

Fatigue Tests of Pretensioned Girders With Blanketed and Draped Strands



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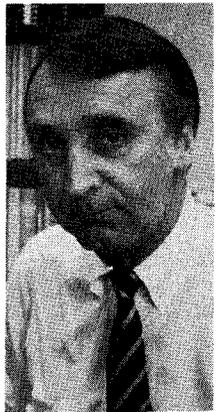
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Based on comparative fatigue tests of full-sized Type II AASHTO-PCI girders, it is shown that, to control stresses in the end regions of pretensioned bridge members, straight strands having unbonded blanketed lengths at the ends of girders can be used effectively and economically as an alternative to draped strands.

Use of draped strands in pretensioned girders can present problems for designers, fabricators and inspectors. In some plants, the tensioning procedure is time consuming, costly, and may leave doubt as to the actual prestress level obtained throughout the length of the strand. Draping of strands can be avoided by using straight tendons having unbonded "blanketed" lengths at the ends of girders.

Test Program

An experimental investigation was carried out at the Construction Technology Laboratories of the Portland Cement Association to determine the effect of repetitive loading on the behavior and strength of girders with blanketed strands.

Six full-sized Type II AASHTO-PCI girders, each 50 ft (15.24 m) long, were tested. Two girders contained draped strands. The other four had straight strands with four tendons blanketed at each end. Strands were $\frac{7}{16}$ -in. (11.1 mm) diameter, Grade 250, stress

relieved, and had a brown surface rust.

Controlled variables in the test program were load level, development length, and use of ties to confine the concrete in the stress transfer region of the blanketed strands. All specimens were cracked prior to fatigue loading.

The test program called for 5 million cycles of loading between dead load and dead load plus live load. Static tests to full dead load plus live load were performed before cyclic loading and after 1, 2½ and 5 million cycles. At the completion of 5 million cycles, the girders were tested to destruction under static load.

This paper summarizes the experimental investigation¹ and presents the results of the tests.

Conclusions

The results of the fatigue tests of this investigation indicate the following:

1. In prestressed bridge girders, concrete stresses may be controlled by either draping strands or, alternatively, using straight

Synopsis

In pretensioned girders, draping of strands can be avoided by using straight tendons having unbonded "blanketed" lengths at the ends of girders. An experimental investigation was carried out at the Construction Technology Laboratories of the Portland Cement Association to determine the effect of repetitive loading on the behavior and strength of girders with draped and blanketed strands.

Six full-sized Type II AASHTO-PCI girders were tested. Two girders contained draped strands. The other four had straight strands with four tendons blanketed at each end. The effects of load level, development length and confining ties were investigated.

The test program called for 5 million cycles of loading followed by a static test to destruction. This paper presents results of the investigation and shows that blanketed strands can be used successfully if adequate strand development length is provided.

One development length, as presently defined by AASHTO Specifications, was adequate for girders where tension was not allowed in the precompressed tensile zone. Fatigue fracture of strands was observed in pre-cracked girders where load level produced tensile stress in the precompressed concrete.

strands having unbonded "blanketed" lengths at the ends of girders.

2. For similar loading conditions, the behavior and strength

were the same for girders having either blanketed or draped strands.

3. The fatigue life of specimens designed for a maximum tensile stress of $6\sqrt{f'_c}$ psi ($0.5\sqrt{f'_c}$ MPa) under full service load was significantly less than that of specimens designed for zero tension.

4. In specimens designed for zero tension in the concrete under service load condition, and having blanketed strands designed for one development length, l_d , as defined in the 1977 AASHTO Specifications, Section 1.6.18, or in ACI 318-77, Section 12.10.1, the behavior and strength of the specimens with blanketed strands were similar to those of girders with draped strands.

5. In the specimen designed for a maximum tensile stress of $6\sqrt{f'_c}$ psi ($0.5\sqrt{f'_c}$ MPa) in the uncracked concrete under full service load and having blanketed strands designed for twice the development length, $2l_d$, only small slip of the strands occurred. This indicated adequate bond of the blanketed strands for about 3 million cycles of repetitive loading.

6. In the specimen designed for a maximum tensile stress of $6\sqrt{f'_c}$ psi ($0.5\sqrt{f'_c}$ MPa) in the uncracked concrete under full service load and having blanketed strands designed for one development length, l_d , blanketed strands slipped indicating bond fatigue occurred.

7. In the three specimens where cyclic loading produced tension of $6\sqrt{f'_c}$ psi ($0.5\sqrt{f'_c}$ MPa) in the concrete at midspan, fatigue fracture of the strands occurred at about 3 million cycles of repeti-

tive loading. These specimens included a control girder with draped strands. Therefore, blanketing did not cause fatigue of strands. Calculated minimum and maximum stresses in the strands at a crack located at midspan were 142 and 151 ksi (980 and 1040 MPa), respectively.

8. Use of ties to confine the concrete in the stress transfer region of blanketed strands in one specimen did not provide any substantial improvement in the behavior of that specimen.

Recommendations

Based on the test results and the conclusions outlined above, it is recommended that:

1. In bridge girders, blanketed strands may be used as an alternative for draped strands.

2. In bridge girders designed for no allowable tension in the concrete under service load conditions, the blanketed length of strands may be calculated allowing for one development length, l_d , as defined by 1977 AASHTO Specifications, Section 1.6.18, or ACI 318-77, Section 12.10.1. In most girders, the development length can be greater than l_d without exceeding the allowable concrete stress at the ends of the girders.* A length greater than l_d will result in less length of strand to be blanketed and, therefore, would be more economical to manufacture.

*To provide the required development length in the test girders, maximum strand length was blanketed. In practice, a shorter length of strand may be blanketed. For further discussion, see "Minimum Blanketed Length" in Appendix B.

PCI Bridge Committee Statement

The Bridge Committee of the Prestressed Concrete Institute has reviewed this report and wishes to make the following comments:

1. Results of this one series of tests should not be used as a basis for making radical changes in current design criteria that are based on numerous other tests. Consideration should be given to the large number of cycles of full load that were required to cause a strand fatigue failure, the details of the tests which were not designed to evaluate strand fatigue properties and to the artificially formed cracks at which the failures occurred.

2. Possible significance of the test results has been studied and an investigation of fatigue in prestressed concrete bridge members designed to current criteria is being considered by the AASHTO Subcommittee on Bridges and Structures.

3. The last paragraph of Section 1.6.18 of the 1977 AASHTO Specifications, and Section 12.10.3 of ACI 318-77 should be revised to read:

"Where strand is debonded at the end of a member and tension at service load is allowed in the precompressed tensile zone, development length specified above shall be doubled."

In most girders, development length can be greater than $2l_d$ without exceeding the allowable concrete stress at the end of the girders. A length greater than $2l_d$

will result in less length of strand to be blanketed and, therefore, would be more economical to manufacture.

4. Use of ties to confine the concrete in the stress transfer region of blanketed strands is not necessary.

5. When tension is allowed in the concrete under service load conditions, design of the girders to prevent strand fatigue should be a consideration.

6. Further research is needed to determine the fatigue properties of prestressing strands, as well as the level of tension in the concrete at which pretensioned girders would be able to withstand traffic loading without strand fatigue during their design service life.

7. Research is needed to determine how far strands should be extended beyond the point where they are not needed. In reinforced concrete members, the cut-off point of a reinforcing bar is governed by either a development length measured from the critical section, or a minimum distance (e.g., 15 bar diameters)² measured from the theoretical point where the bars are not needed. Presently, specifications do not provide the designer with a minimum distance criterion for prestressing steel.

Introduction

In the design of prestressed girders, economy and efficiency dictate that at midspan the eccentricity of the prestressing force be as large as possible. However, if the eccentricity and mag-

nitude of the prestressing force are kept constant over the full length of the girder, the allowable stresses may be violated at the ends of the girder. Either too much tension in the top fibers or too much compression in the bottom fibers may occur.

For this reason, in practice, some strands are draped at the ends of the girder. This is the prevalent means of controlling the end stresses and confining them to within the allowable values.

Draping of strands in pretensioned beams can present problems for designers, fabricators, and inspectors in some plants. The tensioning procedure is time consuming, costly and leaves doubt as to the actual prestress level obtained throughout the length of the strand.

Draping of strands can be avoided by using straight tendons, so long as the concrete stresses at the end of the beams remain within the allowable limits. This requirement can be satisfied by controlling the magnitude of the prestressing force at the end regions of the beams rather than the eccentricity of the force.

The magnitude of the prestressing force at the ends of the beams can be decreased by preventing bond of some strands with the concrete near the ends of the girder. This is achieved by using the "blanketing" concept. This can be done by greasing the strand, coating it with retarder or covering it with plastic tubing. Greasing or using a retarder is risky because of the possibility of affecting other than the specified strands. Plastic tubing is preferred because it also provides an easy means of inspection.

Use of blanketed strands seems to be an easy solution to avoid draping strands. The blanketing technique has been tested previously.^{3,4} Before adopting this technique some caution is necessary since the recent ACI Building Code⁵ and AASHTO Spec-

Table 1. Details of test specimens.

