Flexural Strength of Prestressed Concrete Sections by Programmable Calculator

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It has always been recognized that the approximate Eq. (18-3) in ACI 318-77[1] for the stress in bonded prestressing steel at flexural ultimate, f_{ps}, was in general conservative. A more precise value for f_{ps}, and hence for the flexural strength, can be obtained by solving the equations of equilibrium of forces and of compatibility of strains across the section, making use of the actual stress-strain relationship for the prestressing steel. Until the advent of the computer and the programmable calculator, this was a time-consuming process and hence was little used.

In a 1977 paper, Naaman[2] discussed the more precise approach. He showed how more accurate values of flexural strength could readily be obtained using a computer to solve the equations of equilibrium and compatibility, if the stress-strain curve for the prestressing steel were expressed algebraically.

Since the time of Naaman's studies, there has been an explosion in the availability and the capabilities of small programmable electronic calculators. These machines now make it feasible and economic for any engineer to use the more precise method of calculation for flexural strength, with consequent economic advantages.

Prestressing Steel, Stress-Strain Curve

The stress-strain curve for prestressing steel typically consists of a linear part (corresponding to the region of elastic behavior), a sharply curved part in the vicinity of the nominal yield point stress, and an al-

The author presents calculator programs for the flexural strength of bonded prestressed or partially prestressed concrete sections of general form. The calculations make use of the actual stress-strain properties of the prestressing steel and in general result in less conservative values for flexural strength than are obtained through use of the approximate Eq. (18-3) of the ACI Building Code. Several detailed design examples show the application of the calculator programs.

most linear, slightly strain-hardening part reaching to failure.

Naaman represented the stress-strain curve by three separate equations, corresponding to the three parts of the curve identified above. He used two different linear equations for the first and third parts, and a fourth degree polynomial for strains between 0.006 and 0.014. The eight coefficients of these equations were determined using the computer, so as to give the best fit.

This approach to algebraic representation of the steel stress-strain curve does not lend itself to use with a small calculator. Other possibilities were therefore investigated.

It has been found possible to represent the stress-strain curve of prestressing steel very closely using a single equation of the form:

$$f_s^* = (1 - Q)\frac{\epsilon^*}{(1 + \epsilon^{*R})^{1/R}} + Q\epsilon^* \quad (1)$$

where $f_s^* = f_s/f_{so}$ and $\epsilon^* = \epsilon/\epsilon_o$.

In the above, f_s is the stress at a strain ϵ, f_{so} and ϵ_o are a reference stress and its corresponding strain.

This form of equation was used by Menegotto and Pinto[3] to represent the stress-strain relationship for reinforcing bars subjected to cyclic loading beyond their yield point. The curve represented by Eq. (1) is shown in Fig. 1. The first part of the curve is linear with a slope of unity; the second part is curved, the radius of curvature becoming larger as the coefficient R decreases; and the third part is linear with a slope equal to the coefficient Q.

This form of curve has been adapted to represent the stress-strain curve of prestressing steel by making:

$$f_{so} = Kf_{py} \text{ and } \epsilon_o = Kf_{py}/E$$

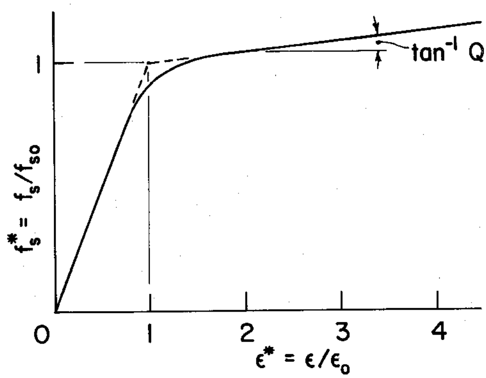


Fig. 1. Curve corresponding to Eq. (1).

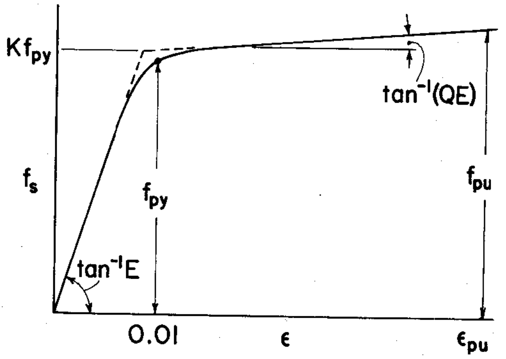


Fig. 2. Typical stress-strain curve for prestressing steel as represented by Eq. (2).

where

K = a coefficient
f_{py} = specified yield strength of prestressing steel
E = modulus of elasticity of prestressing steel

The equation then becomes:

$$f_s = \epsilon E \left[Q + \frac{1 - Q}{\left\{ 1 + \left(\frac{\epsilon E}{K f_{pu}} \right)^R \right\}^{1/R}} \right] \qquad (2)$$

where

$$Q = \left(\frac{f_{pu} - K f_{py}}{\epsilon_{pu} E - K f_{py}} \right)$$

The coefficient R is determined by solving Eq. (2) for the condition $f_s = f_{py}$ when $\epsilon = 0.010$. The values f_{pu} and ϵ_{pu} are the specified tensile strength and the strain at tensile failure of the prestressing steel, respectively.

The curve corresponding to Eq. (2) is shown in Fig. 2. The initial slope is equal to the modulus of elasticity, the curve passes through the nominal yield point and the final slope corresponds to the measured strain-hardening behavior of the steel.

Eq. (2) can be made to correspond very closely to actual stress-strain curves, as shown in Fig. 3, by use of appropriate values for the coefficients K, Q, and R. Using Eq. (2) it has been found possible to calculate the stress corresponding to a given strain to within 1 percent or less, for a number of actual stress-strain curves.

If the complete stress-strain curve for the steel is available, K is determined as follows:

Produce the two linear parts of the stress-strain curves until they meet. The stress corresponding to the point of intersection is Kf_{py}. Hence, obtain K. If the complete steel stress-strain curve is not available, a reasonable value to assume for K in the case of seven-wire strand is 1.04.

In Appendix A are given the user instructions and the listing of a program for the Hewlett Packard HP-67/97 programmable calculators, which will:

1. Determine the values of Q and R for the stress-strain curve for a particular steel, given f_{py}, f_{pu}, E, ϵ_{pu}, and K.
2. Calculate the stress and the force in a tendon of cross section A_s, for any given strain ϵ. (Agreement between the calculated and experimental stress-strain relationships can therefore be checked readily.)
3. Calculate the strain ϵ_{se} corresponding to the effective prestress f_{se} in a particular prestressed beam, and store as needed for a subsequent flexural strength calculation.

The solution of Eq. (2) to obtain R is carried out using an adaptation of Newton's method.[4] This involves an iterative procedure and typically takes about 45 seconds. For this program to work, Kf_{py} must be less than f_{pu} and greater than about 0.75 f_{pu}. This covers the characteristics of prestressing steels meeting the relevant ASTM specifications.

