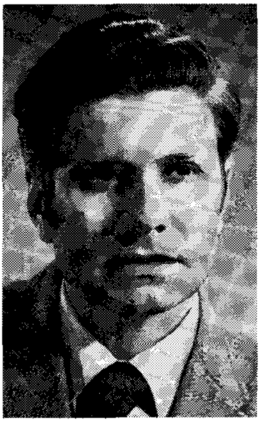
Design proposals for reinforced concrete corbels



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This paper presents design proposals for reinforced concrete corbels, based upon conclusions drawn from recent experimental studies of the behavior of reinforced concrete corbels[1] and of shear transfer across a plane which is also subject to moment and direct tension.[2]

General philosophy

The proposals for corbel design presented here, were developed so as to be compatible with the safety provisions and the design provisions for flexure and shear transfer contained in the ACI Building Code (ACI 318-71).[3] They are proposed for corbels with shear-span-to-depth ratios of unity or less, subject to combinations of vertical and horizontal loads in which the horizontal load is equal to or less than the vertical load.

It was considered desirable that the design proposals be based on a simple mechanical model of behavior for the corbel, which designers could readily visualize and use.

It is therefore proposed that the design of corbels to resist a combination of vertical and horizontal loads be based upon satisfaction of the laws of statics, when the corbel is considered as a "free body" cut from the column at the corbel-column interface, as shown in Figs. 1a and 1b.

Simple design proposals for normal weight and lightweight reinforced concrete corbels are presented, based on previously reported experimental studies.

A "Model Code Clause" embodying the design proposals is detailed, together with a step-by-step procedure for practical application.

Design examples are included for both normal weight and lightweight concrete corbels using both the ACI 318-71 shear-friction provisions and the modified shear-friction theory.

An Appendix section contains charts to facilitate proportioning of the corbel reinforcement.

Also included is a programable calculator program for designing reinforced concrete corbels.

Use of the design proposals can lead to savings in reinforcement, particularly if the modified shear-friction theory is used for shear design.

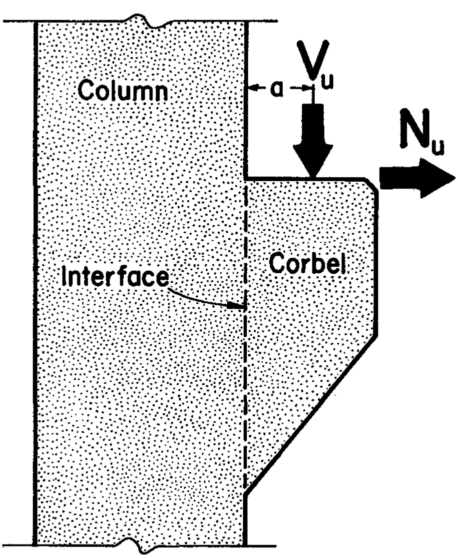

Fig. 1a. Typical corbel.

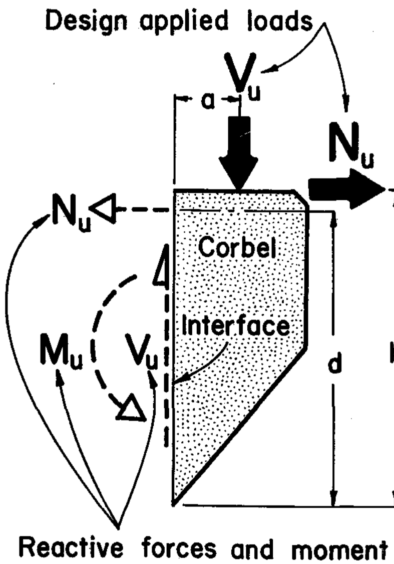

Fig. 1b. The corbel as a "free body."

This approach has been demonstrated[1] to be valid, subject to the provision of sufficient horizontal stirrups in the corbel, to prevent a premature diagonal tension failure of the corbel.

The design of a corbel now reduces to the calculation of the required amounts of reinforcement so that the interface plane can carry the reactive forces and moments V_u, N_u, and M_u shown in Fig. 1b, and the calculation of the necessary amount of horizontal stirrup reinforcement to prevent a premature diagonal tension failure of the corbel.

For static equilibrium of the corbel, the reactive forces V_u and N_u must be equal to the design vertical and horizontal loads, V_u and N_u, respectively. In addition, the reactive moment M_u must be equal to $[V_u a + N_u(h-d)]$.

Conclusions from previous investigation

The experimental study already reported[1,2] has demonstrated the following:

1. The flexural capacity of the interface may be calculated using the provisions of Section 10.2 of ACI 318-71.

2. The direct force N_u may be resisted by providing additional reinforcement having a yield strength equal to N_u.

3. The shear capacity of the interface may be calculated using the shear-friction provisions of Section 11.15 of ACI 318-71,* or using the "modified shear-friction" equations previously proposed.[1,2,4]

4. A premature diagonal tension failure of the corbel will not occur if closed stirrups or ties parallel to the main tension reinforcement are provided, having a total yield strength equal to one-half the yield strength of the reinforcement required to resist the moment M_u or one-third the yield strength of the reinforcement required to resist the shear V_u, whichever is the greater. This reinforcement is to be uniformly distributed within the two-thirds of the effective depth of the interface adjacent to the main reinforcement.

If the yield point of the stirrups is equal to that of the main tension reinforcement, the required total cross-directional area of the horizontal stirrups A_h is simply given by

$$A_h = 0.50(A_s - A_t)$$

where

A_s = total area of main tension reinforcement

A_t = area of reinforcement provided to resist N_u

* The provisions of Section 11.15 of ACI 318-71 relate to normal weight concrete only. Based on previously reported studies,[1,4] it is proposed that for corbels made of lightweight concrete, the values of μ given in Section 11.15 be multiplied by 0.75 for all-lightweight concrete weighing at least 92 lb per cu ft and 0.85 for sanded lightweight concrete weighing at least 105 lb per cu ft and in addition that v_u shall not be taken to be greater than $(0.2 - 0.07a/d)f_c'$ nor $(800 - 780a/d)$ psi for all-lightweight concrete, and $(0.2 - 0.07a/d)f_c'$ nor $(1000 - 350a/d)$ psi for sanded lightweight concrete.

PROPOSED MODEL CODE CLAUSE

Based on the design philosophy discussed above, the following is proposed as a replacement for the existing Section 11.14 of ACI 318-71. Design for shear is based on the existing shear-friction provisions of Section 11.15, modified as necessary for the case of lightweight concrete.

11.14—Special provisions for brackets and corbels

11.14.1—These provisions apply to brackets and corbels having a shear-span-to-depth ratio, a/d of unity or less, which are subjected to a design horizontal tensile force N_u less than or equal to the design shear force V_u. The distance d shall be measured at a section adjacent to the face of the support.

11.14.2—The section adjacent to the face of the support shall be designed to resist simultaneously, a design shear V_u, a design moment $[V_u a + N_u(h-d)]$, and a design horizontal tensile force N_u.

11.14.2.1—The reinforcement A_{vf} required to resist the design shear shall be calculated using the design provisions of Section 11.15.*

11.14.2.2—The reinforcement A_f required to resist the design moment shall be calculated using the design provisions of Section 10.2.

11.14.2.3—The reinforcement A_t required to resist the design tensile force N_u shall be taken as $N_u/(\phi f_y)$. The design tensile force N_u shall not be taken as less than $0.2V_u$ unless special provisions are made to avoid tension forces due to restrained shrinkage and creep, so that the member is subject to shear and moment only. The tensile force N_u shall be regarded as a live load even when it results from creep, shrinkage, or temperature change.

