Proposed Revisions to Shear-Friction Provisions

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Based on an analysis of the most recent research, the author proposes suggested revisions to the shear-friction provisions contained in the *PCI Manual on Design of Connections for Precast Prestressed Concrete.* An accompanying commentary explains the proposed revisions.

Since its introduction about 10 years ago, the shear-friction concept has been widely used as a relatively simple analytical tool for the design of various types of connections in precast prestressed concrete structures.

Considerable amount of research during the past 5 years on this subject shows that the current shear-friction provisions lead to conservative designs. This conclusion is also confirmed by experience of the past 10 years.

Based on a review and analysis of this new research information, this report presents a suggested revision to the shear-friction provisions contained in Section 2.2 of the *PCI Manual on Design of*

*Connections for Precast Prestressed Concrete.**

The suggested revisions are considered to be more realistic and generally lead to more economical designs.

This report was prepared by the writer as an assignment from the PCI Committee on Connections Details. It has been reviewed and commented on by the Committee and, at its recommendation, is being published in the PCI JOURNAL for input from the profession at large. Many helpful comments were made by the committee members and the writer gratefully acknowledges their contribution. Thanks are also due to Young Lee, graduate assistant at the University of Wisconsin-Milwaukee, for his capable help in preparing this paper.

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*PCI Committee on Connection Details, *PCI Manual on Design of Connections for Precast Prestressed Concrete*, Prestressed Concrete Institute, Chicago, 1973, p. 22.

PROPOSED REVISIONS

2.2 SHEAR-FRICTION

The shear-friction concept is useful in assessing the safe ultimate strength of many different types of connections in precast prestressed concrete structures.

The basic premise in applying the shear-friction concept is that by placing reinforcement across the anticipated crack or failure plane shear resistance capability is produced at the crack interface. Under ultimate conditions, the reinforcement develops a tensile force equal to $A_s f_y$, where A_s is the area of the non-prestressed steel reinforcement crossing the interface and f_y is the specified yield strength of the steel.

The component of the steel force normal to the crack interface produces an equal compressive force on the concrete, which, in conjunction with friction analogy, results in shear resistance at the interface. Fig. 1 (on the next page) shows schematically the principle.

For ultimate shear across any crack plane, the reinforcement normal to the plane can be calculated by:

$$A_{vf} = \frac{V_u}{\phi f_{yv} \mu_e} \qquad (2.1)$$

where

V_u = ultimate shear force
f_{yv} = specified yield strength of shear-friction reinforcement
ϕ = capacity reduction factor (where ϕ equals 0.85 for shear)
μ_e = effective coefficient of friction

The effective coefficient of friction is calculated from:

$$\mu_e = \frac{1000\, C_s^2\, \mu}{v_u} \qquad (2.2)$$

where

C_s = constant used for effect of concrete density. Note that:
C_s = 1.0 for normal weight concrete
C_s = 0.85 for sand-lightweight concrete
C_s = 0.75 for all-lightweight concrete
v_u = nominal ultimate shear stress along interface. Note that $v_u = V_u / A_{cr}$, where A_{cr} is the

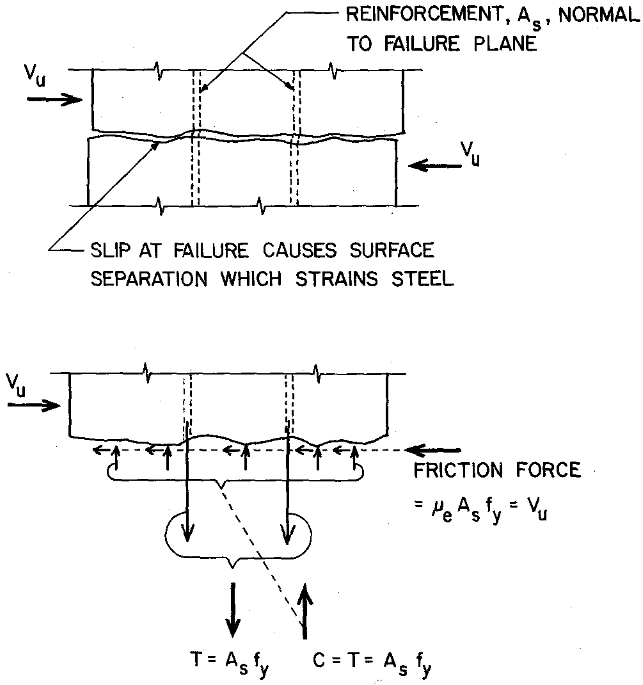

Fig. 1. Schematic diagram of shear-friction principle.