Specimen No.	Stress* level	Development length	Confinement reinforcement
G11 G13 G10	} Tension in bottom fiber $6\sqrt{f'_c}$ psi	l_d	No
		$2l_d$	No
		Draped	No
G14 G12 G10-A	} No tension under service load	l_d	Yes
		l_d	No
		Draped	No

* Maximum concrete stress at midspan under service load.
 $6\sqrt{f'_c}$ psi = $0.5\sqrt{f'_c}$ MPa

ifications² permit tension in the precompressed concrete fibers under service load conditions.

Cracks may occur in bridge girders due to service loads or due to overloads. Then, under repetitive passage of traffic, there is a possibility that debonding of the blanketed strands may spread on either side of the cracks. This causes a bond fatigue failure.⁶⁻⁸

Blanketed strands can be more prone to such failure. However, if the strands are adequately anchored bond fatigue can be avoided. What constitutes good anchorage and what constitutes adequate development length for the strands are two questions that resulted in the research program described in this paper.

Therefore, the aims of this investigation were to:

1. Determine whether tension in the concrete under service load condition affects the development length.
2. Determine whether one or two development lengths, l_d , (as defined by 1977 AASHTO Specifications, Section 1.6.18, or ACI 318-77, Section 12.10.1) are required for blanketed strands.
3. Determine whether ties to confine the concrete in the stress transfer region of blanketed strands are beneficial.

Description of Specimens

The research program was an extension of work done at Tulane University for the State of Louisiana.⁴ Therefore, it was desirable to adopt the same sized specimens. The same number, size, and grade of strands were also used.

Six full-sized Type II AASHTO-PCI specimens were tested in this investigation. Table 1 summarizes the test variables. Cross sections, reinforcing details and the loading pattern are shown in Fig. 1. Design calculations are summarized in Appendix B.

Design Criteria

Test specimens were designed according to the 1977 AASHTO Specifications.²

Concrete strength at transfer of prestress, was assumed to be at least 4000 psi (27.6 MPa). The concrete design strength at 28 days was taken as 5000 psi (34.5 MPa).

Strands used were $\frac{7}{16}$ -in. (11.1 mm) diameter with a nominal strength of 250 ksi (1724 MPa). Effective bed pull was based on 70 percent of the nominal ultimate strength of the strand. Prestress losses including elastic shortening at time of test were assumed to be 20 percent for all girders.

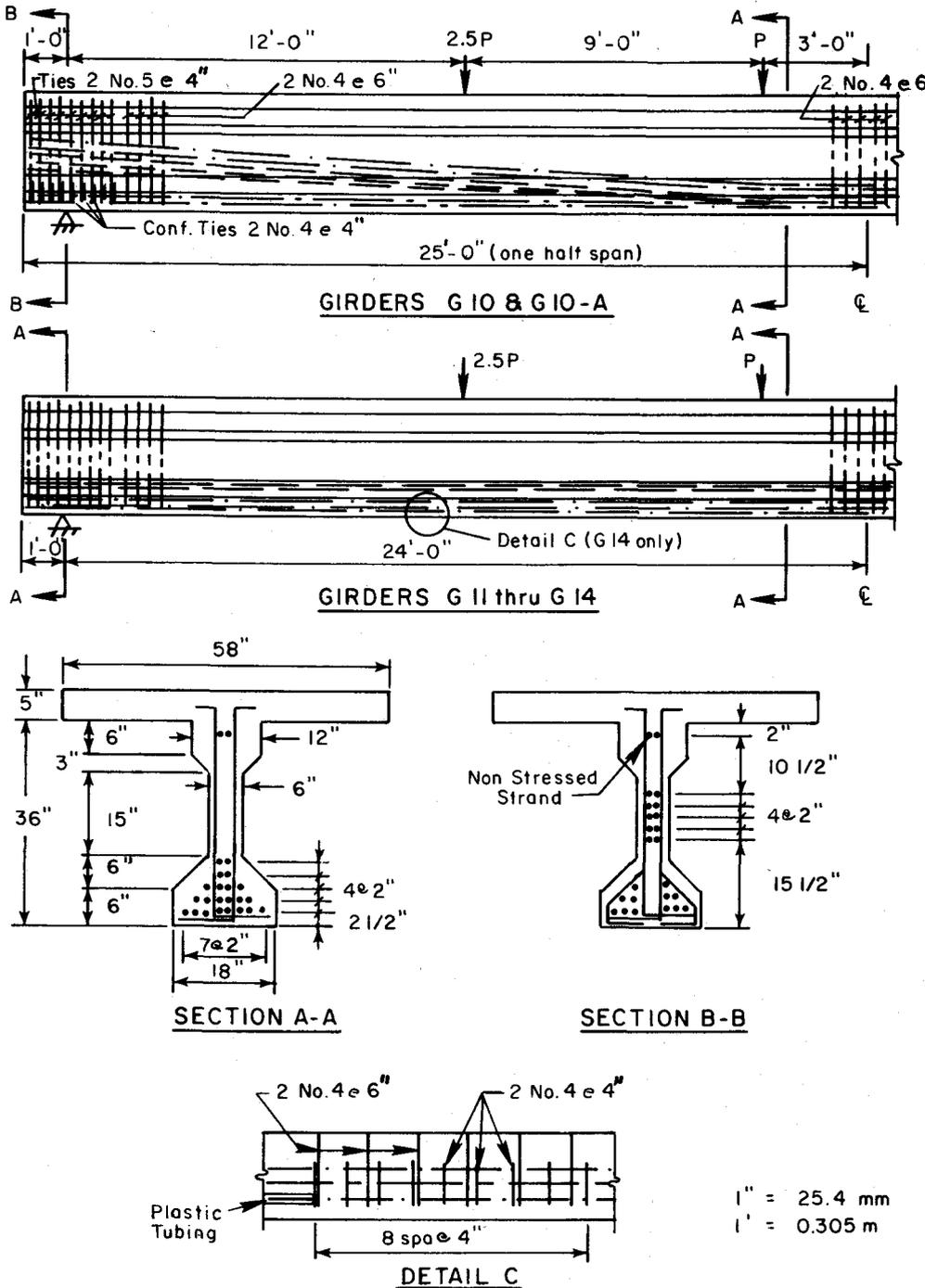


Fig. 1. Cross section and reinforcing details of test specimens.

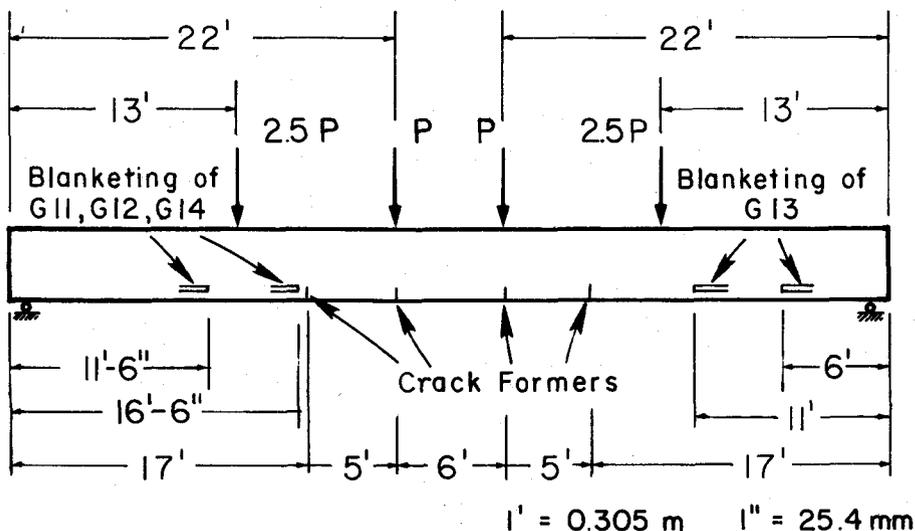


Fig. 2. Location of blanketing tubing and crack formers relative to position of loading points.

Number of Blanketed Strands

A minimum number of strands were blanketed to ensure that the top and bottom concrete stresses of the end regions remained within the allowable values. At each end, four strands were blanketed in all girders having straight tendons only.

Blanketing Location

Blanketing location should satisfy two distinct criteria:

1. Adequate development length should be provided. This condition yields the maximum blanketed length.
2. At transfer, top and bottom concrete stresses of the end regions should remain within the allowable values. This condition yields the minimum blanketing length.

In this investigation, the blanketing location was determined using the first criterion. The procedure is similar to that for stopping reinforcing bars in reinforced concrete members.

One development length was calculated² to be 5 ft 6 in. (1.68 m). Location of blanketing tubing relative to the position of the applied loads is shown in Fig. 2. Strands were blanketed in pairs to maintain symmetry. Only strands in the bottom layer were blanketed.

Development length was measured from the locations where the strands were required to exhibit their full strength. To force the cracks to occur at these critical locations, crack forming devices were placed in the test girders at the locations shown in Fig. 2. The crack formers consisted of $2\frac{3}{16} \times 17\frac{7}{8}$ -in. (55 x 454 mm) pieces of $\frac{1}{16}$ -in. (1.6 mm) thick sheet metal.