11.14.2.4—The area of main tension reinforcement A_s shall be made equal to $(A_f + A_t)$ or $(2A_{vf}/3 + A_t)$, whichever is the greater.

11.14.2.5—The main tension reinforcement shall be anchored as close to the outer face of the corbel as cover requirements permit, by welding a bar of equal diameter across the ends of the main reinforcing bars, or by some other means of positive anchorage.

11.14.3—Closed stirrups or ties parallel to the main tension reinforcement, having a total cross-sectional area A_h not less than $0.50(A_s - A_t)$ shall be uniformly distributed within two-thirds of the effective depth adjacent to the main tension reinforcement.

11.14.4—The ratio $\rho = A_s/(bd)$ shall be not less than $0.04(f_c'/f_y)$.

11.14.5—The depth of the corbel or bracket at the outside edge of the bearing area shall be not less than one-half the effective depth of the corbel or bracket at the section adjacent to the face of the support.

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* For corbels made of lightweight concrete, the values of μ given in Section 11.15 shall be multiplied by 0.75 for all-lightweight concrete weighing at least 92 lb per cu ft and 0.85 for sanded lightweight concrete weighing at least 105 lb per cu ft and, in addition, v_u shall not be taken to be greater than
$(0.2 - 0.07a/d)f_c'$ nor $(800 - 780a/d)$ psi for all-lightweight concrete, and
$(0.2 - 0.07a/d)f_c'$ nor $(1000 - 350a/d)$ psi for sanded lightweight concrete.

DESIGN PROCEDURE SATISFYING MODEL CODE

1. Select tentative proportions for the corbel, checking that a/d is not more than 1.0, and that v_u is not more than:

(a) $0.2f_c'$ nor 800 psi for normal weight concrete;

(b) $(0.2 - 0.07a/d)f_c'$ nor $(800 - 280a/d)$ psi for all-lightweight concrete; or

(c) $(0.2 - 0.07a/d)f_c'$ nor $(1000 - 350\ a/d)$ psi for sanded lightweight concrete.

2. Calculate the area of reinforcement A_{vf} needed across the shear plane to carry shear, using Eq. (11-30) of ACI 318-71:

$$A_{vf} = \frac{V_u}{\phi f_y \mu}$$

where $\phi = 0.85$.

For corbels cast monolithically with the column:

$\mu = 1.40$ for normal weight concrete

$\mu = 1.4(0.85) = 1.19$ for sanded lightweight concrete (unit weight not less than 105 lb per cu ft)

$\mu = 1.4(0.75) = 1.05$ for all-lightweight concrete (unit weight not less than 92 lb per cu ft)

3. Estimate the distance $(h - d)$ from the top face of the corbel bearing plate to the centroid of the main tension reinforcement (see Fig. 2), and calculate the design ultimate moment the corbel-column interface must resist:

$$\text{Req. } M_u = V_u a + N_u(h - d) \quad (1)$$

4. Calculate the area of reinforcement A_f necessary to provide the resistance moment M_u, using the provisions of Section 10.2 of ACI 318-71 and a capacity reduction factor ϕ of 0.9.*

5. Calculate the area of reinforcement A_t necessary to resist the horizontal force N_u, using:

$$A_t = \frac{N_u}{\phi f_y} \quad (2)$$

where ϕ is 0.85.

6. Check whether A_f is greater than $2A_{vf}/3$. If A is greater than $2A_{vf}/3$, calculate the total area of main tension reinforcement, using:

$$A_s = A_f + A_t \quad (3)$$

If A_f is less than $2A_{vf}/3$, calculate the total area of main tension reinforcement, using:

$$A_s = 2A_{vf}/3 + A_t \quad (4)$$

7. Check that $\rho = A_s/(bd)$ is not less than $0.04(f_c'/f_y)$.

8. Calculate the total cross-sectional area of horizontal stirrup reinforcement A_h, making A_h equal to $0.50\ (A_s - A_t)$. Distribute this reinforcement in the two-thirds of the effective depth adjacent to the main tension reinforcement.

9. Recheck the dimensions of the corbel and in particular compute the depth of the outer face of the corbel in accordance with Section 11.14.5 of Model Code Clause.

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* It can be shown that A_f/bd is less than $0.75\rho_b$ for the worst case of $v_u = 0.2f_c'$, $a/d = 1.0$, and $N_u/V_u = 1.0$, if $(h\text{-}d)/d$ is assumed equal to 1/8, $f_c' \leqslant 6000$ psi, and $f_y \leqslant 60$ ksi. (The reinforcement ratio ρ actually equals $0.70\rho_b$ for this limiting case.)

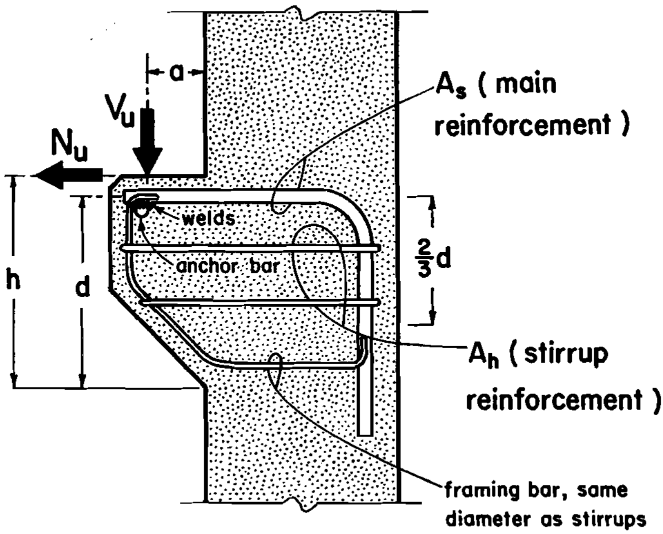

Fig. 2. Typical corbel reinforcement.

ALTERNATE DESIGN METHOD FOR SHEAR

The design of the interface for shear may alternatively be based on the following "modified shear-friction" equations previously proposed.[1,2,4]

1. For normal weight concrete:

$$v_u = \frac{V_u}{\phi bd} = 0.8\rho_v f_y + 400 \text{ psi} \quad (5)$$

but not more than $0.3f_c'$.

2. For all-lightweight concrete having a unit weight of not less than 92 lb per cu ft:

$$v_u = \frac{V_u}{\phi bd} = 0.8\rho_v f_y + 200 \text{ psi} \quad (6)$$

but not more than $(0.2 - 0.07a/d)f_c'$ nor $(800 - 280a/d)$ psi.

3. For sanded-lightweight concrete having a unit weight of not less than 105 lb per cu ft:

$$v_u = \frac{V_u}{\phi bd} = 0.8\rho_v f_y + 250 \text{ psi} \quad (7)$$

but not more than $(0.2 - 0.07a/d)f_c'$ nor $(1000 - 350a/d)$ psi.,

where $\rho_v = A_{vf}/(bd)$ must be not less than $200/f_y$ in all cases.

For design purposes Eqs. (5), (6), and (7) may be transposed as follows, where V_u is in lb and f_y is in psi.