Table 1. Shear-friction coefficients, μ, and maximum shear stress values, v_u, for various interface conditions.

Crack Interface Condition	Recommended μ	Maximum v_u, psi
① Concrete to concrete, cast monolithically	1.4	$0.30f'_c C_s^2 \leq 1200\ C_s^2$
② Concrete to hardened concrete, ¼ in. roughness	1.0	$0.25f'_c C_s^2 \leq 1000\ C_s^2$
③ Concrete to concrete, smooth interface	0.4	$0.15f'_c C_s^2 \leq 600\ C_s^2$
④ Concrete to steel	0.6	$0.20f'_c C_s^2 \leq 800\ C_s^2$

area of shear crack interface.

μ = coefficient of static friction applicable to reinforced concrete shear interface. For usual design conditions, the recommended values given in Table 1 may be used.

Unless specified otherwise, a minimum area of shear-friction reinforcement, A_{vf}, equivalent to $\rho_{vf} f_{yv}$ = 120 psi shall be provided across the crack interface where ρ_{vf} is the ratio of A_{vf}/A_{cr}.

COMMENTARY

2.2 SHEAR-FRICTION

In rewriting Section 2.2, it appeared desirable to retain the original Mast[1] concept and the current as well as the proposed ACI Code approach regarding shear-friction. Use of an *apparent* or *effective* coefficient of friction is made to estimate the shear resistance at an interface. The shear-friction approach is simple conceptually, lends itself to design use easily, and the design profession has become familiar with it progressively since 1968.

The findings of the comprehensive research work on shear transfer by Prof. Mattock and his associates as well as others can be taken into account by redefining the effective coefficient of friction, μ_e, to be used in conjunction with the basic shear-friction equation [Eq. (2.1)].

There can be numerous expressions for μ_e which would fit the research data within an acceptable degree of design accuracy. For example, for normal weight concrete and a monolithic interface condition, the following equations have been obtained:

Mattock[2]

$$v_u = 0.8\,\rho_v f_y + 400$$

$$\mu_e = \frac{0.8\,v_u}{v_u - 400} = \frac{400}{\rho_v f_y} + 0.8$$

Birkeland[3]

$$v_u = 33.5\sqrt{\rho_v f_y}$$

$$\mu_e = \frac{1122}{v_u} = \frac{33.5}{\sqrt{\rho_v f_y}} \quad (\mu = 1.4)$$

Raths[4]

$$v_u = 37.42\sqrt{\rho_v f_y}$$

$$\mu_e = \frac{1400}{v_u} = \frac{37.42}{\sqrt{\rho_v f_y}} \quad (\mu = 1.4)$$

Plots of these equations and also the current ACI and PCI equations are transposed on available test data[7,8,9] for normal weight concrete (see Figs. 2, 3, and 4 on the following pages).

Note that the symbol σ_n in Figs. 2 and 4 denotes the stress produced by externally applied force normal to the shear plane and is taken positive when compression and negative when tension.

Mattock *et al.*[9] have shown that it is appropriate to add the normal stress, σ_n, to the reinforcement parameter ρf_y. In other words, an externally applied tension force requires reinforcement in addition to the calculated shear-friction reinforcement. Note, however, that when the force is compressive, the calculated shear-friction reinforcement may be reduced accordingly.

Despite the dimensional ambiguity of Birkeland's and Raths' equations, their parabolic form is quite simple to work with, particularly as extended by Raths to include the effect of concrete density and the currently used shear-friction coefficients. Expressing μ_e as a function of v_u, rather than $\rho_v f_y$, is preferable for relating to maximum values of v_u.

Thus, the Raths' expression:

$$\mu_e = \frac{1000\,C_s^2\,\mu}{v_u}$$

is selected to evaluate the effective coefficient of friction.