Confining Ties

Ties to confine the concrete in the end transfer region of all girders conformed with the Louisiana "typical details for prestressed concrete girder construction." The same confining tie details were used at the stress transfer regions of the blanketed strands of Specimen G14.

Table 2. Calculated fatigue loads and stress levels.

Applied load, moment, and stress	Specimen No.	
	G11, G13, G10	G14, G12, G10-A
Applied load, kips, P_{min}	4.1	0.6
P_{max}	14.6	10.9
Moment (constant moment zone), kip-in. min.*	4863	2677
max.	11283	9064
Strand stress† (bottom layer), ksi, min.	142.6	140.4
max.	151.0	146.5
Strand stress range, ksi	8.4	6.1
Midspan bottom fiber concrete stress at P_{max}	$6\sqrt{f'_c}$ psi	0

* Includes self weight moment of 2370 kip-in. (268 kN.m).

† Strand stresses are calculated assuming 20 percent loss of prestress and based upon actual material properties.

1 kip = 4.45 kN, 1 kip-in. = 0.113 kN·m; 1 ksi = 6.895 MPa;

$6\sqrt{f'_c}$ psi = $0.5\sqrt{f'_c}$ MPa.

Range of Fatigue Loads

Choice of the range of loads for the fatigue testing of Specimens G11, G13, and G10 was governed by the following criteria:

1. The dead load moment, M_D , plus the live load moment, M_L , caused a tensile stress of $6\sqrt{f'_c}$ psi ($0.5\sqrt{f'_c}$ MPa) in the bottom concrete fibers at midspan.
2. The factored dead load plus live load moments equaled the nominal flexural strength, M_U .

Therefore,

$$M_U = 1.3M_D + 2.167M_L.$$

These two conditions yielded values for M_D and M_L for Specimens G11, G13, and G10. The minimum load, P_{min} , and maximum load, P_{max} , applied during the repetitive load test were determined from M_D and M_L . Load P_{min} and specimen self weight caused a moment M_D . Load P_{max} and specimen self weight caused a moment $M_D + M_L$.

For Specimens G14, G12, and G10-A, the maximum service load, P_{max} , was chosen to correspond to a midspan bottom concrete fiber stress

of zero tension. The minimum load, P_{min} , was chosen to be a nominal 600 lbs (2.7 kN) required to keep the rams in contact with the top of the girders. Table 2 summarizes calculated values for range of fatigue loads, the corresponding midspan moments, stresses and stress range in these strands. Note that the loads P_{min} and P_{max} were the inner point loads of Fig. 2. The outer point loads were 2.5 times the magnitude of the listed loads.

Fatigue Analysis

Present Codes⁵ and Specifications² do not provide the designer with guidance concerning allowable stress range to prevent fatigue failure of prestressing strands. In a report⁸ published in 1974, ACI Committee 215, Fatigue of Concrete, recommended that at 2 million cycles:

“The stress range in prestressed reinforcement that may be imposed on minimum stress levels up to 60 percent of the tensile strength shall not exceed 10 percent of the ten-

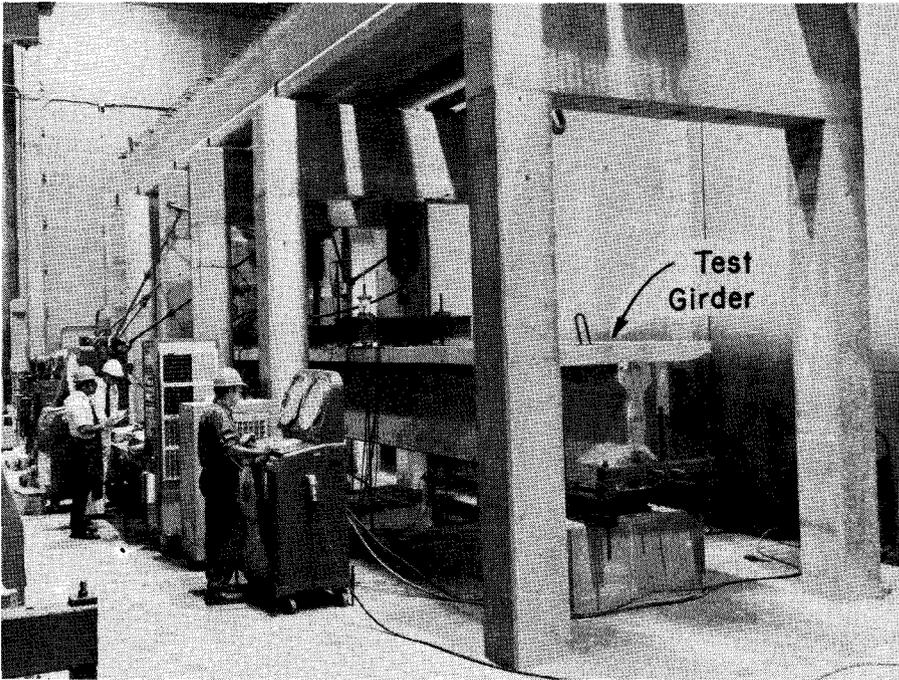


Fig. 3. Test setup at Construction Technology Laboratories at PCA.

sile strength for strands and bars.”

Similar recommendations are given in the more recent ACI Committee 443 Report, “Analysis and Design of Reinforced Concrete Bridge Structures.”⁹

Strands in the present investigation were Grade 250. Therefore, the calculated stress levels listed in Table 2 are much smaller than the allowable stresses recommended by ACI Committees 215 and 443.

Test Program

This section describes the test setup, instrumentation, and test procedure.

Test Setup

Two loading systems were used to test each specimen. The first was ba-

sically for dynamic loading, and the second for static tests to destruction. The second system is shown in Fig. 3.

Dynamic loading was applied through four rams. Each ram was secured to a concrete frame prestressed to the laboratory floor. In Fig. 3, the stems of two rams are in the retracted position.

During testing, ends of the specimens were supported on 5-in. (127 mm) diameter rollers located between two steel plates. A jig was erected at each support to prevent the girder from rolling longitudinally during dynamic testing.

To accommodate the large deflections required to test the specimens to destruction, the test setup was modified. The second loading system consisted of cross heads and tie rods. Loads were applied by hydraulic rams reacting against the underside of the test floor.¹⁰

Instrumentation

Instrumentation was first installed during manufacture of the girders in the plant. Quantities measured included prestressing force, strand strains, confining tie strains, and camber.

In the laboratory, measurements of applied loads, deflections, strand strains, confining tie strains, and strand slip were recorded during the static tests.

During the dynamic tests, strand slip and number of cycles of repetitive loading were recorded.

Test Procedure

Testing started with 3 cycles of static loading between P_{min} and P_{max} . These predetermined loads are listed

in Table 2. To ensure that Specimens G14, G12 and G10-A would crack, they were loaded up to $P_{max} = 14.6$ kips (65 kN) during the static tests only.

Repetitive loading was applied at the testing machine rate of 265 cycles per min. This is within the frequency range considered "desirable" by ACI Committee 215.⁸ The large mass of the girder necessitated a dynamic correction. This correction was accounted for by controlling the loads such that the deflections produced due to cyclic loading corresponded with the measured minimum and maximum static deflections.

To determine the effect of cyclic loading on the girder's response, dynamic loading was interrupted at 1 million and 2.5 million cycles to per-

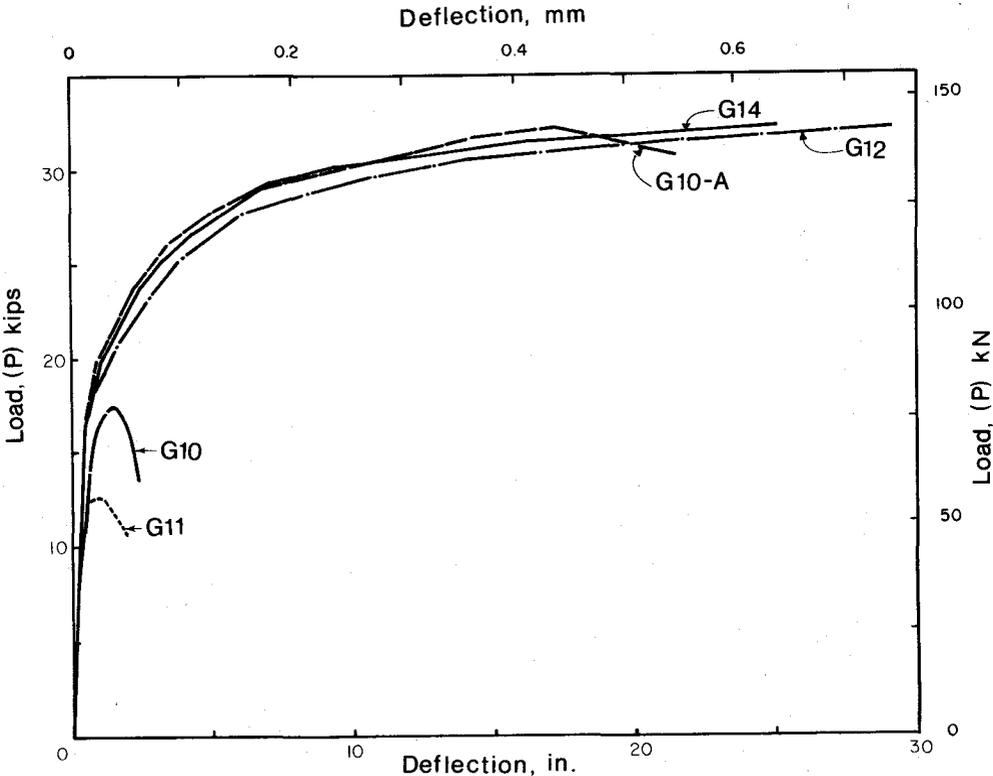


Fig. 4. Load versus midspan deflection envelopes for static tests to destruction.

mit a static test to be conducted between P_{min} and P_{max} . After 5 million cycles, the repetitive loading was stopped. The loading system was then modified and the specimen loaded statically in increments to destruction.