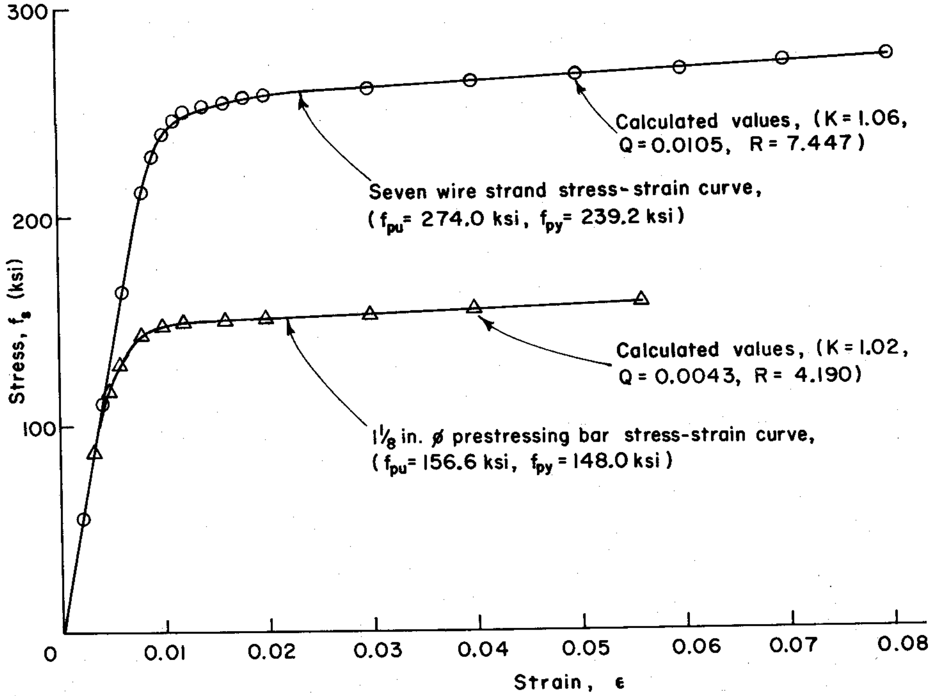


Fig. 3. Comparison of calculated values of f_s and ϵ with experimental stress-strain curves.

Calculation of Flexural Strength

The calculation of flexural strength is based on Section 10.2—Design Assumptions, of ACI 318-77; except that the stress in the reinforcement is assumed to be related to the strain in accordance with Eq. (2) above. Both the prestressed and the non-prestressed reinforcement is assumed to have the same stress-strain curve. Such a situation arises when seven-wire strand is used as both prestressed and non-prestressed reinforcement.

The stress and strain conditions in a bonded prestressed concrete beam are shown in Fig. 4. A single-tee section with a tapering flange and fillets at the web-flange junction is shown, but the equations developed and the calculator program based on them are applicable to a variety of beam sections, as shown later.

Use is made of the equivalent rectangular stress distribution defined in Section 10.2.7 of ACI 318-77 when calculating the resultant compression force in the concrete at ultimate, and its center of action.* The width of the section at the bottom edge of the equivalent rectangular stress distribution is designated as b', i.e., at distance "a" from the maximum compression fiber.

The resultant concrete compression force C for various possible locations of the neutral axis is given by:

*The applicability of the equivalent rectangular stress distribution to non-rectangular sections has been verified experimentally.[5,6]

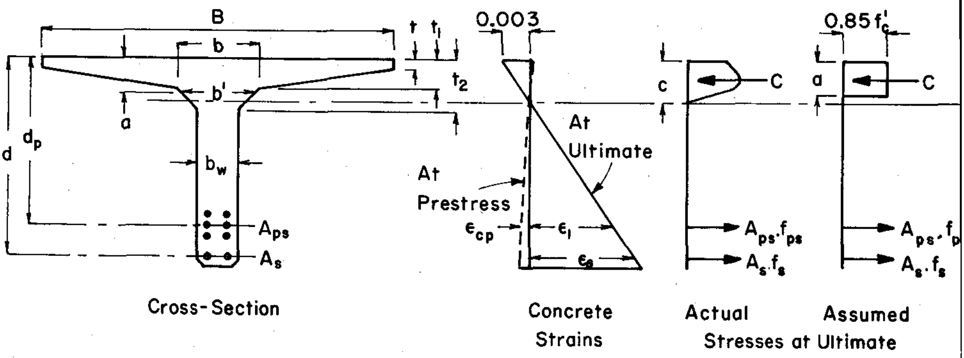


Fig. 4. Stress and strain conditions in a prestressed single-tee.

(a) $a \leqslant t$; $\quad C = 0.85 f_c' \, Ba$ (3)

(b) $t < a < t_1$; $\quad C = 0.85 f_c'\left[A_1 + (a-t)\left\{B - \left(\frac{B-b}{2}\right)\left(\frac{a-t}{t_1-t}\right)\right\}\right]$ (4)

(c) $t_1 < a < t_2$; $\quad C = 0.85 f_c'\left[A_2 + (a-t_1)\left\{b - \left(\frac{b-b_w}{2}\right)\left(\frac{a-t_1}{t_2-t_1}\right)\right\}\right]$ (5)

(d) $t_2 < a$; $\quad C = 0.85 f_c'\left[A_3 + (a-t_2)b_w\right]$ (6)

$$\text{where } A_1 = Bt \text{ and } A_2 = A_1 + \left(\frac{B+b}{2}\right)(t_1 - t)$$

$$\text{and } A_3 = A_2 + \left(\frac{b+b_w}{2}\right)(t_2 - t_1)$$

Since the reinforcement is bonded to the concrete, the change in strain in the reinforcement can be assumed equal to the change in strain in the adjacent concrete. The strain in the prestressed reinforcement at flexural ultimate ϵ_{ps}, is therefore given by:

$$\epsilon_{ps} = \epsilon_{se} + \epsilon_{cp} + \epsilon_1$$
$$\epsilon_{ps} = \epsilon_{se} + \epsilon_{cp} + 0.003(d_p - c)/c \quad (7)$$

where

ϵ_{se} = strain in reinforcement A_{ps} due to effective prestress f_{se}

and

ϵ_{cp} = compressive strain in concrete adjacent to A_{ps}, due to prestress

The strain ϵ_{cp} is very small compared to ϵ_{se} and ϵ_1, and for simplicity may be neglected. Hence we may write:

$$\epsilon_{ps} = \epsilon_{se} + 0.003(d_p - c)/c \quad (8)$$

The strain in the unprestressed reinforcement at flexural ultimate ϵ_s, is given by:

$$\epsilon_s = 0.003(d - c)/c \quad (9)$$

The stresses f_{ps} and f_s in the prestressed and unprestressed reinforcement at flexural ultimate, can be obtained by substituting the strains ϵ_{ps} and ϵ_s in Eq. (2). The total tensile force in the section at flexural ultimate is then given by:

$$T = A_{ps}f_{ps} + A_s f_s \quad (10)$$

Equilibrium of forces in the section requires that:

$$T = C \quad (11)$$

Solution of this equation yields the depth of the neutral axis at failure, c. Unfortunately, because of the variation in the width of the section and the complex stress-strain equation for the reinforcement, it is not possible to obtain a closed form solution for this equation. However, "c" can be evaluated by an iterative process, with the help of a programmable calculator.

In Appendix B are given the user instructions and the listing of a program for the HP-67/97 programmable calculators, Part 1 of which will calculate the neutral axis depth at flexural ultimate, using the following iterative process:

(a) Assume an initial value for "c."
(b) Calculate the strains in the reinforcement using Eqs. (8) and (9).
(c) Calculate the stresses in the reinforcement using Eq. (2) and hence the total tensile force in the reinforcement, $(A_{ps}f_{ps} + A_s f_s)$.
(d) Calculate the resultant concrete compression force C, using Eqs. (3), (4), (5), or (6), as appropriate.
(e) Compare the calculated resultant compressive force C with the resultant tensile force $(A_{ps}f_{ps} + A_s f_s)$. If they are not within 1 percent of being equal, adjust the assumed value of "c" and recycle as often as necessary until this degree of agreement is reached. When the compressive and tensile forces are sufficiently close, the corresponding value of "c" may be assumed to be the neutral axis depth at flexural ultimate.