Mattock[5] has suggested that a coefficient of about 900 (instead of 1000) in Raths expression might be more appropriate for a better correlation with the "mean value" of the test data. (The writer believes that a coefficient of 1000 provides a reasonable compromise because this figure represents a safe lower bound for the uncracked specimen data, whereas the mean

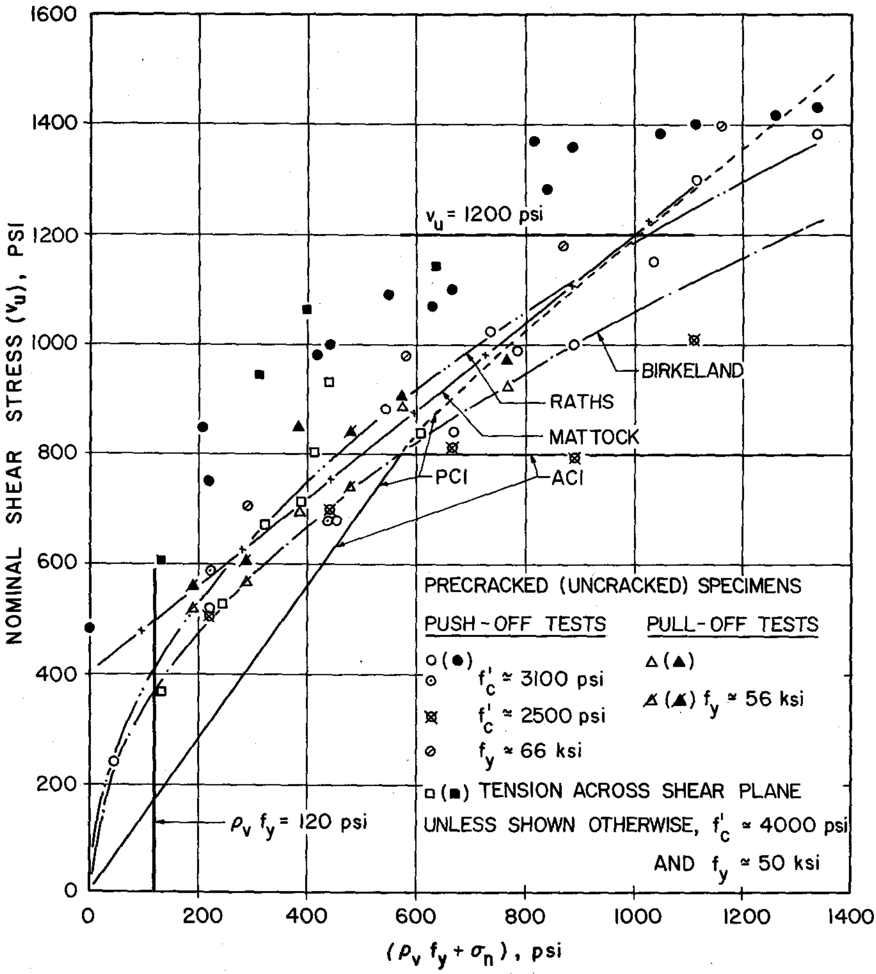

Fig. 2. Nominal shear stress versus $\rho_v f_y + \sigma_n$ (normal weight concrete).

value referred to above, represents the precracked specimen data.)

For lightweight concretes, a comparison of Mattock's[6] and Raths'[4] equations (in terms of v_u versus $\rho_v f_y$) is shown in Figs. 5 and 6.

Examination of Figs. 2, 5, and 6 (and also Figs. 3 and 4) suggests that, within the usual design range, a detailed comparison of differences between Mattock's and Raths' proposals is hardly warranted. The differences are small and from a design viewpoint are likely to be insignificant.

Regarding the minimum reinforcement for shear-friction, the values suggested for $\rho_v f_y$ are: 100 psi (Raths),[4] 200 psi (Mattock),[8] and 0.04 f'_c = 160 psi for f'_c = 4000 psi (ACI 318-71). These values (using Raths curve, see Fig. 2) correspond to v_u = 374 psi, 528 psi, and 473 psi, respectively.