Test Results

Detailed test results are given elsewhere.¹ The following is a summary of material properties and behavior of specimens.

Material Properties

The concrete compressive strength was determined from 6 x 12-in. (152 x 305 mm) concrete cylinders. At test time, the concrete strength of the girders ranged between 5900 and 7600 psi (40.7 and 52.4 MPa). Strength of the deck concrete was between 5000 and 6000 psi (34.5 and 41.4 MPa).

All strands were Grade 250, stress-relieved. They were manufactured in Japan. Strand properties were provided by the manufacturer. Breaking strength varied between 28.5 and 29.1 kips (127 and 129 kN). Yield strength at 1 percent elongation varied between 25.7 and 26.8 kips (114 and 119 kN). Modulus of elasticity ranged between 28,000 and 28,430 ksi (193 and 196 kN/mm²). Cross-sectional area of strands was 0.109 in.² (70 mm²).

Coupons from the same strands used in the girders were tested in the laboratory. These coupons were instrumented with strain gages similar to those used in instrumenting strands in the girders. The modulus of elasticity was found to be 33,400 ksi (231 kN/mm²). It was used to convert the measured strand strains into strand stresses. The measured modulus was high because the strain gages were placed along a wire, i.e., along a spiral.

When unrolled from their coils, the strand had a shiny surface. For about 10 days the strands were exposed to high humidity due to rain and curing steam from adjacent prestressing beds. This resulted in brown surface rusting of the strands.

Behavior of Specimens

The following is a description of the behavior of each specimen. Specimens are discussed in the sequence of testing.

Specimen G11—For Specimen G11, increasing slip of the blanketed strands was measured as the dynamic test progressed. After 3.78 million cycles, fatigue loading was stopped because of the formation of a large crack at an outer crack former. The stiffness of specimen had decreased considerably. The specimen was then unloaded and the loading system modified in preparation for the final static test to destruction.

During the test to destruction, sudden fracture of the specimen occurred at a load very close to the specified service load level. All distress occurred at the section where the large crack had formed. No other cracks appeared. Only two of the 22 strands did not fracture. These were blanketed over a longer length and slipped inside the end portion of the girder. A plot of the applied load versus mid-span deflection is shown in Fig. 4.

Specimen G13—After 3.2 million cycles of repetitive loading, a large crack had extended high into the web at one of the inner crack formers of Specimen G13. Slip of the blanketed strands was negligible. The stiffness of the girder had decreased when the crack formed.

Specimen G13 was cut open at the critical section to inspect the strands. Six strands were found to be fully fractured in fatigue. Six strands had one to five of the seven wires frac-

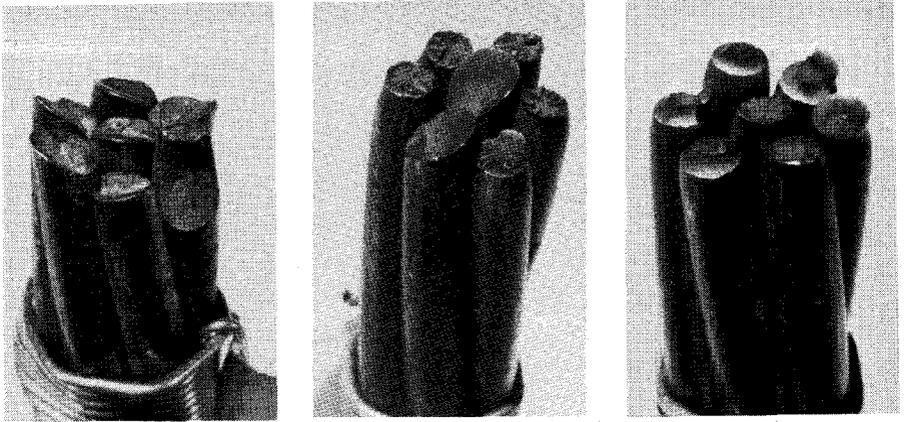


Fig. 5. Fatigue and tension fracture surfaces of strands: left, fatigue fracture; center, fatigue and tension fractures; right, cup and cone tension fracture.

tured in fatigue. Only 10 of the 22 strands had no visible evidence of fatigue.

While removing the concrete cover at the critical section, the position of the crack former with respect to the bottom layer of strands was carefully observed. Outside strands of the bottom layer were bearing against the crack former while the intermediate strands were clear. Of the two strands, one was intact, and the other had fatigue fracture about 1.5 in. (38 mm) away from the crack former.

To determine whether fatigue had affected the properties of the intact strands that crossed the critical section, coupons were extracted from the girder. These were tested statically in tension. The breaking strength of these strands corresponded to the manufacturer's strand strength.

To obtain further information from Specimen G13, the strands at the inner crack former at the opposite end were exposed. A crack of limited height had formed at this section. There were no external visible signs of damage. After exposing the tendons, it was found that one strand of the second layer and one wire from a

bottom layer strand were fractured in fatigue.

Specimen G10—Behavior of Specimen G10 was very similar to that of the two previous specimens. After 3.63 million cycles, the dynamic loading was intentionally stopped after observing formation of a large crack at an inner crack former, at the location of a hold-down device.

Specimen G10 was then loaded statically to destruction. It fractured prematurely and suddenly as illustrated by the applied load versus mid-span deflection curve of Fig. 4. The failure was concentrated at the critical section. No new cracks appeared along the beam.

After separating the two segments of the girder, the fractured surface of all strands was inspected. It was possible to identify the strands that failed due to fatigue and the ones that failed due to tension. The two modes of fracture are very distinct as illustrated by Fig. 5. Six strands were found to be fully fractured in fatigue. Eight strands had one to four of the seven wires fractured in fatigue.

As observed earlier in Specimen G13, it was noticed that the two outer

strands of the bottom layer were touching the crack former while all the intermediate ones were clear. The outer strands had fractured in tension. It is interesting to note that the hold-down device did not seem to be the cause of fatigue of strands. The distribution of fatigued wires and strands appeared to be random.

Specimen G14—Specimen G14 survived 5 million cycles. Only small cracks had formed at the four crack formers. Strand strains and confining tie strains remained stable during the test. Slip of the blanketed strands did not exceed 0.0045 in. (0.11 mm).

Specimen G14 was then loaded statically to destruction. Fig. 4 illustrates the applied load versus midspan deflection curve. The specimen exhibited ductile behavior. Uniformly spaced cracks formed over the center 28 ft (8.5 m) of the specimen. After reaching a midspan deflection of 28 in. (0.7 m), the specimen fractured. The measured strength exceeded that calculated by 4 percent.

During the initial and intermediate static tests, the gages attached to the confining ties did not record any significant strains. It was only during the final static test, after closely spaced cracks were opening, that some gages recorded strains.

Specimen G12—Specimen G12 was similar to Specimen G14 in every respect except that it did not contain confining ties in the stress transfer regions of the blanketed strands. Specimen G12 was tested in a similar manner and the response was similar. The measured strand slip was a little larger.

During the static test to destruction, Specimen G12 exhibited very ductile behavior as illustrated by the applied load versus midspan deflection curve of Fig. 4. The test was stopped after a midspan deflection of 31 in. (0.8 m) was reached. Measured strength ex-

ceeded that calculated by about 2 percent.

Specimen G10-A—Specimen G10-A had draped strands and was similar to Specimen G10. The only difference was the stress level during cycle loading. Under cyclic loading, no tension was allowed in the bottom fibers at midspan.

Applied load versus midspan deflection curve during the final static test is shown in Fig. 4. The strength of Specimen G10-A exceeded that calculated by about 4 percent.

Analysis of Test Results

Based on the data collected from each test, comparisons of performance of the specimens are given in this section. Significant test observations are also discussed.