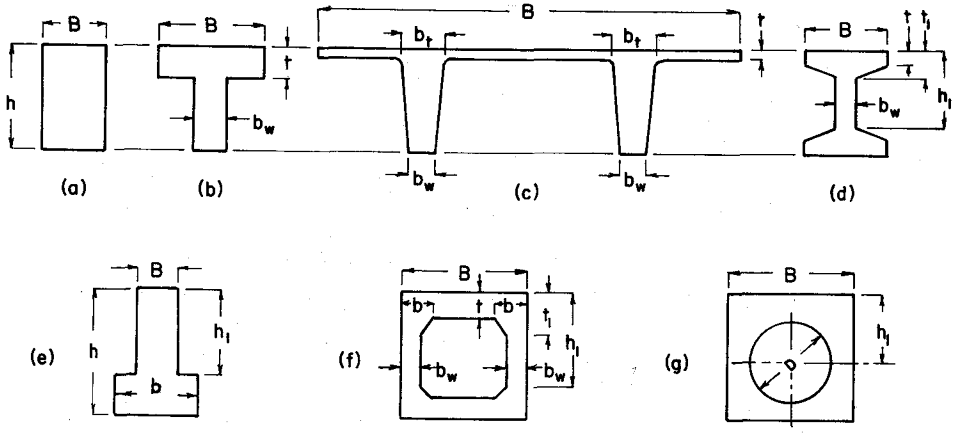
When the neutral axis depth has been obtained, the flexural strength may be calculated using whichever of the following equations is appropriate.

(a) $a < t$;
$$M_n = C\left(d_p - \frac{a}{2}\right) + A_s f_s(d - d_p) \quad (12)$$

(b) $t < a < t_1$;
$$M_n = 0.85f'_c\left[A_4 + \left(\frac{B+b'}{2}\right)\left(a-t\right)\left\{d_p - t - \left(\frac{a-t}{3}\right)\left(\frac{B+2b'}{B+b'}\right)\right\}\right] + A_s f_s(d - d_p) \quad (13)$$

$$\text{where } b' = \left\{B - \frac{(B-b)(a-t)}{(t_1 - t)}\right\} \quad (13A)$$

(c) $t_1 < a < t_2$;
$$M_n = 0.85f'_c\left[A_5 + \left(\frac{b+b'}{2}\right)\left(a-t_1\right)\left\{d_p - t - \left(\frac{a-t_1}{3}\right)\left(\frac{b+2b'}{b+b'}\right)\right\}\right] + A_s f_s(d - d_p) \quad (14)$$



Program Use Step	Dimension to be Entered							
	Single-Tee	(a) Rect.	(b) T-beam	(c) Dbl.-Tee	(d) I (1)	(e) Inv.-Tee	(f) Box A (1)	(g) Box B (2)
3E	B	B	B	B	B	B	B	B
3F	b	*	b_w	$2b_t$	b_w	b	2b	(B - 0.3D)
3G	b_w	*	b_w	$2b_w$	b_w	b	$2b_w$	(B - D)
3H	t	h	t	t	t	h_1	t	$(h_1 - 0.5D)$
3J	t_1	*	t	t	t_1	h_1	t	$(h_1 - 0.5D)$
3K	t_2	*	h	h	h_1	h	t_1	$(h_1 - 0.2D)$

*No entry (1) $a \not> h_1$ (2) Very nearly, $a \not> 1.2h_1$

Fig. 5. Use of program for various sections.

$$\text{where } b' = \left\{ b - \frac{(b - b_w)(a - t_1)}{(t_2 - t_1)} \right\} \tag{14A}$$

(d) $t_2 < a$;

$$M_n = 0.85f'_c\left[A_6 + (a - t_2)(b_w)\left\{d_p - \left(\frac{t_2 + a}{2}\right)\right\}\right] + A_s f_s(d - d_p) \tag{15}$$

$$\text{where } A_4 = (Bt)(d_p - t/2) \tag{16}$$

$$A_5 = A_4 + \left(\frac{B + b}{2}\right)\left(t_1 - t\right)\left\{d_p - t - \left(\frac{t_1 - t}{3}\right)\left(\frac{B + 2b}{B + b}\right)\right\} \tag{17}$$

$$A_6 = A_5 + \left(\frac{b + b_w}{2}\right)\left(t_2 - t_1\right)\left\{d_p - t_1 - \left(\frac{t_2 - t_1}{3}\right)\left(\frac{b + 2b_w}{b + b_w}\right)\right\} \tag{18}$$

Part 2 of the calculator program given in Appendix B calculates the nominal flexural strength M_n and the design flexural strength ϕM_n, using whichever of the above equations is appropriate. The program also prints out the reinforcement stresses at flexural ultimate and indicates whether the prestressed reinforcement has yielded. This corresponds to checking whether ω_p, $(\omega + \omega_p)$ or $(\omega_w + \omega_{pw})$ is less than 0.30, as required by ACI 318-77.

Although the program was written initially for the single-tee section shown in Fig. 4, it can also be used for a variety of other sections, such as those shown in Fig. 5. All that is necessary is to make the substitutions shown in Fig. 5 when entering the section dimensions in User Instruction Steps 3E through 3K (Table B1).

Examples of Use of the Programs

In the following examples, the calculator "print out" is shown in *italics.*

Example 1

Calculate the coefficients of the stress-strain curve equation, and ϵ_{se} corresponding to f_{se} = 160 ksi, given:

f_{pu} = 270 ksi f_{py} = 240 ksi
K = 1.04 E = 28,000 ksi
ϵ_{pu} = 0.05 A_s = 0.153 in.²(½-in. ϕ strand)

1. Load "Prestressing Steel, Stress-Strain Curve" program.
2. Switch at "MAN,"—No input data record.
3. Store data using keystrokes:

   240 [STO] [A] , [STO] [B] , 270 [STO] [C] ,
   0.05 [STO] [D] , 28,000 [STO] [E] , 0.153 [STO] [4] .

4. Press [A] → *0.0177* (coefficient Q)
5. Press [B] → *7.9694* (coefficient R)
6. Check stress at ϵ = 0.01 (i.e., f_{py}), using keystrokes

   [C] 0.01 [R/S] → *0.0100* (ϵ)
   240.0 (f_s, ksi) O.K.
   36.72 (F_s, kip)

7. For any other strain "ϵ," use keystrokes:

   ϵ [R/S] → ϵ
   f_s (ksi)
   F_s (kip)

8. Calculate ϵ_{se} for f_{se} = 160 ksi, using keystrokes:

   [E] 160 [R/S] → *160.0* (f_{se}, ksi)
   0.0057 (ϵ_{se})

Example 2

Calculate the flexural strength of an 8ST 36 Single-Tee, prestressed with 18, ½-in. ϕ, 270K strands with an effective depth of 31 in. In addition, six unprestressed ½-in. ϕ, 270K strands are provided, with an effective depth of 33.5 in. The effective prestress in the strands is 160 ksi and the properties of the strand are as in Example 1. The concrete strength is 5000 psi.

Assume that the "Prestressing Steel, Stress-Strain Curve" program has just been run, so that the data relating to the steel are in the correct locations.