It appears reasonable to specify a minimum $\rho_v f_y$ that would result in v_u at least equal to the shear resistance due to cohesion of concrete, namely, 400 psi as included in Mattock's ex-

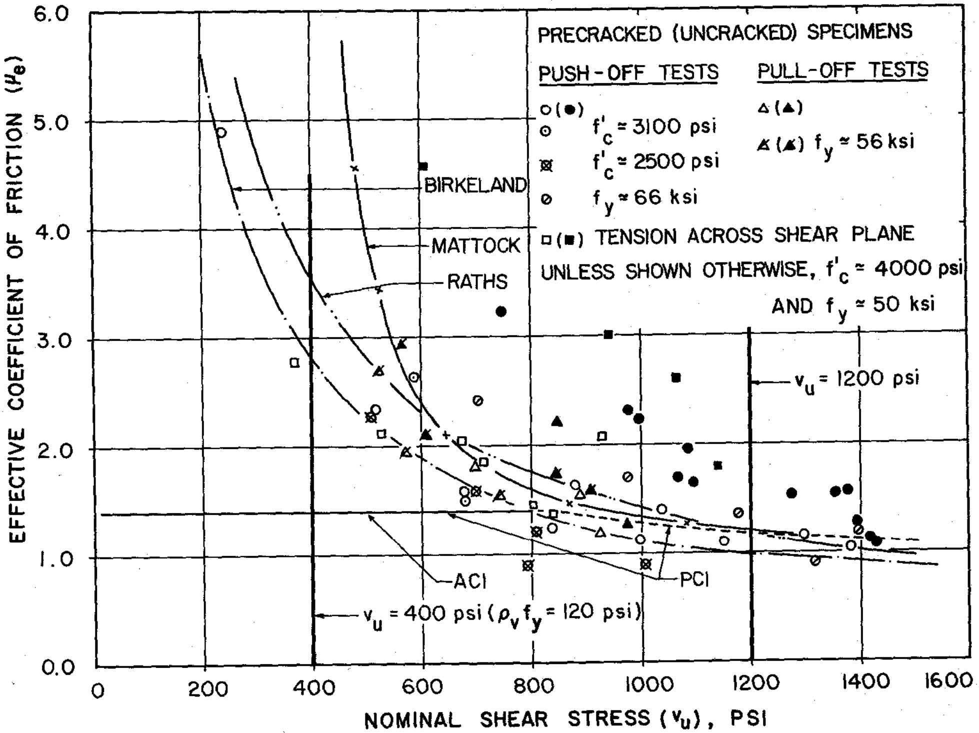

Fig. 3. Effective coefficient of friction versus nominal shear stress (normal weight concrete).

pression. Thus, a $\rho_v f_y$ = 120 psi would be reasonable and is the value proposed. The use of the ACI limit of $\rho_v f_y = 0.04 f'_c$ would become prohibitive for high strength concretes.

Table 2 is an expanded version of Table 1 and is included here to show the variation in values of μ_e and maximum v_u. For crack interface condition (3), the present value is $\mu = \mu_e = 0.7$. Raths[4] has suggested use of $\mu = 0.6$ and μ_e evaluated from Eq. (2.2). Mattock[5] has noted that $\mu = 0.6$ is unconservative. Review of Fig. 7 (taken from Ref. 4) suggests that $\mu = 0.4$, resulting in μ_e = 0.85 to 0.66, might be appropriate. For crack interface condition (4), $\mu = 0.6$ results in μ_e = 2.24 to 0.75.

The current PCI Connections Manual uses $\mu_e = \mu = 1.0$. The AISC approach to composite shear design translates to using $\mu_e = 1.0$ (varying between 0.96 to 1.1 depending on the strength of concrete) for full composite action. Based on the behavior of μ_e for concrete to concrete interface conditions, it would be reasonable to expect that μ_e, for steel to concrete interface, would also be relatively high for low values of $\rho_v f_y$ and would decrease as $\rho_v f_y$ increases.