Level of Fatigue Loading

In Specimens G11, G13 and G10, the higher limit of repetitive loading corresponded to a tensile stress of $6\sqrt{f'_c}$ psi ($0.5\sqrt{f'_c}$ MPa) in the bottom concrete fibers at midspan. The specimens were cracked prior to fatigue loading. In these three specimens, strands fractured due to fatigue after 3.2 to 3.7 million cycles.

In Specimens G14, G12 and G10-A, the repetitive loading did not cause tension in the concrete. All three specimens survived 5 million cycles of repetitive loading. In subsequent static tests to destruction, all three specimens exhibited ductile behavior as illustrated by Fig. 4. Strength of Specimens G14, G12 and G10-A exceeded that calculated by 2 to 4 percent.

Strand Stress Range

Stress range is defined as the difference between maximum and mini-

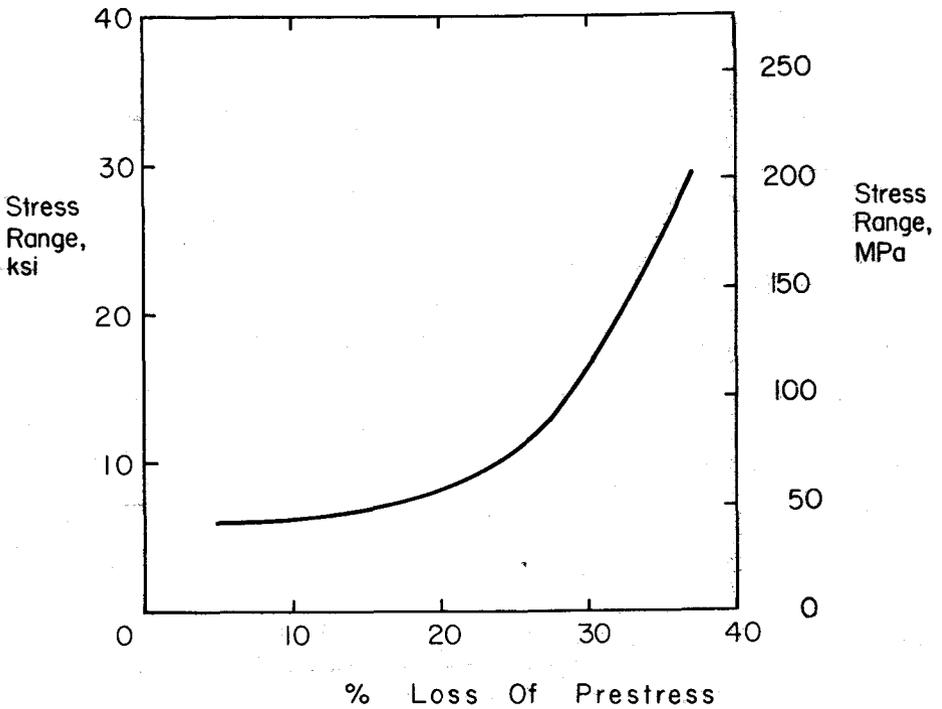


Fig. 6. Calculated stress range versus percentage loss of prestress.

imum strand stresses corresponding to the respective maximum and minimum loads. Stress range was calculated through strain compatibility, equilibrium of internal forces and a knowledge of the actual material properties.

Stress range was also measured by means of strain gages applied to the strands. To eliminate the time-dependent effects of creep and shrinkage, and any possible temperature effects, the stress range of each static test was used for comparison purposes.

Table 3a lists the highest measured range of strand strain corresponding to maximum and minimum inner loads of 14.6 and 4.1 kips (65 and 18.2 kN), respectively. Measured strand strains were converted to stresses using the experimentally determined modulus of elasticity (Table 3b). Specimen

G10-A was not instrumented for strand strains.

Calculations for internal stresses indicate that stress range is a function of the effective prestress. For example at a 21 percent loss of prestress, calculated stress range corresponding to maximum and minimum inner loads of 14.6 kips and 4.1 kips (65 and 18.2 kN) is 8.4 ksi (58 MPa). At 27 percent loss, the calculated stress range corresponding to the same loads is 12 ksi (82.7 MPa). The stress range as affected by prestress is shown in Fig. 6.

For all specimens, measured stress range increased with increase of the cycles of repetitive loading as shown in Table 3b. However, in Specimens G11, G13 and G10, the rate of increase in stress range was much higher. These three specimens were subjected to higher load levels. Strands of the above three specimens

Table 3a. Measured strains.

Number of cycles	Strain, millionths				
	G11	G13	G10	G14	G12
1	318	369	255	352	379
2	314	380	223	—	—
3	320	383	237	—	—
1.0×10^6	424	412	583	386	426
2.5×10^6	544	602	569	382	462
5.0×10^6	—	—	—	400	464

* $P_{min} = 4.1$ kips; $P_{max} = 14.6$ kips.

Table 3b. Corresponding stress range.†

Number of cycles	Stress, ksi				
	G11	G13	G10	G14	G12
1	10.6	12.3	8.5	11.8	12.7
2	10.5	12.7	7.5	—	—
3	10.7	12.8	7.9	—	—
1.0×10^6	14.2	13.8	19.5	12.9	14.2
2.5×10^6	18.2	20.1	19.0	12.8	15.5
5.0×10^6	—	—	—	13.4	15.5

† Based on $E = 33,445$ ksi

1 kip = 4.45 kN; 1 ksi = 6.895 MPa.

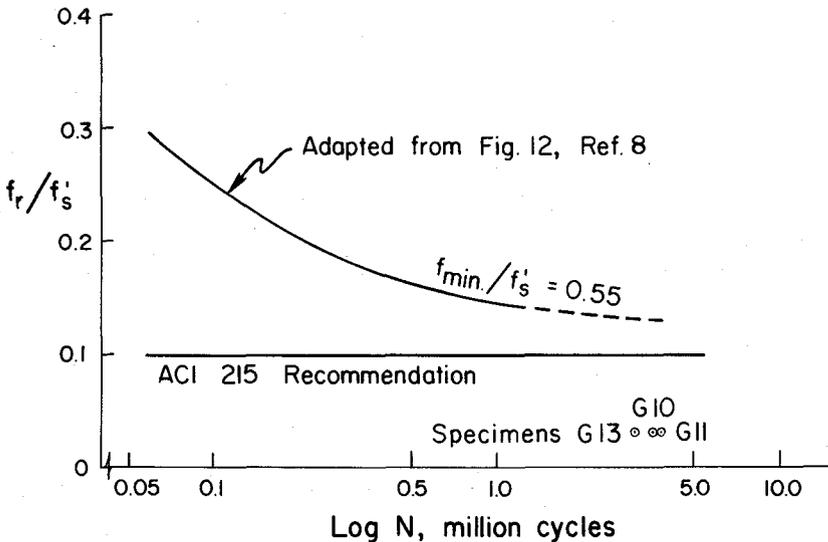


Fig. 7. S-N curve for 7/16-in. diameter strands.

fractured due to fatigue between 3.2 and 3.7 million cycles of repetitive loading.

Fig. 7 is a plot of calculated stress range in the strands at 20 percent loss of prestress versus fatigue life of Specimens G11, G13 and G10. The S-N curve was adapted from Fig. 12 of a report prepared by ACI Committee 215, Fatigue of Concrete.⁸ Their curve was obtained through a regression analysis of fatigue test results.¹¹

It can be seen that the fatigue strength of Specimens G11, G13 and G10 was lower than previous fatigue test results. Very recently, fatigue tests of full-sized bridge girders were conducted in England. Fatigue of strands was observed at 3 million cycles.¹²

Slip of Blanketed Strand

Strand slip measured with a dial gage at the end of the girder provided a good indication of the effectiveness of development length. Fig. 8 illustrates the load versus slip recorded during static tests. For each girder, data from the blanketed strand that had the largest slip is plotted.

For Specimen G11, as shown in Fig. 8a, slip increased with repeated load; this denoted bond fatigue.^{6,7} When Specimen G11 was loaded to destruction, two strands slipped inside the end portion of the girder.

Fig. 8b records the response of Specimen G13. Slip stabilized after the initial elastic slip. Even after 3.2 million cycles, the increase in slip was negligible denoting good anchorage of the blanketed strands. Specimen G13 had double the development length specified in ACI 318-77, Section 12.10.1.

Figs. 8c and 8d, Specimens G14 and G12 respectively, show that strand slip in Specimen G14 was smaller than in Specimen G12. This suggests some beneficial effect of the extra confining ties. However, the effect

was not significant enough to justify the use of the ties.

Slip in Specimen G14 also remained smaller during the static test to destruction, as shown in Fig. 9. Maximum strand movements of Specimens G14 and G12 were small up to 5 million cycles. The small slip did not affect the strength and behavior of the two beams.

Effect of the number of cycles of repetitive loading on strand slip is illustrated in Fig. 10. For each specimen, the strand with the largest slip is plotted. This plot denotes rapid bond deterioration of Specimen G11.