1. Load "Flexural Strength of P.S.C. Beam, (Part 1)," program.
2. Switch at "MAN,"—No input data record.
3. Store data using keystrokes:

31.00 (d_p)	[STO]	[4] ,	33.50 (d)	[STO]	[5] ,
2.754 (A_{ps})	[STO]	[6] ,	0.918 (A_s)	[STO]	[7] ,
[f]	[P⇄S]				
96.00 (B)	[STO]	[0] ,	14.00 (b)	[STO]	[1] ,
8.00 (b_w)	[STO]	[2] ,	1.50 (t)	[STO]	[3] ,
4.50 (t_1)	[STO]	[4] ,	7.50 (t_2)	[STO]	[5] ,
5.00 (f'_c)	[STO]	[6] ,	0.85	[STO]	[7] ,
[f]	[P⇄S]				

4. Calculate c, C, and T. (Initial estimate of c = 2 in., say.) Use keystrokes:

2.00 {c(initl.)} [A]	→	*3.08*	(c, in.)
		951.74	(C, kip)
		958.74	(T, kip)

5. Load "Flexural Strength of P.S.C. Beam (Part 2)," program.
6. Store f_{py} in R8, using keystrokes:

   240 [STO] [8]

7. Calculate f_{ps}, f_s, M_n, and ϕM_n.

Press [A]	→	*261.50*	(f_{ps}, ksi)
		259.87	(f_s, ksi)
		28976.58	(M_n, kip-in.)
		2414.72	(M_n, kip-ft)
		26078.92	(ϕM_n, kip-in.)
		2173.24	(ϕM_n, kip-ft)

8. Store $E/(Kf_{py})$ in R8 in preparation for analyzing next section, using keystrokes:

   112.18 [STO] [8]   *112.18* (E/Kf_{py})

Example 3

Calculate the flexural strength of a 24IT48 Inverted Tee, prestressed with 20, ½-in. ϕ, 270K strands with an effective depth of 45 in. The effective prestress in the strands is 160 ksi and the properties of the strand are as in Example 1. The concrete strength is 5500 psi.

1. Load "Flexural Strength of P.S.C. Beam (Part 1)," program.
2. Say switch at "NORM" for record of input data.
3. Store data using keystrokes:

45.0 (d_p)	[STO]	[4]	→	*45.00 STO4*
3.06 (A_{ps})	[STO]	[6]	→	*3.06 STO6*
0 (A_s)	[STO]	[7]	→	*0.00 STO7*
[f] [P⇄S]			→	*P⇄S*
12 (B)	[STO]	[0]	→	*12.00 STO0*
24 (b)	[STO]	[1]	→	*24.00 STO1*
	[STO]	[2]	→	*STO2*
32 (h_1)	[STO]	[3]	→	*32.00 STO3*
	[STO]	[4]	→	*STO4*
48 (h)	[STO]	[5]	→	*48.00 STO5*
5.5 (f'_c)	[STO]	[6]	→	*5.50 STO6*
0.85	[STO]	[7]	→	*0.85 STO7*
[f] [P⇄S]				*P⇄S*

4. Calculate c, C, and T. (Initial estimate of c = 12 in., say.) Use keystrokes:

12.00 {c(initl.)}	[A]	→	*12.00 GSBA*
			17.10 (c, in.)
			743.42 (C, kip)
			745.59 (T, kip)

5. Load "Flexural Strength of P.S.C. Beam, (Part 2)," program.
6. Store f_{py} in R8, using keystrokes:

240	[STO]	[8]	*240.00 STO8*

7. Calculate f_{ps}, M_n, and ϕM_n.

Press [A]	→	*GSBA*
		243.66 (f_{ps}, ksi)
		28528.22 (M_n, kip-in.)
		2377.35 (M_n, kip-ft)
		25675.40 (ϕM_n, kip-in.)
		2139.62 (ϕM_n, kip-ft)

8. Store $E/(Kf_{py})$ in R8 in preparation for analyzing next section, using keystrokes:

112.18	[STO]	[8]	*112.18* (E/Kf_{py})

Example 4

Calculate the flexural strength of an 8DT32B Double Tee, prestressed with 28, ½-in. ϕ, 270K strands with an effective depth of 28.5 in. The effective prestress in the strands is 160 ksi and the properties of the strand are as in Example 1. The concrete strength is 6000 psi.

1. Load "Flexural Strength of P.S.C. Beam (Part 1)," program.
2. Say switch at "NORM" for record of input data.
3. Store data using keystrokes:

28.5 (d_p)	[STO]	[4]	→	*28.50 STO4*
4.28 (A_{ps})	[STO]	[6]	→	*4.28 STO6*
0 (A_s)	[STO]	[7]	→	*0.00 STO7*
[f] [P⇄S]			→	*P⇄S*
96 (B)	[STO]	[0]	→	*96.00 STO0*
19.50 ($2b_t$)	[STO]	[1]	→	*19.50 STO1*
12.00 ($2b_w$)	[STO]	[2]	→	*12.00 STO2*
2.00 (t)	[STO]	[3]	→	*2.00 STO3*
	[STO]	[4]	→	*STO4*
32.00 (h)	[STO]	[5]	→	*32.00 STO5*
6.00 (f'_c)	[STO]	[6]	→	*6.00 STO6*
0.85	[STO]	[7]	→	*0.85 STO7*
[f] [P⇄S]				*P⇄S*

4. Calculate c, C, and T. (Initial estimate of c = 3 in., say.) Use keystrokes:

3.00 {c(initl.)} [A]   *3.00 GSBA*

4.14 (c, in.)
1088.66 (C, kip)
1098.88 (T, kip)

5. Load "Flexural Strength of P.S.C. Beam, (Part 2)," program.
6. Store f_{py} in R8, using keystrokes:

240 [STO] [8]   *240.00 STO8*

7. Calculate f_{ps}, M_n, and ϕM_n

Press [A]   *GSBA*

256.75 (f_{ps}, ksi)
29768.16 (M_n, kip-in.)
2480.68 (M_n, kip-ft)
26791.35 (ϕM_n, kip-in.)
2232.61 (ϕM_n, kip-ft)

Concluding Remarks

Calculator programs for determining the flexural strength of bonded prestressed concrete beams, and the equations on which they are based, have been presented. The programs are applicable to most beam cross sections in current use. They make use of the actual stress-strain curve of the prestressing steel and hence in many cases yield a less conservative value of flexural strength than is obtained when use is made of Eq. (18-3) of ACI 318-77. Although the programs apply specifically to the Hewlett Packard, HP-67/97 calculators, they could readily be adapted to other types of programmable calculators.

Metric (SI) Unit Equivalents

1 in. = 25.4 mm
1 ft = 0.305 m
1 in.^{2} = 645.16 mm^2
1 kip = 4.448 kN
1 psi = 6.895 kPa
1 ksi = 6.895 MPa
1 in.-lb = 0.113 N • m
1 ft-lb = 1.356 N • m
1 kip-in. = 113 N • m
1 kip-ft = 1.356 kN • m
1 $\sqrt{f'_c}$ psi = 0.083036 $\sqrt{f'_c}$ MPa

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NOTE

Appendices A through D, on the following pages, give user instructions and program listings for the programs described in the article, a summary of the features of the HP67/97 calculator, and a glossary of notation.

APPENDIX A — USER INSTRUCTIONS AND PROGRAM LISTING FOR PRESTRESSING STEEL STRESS-STRAIN CURVE PROGRAM

Table A1. User Instructions for "Prestressing Steel, Stress-Strain Curve" Program.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Load side 1 and side 2				
2	Switch to "NORM" if data input record is required				
3	Store f_{py} in A	f_{py} (ksi)	STO	A	
	K in B	K	STO	B	
	f_{pu} in C	f_{pu} (ksi)	STO	C	
	ε_{pu} in D	ε_{pu}	STO	D	
	E in E	E (ksi)	STO	E	
	A_s in R_4	A_s (in.^{2})	STO	4	
	(If value of K is unknown, use 1.04. A_s = area of strand, bar or wire.)				
4	Calculate coefficient Q		A		Q
5	Calculate coefficient R		B		R
6	To calculate f_s and F_s for a given ε		C		
6A	Enter ε	ε	R/S		ε f_s (ksi) F_s (kip)
7	For other values of ε repeat step 6A				
8	To calculate ε_{se} (strain due to the effective prestress f_{se}, ksi)		E		
8A	Enter f_{se}	f_{se} (ksi)	R/S		f_{se} (ksi) ε_{se}
	(Step 8A also stores (E + ε_{se}) in R1 and f_{pu} in R0, and step 5 stores E/(Kf_{py}) in R8, for use in prestressed beam flexural strength program.)				