Review of the literature on steel to concrete composite tests suggests that an exact quantification of μ_e is quite difficult, because of the type and range of parameters considered. An approximate analysis of 20 of the beams tested by Slutter and Driscoll[10] indicates that the range of μ_e is 3.1 to 0.81 corresponding to $\rho_v f_y$ = 175 to 659 psi. Thus, $\mu = 0.6$, resulting in

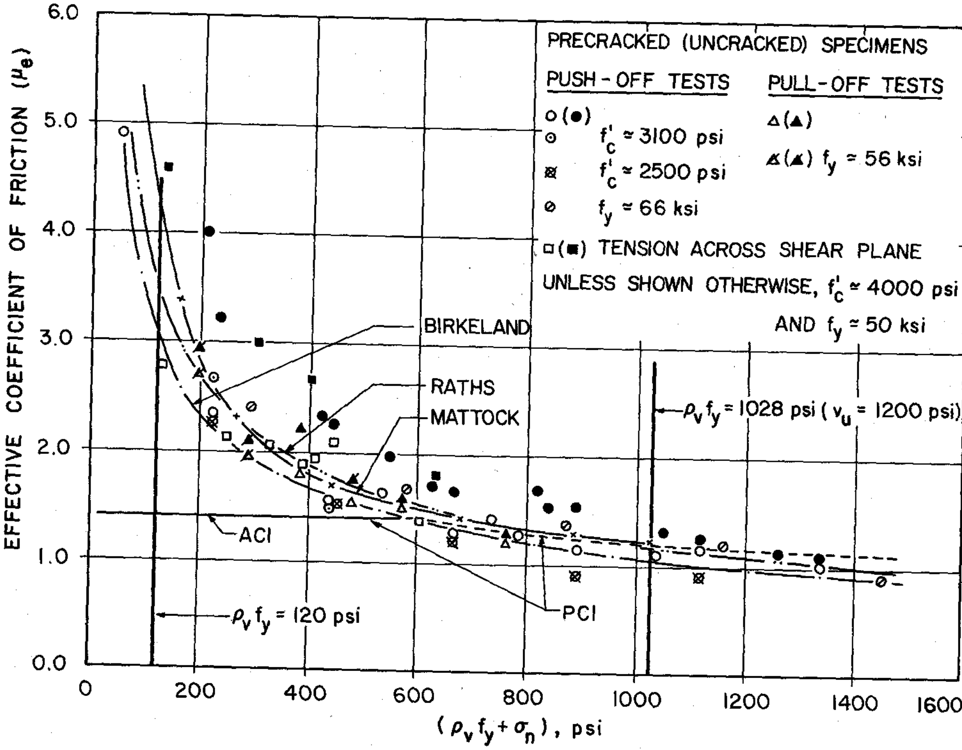

Fig. 4. Effective coefficient of friction versus $\rho_v f_y + \sigma_n$ (normal weight concrete).

Table 2. Expanded Table 1—Shear-friction coefficients, μ, and maximum shear stress values, v_u, for various interface conditions and different types of concrete.

Crack interface condition	μ	μ_e (normal weight concrete)		max. v_u* psi								
		corresp. to min. ρf_y	corresp. to max. v_u	$f'_c \geq 4000$ psi			f'_c = 3000 psi			f'_c = 2500 psi		
				Normal weight	Sand light weight	All light weight	Normal weight	Sand light weight	All light weight	Normal weight	Sand light weight	All light weight
①	1.4	3.42	1.17	1200	867	675	900	650	506	750	541	422
Mattock's values for max. v_u				1200	800-1000	800	900	600	600	750	500	500
②	1.0	2.89	1.0	1000	723	563	750	542	422	625	451	352
③	0.4	1.83	0.66	600	433	338	450	325	253	375	271	211
④	0.6	2.24	0.75	800	578	450	600	434	338	500	361	281

* Listing of max. v_u values corresponding to f'_c = 3000 psi and 2500 psi is for a comparison with test data plotted in Figs. 2 thru 5 and should not be construed as state of the art with respect to concrete strength.

① : Concrete to concrete cast monolithically.

② : Concrete to hardened concrete, ¼ in. roughness.

③ : Concrete to concrete, smooth interface.

④ : Concrete to steel.