1. For normal weight concrete:

$$A_{vf} = (V_u/\phi - 400bd)/(0.8f_y) \quad (5A)$$

2. For all-lightweight concrete:

$$A_{vf} = (V_u/\phi - 200bd)/(0.8f_y) \quad (6A)$$

3. For sanded lightweight concrete:

$$A_{vf} = (V_u/\phi - 250bd)/(0.8f_y) \quad (7A)$$

In all cases, A_{vf} must be not less

than $200bd/f_y$, and the upper limits to the value of v_u must also be observed.

If V_u is given in kips and f_y in ksi, these equations may be restated as follows:

$$A_{vf} = [V_u/(0.8\phi) - Kbd]/f_y \quad (8)$$

but not less than $0.2bd/f_y$

where

$K = 0.50$ for normal weight concrete

or

$K = 0.25$ for all-lightweight concrete

or

$K = 0.31$ for sanded lightweight concrete

Note: When this method of design for shear is used and the design (ultimate) shear stress exceeds 1000 psi, then if a/d exceeds 0.6, a check must be made that $A_f/(bd)$ is less than $0.75\rho_b$.

Advantages of Proposed Design Method

An important advantage of the design method proposed is its simplicity of concept and the avoidance of complicated empirical equations. This enables the engineer to develop a feel for the way the corbel resists forces and hence for the reasonableness of his designs.

Use of the design proposals can lead to savings in reinforcement, particularly if the modified shear-friction theory is used for shear design.

Also, higher design shear stresses can be used than currently allowed by ACI 318-71, if the modified shear-friction theory is used for shear design. Although not always desirable, this can be convenient in certain circumstances.

DESIGN EXAMPLES

In this section two fully-worked design examples are presented applying the preceding design proposals: (1) using a normal weight concrete corbel and (2) a lightweight concrete corbel.

The problem in Example 1 is approached employing two methods. In the first method the corbel is assumed to be moderately reinforced and in the second the corbel is designed to have a specified overall depth.

In both Examples 1 and 2 the problem is solved using the ACI 318-71 shear-friction provisions and the proposed modified shear-friction theory and in each case the reinforcing steel requirements compared.

Finally, the last part in this section contains some practical comments on the reinforcing steel details.

EXAMPLE 1
(Normal weight concrete)

Design a corbel (see Fig. 3) which is to project from the face of a 14 x 14-in. column and carry the following loads:

(a) Vertical dead load of 32 kips.

(b) Vertical live load of 30 kips.

(c) Horizontal force of 24 kips due to restraint of beam creep and shrinkage deformations.

Assume normal weight concrete with $f_c' = 5000$ psi and let the yield strength of the reinforcing steel be $f_y = 60$ ksi.

Design loads (ultimate)

$$V_u = 1.4V_D + 1.7V_L = 1.4(32) + 1.7(30) = 95.8 \text{ kips}$$

$$N_u = 1.7N = 1.7(24) = 40.8 \text{ kips}$$

Using a 14 x 4 x 1-in. bearing plate, check the bearing stress:

$$f_b = \frac{V_u}{\phi bw} = \frac{95{,}800}{0.7 \times 14 \times 4} = 2444 \text{ psi}$$

This stress is satisfactory since it is less than the allowable stress:

$$0.5f_c' = 0.5(5000) = 2500 \text{ psi}$$

Following the recommendation of the *PCI Design Handbook*,[5] assume that the vertical load acts at the outer third point of the bearing plate. A 1-in. gap is assumed between the rear edge of the bearing plate and the column face.

Hence, the shear span is $a = 1 + \frac{2}{3}(4) = 3.67$ in.

1. Moderately reinforced corbel

For a moderately reinforced corbel in which the reinforcement should be reasonably easy to place, assume that the nominal shear stress, v_u, is about 600 psi. The depth of the corbel can then be found from:

$$v_u = \frac{V_u}{\phi bd} = \frac{95{,}800}{0.85(14)(d)} \approx 600 \text{ psi}$$

from which

$$d = \frac{95{,}800}{0.85(14)(600)} \approx 13.4 \text{ in.}$$

Let the total depth of the corbel, h, be 15 in. Then assuming that we have a 1-in. cover and are using #8 bars:

$$d = 15 - 1 - \tfrac{1}{2} = 13.5 \text{ in.}$$
and $a/d = 0.27$

(a) Using the ACI 318-71 shear-friction provisions

The area of shear transfer reinforcement can be found from:

$$A_{vf} = \frac{V_u}{\phi f_y \mu}$$

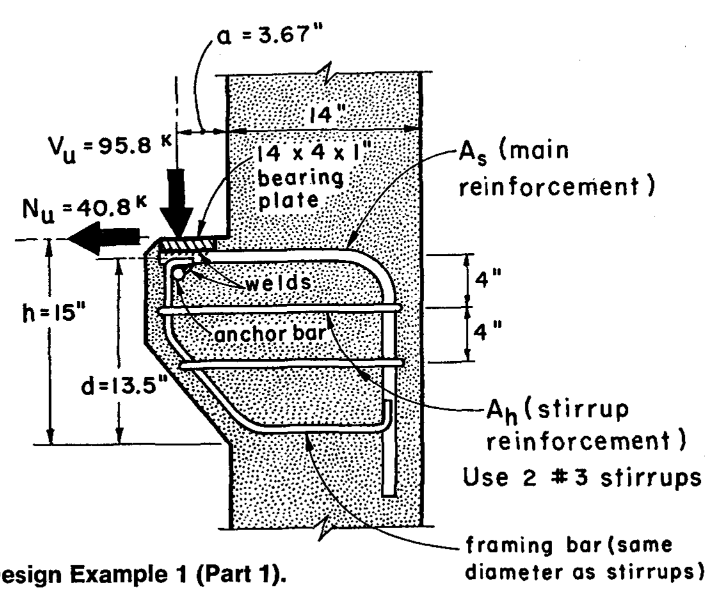

Fig. 3. Design Example 1 (Part 1).

$$A_{vf} = \frac{95.8}{0.85(60)(1.4)}$$
$$= 1.34 \text{ sq in.}$$

Therefore, $\frac{2}{3}A_{vf} = 0.89$ sq in.

The required moment capacity is

$$M_u = V_u a + N_u(h - d)$$
$$= 95.8(3.67) + 40.8(15 - 13.5)$$
$$= 351.6 + 61.2$$
$$= 412.8 \text{ in.-kips}$$

Assume that the depth of the rectangular stress block, x, equals 0.5 in. Then the area of flexural reinforcement is

$$A_f = \frac{M_u}{\phi f_y(d - x/2)}$$
$$= \frac{412.8}{0.9(60)(13.5 - 0.5/2)}$$
$$= 0.58 \text{ sq in.}$$

which is less than $\frac{2}{3}A_{vf}$ (0.89 sq in.).

Check the stress block depth:

$$x = \frac{0.58(60)}{0.85(5)(14)}$$
$$= 0.6 \text{ in.} \ (\approx 0.5 \text{ in., ok})$$

The area of horizontal reinforcement is

$$A_t = \frac{N_u}{\phi f_y} = \frac{40.8}{0.85(60)} = 0.80 \text{ sq in.}$$

Since $\frac{2}{3}A_{vf}$ is greater than A_f, the total area of main tension reinforcement is

$$A_s = \tfrac{2}{3}A_{vf} + A_t$$
$$= 0.89 + 0.80 = 1.69 \text{ sq in.}$$

Use = 3 #7 bars (1.80 sq in.).

The total area of shear reinforcement is

$$A_h = \tfrac{1}{2}(A_s - A_t)$$
$$= \tfrac{1}{3}A_{vf} = 0.45 \text{ sq in.}$$

Use 2 #3 stirrups (0.44 sq in.).