Table A2. Part (a) of Program Listing for "Prestressing Steel, Stress-Strain Curve" Program.

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11	Calculate Q		RCLA	36 11	f_{pu}
	RCLC	36 13			RCLB	36 12	K
	RCLB	36 12			x	-35	
	RCLA	36 11		060	÷	-24	
	x	-35			STO8	35 08	$E/(Kf_{py})$ in R8
	-	-45	$(f_{pu} - Kf_{py})$		0	00	
	RCLD	36 14			RTN	24	
	RCLE	36 15			*LBL1	21 01	Calculate f(R) $= 0.01Q - f_{py}/E + \frac{0.01(Q-1)}{\left\{1+\left(\frac{.01E}{Kf_{py}}\right)^R\right\}^{1/R}}$
	x	-35	$\varepsilon_{pu}E$		.	-62	
010	RCLB	36 12			0	00	
	RCLA	36 11			1	01	
	x	-35			RCLE	36 15	
	-	-45	$(\varepsilon_{pu}E - Kf_{py})$		x	-35	
	÷	-24		070	RCLA	36 11	f_{py}
	PRTX	-14	Q		÷	-24	
	STO2	35 02			RCLB	36 12	K
	RTN	24			÷	-24	$0.01E/(Kf_{py})$
	*LBLB	21 12	Calculate R		RCL3	36 03	R
	0	00			Y^x	31	$(\quad)^R$
020	STO7	35 07			1	01	
	STO8	35 08			+	-55	$\{1 + (\quad)^R\}$
	3	03	Initial estimate of R		RCL3	36 03	R
	STO3	35 03			1/X	52	
	2	02	Compute initial correction	080	Y^x	31	$\{1 + (\quad)^R\}^{1/R}$
	÷	-24			1	01	
	STO8	35 08			RCL2	36 02	Q
	GTO1	22 01	Go to f(R)		-	-45	(1 - Q)
	*LBL3	21 03			X⇄Y	-41	
	X=0?	16-43	If f(R) = 0 go to print out		÷	-24	
030	GTO4	22 04			.	-62	
	RCL7	36 07	If f(R) ≠ 0 exchange f(R) (previous) with f(R) (new)		0	00	
	X⇄Y	-41			1	01	
	STO7	35 07			x	-35	$.01(1-Q)/\{\quad\}^{1/R}$
	-	-45		090	RCL2	36 02	Q
	1/X	52	Compute new correction		.	-62	
	RCL7	36 07			0	00	
	x	-35			1	01	
	RCL8	36 08			x	-35	0.01Q
	x	-35			+	-55	
040	STO8	35 08			RCLA	36 11	
	ST-3	35-45 03	Subtract from R		RCLE	36 15	
	RCL3	36 03	R		÷	-24	f_{py}/E
	÷	-24			-	-45	f(R)
	ABS	16 31	If correction is less than 0.1% of R go to print out; if not, go to 1 and make another iteration.	100	GTO3	22 03	
	.	-62			*LBLC	21 13	Calculate f_s
	0	00			DSP4	-63 04	
	0	00			R/S	51	Enter ε
	1	01			PRTX	-14	
	X>Y?	16-34			ENT↑	-21	
050	GTO4	22 04			ENT↑	-21	
	GTO1	22 01			RCL8	36 08	
	*LBL4	21 04	Recall final value of R and print.		x	-35	$(\varepsilon E/Kf_{py})$
	RCL3	36 03			RCL3	36 03	R
	PRTX	-14		110	Y^x	31	$(\quad)^R$
	SPC	16-11			1	01	
	RCLE	35-15	E		+	-55	$\{1 + (\quad)^R\}$

REGISTERS

0 f_{pu}	1 E + ε_{se}	2 Q	3 R	4 A_s	5	6	7 T/S	8 T/S	9
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A f_{py}	B K	C f_{pu}	D ε_{pu}	E E	I				

Table A3. Part (b) of Program Listing for "Prestressing Steel, Stress-Strain Curve" Program.

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
	RCL3	36 03					
	1/X	52	1/R	170			
	Y^X	31					
	1/X	52	$1/\{1 + (\ )^R\}^{1/R}$				
	1	01					
	RCL2	36 02	Q				
	-	-45					
120	x	-35	$(1-Q)/\{1+(\ )^R\}^{1/R}$				
	RCL2	36 02	Q				
	+	-55					
	x	-35	$\varepsilon[Q+(\ )/\{\ \}^{1/R}]$				
	RCLE	36 15	E	180			
	x	-35	f_s				
	DSP1	-63 01					
	PRTX	-14					
	RCL4	36 04	A_s				
	x	-35					
130	DSP2	-63 02					
	PRTX	-14	$F_s = A_s f_s$				
	SPC	16-11					
	0	00					
	GTOC	22 13		190			
	*LBLE	21 15	Calculate ε_{se}				
	0	00					
	R/S	51	Enter f_{se}				
	DSP1	-63 01					
	PRTX	-14					
140	DSP4	-63 04					
	RCLE	36 15	E				
	÷	-24					
	PRTX	-14	ε_{se}				
	SPC	16-11		200			
	RCLE	36 15	E				
	+	-55					
	STO1	35 01	$(E + \varepsilon_{se})$ in R1				
	RCLC	36 13					
	STO0	35 00	f_{pu} in R0				
150	GTOE	22 15					
				210			
160							
				220			

LABELS					FLAGS	SET STATUS		
A Q	B R	C f_s & F_s	D -	E ε_{se}	0	FLG. ON OFF	TRIG	DISP
a	b	c	d	e	1	0 □ □	DEG □	FIX □
0	1 f(R)	2	3 ✓	4 ✓	2	1 □ □	GRAD □	SCI □
5	6	7	8	9	3	2 □ □	RAD □	ENG □
						3 □ □		n = 4

APPENDIX B — USER INSTRUCTIONS AND PROGRAM LISTING FOR FLEXURAL STRENGTH OF P.S.C. BEAM

Table B1. User Instructions for "Flexural Strength of P.S.C. Beam (Part 1)."

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Load side 1 and side 2				
2	Switch to "NORM" if data input record is required				
3	Store:-				
3A	Effective depth of prestressing steel	d_p (in.)	STO	4	
3B	Effective depth of unprestressed steel	d (in.)	STO	5	
3C	Area of prestressing steel	A_{ps} (in.^{2})	STO	6	
3D	Area of unprestressed steel (Exchange registers)	A_s (in.^{2})	STO f	7 P⇄S	
3E	Flange width B	B (in.)	STO	0	
3F	Fillet width b	b (in.)	STO	1	
3G	Web width b_w	b_w (in.)	STO	2	
3H	Flange thickness t	t (in.)	STO	3	
3J	Flange thickness t_1	t_1 (in.)	STO	4	
3K	Depth t_2	t_2 (in.)	STO	5	
3L	Concrete compressive strength f'_c	f'_c (ksi)	STO	6	
3M	Coefficient 0.85	0.85	STO	7	
	(Exchange registers)		f	P⇄S	
4	If "Prestressing Steel, Stress-Strain Curve" program has <u>not</u> just been used to determine Q, R, ε_{se}, etc. Store:				
4A	Steel ultimate strength	f_{pu} (ksi)	STO	0	
4B	E + ε_{se} , (E in ksi, and ε_{se} = strain in steel due to effective prestress	E + ε_{se}	STO	1	
4C	Coefficient Q for steel stress-strain equation	Q	STO	2	
4D	Coefficient R for steel stress-strain equation	R	STO	3	
4E	$E/(Kf_{py})$, (E and f_{py} in ksi, f_{py} is steel stress at 0.01 strain. (These items of data are stored automatically by the "Prestressing Steel, Stress-Strain Curve" program.)	$E/(Kf_{py})$	STO	8	
5	Calculate neutral axis depth c, etc. Enter initial estimate of c, (in.). (Calculation time typically 1 to 1½ minutes.)	c (init.) (in.)	A		c (in.) C (kip) T (kip)

Table B2. User Instructions for "Flexural Strength of P.S.C. Beam (Part 2)."