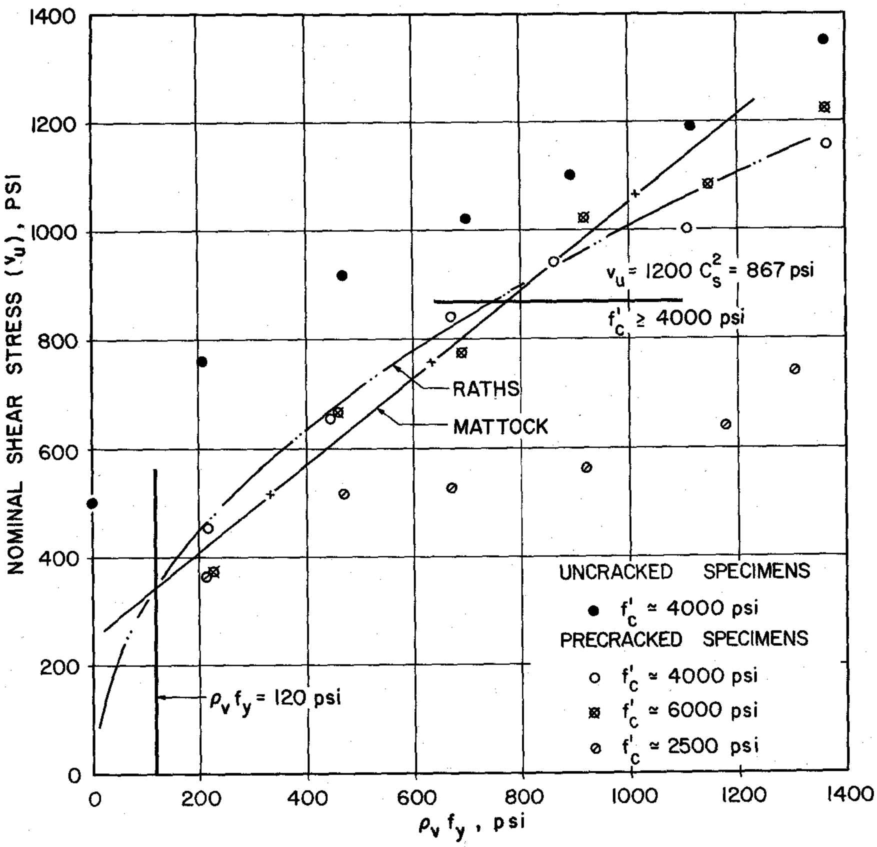

Fig. 5. Nominal shear stress versus $\rho_v f_y$ (sand-lightweight concrete).

μ_e = 2.24 to 0.75, is proposed as a reasonable value.

The maximum v_u values given in Table 2 are based on a review of Mattock's[6, 8] and Raths'[4] proposals and also on an examination of the test data with respect to variation in v_u versus $\rho_v f_y$. As shown in Figs. 2, 5, and 6 (noticeably in Fig. 2), beyond a certain $\rho_v f_y$ value, there appears to be a sharper decrease in the gain in v_u. The maximum v_u values should be lower than the values at which this trend is observed.*

A comparison of maximum v_u values with Mattock's proposals for a monolithic interface is shown in Table 2. It is noted that the limits for v_u indicated are for situations where the a/d effect is small. Mattock[11] has shown that the a/d ratio is an influencing parameter in the determination of v_u limits. However, this influence would be more appropriately included when establishing limits for individual connection types such as the corbel.

*From a production viewpoint, it may be necessary to lower the maximum v_u value further in order to avoid unmanageable reinforcement conjestion in connection areas. For example, for crack interface condition (1), a maximum $v_u \leq 1000\,C_s^2$ might be more appropriate.