The horizontal stirrup reinforcement is to be placed within the two-thirds of the effective depth adjacent to the main reinforcement, A_s.

Therefore, the maximum stirrup spacing equals $\frac{1}{2}(\frac{2}{3})(13.5) = 4.5$ in.

Use a 4-in. stirrup spacing.

Minimum depth of corbel at outside edge of bearing plate:

$$0.5(13.5) = 6.75 \text{ in.}$$

Make depth of outer face of corbel 8 in.

(b) Alternate design using modified shear-friction equation

The area of shear transfer reinforcement is found from:

$$A_{vf} = \left[\frac{V_u}{0.8\phi} - 0.5bd\right]/f_y$$

but not less than $0.2bd/f_y$.

$$A_{vf} = \left[\frac{95.8}{0.8(0.85)} - (0.5)(14)(13.5)\right]/60$$
$$= 0.77 \text{ sq in.}$$

but not less than

$$0.2(14)(13.5)/60 = 0.63 \text{ sq in.}$$

Therefore, $A_{vf} = 0.77$ sq in. controls.

Now, $\frac{2}{3}A_{vf} = 0.51$ sq in., i.e., less than A_f (0.58 sq in.).

Total area of tension reinforcement:

$$A_s = A_f + A_t$$
$$= 0.58 + 0.80$$
$$= 1.38 \text{ sq in.}$$

Use 2 #6 plus 1 #7 bars (1.48 sq in.).

Total area of shear reinforcement:

$$A_h = 0.5(A_s - A_t) = 0.5A_f$$
$$= 0.5(0.58) = 0.29 \text{ sq in.}$$

Use 2 #3 stirrups (0.44 sq in.) at 4-in. centers.

2. Corbel with specified depth

Design a corbel to have a specified shallow overall depth of 8 in.

Therefore, $d = 8 - 1 - \frac{1}{2} = 6.5$ in. and $a/d = 0.56$.

$$v_u = \frac{V_u}{\phi bd}$$

$$v_u = \frac{95{,}800}{0.85(14)(6.5)}$$
$$= 1239 \text{ psi}$$

i.e., less than $0.3f_c'$ (1500 psi), ok.

Using the modified shear-friction equation:

$$A_{vf} = \left[\frac{V_u}{0.8\phi} - 0.5bd\right]/f_y$$

$$= \left[\frac{95.8}{0.8(0.85)} - 0.5(14)(6.5)\right]/60$$

$$= 1.59 \text{ sq in.}$$

$\frac{2}{3}A_{vf} = 1.06$ sq in.

Assume that the depth of the rectangular stress block, $x = 1.5$ in. Then:

$$A_f = \frac{M_u}{\phi f_y(d - x/2)}$$

$$= \frac{412.8}{0.9(60)(6.5 - 1.5/2)}$$

$$= 1.33 \text{ sq in.}$$

which is greater than $\frac{2}{3}A_{vf}$ (1.06 sq in.).

Check the depth of stress block:

$$x = \frac{1.33(60)}{0.85(5)(14)}$$

$$= 1.3 \text{ in.} (\approx 1.5 \text{ in.}), \text{ ok}$$

Check the reinforcement ratio:

$$\rho_f = A_f/bd$$
$$= 1.33/[(14)(6.5)]$$
$$= 0.0146$$

which is less than $0.75\rho_b$ (0.025), ok.

Total area of tension reinforcement:

$$A_s = A_f + A_t$$
$$= 1.33 + 0.80$$
$$= 2.13 \text{ sq in.}$$

Use 3 #8 bars (2.37 sq in.)

Total area of shear reinforcement:

$$A_h = 0.5(A_s - A_t) = 0.5A_f$$
$$= 0.5(1.33) = 0.67 \text{ sq in.}$$

Use 2 #4 stirrups (0.80 sq in.)

Maximum stirrup spacing:

$\frac{1}{2}(\frac{2}{3})(6.5) = 2.17$ in.

Use 2-in. spacing

Minimum depth of corbel at outside edge of bearing plate:

$0.5(6.5) = 3.25$ in.

Make depth of outer face of corbel 4 in.

EXAMPLE 2
(Lightweight concrete)

Assume the same dimensions and loads as for Example 1, but instead of normal weight concrete use an all-lightweight concrete with $f_c' = 4000$ psi. Let $f_y = 60$ ksi.

As before:

$V_u = 95.8$ kips and $N_u = 40.8$ kips

Using a 14 x 5-in. bearing plate:

$$f_b = \frac{V_u}{\phi bw}$$

$$= \frac{95{,}800}{0.7(14)(5)}$$

$$= 1955 \text{ psi}$$

i.e., less than $0.5f_c'$ (2000 psi), ok.

$a = 1 + \frac{2}{3}(5) = 4.33$ in.

As before, try $h = 15$ in. with $d =$ 13.5 in.

Therefore, $a/d = 0.32$.

The maximum nominal shear stress for all-lightweight concrete is found from

$$\text{max. } v_u = [0.2 - 0.07(a/d)]f_c'$$

but not greater than $[800 - 280(a/d)]$

For $f_c' = 4000$ psi:

$$v_u = 800 - 280(0.32)$$
$$= 710 \text{ psi}$$

The calculated shear stress is

$$v_u = \frac{V_u}{\phi bd}$$

$$= \frac{95{,}800}{0.85(14)(13.5)}$$

$$= 596 \text{ psi}$$

i.e., less than the maximum v_u (ok).

(a) Using the ACI 318-71 shear-friction provisions modified for all-lightweight concrete as proposed

That is, $\mu = 0.75(1.4) = 1.05$

Therefore, the area of shear transfer reinforcement is

$$A_{vf} = \frac{V_u}{\phi f_y \mu}$$

$$= \frac{95.8}{0.85(60)(1.05)}$$

$$= 1.79 \text{ sq in.}$$

$$\tfrac{2}{3}A_{vf} = 1.19 \text{ sq in.}$$

The required moment capacity is

$$\begin{aligned} M_u &= V_u a + N_u(h - d) \\ &= 95.8(4.33) + 40.8(1.5) \\ &= 476 \text{ in.-kips} \end{aligned}$$

Assume that the depth of the rectangular stress block, $x = 0.7$ in.

The area of flexural reinforcement is

$$A_f = \frac{M_u}{\phi f_y(d - x/2)}$$

$$= \frac{476.0}{0.9(60)(13.5 - 0.7/2)}$$

$$= 0.67 \text{ sq in.}$$

i.e., less than $\tfrac{2}{3}A_{vf}$ (1.19 sq in.) ok.

Check the stress block depth:

$$x = \frac{0.67(60)}{0.85(5)(14)}$$

$$= 0.7 \text{ in. (ok)}$$

The area of horizontal reinforcement is

$$A_t = \frac{N_u}{\phi f_y} = \frac{40.8}{0.85(60)} = 0.80 \text{ sq in.}$$

Since $\tfrac{2}{3}A_{vf}$ is greater than A_f, the total area of main tension reinforcement is

$$\begin{aligned} A_s &= \tfrac{2}{3}A_{vf} + A_t \\ &= 1.19 + 0.80 \\ &= 1.99 \text{ sq in.} \end{aligned}$$

Use 2 #9 bars (2.00 sq in.)

Total area of shear reinforcement:

$$\begin{aligned} A_h &= 0.5((A_s - A_t) \\ &= \tfrac{1}{3}A_{vf} = 0.60 \text{ sq in.} \end{aligned}$$

Use 3 #3 stirrups (0.66 sq in.)