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Load side 1 and side 2 after running Part 1 of this program.				
2	Store f_{py} in R8	f_{py} (ksi)	STO	8	
3	Calculate		A		f_{ps} (ksi) f_s (ksi)
	Prints 0.00 after printing stresses if section is over-reinforced, i.e., if $f_{ps}<f_{py}$.				M_n (k.in.) M_n (k.ft) ϕM_n (k.in.) ϕM_n (k.ft)
4	If program is to be used to analyze another section using the same steel, $E/(Kf_{py})$ must be stored in R8 before using Part 1 of program again. Steps 4A through 4E may now be omitted when using Part 1 of program.	$E/(Kf_{py})$	STO	8	

Table B3. Part (a) of Program Listing for "Flexural Strength of P.S.C. Beam (Part 1)."

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLb	21 16 12	Calculate A_1		RCL3	36 03	R
	RCL0	36 00	B		Y^X	31	$(\ )^R$
	RCL3	36 03	t		1	01	
	x	-35	A_1	060	+	-55	$\{1 + (\ )^R\}$
	RTN	24			RCL3	36 03	
	*LBLc	21 16 13	Calculate A_2		1/X	52	1/R
	GSBb	23 16 12	A_1		Y^X	31	$\{1 + (\ )^R\}^{1/R}$
	RCL0	36 00	B		1/X	52	$1/\{ \ \}^{1/R}$
	RCL1	36 01	b		1	01	
010	+	-55			RCL2	36 02	Q
	2	02			-	-45	
	÷	-24	$(B+b)/2$		x	-35	$(1-Q)/\{ \ \}^{1/R}$
	RCL4	36 04	t_1		RCL2	36 02	Q
	RCL3	36 03	t	070	+	-55	
	-	-45			x	-35	$\varepsilon[Q + (1-Q)/\{ \ \}^{1/R}]$
	x	-35	$(t_1 - t)\left(\frac{B+b}{2}\right)$		RCL1	36 01	E
	+	-55	A_2		x	-35	f_s(calc.)
	RTN	24			RCL0	36 00	f_{pu}
	*LBLd	21 16 14	Calculate A_3		X≤Y?	16-35	If $f_{pu} \le f_s$(calc.)
020	GSBc	23 16 13	A_2		RTN	24	$f_s = f_{pu}$
	RCL1	36 01	b		X⇄Y	-41	if not, $f_s = f_s$(calc.)
	RCL2	36 02	b_w		RTN	24	
	+	-55			*LBLE	21 15	Calc. 0.003(d - c)/c
	2	02		080	RCLI	36 46	c
	÷	-24	$(b + b_w)/2$		-	-45	(d - c)
	RCL5	36 05	t_2		RCLI	36 46	c
	RCL4	36 04	t_1		÷	-24	(d - c)/c
	-	-45			.	-62	
	x	-35	$(t_2 - t_1)\left(\frac{b+b_w}{2}\right)$		0	00	
030	+	-55	A_3		0	00	
	RTN	24			3	03	
	*LBLe	21 16 15	Calc. ( ){ ( )( )}		x	-35	0.003(d - c)/c
	RCL8	36 08	a \in Eq. 4 & 5		RTN	24	
	RCLC	36 13	C = t_1 or t	090	*LBLA	21 11	Calc. c etc.
	-	-45			STOI	35 46	Initl. c in I
	ST09	35 09			RCL4	36 04	d_p
	RCLD	36 14	D = t_1 or t_2		GSBE	23 15	$0.003(d_p - c)/c$
	RCLC	36 13	C		RCL1	36 01	
	-	-45			FRC	16 44	ε_{se}
040	÷	-24	(a-C)/(D-C)		+	-55	ε_{ps}
	RCLB	36 12	B = b or b_w		GSB1	23 01	f_{ps}
	RCLA	36 11	A = B or b		RCL6	36 06	A_{ps}
	-	-45			x	-35	$T_1 = A_{ps} f_{ps}$
	x	-35		100	ST09	35 09	
	2	02			RCL7	36 07	A_s
	÷	-24	$-\left(\frac{A-B}{2}\right)\left(\frac{a-C}{D-C}\right)$		X=0?	16-43	If $A_s = 0$
	RCLA	36 11			GT02	22 02	go to 2
	+	-55			RCL5	36 05	if not, RCL d
	RCL9	36 09			GSBE	23 15	0.003(d - c)/c
050	x	-35	(a-c){A - ( )( )}		GSB1	23 01	f_s
	RTN	24			RCL7	36 07	A_s
	*LBL1	21 01	Calculate f_s, ε in		x	-35	$T_2 = A_s f_s$
	ENT↑	-21	X at start		*LBL2	21 02	
	ENT↑	-21		110	STOE	35 15	
	RCL8	36 08	(E/Kf_{py})		ST+9	35-55 09	$T = T_1 + T_2$
	x	-35	$(\varepsilon E/Kf_{py})$		P⇄S	16-51	Exch. Reg.

REGISTERS

0 f_{pu}	1 $E + \varepsilon_{se}$	2 Q	3 R	4 d_p	5 d	6 A_{ps}	7 A_s	8 E/Kf_{py}	9 T
S0 B	S1 b	S2 b_w	S3 t	S4 t_1	S5 t_2	S6 f'_c	S7 0.85	S8 a	S9 T/S
A T/S	B T/S	C T/S	D T/S	E T/S	I c				

Table B4. Part (b) of Program Listing for "Flexural Strength of P.S.C. Beam (Part 1)."

STEP	KEY ENTRY	KEY CODE	COMMENTS
	4	04	
	RCL6	36 06	f'_c
	X>Y?	16-34	If f'_c>4 ksi
	GT04	22 04	go to 4
	RCL7	36 07	If not, β_1=.85
	GT05	22 05	
	*LBL4	21 04	(f'_c in X)
120	8	08	
	X≤Y?	16-35	If f'_c>8 ksi
	GT06	22 06	go to 6
	4	04	If not, calc. β_1
	RCL6	36 06	
	-	-45	$-(f'_c - 4)$
	2	02	
	0	00	
	÷	-24	$-.05(f'_c - 4)$
	RCL7	36 07	0.85
130	+	-55	β_1
	GT05	22 05	
	*LBL6	21 06	
	.	-62	
	6	06	
	5	05	β_1 = .65
	*LBL5	21 05	
	RCLI	36 46	c
	x	-35	a=β_1c
	ST08	35 08	
140	RCL3	36 03	t
	X⇄Y	-41	
	X>Y?	16-34	If a>t
	GT07	22 07	go to 7
	RCL0	36 00	B
	x	-35	Ba
	GT08	22 08	
	*LBL7	21 07	(a in X)
	RCL4	36 04	t_1
	X⇄Y	-41	
150	X>Y?	16-34	If a>t_1
	GT09	22 09	go to 9
	RCL0	36 00	
	STOA	35 11	B in RA
	RCL1	36 01	
	STOB	35 12	b in RB
	RCL3	36 03	
	STOC	35 13	t in RC
	RCL4	36 04	
	STOD	35 14	t_1 in RD
160	GSBe	23 16 15	( ){ ( )( )}
	GSBb	23 16 12	A_1
	+	-55	[A_1 + ( ){ ( )( )}]
	GT08	22 08	
	*LBL9	21 09	(a in X)
	RCL5	36 05	t_2
	X⇄Y	-41	
	X>Y?	16-34	If a>t_2
	GT00	22 00	go to 0
	RCL1	36 01	
170	STOA	35 11	b in RA
	RCL2	36 02	
	STOB	35 12	b_w in RB
	RCL4	36 04	
	STOC	35 13	t_1 in RC
	RCL5	36 05	
	STOD	35 14	t_2 in RD
	GSBc	23 16 13	A_2
	GSBe	23 16 15	( ){ ( )( )}
	+	-55	[A_2 + ( ){ ( )( )}]
180	GT08	22 08	
	*LBL0	21 00	(a in X)
	GSBd	23 16 14	A_3
	RCL8	36 08	a
	RCL5	36 05	t_2
	-	-45	
	RCL2	36 02	b_w
	x	-35	
	+	-55	[$A_3 + (a - t_2)(b_w)$]
	*LBL8	21 08	
190	RCL6	36 06	f'_c
	RCL7	36 07	0.85
	x	-35	
	x	-35	C = .85f'_c[ ]
	P⇄S	16-51	Exch. Reg.
	STOD	35 14	C in RD
	RCL9	36 09	T
	-	-45	
	RCLD	36 14	C
	÷	-24	(C - T)/C
200	CHS	-22	
	STOA	35 11	-(C - T)/C in RA
	ABS	16 31	|(C - T)/C|
	.	-62	
	0	00	
	1	01	
	X>Y?	16-34	If |(C - T)/C| is
	GTOD	22 14	<.01, go to D
	RCLA	36 11	If not, RCL
	.	-62	-(C - T)/C
210	7	07	
	x	-35	K_1 = -.7(C - T)/C
	1	01	
	+	-55	(1 + K_1)
	RCLI	36 46	Init. val. of c
	x	-35	New est. of c
	GTOA	22 11	
	*LBLD	21 14	
	RCLI	36 46	c
	PRTX	-14	Print c
220	RCLD	36 14	C
	PRTX	-14	Print C
	RCL9	36 09	T
	PRTX	-14	Print T
	RTN	24	Stop

