersected by a line running upwards at 45 deg from the bottom corner of the beam.

General Comments

The measured ultimate shear, V_n(test), was greater than V_n(calc.) for all of Specimens 3A, 3B, 4A, and 4B. Also, the shear at yield of the main dapped-end reinforcement was satisfactorily high for these same specimens.

The crack control provisions of ACI 318-77 are equivalent to allowing maximum crack widths of 0.013 and 0.016 in. (0.33 and 0.41 mm) for exterior and interior exposures, respectively. The maximum crack widths for Specimens 4A and 4B are well below these allowable values, and the loading used for these specimens is probably more representative of actual practice than that used for the other tests.

If the crack widths had been measured at a service load related to the nominal yield strength of 60 ksi (414 MPa) (as is the case in practical design), rather than to the actual yield strengths of up to 70 ksi (483 MPa), it is probable that the maximum crack widths in Specimens 3A and 3B would not have exceeded the allowable width for exterior exposure.

Conclusions from Tests

The following conclusions may be derived from the test program:

1. The reduced depth part of the dapped end may be designed as if it were a corbel, using the design proposals of Mattock,[3] providing the shear span "a" used in design is taken equal to the distance from the center of action of the vertical load to the center of gravity of the hanger reinforcement A_{vh}. (Note that for the corbel design proposals to be used, a/d must be ≤ 1.0.)

2. A group of closed stirrups A_{vh}, having a yield strength $A_{vh}f_y$ not less than V_u/ϕ, should be provided close to the end face of the full-depth beam to resist the vertical component of the inclined compression force in the nib. This reinforcement must be positively anchored at both top and bottom by wrapping around longitudinal reinforcing bars.

3. The full-depth part of the beam should be designed so as to satisfy moment and force equilibrium requirements across inclined cracks AY and BZ in Fig. 2, in addition to carrying out the usual design of sections normal to the longitudinal axis of the beam for flexure, shear, and axial force.

4. The main nib reinforcement A_s should be provided with a positive anchorage as close to the end face of the beam as possible. These bars should also extend into the beam a distance $(H - d + l_d)$ beyond the re-entrant corner, so that they can develop their yield strength where intersected by a diagonal tension crack originating at the bottom corner of the beam. [Note that if these bars A_s are 12 in. (305 mm) or more above the bottom of the beam they are "top bars" according to the ACI Code[5] and l_d must be increased accordingly.]

5. The horizontal stirrups A_h should be positively anchored near the end face of the beam by wrapping around vertical bars in each corner (as shown in Figs. 2 and 6). They should project beyond the re-entrant corner a distance 1.7 l_d as recommended in the *PCI Design Handbook.*[6]

1978 PCI Design Recommendations

The design provisions for dapped-end beams contained in the *PCI Design Handbook* were revised in the 1978 edition.[6] With reference to Fig. 2, they now require the following steps in design:

1. Design of the nib for flexure and axial force. The critical section for flexure being taken to be the interface between the nib and the full depth beam, i.e., $a = l_v$.

2. Design of the interface between the nib and the full-depth beam for direct shear. (One-third of the shear-friction reinforcement A_{vf} to be provided as horizontal stirrups A_h. The remaining shear-friction reinforcement to be combined with the reinforcement for the direct force N_u to give A_s, if ⅔ A_{vf} is greater than the reinforcement required for flexure A_f.)

3. Design of the hanger reinforcement A_{vh} to carry the total shear V_u across the diagonal tension crack originating at the re-entrant corner A.

4. Design of the nib for diagonal tension, providing vertical stirrups (total area A_v) in the nib in addition to the horizontal stirrups A_h, so as to satisfy the following equation:

$$V_u \geq \phi V_n = \phi(A_v f_y + A_h f_y + 2\lambda b d \sqrt{f_c'})$$

where

λ = 1.0 for normal weight concrete
λ = 0.85 for sanded-lightweight concrete
λ = 0.75 for all-lightweight concrete
d = effective depth of A_s in the nib

It is also specified that A_v is to be not less than one-half the total reinforcement required according to this equation.

5. Design of bearing for V_u acting on the nib.

The PCI design provisions specify that both the main dapped-end reinforcement A_s and the horizontal stirrups A_h should extend a distance 1.7 l_d into the beam beyond the interface between the nib and the full-depth beam. The main reinforcement is also to be positively anchored at the end of the beam by welding to a cross-bar, angle or plate.