Maximum stirrup spacing:

$$\tfrac{1}{3}(\tfrac{2}{3})(13.5) = 3.0 \text{ in.}$$

Use 3-in. spacing

Minimum depth of corbel at outside edge of bearing plate:

$$0.5(13.5) = 6.75 \text{ in.}$$

Make depth of outer face of corbel 8 in.

(b) Alternate design using modified shear-friction equation

$$A_{vf} = \left[\frac{V_u}{0.8\phi} - 0.25bd\right]$$

but not less than $0.2bd/f_y$.

$$A_{vf} = \left[\frac{95.8}{0.8(0.85)} - 0.25(14)(13.5)\right]/60$$

$$= 1.56 \text{ sq in.}$$

but not less than

$$0.2(14)(13.5)/60 = 0.63 \text{ sq in.}$$

Therefore, $\tfrac{2}{3}A_{vf} = \tfrac{2}{3}(1.56) = 1.04$ sq in. i.e., greater than A_f (0.67 sq in.).

Main tension reinforcement:

$$\begin{aligned} A_s &= \tfrac{2}{3}A_{vf} + A_t \\ &= 1.04 + 0.80 \\ &= 1.84 \text{ sq in.} \end{aligned}$$

Use 2 #8 plus 1 #5 bars (1.89 sq in.)

or 2 #9 bars (2.00 sq in.)

Total shear reinforcement:

$$\begin{aligned} A_h &= 0.5(A_s - A_t) \\ &= \tfrac{1}{3}A_{vf} = 0.52 \text{ sq in.} \end{aligned}$$

Use 3 #3 stirrups (0.66 sq in.)

at 3-in. centers

Reinforcing Steel Details

In all corbels, the main reinforcement A_s must be anchored as close to the outer corner of the corbel as cover requirements will permit, by welding a reinforcing bar of equal diameter across the ends of the main reinforcing bars, as indicated in Fig. 3 or by the use of some other form of positive anchorage.

The welds must be sized so as to be able to transfer to the transverse reinforcing bar, a force equal to the yield strength of the main reinforcing bar.

The main reinforcement must also be anchored within the column, by being extended a distance beyond the corbel-column interface at least equal to the development length for the size of reinforcing bar used.

The minimum radius that reinforcing bars may be bent, *must* be taken into account when detailing corbel reinforcement.

The bearing plate within the corbel should either be welded to the main reinforcement or be anchored adequately within the body of the corbel, so as to be able to transfer the horizontal force N_u to the main reinforcement.

DESIGN AIDS

Two design aids are provided.

Figs. A1 through A5 of Appendix A are design charts to facilitate the sizing of the main tension and stirrup reinforcement of normal weight concrete corbels, when modified shear-friction is used for shear design.

Appendix B contains programs for the Hewlett Packard HP-65 programable pocket calculator, which calculate corbel reinforcement using either the ACI 318-71 shear-friction provisions or modified shear-friction for shear design.

Use of Design Charts for Normal Weight Concrete Corbels

Design charts for

$$\rho f_y = \frac{A_s f_y}{bd} \text{ and } \rho_h f_y = \frac{A_h f_y}{bd}$$

are given for values of N_u/V_u of 0.2, 0.4, 0.6, 0.8, and 1.0. Values of ρf_y and $\rho_h f_y$ for other values of N_u/V_u can be obtained by linear interpolation.

Use of these charts in design is as follows:

1. Select tentative proportions for the corbel, checking that a/d is not more than 1.0 and that v_u is not more than $0.3f_c'$.
2. Enter the chart on the horizontal axis, at the value of a/d for the corbel proportion tentatively selected.
3. Proceed vertically to the curve for the value of v_u corresponding to the corbel dimensions and the design shear. (If necessary interpolate between curves.)
4. Proceed horizontally to the ρf_y axis and read the required ρf_y. (Check that ρf_y is not less than $0.04f_c'$.)
5. Continue horizontally to the left to the appropriate curve for v_u.
6. Travel vertically downward to the $\rho_h f_y$ axis and read the required $\rho_h f_y$.

Application of Design Charts to Design Example 1(b)

$N_u = 40.8$ kips; $V_u = 95.8$ kips.
Therefore, $N_u/V_u = 0.426$.
$a = 3.67$ in.; $d = 13.5$ in.
Therefore, $a/d = 0.27$.

$$v_u = \frac{V_u}{\phi bd} = \frac{95,800}{0.85(14)(13.5)}$$

$$= 596 \text{ psi } (\approx 600 \text{ psi})$$

1. On chart for $N_u/V_u = 0.4$ (Fig. A2)

Enter at $a/d = 0.27$, proceed vertically to $v_u = 600$-psi curve, then horizontally to read $\rho f_y = 0.42$ ksi. Continue left horizontally to $v_u = 600$-psi line, then vertically and read $\rho_h f_y = 0.09$ ksi.

2. On chart for $N_u/V_u = 0.6$ (Fig. A3)

Enter at $a/d = 0.27$, proceed vertically to $v_u = 600$-psi curve, then horizontally to read $\rho f_y = 0.56$ ksi. Continue left horizontally to $v_u = 600$-psi line, then vertically and read $\rho_h f_y = 0.10$ ksi.

Then, for actual N_u/V_u of 0.426:

$$\rho f_y = 0.42 + \frac{0.026}{0.20}(0.56 - 0.42)$$

$$= 0.44 \text{ ksi}$$

Therefore, the total tension reinforcement is

$$A_s = (\rho f_y)bd/f_y$$
$$= (0.44)(14)(13.5)/60$$
$$\underline{A_s = 1.39 \text{ sq in.}}$$

$$\rho_h f_y = 0.09 + \frac{0.026}{0.20}(0.10 - 0.09)$$

$$= 0.091 \text{ ksi}$$

Therefore, the total shear reinforcement is

$$A_h = (\rho_h f_y)\, bd/f_y$$
$$= (0.091)(14)(13.5)/60$$
$$\underline{A_h = 0.29 \text{ sq in.}}$$

These reinforcement areas, of course, agree with the values previously obtained [see Example 1 (Part b)].

Use of HP-65 Calculator Programs

Appendix B contains programs (see Tables B2, B3, and B4) for computing corbel reinforcement using the Hewlett Packard HP-65 programable calculator.

Program Cards 1 and 2 are used if the ACI 318-71 shear-friction provisions are to be used for shear design.

Program Cards 1 and 3 are used if modified shear-friction is to be used for shear design.

Operating instructions for both programs are shown in Table B1. The programs are applicable to normal weight, all-lightweight, and sanded lightweight concrete corbels.

The programs calculate the following quantities:

A_f, $(2/3)A_{vf}$, A_t, A_s, and A_h

The following checks are incorporated in the programs:

1. That A_f/bd is not greater than $0.75\ \rho_b$.
2. That A_s/bd is not less than $0.04f_c'/f_y$.
3. That A_{vf} is not less than $0.2\ bd/f_y$ when modified shear-friction is used.

The user must independently check that the shear stress v_u, corresponding to the chosen corbel dimensions, is less than the maximum allowed for the type of concrete and for the method of shear design being used.