LABELS

A Calc.	B —	C —	D √	E √
a	b √	c √	d √	e √
0 √	1 √	2	3	4 √
5 √	6 √	7 √	8 √	9 √

FLAGS

0	
1	
2	
3	

SET STATUS

FLG.	ON	OFF	TRIG		DISP	
0	□	□	DEG	□	FIX	☒
1	□	□	GRAD	□	SCI	□
2	□	□	RAD	□	ENG	□
3	□	□			n =	2

Table B5. Part (a) of Program Listing for "Flexural Strength of P.S.C. Beam (Part 2)."

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11	Calculate
	SPC	16-11	
	RCL9	36 09	T
	RCLE	36 15	$A_s f_s$
	-	-45	$A_{ps} f_{ps}$
	RCL6	36 06	A_{ps}
	÷	-24	
	PRTX	-14	f_{ps}
	STOA	35 11	
010	RCLE	36 15	$A_s f_s$
	RCL7	36 07	A_s
	X=0?	16-43	If $A_s = 0$
	GTO4	22 04	go to 4
	÷	-24	f_s
	PRTX	-14	
	*LBL4	21 04	
	RCLA	36 11	f_{ps}
	RCL8	36 08	f_{py}
	X≤Y?	16-35	If $f_{py} \le f_{ps}$
020	GTO3	22 03	go to 3
	0	00	If not,
	PRTX	-14	print 0.00
	*LBL3	21 03	
	SPC	16-11	
	GTOB	22 12	
	*LBLa	21 16 11	Calc. $A_s f_s(d - d_p)$
	RCL5	36 05	d
	RCL4	36 04	d_p
	-	-45	
030	RCL9	36 09	$A_s f_s$
	x	-35	$A_s f_s(d - d_p)$
	RTN	24	
	*LBLb	21 16 12	Calc. ( )( ){ }
	RCLA	36 11	Eq. 16,17,20,21
	RCLB	36 12	A = B,b,B or b
	2	02	B = b',b',b or b_w
	x	-35	
	+	-55	(A + 2B)
	RCLA	36 11	
040	RCLB	36 12	
	+	-55	
	÷	-24	(A + 2B) / (A + B)
	RCLD	36 14	D = t, t_1, t or t_1
	RCLC	36 13	C = a, a, t or t_2
	-	-45	
	3	03	
	÷	-24	
	x	-35	$\left(\frac{D-C}{3}\right)\left(\frac{A+2B}{A+B}\right)$
	RCLD	36 14	
050	-	-45	
	RCLE	36 15	
	+	-55	$\{d_p - D - (\ )(\ )\}$
	RCLC	36 13	
	RCLD	36 14	
	-	-45	
	x	-35	(C - D){ }
	2	02	
	÷	-24	$\left(\frac{C-D}{2}\right)\{\ \}$
	RCLA	36 11	
060	RCLB	36 12	
	+	-55	(A + B)
	x	-35	( )( ){ }
	RTN	24	
	*LBLc	21 16 13	Calculate A_4
	RCL3	36 03	t
	2	02	
	÷	-24	
	CHS	-22	-t/2
	RCLE	36 15	d_p
070	+	-55	$(d_p - t/2)$
	RCL0	36 00	B
	x	-35	
	RCL3	36 03	t
	x	-35	A_4
	RTN	24	
	*LBLd	21 16 14	Calculate A_5
	RCL0	36 00	
	STOA	35 11	B in RA
	RCL1	36 01	
080	STOB	35 12	b in RB
	RCL4	36 04	
	STOC	35 13	t_1 in RC
	RCL3	36 03	
	STOD	35 14	t in RD
	GSBb	23 16 12	( )( ){ }
	GSBc	23 16 13	A_4
	+	-55	A_5
	RTN	24	
	*LBLe	21 16 15	Calculate A_6
090	RCL1	36 01	
	STOA	35 11	b in RA
	RCL2	36 02	
	STOB	35 12	b_w in RB
	RCL5	36 05	
	STOC	35 13	t_2 in RC
	RCL4	36 04	
	STOD	35 14	t_1 in RD
	GSBb	23 16 12	( )( ){ }
	STOI	35 46	
100	GSBd	23 16 14	A_5
	RCLI	36 46	
	+	-55	A_6
	RTN	24	
	*LBLE	21 15	Calculate b'
	RCL8	36 08	a
	RCLD	36 14	D = t or t_1
	-	-45	
	RCLA	36 11	A = B or b
	RCLB	36 12	B = b or b_w
110	-	-45	
	x	-35	(A - B)(a - D)
	RCLC	36 13	C = t_1 or t_2

REGISTERS

0 f_{pu}	1 E + ε_{se}	2 Q	3 R	4 d_p	5 d	6 A_{ps}	7 A_s	8 f_{py}	9 T/S
S0 B	S1 b	S2 b_w	S3 t	S4 t_1	S5 t_2	S6 f'_c	S7 0.85	S8 a	S9 T/S
A T/S	B T/S	C T/S	D T/S	E T/S	I T/S				

Table B6. Part (b) of Program Listing for "Flexural Strength of P.S.C. Beam (Part 2)."