These revised provisions are a distinct improvement on those contained in the 1971 *PCI Design Handbook;* i.e., they recognize that the reinforcement A_{vh} close to the re-entrant corner A must be designed to carry the total shear V_u. The corresponding reinforcement in the 1971 *PCI Design Handbook* was, in effect, only designed to carry a force V_u/μ^2 in the case of vertical load only, i.e., $0.5V_u$ for $\mu = 1.4$.

However, in view of the behavior of Specimens 1A and 1B which were designed for flexure as specified in the 1978 PCI provisions, it appears that flexural design of the nib according to the 1978 *PCI Design Handbook* is inadequate.

The behavior of Specimens 3A, 3B, 4A, and 4B indicates that the critical section for flexure should be taken at the center of gravity of the hanger reinforcement; i.e., "*a*" should be the distance from the center of support to the center of gravity of the hanger reinforcement.

The 1978 *PCI Design Handbook* indicates that the main dapped-end reinforcement A_s should extend a distance 1.7 l_d into the beam beyond the re-entrant corner. Depending upon the proportions of the beam and the size of reinforcing bar used for A_s, this distance may be too short.

These bars should extend a distance not less than their development length beyond Point "c" in Fig. 2. This is necessary so that the yield strength of these bars can be developed where the diagonal tension crack originating at the bottom corner of the beam crosses them. This is required in order to satisfy moment equilibrium about the top of this crack (i.e., Point A in Fig. 2).

It should also be noted that in many cases these bars will be more than 12 in. (305 mm) from the bottom of the beam, and that they are consequently "top bars" when calculating their required development length according to ACI 318-77.[5]

The 1978 PCI design provisions require the provision of both horizontal and vertical stirrups in the nib, and also limit the shear stress in the nib ($V_u/\phi bd$) to $8\sqrt{f'_c}$. The tests reported here indicate that, if a/d is ≤ 1.0, horizontal stirrups alone are satisfactory, providing their total cross section is made not less than the greater of $A_f/2$ and $A_{vf}/3$, as recommended for corbel design by Mattock.[3]

Using this amount of horizontal stirrups in the nib, the average ultimate shear stress for Specimens 3A, 3B, 4A, and 4B was $11.4\sqrt{f'_c}$. The limitation of $8\sqrt{f'_c}$ appears to be conservative if the stirrup reinforcement recommended in this paper is used.

If a/d is greater than 1.0 then both vertical and horizontal stirrups should be provided in the nib. It is recommended that they be designed using the procedures proposed for deep beam design by ACI Committee 426.[8,9] Using the notation of this paper, these procedures may be expressed as follows for the case of a nib:

$$V_n = V_u/\phi \ngtr V_c + V_s + V_h$$

where

V_s = shear carried by vertical stirrups

$V_s = A_v f_y(1 - 0.5d/a)$, but $\ngtr A_v f_y d/a$

A_v = total area of vertical stirrups in nib (not less than $50\ ba/f_y$) with spacing of vertical stirrups not more than $d/4$.

V_h = shear carried by horizontal stirrups

$V_h = A_h f_y(1.5 - a/d)$, but $\ngtr A_h f_y$

$V_s + V_h \ngtr 8\,bd\,\lambda\sqrt{f'_c}$

V_c = shear carried by concrete $= (3.5\,d/a)bd\,\lambda\sqrt{f'_c}$

$V_c + V_s + V_h \ngtr 0.2\,bd f'_c$ nor $800\,bd$ (lbs)

The expression for V_c is an approximation to Eq. (19) of Reference 8, for purpose of simplification.

Fig. 5.9.1 of the 1978 *PCI Design Handbook* does not indicate that both the vertical and horizontal stirrups should be positively anchored at their outer ends by wrapping around reinforcing bars running at right angles to the stirrups. This positive anchorage is essential if the stirrups are to develop their yield strength. If vertical stirrups are used in the nib, they must wrap around the main nib reinforcement A_s or be welded to the bearing plate if they are to be effective.