REFERENCES

1. Mattock, A. H., Chen, K. C., and Soongswang, K., "The Behavior of Reinforced Concrete Corbels," PCI JOURNAL, V. 21, No. 2, March-April 1976, pp. 52-77.
2. Mattock, A. H., Johal, L. P., and Chow, C. H., "Shear Transfer in Reinforced Concrete with Moment or Tension Acting Across the Shear Plane," PCI JOURNAL, V. 20, No. 4, July-August 1975, pp. 76-93.
3. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-71)," American Concrete Institute, Detroit, Michigan, 1971.
4. Mattock, A. H., Li, W. K., and Wang, T. C., "Shear Transfer in Lightweight Reinforced Concrete," PCI JOURNAL, V. 21, No. 1, January-February 1976, pp. 20-39.
5. *PCI Design Handbook*, Prestressed Concrete Institute, Chicago, Illinois, 1971.

APPENDIX A—REINFORCED CONCRETE CORBEL DESIGN CHARTS

Figures A1 through A5 relate a/d, v_u, ρf_y, and $\rho_h f_y$, for values of N_u/V_u of 0.2, 0.4, 0.6, 0.8, and 1.0, when modified shear-friction is used in design for shear of normal weight concrete corbels.

The horizontal parts of the curves relating a/d and ρf_y, correspond to the value of ρf_y being controlled by design for shear. The upward curving parts of these curves correspond to the value of ρf_y being controlled by design for moment.

The chain lines cutting across the curves relating a/d and ρf_y, indicate the maximum values of ρf_y that can be used for the indicated combinations of reinforcement yield point and concrete compressive strength. These lines correspond to $A_f/(bd)$ being equal to 0.75 ρ_b.

A value of 0.10 was used for the ratio $(h-d)/d$, when calculating the ordinates of the design curves. The value of this ratio is significant only when the value of ρf_y is controlled by design for moment, and N_u/V_u is relatively large. The exact value of $(h-d)/d$ will vary between about 0.05 and 0.15, tending to be smaller than 0.1 for corbels larger than 15 in. deep and larger than 0.1 for corbels smaller than 15 in. deep.

The result is that when moment controls and N_u/V_u is large, the curves will yield slightly conservative values of ρf_y for large corbels and slightly unconservative values of ρf_y for small corbels.

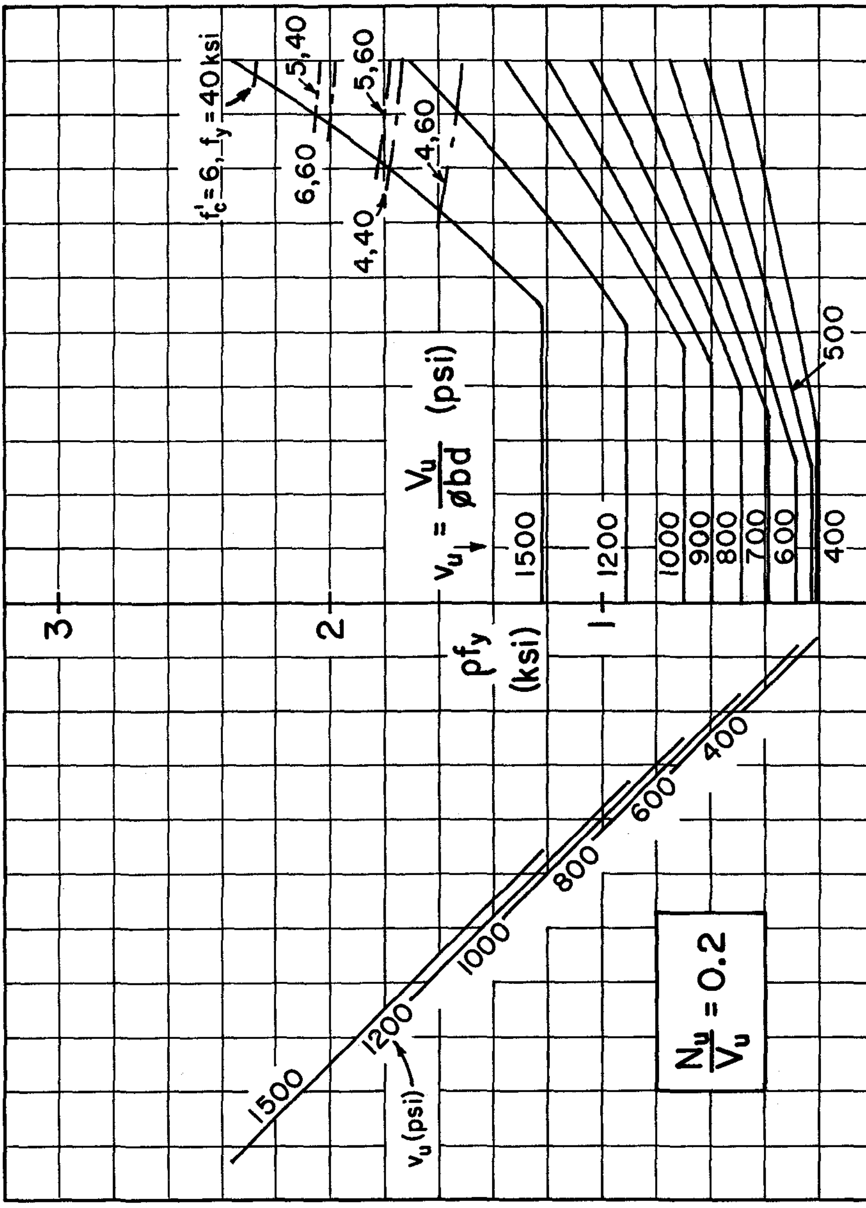

Fig. A1. Design chart for determining shear reinforcement in normal weight ***concrete corbel using modified shear-friction theory ($N_u/V_u = 0.2$).***