It can be seen that for Specimens G12 and G14, magnitudes of movements plotted in Fig. 8 are different from those of Fig. 10. During the repetitive loading of Specimens G14 and G12, the maximum applied inner load, P_{max} , was 10.9 kips (48.5 kN). During static tests, the maximum applied inner load was 14.6 kips (65 kN). Thus, slip measured during static tests was larger.

Development Length

Specimen G13 was designed for two development lengths, $2l_d$. Specimen G11 was designed for one development length. Both specimens were cycled at the higher load level corresponding to a tensile stress of $6\sqrt{f'_c}$ psi ($0.5\sqrt{f'_c}$ MPa) in the concrete at midspan. Both specimens had fatigue fracture of the strands. The main difference in behavior was that, in Specimen G11, slip of the strands kept increasing with cyclic loading as shown in Fig. 10.

A bond fatigue failure of the blanketed strands of Specimen G11 was observed. On the other hand, in Specimen G13, after the initial elastic slip occurred, slip of strands remained virtually unchanged up to 3.2 million cycles of repetitive loading as shown in Fig. 8b. Therefore, twice the development length, $2l_d$, used in Speci-

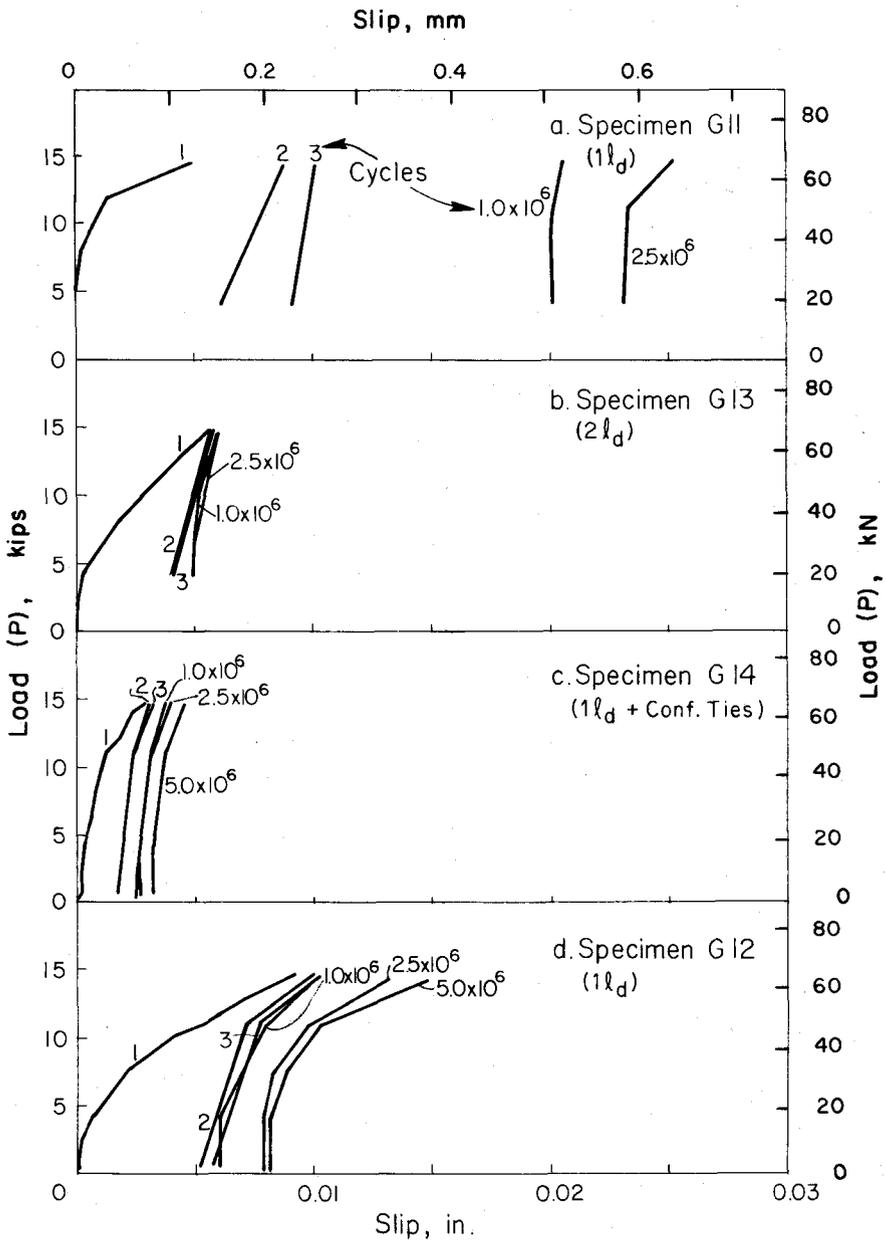


Fig. 8. Measured load versus slip at selected number of cycles.

men G13, provided good anchorage of the blanketed strands.

Specimens G14 and G12 were designed for one development length. They were tested at a load level corre-

sponding to zero tension in the concrete. Both specimens survived 5 million cycles of repetitive loading.

In the subsequent static test to destruction, both specimens reached

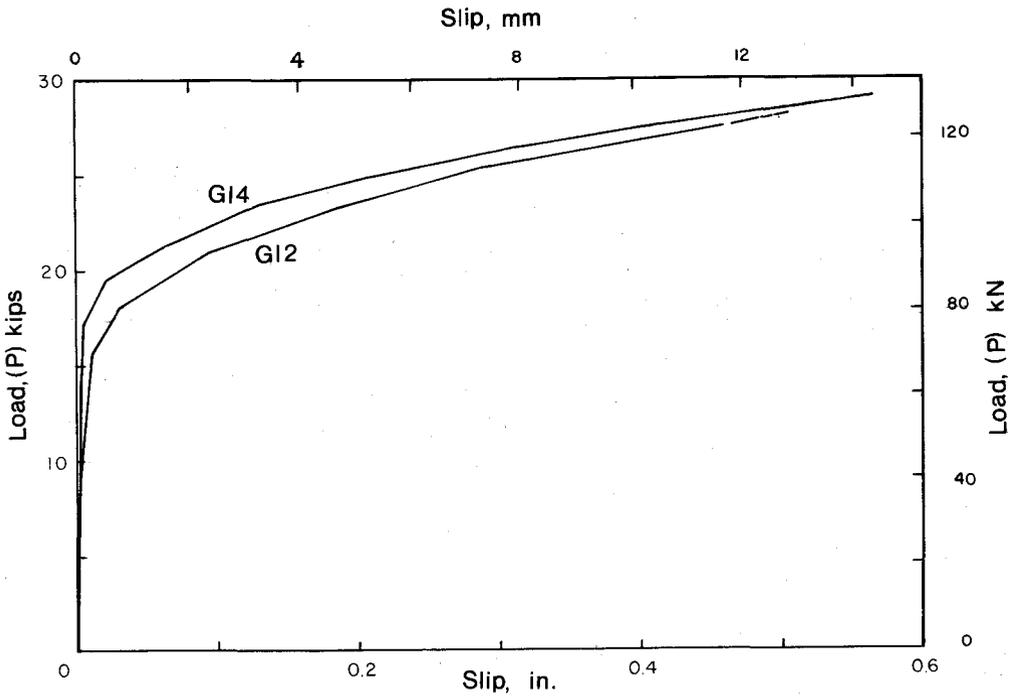


Fig. 9. Load versus strand slip during static test to destruction.

their calculated strength. No bond fatigue of the strands was observed. Therefore, when the cyclic load corresponded to zero tension in the concrete, one development length was adequate.

Effect of Crack Formers

During this investigation, the question arose as to whether the presence of crack formers had any detrimental effect on the performance of the tested specimens, and, whether rubbing of the crack formers against the bottom layer of strands provoked fatigue of the strands.

Confining Ties

Strains measured on confining ties at service load levels were negligible. Significant strains were measured only at very high loads following the formation of large cracks, during the static test to destruction.

Smaller slip of the blanketed strands was measured in Specimen G14, with confining ties than in Specimen G12. However, the behavior and strength of Specimen G14 were not judged to indicate any significant beneficial effect of the confining ties.

Surface Condition of Strands

In the present investigation, strands used in the test specimens had brown surface rust. A strand coupon was inspected visually.¹³ It was judged to be similar to those used in daily production.

It is well known that the surface condition affects the required development length. However, the surface condition of the strands was not one of the control variables of this investigation. It is possible that the surface condition of the strands may have affected both development length and fatigue properties.