No bearing plate is shown in Fig. 5.9.1, although one is shown in Fig. 5.9.2 illustrating the design example. If a horizontal force N_u acts on the dapped end, a bearing plate should be provided. It should either be welded directly to the main nib reinforcement, so as to transfer N_u directly into A_s, or it may be attached to the nib by headed stud shear connectors.

However, in this latter case the studs must lie inside the positive anchorage of the main nib reinforcement. It is also recommended that a bearing plate or armouring angle be welded to the main nib reinforcement, in all cases, if the bearing area extends beyond the positive anchorage of the main nib reinforcement.

Proposed Design Recommendations

On the basis of the test results reported here and the foregoing discussion, it is proposed that dapped-end beams having a/d ratios ≤ 1.0* be designed as follows:

1. Check that shear stress in nib due to factored shear V_u is not more than $0.2f'_c$, i.e., $V_u/(\phi db) \not> 0.2f'_c$. (This limit corresponds to the maximum ultimate shear stress developed in these tests. Future tests may indicate a higher ultimate shear stress is possible).

2. Design the hanger reinforcement A_{vh} to carry the total shear V_u due to factored loads using:

$$A_{vh} = \frac{V_u}{\phi f_y}$$

Provide this reinforcement in the form of closed stirrups and place as close to the re-entrant corner as possible.

3. Calculate the design moment due to factored loads using:

$$M_u = V_u a + N_u(h - d)$$

where a is the distance from the center of action of V_u to the center of gravity of the hanger reinforcement A_{vh}.

Calculate the required flexural reinforcement area A_f so that $\phi M_n \geq M_u$.

4. Calculate reinforcement area A_n to resist horizontal force N_u due to factored loads using:

$$A_n = N_u/\phi f_y$$

5. Calculate reinforcement area A_{vf} to transfer shear across interface between nib and full depth beam. Use "Shear Friction" provisions of ACI 318-77; Section 5.6, Shear Friction, of the 1978 *PCI Design Handbook;* or the following "Modified Shear Friction"[3] equation:

$$A_{vf} = [V_u/(0.8\phi) - Kbd]/f_y$$
but not less than $0.2\, bd\, f_y$

where $K = 0.5$ for normal weight concrete
or $K = 0.25$ for all-lightweight concrete
or $K = 0.31$ for sanded lightweight concrete

In the above equation, V_u is in kips, b and d are in inches and f_y is in kips/in.^{2}.

*See Fig. 2 and Appendix A for definitions of symbols.

6. Check whether $\frac{2}{3}A_{vf}$ is greater than or less than A_f.

If $\frac{2}{3}A_{vf}$ is greater than A_f:

$$A_s = \frac{2}{3}A_{vf} + A_n$$

If $\frac{2}{3}A_{vf}$ is less than A_f:

$$A_s = A_f + A_n$$

7. Provide a positive anchorage for A_s at the outer end. These bars must also extend into the beam a distance $(H - d + l_d)$ beyond the re-entrant corner. (This is to ensure that these bars can develop their yield strength, where intersected by a diagonal tension crack originating at the bottom corner of the beam.)

8. Provide horizontal stirrups in the lower two-thirds of the depth of the nib, having total area:

$$A_h = 0.5(A_s - A_n)$$

(This provision automatically provides the greater of $A_{vf}/3$ and $A_f/2$.)

These stirrups should be closed at their outer end but may be closed or open at the other end. They should project beyond the interface between the nib and the full-depth beam a distance $1.7\, l_d$.

9. The hanger reinforcement must be positively anchored at both top and bottom. At the bottom it should pass around longitudinal reinforcing bars having a total area not less than that of the hanger reinforcement. These longitudinal bars should be positively anchored at their outer ends. They would normally be a continuation of the beam flexural reinforcement in a reinforced concrete beam. In a prestressed concrete beam they should extend into the beam the greater of 1.7 l_d (for the bars) and the development length of the strand.

10. Use strength reduction factor $\phi = 0.85$ in all reinforcement design calculations.

11. Design for bearing as per 1978 *PCI Design Handbook*. The center of action of the vertical force V_u should lie inside the positive anchorage of the main nib reinforcement. If the main nib reinforcement takes the form of a hairpin type bar, then in accordance with the recommendations of ACI Committee 426[8] for corbels, the bearing area should not project beyond the straight portion of the bars.