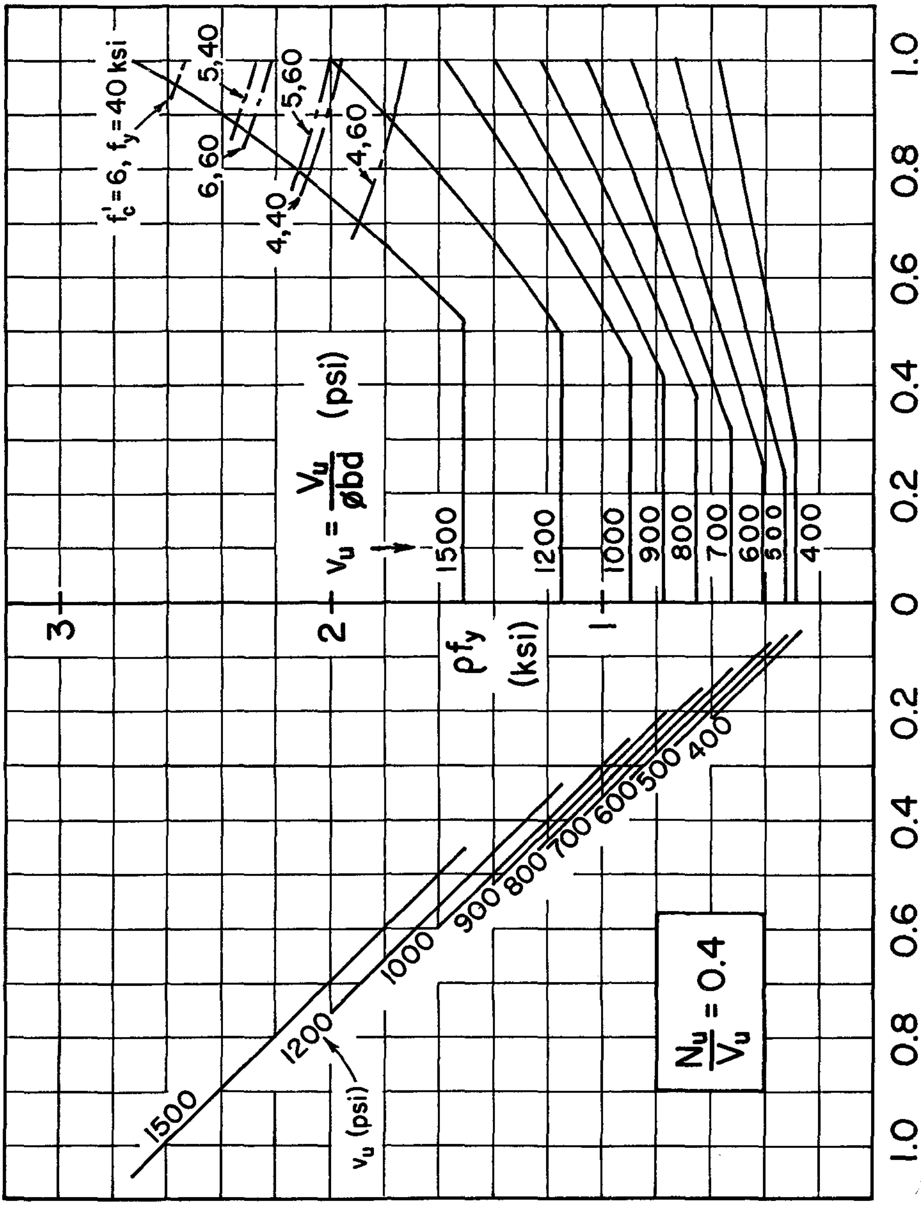

Fig. A2. Design chart for determining shear reinforcement in normal weight concrete corbel using modified shear-friction theory ($N_u/V_u = 0.4$).

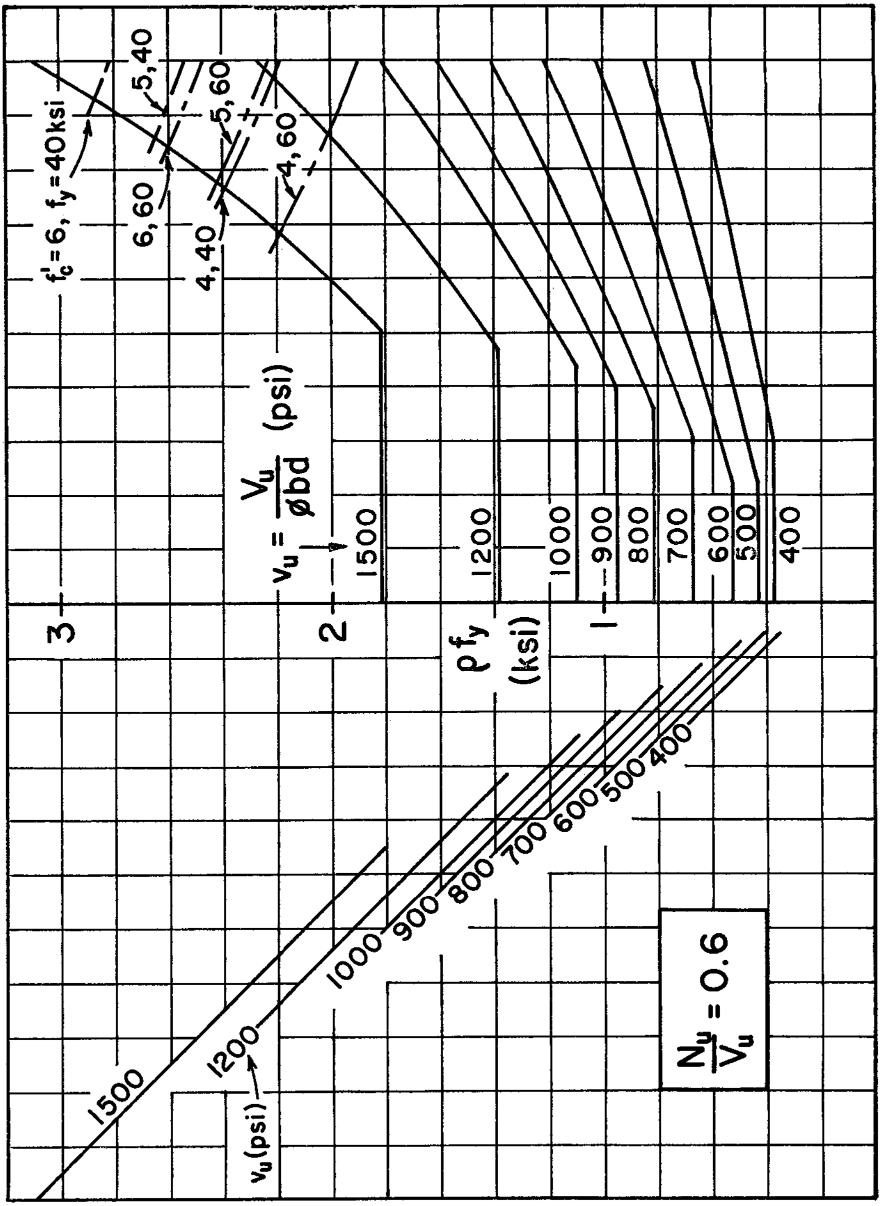

Fig. A3. Design chart for determining shear reinforcement in normal weight concrete corbel using modified shear-friction theory ($N_u/V_u = 0.6$).