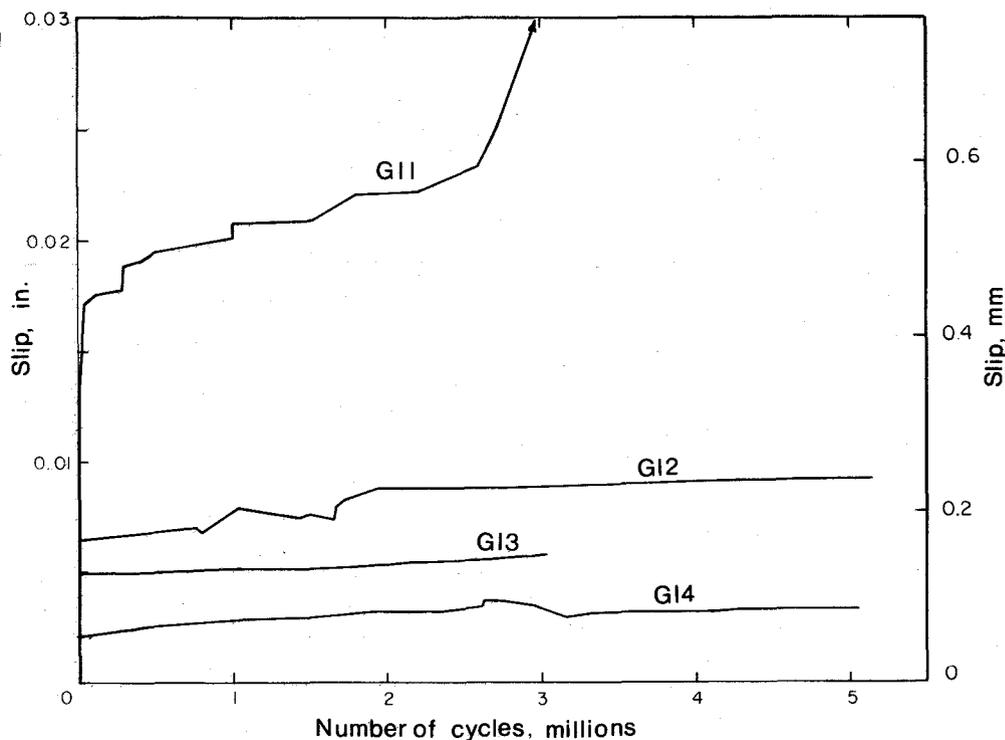


Fig. 10. Variation of slip with applied cycles of repetitive loading.

As discussed earlier, inspection of the fractured sections of Specimens G13 and G10 indicated that only the outer strands of the bottom layer were bearing against the crack formers. The other six strands were clear. In Specimen G13, one of the two outer strands of the bottom layer fractured due to fatigue. However, the fracture occurred at a distance of 1.5 in. (38 mm) away from the crack former.

In Specimen G10, six of the 22 strands fractured due to fatigue. Another eight strands had one or more wires fractured due to fatigue. Neither of the two outer strands of the bottom layer were affected by fatigue. The observed fatigue fractures in Specimens G13 and G10 definitely demonstrated that crack formers did not directly cause the fatigue failures.

Concluding Remarks

The tests of this investigation have confirmed that blanketing of strands is a feasible technique that could lead to safer and more economical manufacturing of prestensioned bridge girders.

The tests have also indicated that fatigue of strands may be an important consideration in prestressed girders designed according to recent Codes where a concrete tensile stress of $6\sqrt{f'_c}$ psi ($0.5\sqrt{f'_c}$ MPa) is permitted under service loads. Present Codes do not provide the designer with guidance regarding fatigue of strands.

Detailed conclusions and recommendations based on this investigation are stated at the beginning of the paper.

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Discussion of this paper is invited.
Please send your comments to PCI
Headquarters by January 1, 1980.

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APPENDIX A—NOTATION

<p>A = area of cross section</p> <p>D = nominal diameter of prestressing steel</p> <p>f'_c = compressive strength of concrete at 28 days</p> <p>f'_{ci} = compressive strength of concrete at time of initial prestress</p> <p>f_{min} = minimum stress in strand</p> <p>f_r = difference between the maximum and minimum stress in the strands in one cycle</p> <p>f'_s = ultimate strength of prestressing steel</p> <p>f_{se} = effective steel prestress after losses</p> <p>f_{su}^* = average stress in prestressing steel at ultimate load</p> <p>I_{xx} = moment of inertia about the</p>	<p style="text-align: right;">centroid of the cross section</p> <p>l_a = development length</p> <p>M_D = total dead load moment</p> <p>M_{D1} = moment due to self weight of girder</p> <p>M_{D2} = moment due to superimposed dead load</p> <p>M_L = moment due to live load</p> <p>M_v = nominal flexural strength of a section</p> <p>N = number of cycles of stress before fracture occurs</p> <p>P_{D2} = concentrated dead load causing moment M_{D2}</p> <p>P_{min} = minimum repetitive load</p> <p>P_{max} = maximum repetitive load</p> <p>S_b, S_t = bottom and top moduli of section</p>
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Table B1. Concrete stresses at end of girder due to initial prestress.

Number of strands	Prestress (kips)	Eccentricity (in.)	Stresses (psi)	
			Top	Bottom
18	320	10.22	+427	-1883
20	352	10.53	+513	-2105
22	383	10.78	+597	-2323

1 kip = 4.45 kN; 1 in. = 25.4 mm; 1 psi = 6.895 kPa.

APPENDIX B—DESIGN CALCULATIONS

Specifications and assumed material properties used in designing the specimens were given in the body of the paper under the heading Design Criteria.

Properties of Section

Properties of the Type II girder and the composite deck-girder cross sections are summarized in Fig. B1.

Initial Stresses at Transfer

Concrete stresses due to initial prestress are listed in Table B1. Effective bed pull was based on 70 percent of the strand strength. Eccentricities for 18 or 20 strands were calculated assuming that strands from the bottom layer were blanketed. Top and bottom concrete stresses were for non-cracked sections.

Number of Blanketed Strands

The allowable tensile stress in the concrete at transfer of prestress was $7.5\sqrt{f'_{ci}}$ psi ($0.625\sqrt{f'_{ci}}$ MPa), i.e., 474 psi (3.27 MPa). The tensile stress corresponding to 18 strands did not exceed the allowable, as shown in Table B1. Therefore, four strands were blanketed.

The strands were blanketed in pairs for symmetry purposes. Therefore, blanketing of the four strands was stopped at two different sections.

Minimum Blanketed Length

The minimum blanketed length corresponded to the section where the sum of concrete stresses due to the initial prestressing force and the dead weight moment was $7.5\sqrt{f'_{ci}}$ psi ($0.625\sqrt{f'_{ci}}$ MPa).

At the section where 20 strands were effective, the top initial concrete stresses exceeded the allowable by

$$513 - 474 = 39 \text{ psi (0.27 MPa).}$$

Therefore, the dead weight moment required to counteract this tension was

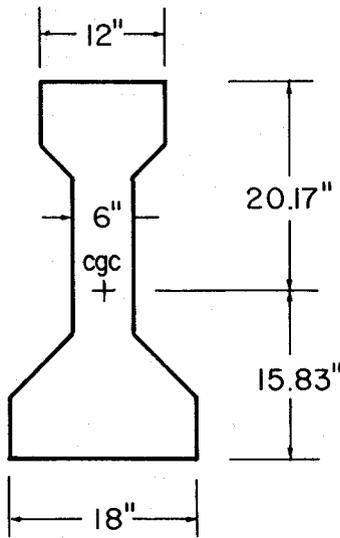
$$39 \times 2527 = 98,553 \text{ lb-in. or } 8.21 \text{ kip-ft}$$

This dead weight moment corresponded to a section located at 0.87 ft (0.27 m) from the ends. Therefore, the minimum blanketed length for four strands was 0.87 ft (0.27 m).

Similarly, the minimum blanketing length for two strands was 2.86 ft (0.87 m) measured from each end. However, as shown later, the blanketed length adopted was determined from development length criteria. This blanketed length was not less than the minimum calculated in this section.

Flexural Strength

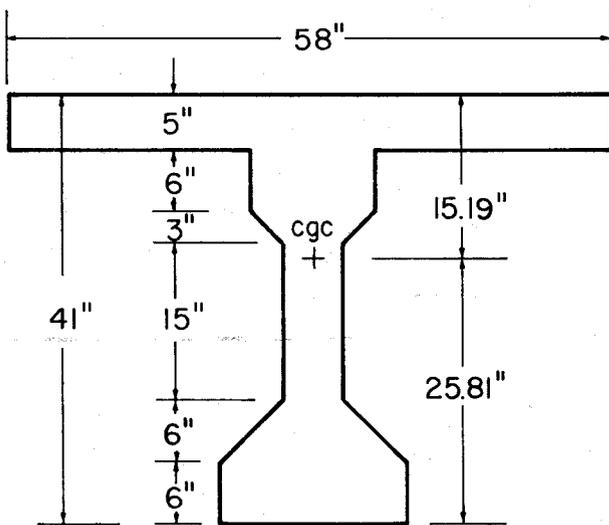
Nominal flexural strength of sections containing 18, 20, and 22 strands are listed in Table B2. These moment capacities were calculated according to the AASHTO Specifications.²



$$\begin{aligned}
 A &= 369 \text{ in.}^2 \\
 &\quad (0.384 \text{ k/ft}) \\
 I_{xx} &= 50979 \text{ in.}^4 \\
 S_t &= 2527 \text{ in.}^3 \\
 S_b &= 3220 \text{ in.}^3
 \end{aligned}$$

$$1'' = 0.0254 \text{ m}$$

$$1 \text{ k/ft.} = 14.6 \text{ kN/m}$$



$$\begin{aligned}
 A &= 659 \text{ in.}^2 \\
 &\quad (0.686 \text{ k/ft}) \\
 I_{xx} &= 135,041 \text{ in.}^4 \\
 S_t &= 8888 \text{ in.}^3 \\
 S_b &= 5233 \text{ in.}^3
 \end{aligned}$$

Fig. B1. Section properties of Type II girder and composite deck-girder.