A bearing plate should be provided if a horizontal force N_u acts on the nib. It should be welded directly to the main nib reinforcement or may be attached to the nib by headed stud shear connector.

In either case, the detail should be designed to transfer the force N_u into the nib. (The studs should lie inside the positive anchorage of the main nib reinforcement.)

A bearing plate or armouring angle should be welded to the main nib reinforcement in all cases, if the bearing area extends beyond the positive anchorage of the main nib reinforcement.

Additional Comments

Although it is necessary to satisfy equilibrium for those parts of the dapped end cut off by the two diagonal tension cracks AY and BZ in Fig. 2, it is not necessary to make an explicit check if the reinforcement is designed as recommended above. In this case, sufficient reinforcement to satisfy these equilibrium requirements is automatically provided, as shown in Appendix B.

It is considered appropriate to use a value of 0.85 for the strength reduction factor ϕ in all calculations because of the uncertainties regarding:

(a) Exact location of center of action of V_u.
(b) Exact value of horizontal force N_u, and
(c) Exact location of critical section for flexure within the end of the beam.

Design Example

Given: A 16RB28 beam with dapped end as shown in Fig. 7. Assume the following data are known:

V_u = 100 kips
N_u = 15 kips
f'_c = 5000 psi (normal weight concrete)
f_y = 60 ksi (all reinforcement)

Required: Design the reinforcement for the 16RB28 beam with dapped end (Fig. 7).

Solution:

1. Shear Stress

Assuming the effective depth of the nib is 15 in., determine the ultimate shear stress from:

$$\begin{aligned} V_u &= V_u/\phi bd \\ &= 100{,}000/(0.85)(16)(15) \\ &= 490 \text{ psi} \\ &< 0.2f'_c = 1000 \text{ psi (ok)} \end{aligned}$$

2. Hanger Reinforcement

$$\begin{aligned} A_{vh} &= V_u/\phi f_y \\ &= 100/(0.85)(60) \\ &= 1.96 \text{ in.}^2 \end{aligned}$$

Use 5 #4 closed ties, A_{vh} = 2.00 in.^{2}

These can be arranged as two, four-legged stirrups and one, two-legged stirrup.

Assuming that the stirrups can be bundled together and providing 1½ in. cover to end face, the distance from re-entrant corner to center of gravity of hanger reinforcement is about 2¾ in.

3. Reinforcement to Resist Flexure

$$\begin{aligned} a &= 4.5 + 2.75 = 7.25 \text{ in.} \\ a/d &= 7.25/15 = 0.48 \text{ (i.e., } < 1.0) \\ M_u &= V_u a + N_u(h - d) \\ &= 100(7.25) + 15(16 - 15) \\ &= 740 \text{ kip-in.} \end{aligned}$$

$$R = \frac{M_u}{\phi b d^2 f'_c} = \frac{740}{0.85(16)(15)^2(5)} = 0.048$$

$$\rho = (f'_c/f_y)(0.85 - \sqrt{0.72 - 1.7R})$$

$$= \left[\frac{5}{60}\right]\left[0.85 - \sqrt{0.72 - 1.7(0.048)}\right]$$

$= 0.0043 < \rho_{max} = 0.0252$ for f'_c = 5000 psi and f_y = 60 ksi.

$\therefore A_f = 0.0043(16)(15) = 1.03$ in.^{2}

4. Reinforcement to Resist Horizontal Force

$$\begin{aligned} A_n &= N_u/\phi f_y \\ &= 15/(0.85 \times 60) \\ &= 0.29 \text{ in.}^2 \end{aligned}$$

5. Shear Transfer Reinforcement

Using Modified Shear-Friction:

$$A_{vf} = \left[\frac{V_u}{0.8\phi} - 0.5bd\right]/f_y$$

but $\nless 0.2bd/f_y$

$$A_{vf} = \left[\frac{100}{0.8(0.85)} - 0.5(16)(15)\right]/60$$

$= 0.45$ in.^{2}

$$\begin{aligned} 0.2bd/f_y &= 0.2(16)(15)/60 \\ &= 0.80 \text{ in.}^2 \end{aligned}$$