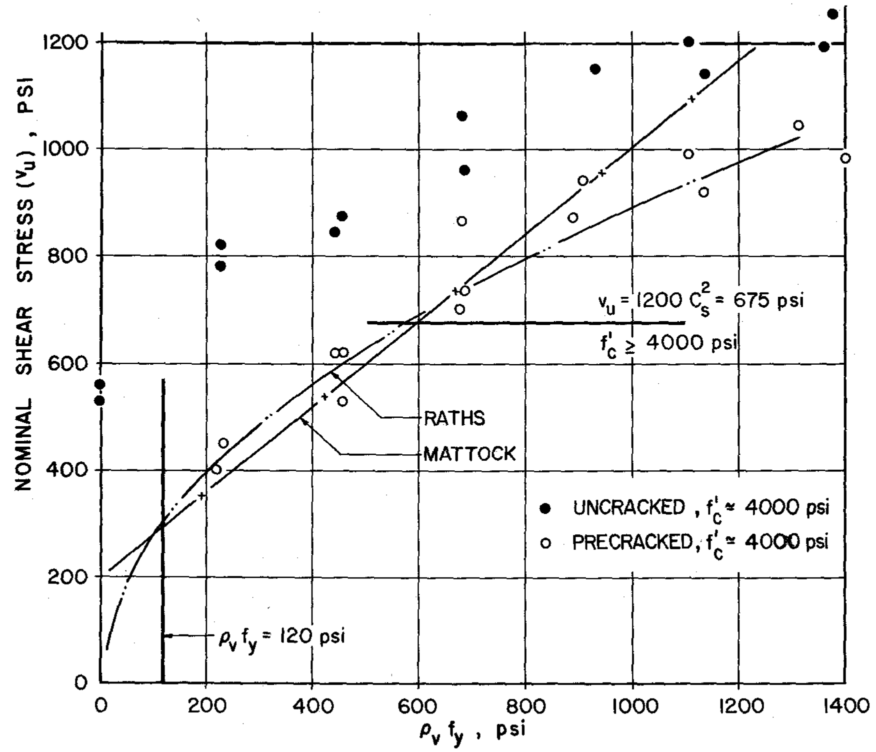

Fig. 6. Nominal shear stress versus $\rho_v f_y$ (all-lightweight concrete).

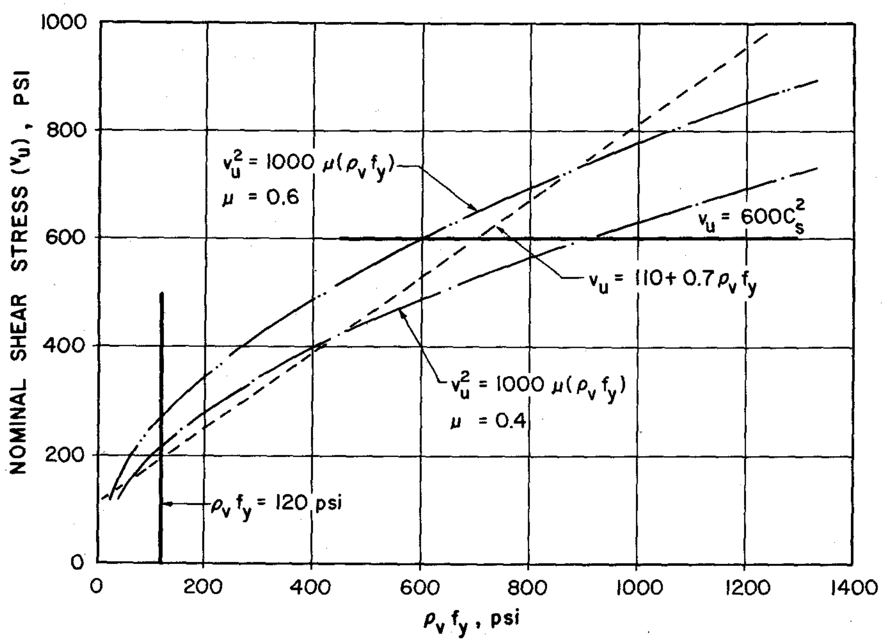

Fig. 7. Nominal shear stress versus $\rho_v f_y$ (smooth interface).

NOTATION

a = shear span
d = depth to centroid of reinforcement
a/d = shear span to depth ratio
A_{cr} = area of shear crack interface
A_v, A_{vf} = area of shear-friction reinforcement
C_s = constant used for effect of concrete density specified
f'_c = specified compressive strength of concrete
f_y, f_{yv} = specified yield strength of shear-friction reinforcement
v_u = nominal ultimate shear stress along interface
V_u = ultimate shear force
μ = coefficient of static friction applicable to reinforced concrete shear interface
μ_e = effective coefficient of friction used in shear-friction equation
ϕ = capacity reduction factor (equal to 0.85 for shear)
σ_n = normal stress produced by externally applied force perpendicular to shear plane
ρ_v , $\rho_{vf} = A_{vf}/A_{cr}$

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Discussion of this report is invited. Please forward your comments to PCI Headquarters by Sept. 1, 1978.