Strength reduction factor was taken as unity.

Critical Sections

Critical sections are discussed in the 1977 AASHTO Specifications,² Section 1.5.13.A and in ACI 318-77,⁵ Section 12.11.2. They are described as follows:

“Critical sections for de-

velopment of reinforcement in flexural members are at points of maximum stress and at points within the span where adjacent reinforcement terminates, or is bent.”

Location of critical sections can be determined either graphically or analytically. Critical sections for 18, 20, and 22 strands were determined

Table B2. Nominal flexural strength.

Number of strands	Area of strands (in. ²)	Effective depth (in.)	Strand Stress at ultimate (ksi)	Flexural strength (kip-ft)
18	1.962	35.39	244.0	1372
20	2.180	35.70	243.4	1530
22	2.398	35.96	242.8	1686

1 in. = 25.4 mm; 1 kip-ft = 2.356 kN.m.

for the load configuration of Fig. B2. These loads correspond to nominal flexural strength at midspan.

For 22 strands, the critical section coincided with the innerpoint loads, i.e., at 21 ft (6.4 m) from the supports or 22 ft (6.71 m) from the end of the girder. The moment gradient due to dead load was ignored in the region between the two inner point loads.

For 20 strands, the critical section was located at 16 ft (4.88 m) from the supports, i.e., at 17 ft (5.18 m) from the end of the girder. Similarly for 18 strands, the critical section was at 12 ft (3.89 m) from the end of the girder.

Development Length

The development length, l_d , was computed² from:

$$\left(f_{su}^* - \frac{2}{3}f_{se}\right) D$$

where

f_{su}^* = average stress in prestressing steel at ultimate load

f_{se} = effective steel prestress after losses

D = nominal diameter of prestressing steel

The average stress in the prestressing steel at ultimate load, f_{su}^* , was 243 ksi (1676 MPa) as shown in Table B2.

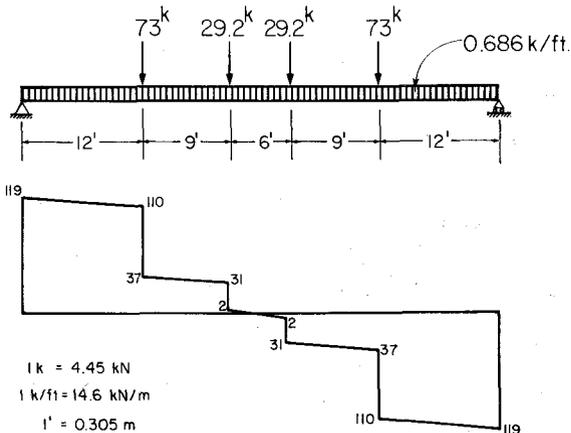


Fig. B2. Loads and shears corresponding to calculated flexural capacity.

Table B3. Blanketed length from end of girder.

Development length	Critical section	Blanketed length
$1/d$	17 ft 0 in.	11 ft 6 in.
	22 ft 0 in.	16 ft 6 in.
$2/d$	17 ft 0 in.	6 ft 0 in.
	22 ft 0 in.	11 ft 0 in.

1 ft = 0.3048 m.

Corresponding to prestress losses of 20 percent, the effective stress in the prestressing steel was 80 percent of the effective bed pull stress in the prestressing steel. Therefore, the effective stress was

$$0.7 \times 0.8 \times 250 = 140 \text{ ksi (965 MPa)}.$$

For a $\frac{7}{16}$ -in. (11.1 mm) strand, the calculated development length was 5 ft 6 in. (1.68 m).

Maximum Blanketed Length

The maximum blanketed length was determined by providing a development length measured from the critical section. This procedure is similar to stopping of reinforcing bars in reinforced concrete members. The ACI Code⁵ specifies that reinforcing bars should be extended a specified "minimum distance" beyond the theoretical cut-off point.

By analogy, in prestressed members, a similar requirement is needed. In fact, this "minimum distance" should be longer because at ultimate, the diagonal compression is flatter due to prestressing. However, presently the codes do not cover such a "minimum distance" for prestressed members.

In the present investigation, it was ensured that the strands were not blanketed at the theoretical cut-off point. Table B3 summarizes the blanketing length corresponding to one and two development lengths.

The theoretical cut-off point for the first two strands was located at 17 ft (5.18 m) from the end of the girder. This was also the critical section for 20 strands. The corresponding blanketed length was 16 ft 6 in. (5.03 m), i.e., smaller, and therefore acceptable.

Shear Reinforcement

Web reinforcement was designed according to the AASHTO² provisions. It consisted of two No. 4 (12 mm) bars spaced at 6 in. (152 mm) on center. Because of the magnitude of the concentrated loads applied to the specimen, the same spacing was used over the full length of the girder. Additional reinforcement to resist splitting was added at the ends of the girders.

Cyclic Loads

The magnitude of the cyclic loads for Specimens G11, G13, and G10 were determined from the following two conditions:

1. The dead load moment M_D , plus the live load moment, M_L , caused a tensile stress of $6\sqrt{f'_c}$ psi ($0.5\sqrt{f'_c}$ MPa) in the bottom concrete fibers at midspan.
2. The factored dead load plus live load moments equaled the nominal flexural strength, M_u . Therefore,

$$M_u = 1.3M_D + 2.167M_L.$$

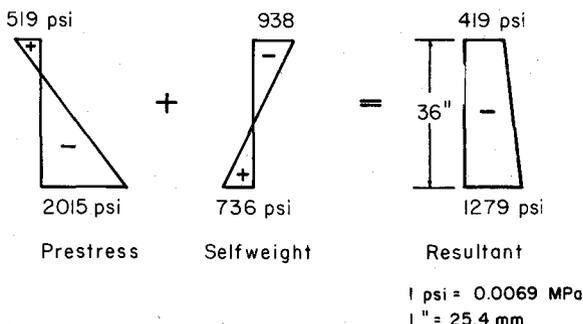


Fig. B3. Effective concrete stresses at midspan.

The dead load moment corresponded to self weight of the girder and superimposed point loads. The magnitude of the superimposed dead and live loads were computed as follows:

Formwork for the concrete deck was supported on the girder. Therefore, the weight of the composite deck-girder was carried by the non-composite section. Midspan concrete stresses due to the effective prestress and due to the self weight of the specimen are shown in Fig. B3.

Under full service load, allowable tensile stress in the concrete at midspan was $6\sqrt{f'_c}$ psi ($0.5\sqrt{f'_c}$ MPa) i.e., 424 psi (2.92 MPa). Superimposed moment in addition to self weight required to reach this tension was

$$\begin{aligned} (1279 + 424) \times 5233 &= \\ 8,912,000 \text{ lb-in. or} & \\ 742.6 \text{ kip-ft (1010 kN}\cdot\text{m)} & \end{aligned}$$

This moment corresponded to the superimposed dead load moment, M_{D2} , and live load moment M_L . The dead weight moment of the girder, M_{D1} , was:

$$\begin{aligned} M_{D1} &= \frac{0.686 \times (48)^2}{8} \\ &= 197.6 \text{ kip-ft} \\ &\quad (268 \text{ kN}\cdot\text{m)} \end{aligned}$$

The above two conditions yielded the equations:

$$M_{D1} + M_{D2} + M_L = 197.6 + 742.6 \text{ kip-ft}$$

and

$$1.3(M_{D1} + M_{D2}) + 2.167 M_L = 1686 \text{ kip-ft}$$

Moment M_{D1} was determined earlier. Solving the two equations, it was found that:

$$M_{D2} = 207.6 \text{ kip-ft (281 kN}\cdot\text{m)}$$

and

$$M_L = 535 \text{ kip-ft (725 kN}\cdot\text{m)}$$

The corresponding inner loads for the load configuration of Fig. 2 were

$$P_{D2} = 4.1 \text{ kips (18.2 kN)}$$

and

$$P_L = 10.5 \text{ kips (46.7 kN)}$$

Therefore, the cyclic loads were

$$P_{min} = 4.1 \text{ kips (18.2 kN)}$$

and

$$P_{max} = 4.1 + 10.5 = 14.6 \text{ kips (65 kN)}$$

These loads are listed in Table 2.

For Specimens G14, G12 and G10-A, maximum cyclic load corresponded to zero tension in the midspan bottom concrete fibers. Required moment for this condition was

$$\begin{aligned} 1279 \times 5233 &= 557.8 \text{ kip-ft} \\ &\quad (758 \text{ kN}\cdot\text{m)}. \end{aligned}$$

The corresponding inner applied load P_{max} was 10.9 kips (48.5 kN).