$\therefore A_{vf} = 0.80$ in.^{2}

6. Main Reinforcement of Nib

$\frac{2}{3}A_{vf} = 0.53$ in.$^2 < A_f = 1.03$ in.^{2}

$$\begin{aligned} \therefore A_s &= A_f + A_n \\ &= 1.03 + 0.29 = 1.32 \text{ in.}^2 \end{aligned}$$

Use 3 #6 bars, A_s = 1.32 in.^{2}

Metric (SI) Unit Equivalents

1 in. = 25.4 mm
1 ft = 0.305 m
1 in.^{2} = 645.16 mm^2
1 psi = 0.006895 MPa
1 ksi = 6.895 MPa
1 lb = 4.448 N
1 kip = 4448 N
1 $\sqrt{f'_c}$ psi = 0.083035 $\sqrt{f'_c}$ MPa

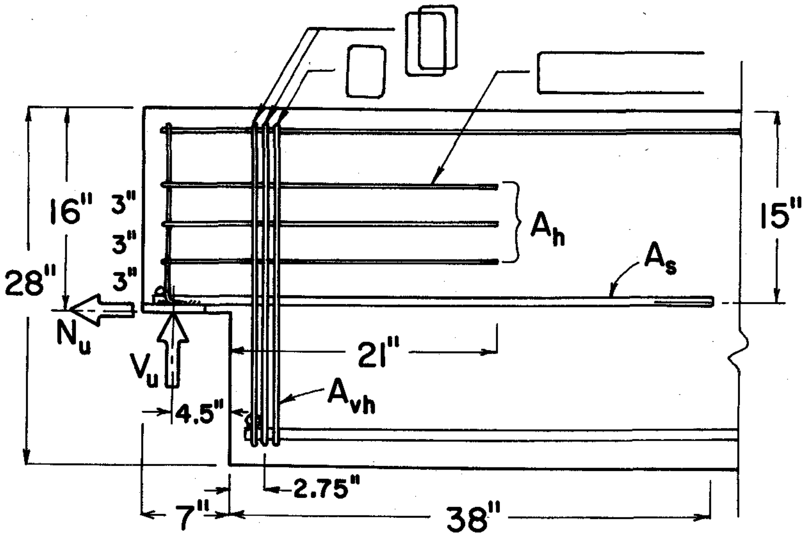

Fig. 7. Design example for dapped-end beam.

7. Anchorage

The main nib reinforcement will be anchored at the outer end by welding a #6 bar transversely across the bars. These bars must be extended a distance beyond Point "c" in Fig. 2 sufficient to develop their yield strength at this point. These are "top bars," being more than 12 in. from the bottom of the beam; hence, the required l_d for f'_c = 5000 psi and f_y = 60 ksi is 1.4(18) = 25 in.

$H - d + l_d = 28 - 15 + 25 = 38$ in.

Therefore, extend the main dapped-end reinforcement A_s a distance 38 in. beyond the re-entrant corner A.

8. Horizontal Stirrups

$A_h = 0.5(A_s - A_n) = 0.5(1.03)$
$= 0.52$ in.²

Use 3 #3 U-bars, A_h = 0.66 in.

$1.7\,l_d = 1.7(12) = 21$ in.

Therefore, extend these stirrups 21 in. beyond the interface between the nib and the full depth beam.

9. Hanger Reinforcement Anchorage

The hanger reinforcement must pass around at least 2.00 in.² of the longitudinal reinforcement at the bottom of the beam. This reinforcement must have a positive end anchorage and must extend at least 1.7 l_d (for the bars) or l_d for the strand if a prestressed beam or would be a continuation of beam flexural reinforcement in a reinforced concrete beam.

10. Strength Reduction Factor

Note that a strength reduction factor $\phi = 0.85$ is used in all the reinforcement design calculations.

11. Bearing

Design for bearing as per the *PCI Design Handbook.*

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APPENDIX A—NOTATION

a = shear span; distance from center of action of shear force V_u to critical section for flexure, in.