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
	RCLD	36 14			STOD	35 14	t_1 in RD
	-	-45	(C - D)	170	GSBE	23 15	b'
	÷	-24	(A - B)(a - D)/(C - D)		RCL8	36 08	
	CHS	-22	-( )( )/( )		STOC	35 13	a in RC
	RCLA	36 11	A		GSBb	23 16 12	( )( ){ }
	+	-55	b'		STO9	35 09	
	STOB	35 12	b' in RB		GSBc	23 16 13	A_4
120	RTN	24			RCL9	36 09	
	*LBLB	21 12	Calculate M_n		+	-55	[A_4 + ( )( ){ }]
	RCLE	36 15	$A_s f_s$		*LBL8	21 08	
	STO9	35 09			RCL7	36 07	0.85
	RCL4	36 04	d_p	180	x	-35	
	STOE	35 15			RCL6	36 06	f'_c
	P⇄S	16-51	Exch. Reg.		x	-35	$0.85f'_c$
	RCL8	36 08	a		GTO9	22 09	
	RCL3	36 03	t		*LBL1	21 01	(a in X)
	X⇄Y	-41			RCL5	36 05	t_2
130	X>Y?	16-34	If a>t		X⇄Y	-41	
	GTO0	22 00	go to 0		X>Y?	16-34	If $a>t_2$
	2	02			GTO2	22 02	go to 2
	÷	-24			RCL1	36 01	
	CHS	-22	-a/2	190	STOA	35 11	b in RA
	RCLE	36 15			RCL2	36 02	
	+	-55	(d_p - a/2)		STOB	35 12	b_w in RB
	RCLD	36 14	C		RCL5	36 05	
	x	-35	C(d_p - a/2)		STOC	35 13	t_2 in RC
	*LBL9	21 09			RCL4	36 04	
140	P⇄S	16-51	Exch. Reg.		STOD	35 14	t_1 in RD
	GSBa	23 16 11	$A_s f_s$(d - d_p)		GSBE	23 15	b'
	+	-55	M_n		RCL8	36 08	a
	STOI	35 46			STOC	35 13	a in RC
	GSB5	23 05	To print out	200	GSBb	23 16 12	( )( ){ }
	RCLI	36 46	M_n		STO9	35 09	
	.	062			GSBd	23 16 14	A_5
	9	09			RCL9	36 09	
	x	-35	ϕM_n		+	-55	[A_5+( )( ){ }]
	*LBL5	21 05			GTO8	22 08	
150	PRTX	-14	Print M, (K.in.)		*LBL2	21 02	
	1	01			RCLE	36 15	d
	2	02			RCL5	36 05	t_2
	÷	-24			RCL8	36 08	a
	PRTX	-14	Print M, (K.ft)	210	+	-55	
	SPC	16-11			2	02	
	RTN	24			÷	-24	
	*LBL0	21 00	(a in X)		-	-45	{d - (t_2 + a)/2}
	RCL4	36 04	t_1		RCL2	36 02	b_w
	X⇄Y	-41			x	-35	
160	X>Y?	16-34	If $a>t_1$		RCL8	36 08	a
	GTO1	22 01	go to 1		RCL5	36 05	t_2
	RCL0	36 00			-	-45	
	STOA	35 11	B in RA		x	-35	(a - t_2)(b_w){ }
	RCL1	36 01		220	STO9	35 09	
	STOB	35 12	b in RB		GSBe	23 16 15	A_6
	RCL4	36 04			RCL9	36 09	
	STOC	35 13	t_1 in RC		+	-55	[A_6 + ( )( ){ }]
	RCL3	36 03			GTO8	22 08	

LABELS					FLAGS	SET STATUS		
A Calc.	B ✓	C ✓	D —	E ✓	0	FLG. ON OFF	TRIG	DISP.
a	b ✓	c ✓	d ✓	e ✓	1	0 ☐ ☐	DEG ☐	FIX ☒
0 ✓	1 ✓	2 ✓	3 ✓	4 ✓	2	1 ☐ ☐	GRAD ☐	SCI ☐
5	6	7	8 ✓	9 ✓	3	2 ☐ ☐	RAD ☐	ENG ☐
						3 ☐ ☐		n = 2

APPENDIX C—SUMMARY OF FEATURES OF THE HP67/97 CALCULATORS

The Hewlett Packard 67/97 calculators employ Reverse Polish Notation. They have 16 primary (direct) storage registers ∅ through 9, A through E and I, as well as 10 secondary registers which are exchanged with the primary registers ∅ through 9 by use of the P ⋚ S key.

Conditional tests such as $X \geqslant Y$? or $X < Y$? execute the next step in memory if the conditional expression is true, or skip one step before resuming execution if the conditional is false.

Other special functions which are available on other calculators or may easily be implemented are the following:

R/S	Run/Stop. Stops program execution for data entry. When key is pressed, begins execution from current step in program.
LBLA	Defines beginning of a routine (here labeled A). When label key (i.e., A in this example) is pressed, execution of program commences at this step.
GTOA	Causes calculator to stop execution and search downwards through program to first designated label, and resume execution there, (i.e., directs branching of program).
GSBA	As GTOA, but at end of subroutine control is returned to next step after GSBA.
RTN	If at end of subroutine, returns control to next step after GSB instruction; otherwise stops execution and returns control to keyboard.
ST−3	Subtracts the current value from memory register 3 and the result is stored in memory 3. Likewise, ST + 9 would add current value to memory register 9.
DSP4	Fixes the display so that 4 digits are displayed to right of decimal point.
FRC	Obtains fractional part of a number.
CHS	Changes sign of current value.
PRTX	Prints the current value.
SPC	Advances the print paper one space.

The above summary is provided to assist in the translation of the programs given in this paper to a form suitable for other calculators.

APPENDIX D—NOTATION

a = depth of equivalent rectangular stress block as defined in Section 10.2.7 of ACI 318-77,[1] in.

A_{ps} = area of prestressed tension reinforcement, in.^{2}

A_s = area of non-prestressed tension reinforcement, in.^{2}

b = width of section at top of fillet (see Fig. 4), in.

b' = width of section at distance "a" from extreme compression fiber, in.

b_w = width of web or stem of section, in.

B = width of flange at extreme compression fiber, in.

c = distance from extreme compression fiber to neutral axis, in.

C = resultant concrete compression force, kips

d = distance from extreme compression fiber to centroid of unprestressed tension reinforcement, in.

d_p = distance from extreme compression fiber to centroid of prestressed tension reinforcement, in.

E = modulus of elasticity of reinforcement, ksi

f'_c = compressive strength of concrete (measured on 6 x 12-in. cylinders), ksi

f_{ps} = stress in prestressed reinforcement at nominal strength, ksi

f_{pu} = specified tensile strength of prestressing steel, ksi

f_{py} = specified yield strength of prestressing steel, ksi

f_s = stress in unprestressed reinforcement at nominal strength, ksi or, stress in reinforcement at strain ϵ

f^*_s = f_s/f_{so}

f_{se} = effective prestress in prestressed reinforcement, ksi

f_{so} = a reference stress (see Fig. 1)

F_s = force in a single strand at strain ϵ, kips

K = a coefficient

M_n = nominal flexural strength of section, in.-lb or ft-lb

Q = $(f_{pu} - Kf_{py})/(\epsilon_{pu}E - Kf_{py})$

R = a coefficient

t = thickness of uniform width part of flange (see Fig. 4)

t_1 = depth to top of fillets (see Fig. 4)

t_2 = depth to bottom of fillets (see Fig. 4)

T = resultant tensile force in reinforcement, kips

β_1 = a/c, factor defined in Section 10.2.7 of ACI 318-77

ϵ = strain

ϵ^* = ϵ/ϵ_o

ϵ_o = a reference strain (see Fig. 1)

ϵ_1 = change in strain in prestressed reinforcement due to loading to nominal flexural strength

ϵ_{cp} = strain in concrete at level of prestressed reinforcement due to prestress

ϵ_{ps} = strain in prestressed reinforcement at nominal flexural strength

ϵ_s = strain in unprestressed reinforcement at nominal flexural strength

ϵ_{se} = strain in prestressed reinforcement due to effective prestress f_{se}

ϵ_{su} = strain in prestressing steel at stress f_{pu}

ϕ = strength reduction factor as per Chapter 9 of ACI 318-77

ω = $(A_s/Bd)f_y/f'_c$

ω_p = $(A_{sp}/Bd)f_{ps}/f'_c$

ω_w, ω_{pw} = reinforcement indices for flanged sections computed as for ω and ω_p, except replace B by b_w, and reinforcement area is that required to develop compressive strength of the web only.