A_f = area of reinforcement required to resist flexure, in.^{2}

A_h = total area of horizontal stirrups, in.^{2}

A_n = area of reinforcement required to resist horizontal force N_u, in.^{2}

A_s = total area of dapped-end main reinforcement, in.^{2}

A_v = area of vertical stirrup reinforcement, in.^{2}

A_{vf} = area of shear transfer reinforcement required to resist shear V_u, in.^{2}

A_{vh} = area of hanger reinforcement, in.^{2}

b = width of nib, in.

d = effective depth of nib, in.

f'_c = specified compressive strength of concrete (measured on 6 x 12-in. cylinders), psi

f_y = specified yield strength of reinforcement, ksi

h = total depth of nib, in.

H = total depth of beam, in.

K = coefficient for type of concrete in modified shear-friction design equation

l_d = reinforcing bar development length, in.

l_v = distance from center of action of shear V_u to interface between nib and full-depth beam, in.

M_n = nominal moment strength, kip-in.

M_u = moment due to factored loads, kip-in.

N_n = nominal strength resisting horizontal loads, kips

N_u = horizontal load due to factored loads, kips

V_c = shear carried by concrete, kips

V_D = shear due to unfactored dead load, kips

V_h = shear carried by horizontal stirrups, kips

V_L = shear due to unfactored live load, kips

V_n = nominal shear strength, kips

V_s = shear carried by vertical stirrups, kips

V_u = total shear due to factored loads, kips

V_y = shear at yield of main dapped-end reinforcement, kips

λ = coefficient for type of concrete in PCI equation for nib shear strength

μ = coefficient of friction used in shear-friction calculations

ρ = A_f/bd

ϕ = strength reduction factor

ϕ = 0.85 for all calculations in dapped-end reinforcement design

APPENDIX B—EQUILIBRIUM OF PARTS OF BEAM CUT OFF BY DIAGONAL TENSION CRACKS

Consider moment equilibrium of parts of the beam cut off by diagonal tension cracks of both Sections AY and BZ (see Fig. 2). Let x be the depth of the center of action of the concrete compression force C in Fig. 3.

1. Beam Section Cut Off by Crack AY

In the case of that part of the beam cut off by Crack AY, consider moment equilibrium about a point at distance x below Point Y.

Moment due to factored loads:

$$M_u = V_u(h + l_v) + N_u(h - x)$$

Resistance moment due to internal forces (not including any forces in stirrups A_h):

$$\phi M_n = \phi A_s f_y(d - x) + \phi A_{vh} f_y(h + l_v - a) \quad \text{or,}$$

$$\phi M_n \geq \phi A_f f_y(d - x) + \phi A_n f_y(d - x) + \phi A_{vh} f_y(h + l_v - a)$$

$$\phi M_n \geq \phi A_f f_y(d - x) + \phi A_n f_y(h - x) - \phi A_n f_y(h - d) + \phi A_{vh} f_y(h + l_v - a)$$

[Since A_s is $\geq (A_f + A_n)$.]

Now, the depth of the concrete compression zone will be the same at Points Y and Z as at the center of gravity of the hanger reinforcement, since it corresponds to the reinforcement force $A_f f_y$ only. Hence, x will be the same at all three locations.

We can therefore write that:

$\phi A_f f_y(d - x)$ = moment in plane at center of gravity of A_{vh}

$$\phi A_f f_y(d - x) = V_u a + N_u(h - d)$$

Also, $\phi A_n f_y(h - x) = N_u(h - x)$

$$\phi A_n f_y(h - d) = N_u(h - d) \quad \text{and,}$$

$$\phi A_{vh} f_y(h + l_v - a) = V_u(h + l_v - a)$$

$$\therefore \phi M_n \geq [V_u a + N_u(h - d)] + N_u(h - x) - N_u(h - d) + V_u(h + l_v - a)$$

$$\phi M_n \geq V_u(h + l_v) + N_u(h - x)$$

i.e. $\phi M_n \geq M_u$

Hence, the resistance moment is equal to or greater than the moment due to factored loads.

2. Beam Section Cut Off by Crack BZ

In the case of that part of the beam cut off by Crack BZ, consider moment equilibrium about a point distance x below Point Z.

Moment due to factored loads:

$$M_u = V_u(H + l_v) + N_u(h - x)$$

Resistance moment due to internal forces:

$$\phi M_n = \phi A_s f_y(d - x) + \phi A_{vh} f_y(H + l_v - a)$$

By a series of substitutions similar to those made in Case 1 above, it can be shown that the resistance moment is equal to or greater than the moment due to the factored loads.