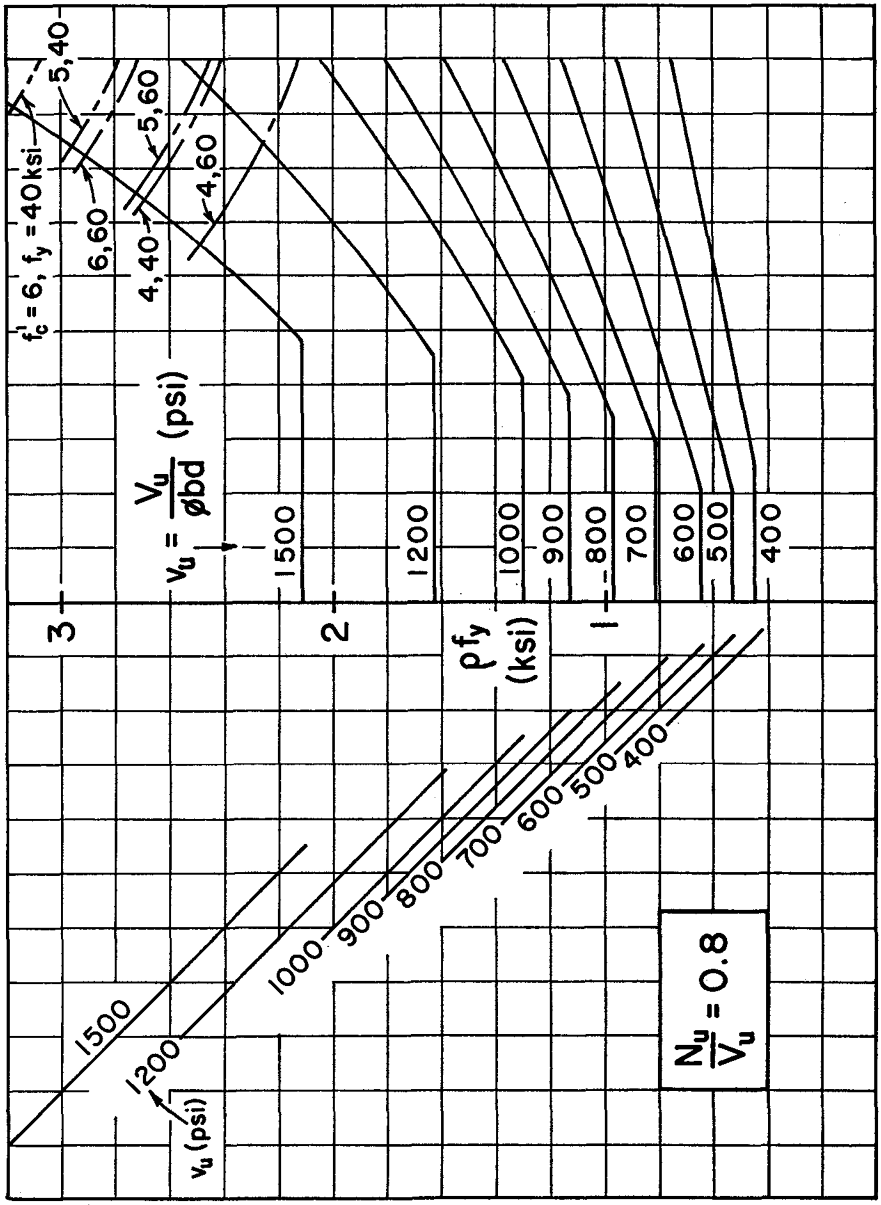

Fig. A4. Design chart for determining shear reinforcement in normal weight concrete corbel using modified shear-friction theory ($N_u/V_u = 0.8$).

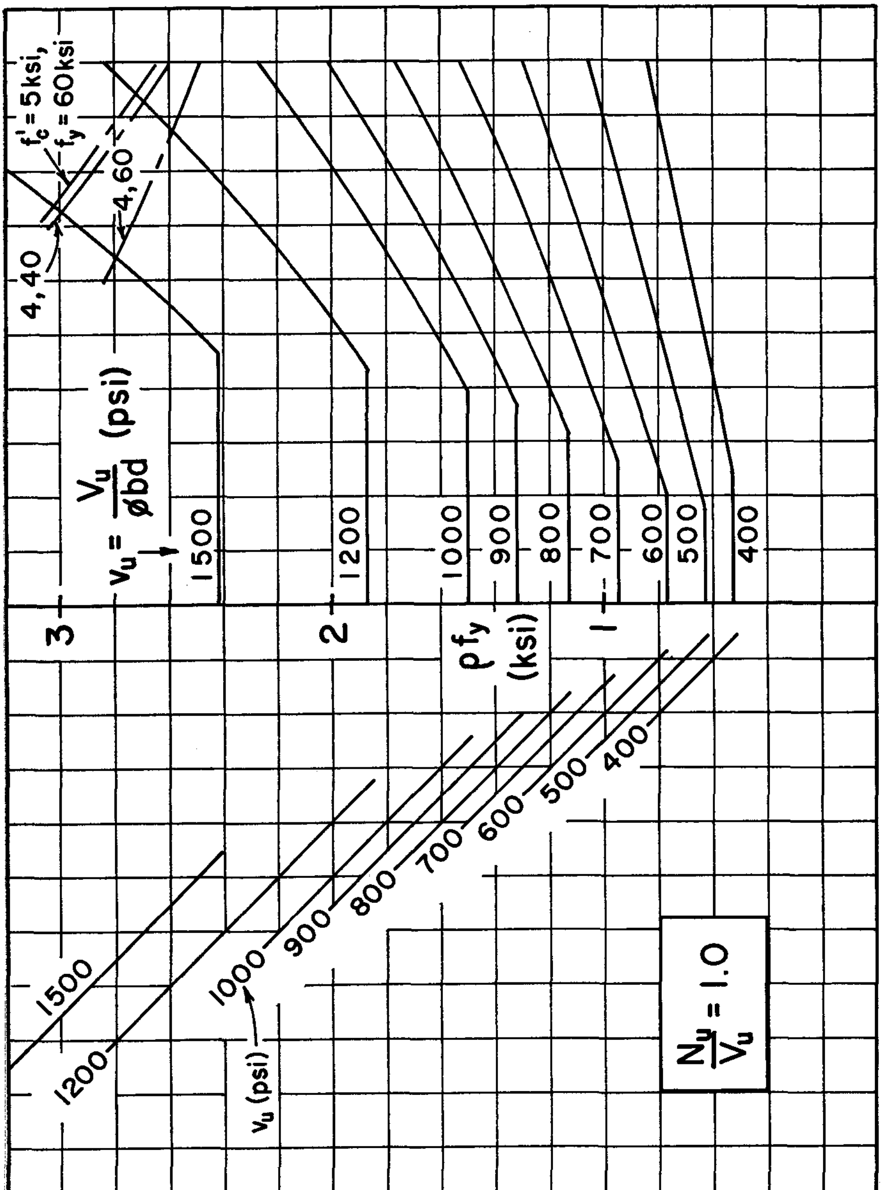

Fig. A5. Design chart for determining shear reinforcement in normal weight concrete corbel using modified shear-friction theory ($N_u/V_u = 1.0$).

APPENDIX B—HP-65 CALCULATOR PROGRAMS FOR REINFORCED CONCRETE CORBEL DESIGN

Table B1 contains detailed operating instructions for the Hewlett Packard HP-65 calculator programs shown in Tables B2, B3, and B4.

The program proceeds by the following steps:

1. A_f is calculated by an iterative process. Initially, the internal lever arm factor j is assumed to be equal to 0.95. The value of A_f required to resist the moment $[V_u a + N_u(h\text{-}d)]$ is calculated using:

$$A_f = \frac{V_u a + N_u(h - d)}{\phi j d f_y}$$

The depth of the flexural compression stress block, and hence the internal lever arm factor j corresponding to A_f is then calculated. This value of j is compared to the assumed value used to calculate A_f and if it is within 0.005, the calculated value of A_f is taken as correct. If not, the calculated value of j is used to recalculate A_f and the process is repeated automatically as many times as may be necessary to achieve the specified precision.

2. The value of $A_f(\max) = 0.75\,\rho_b bd$ is calculated using:

$$0.75\,\rho_b = \frac{0.75(0.85)\beta_1 f_c'}{f_y(1000)}\left(\frac{87}{87 + f_y}\right)$$

$$= \frac{\beta_1 f_c'}{18 f_y(87 + f_y)}$$

where f_c' is in psi and f_y is in ksi.

$A_f(\max)$ and A_f are compared. If A_f is less than $A_f(\max)$, A_f is displayed. If A_f is greater than $A_f(\max)$, 0.00 is displayed. (In this case, the corbel size must be increased.)

3(a) If the ACI 318-71 shear-friction provisions are to be used in design for shear, $(2/3)A_{vf}$ is calculated using:

$$(2/3)A_{vf} = (2/3)\frac{V_u}{\phi f_y}$$

3(b) If modified shear-friction is to be used in design for shear, A_{vf} is calculated using:

$$A_{vf} = \left(\frac{V_u}{0.8\phi} - Kbd\right)/f_y$$

$A_{vf}(\min)$ is also calculated using:

$$A_{vf}(\min) = 0.2bd/f_y$$

A_{vf} and $A_{vf}(\min)$ are compared and the greater of $(2/3)A_{vf}$ and $(2/3)A_{vf}(\min)$ is displayed as $(2/3)A_{vf}$.

4. A_t is calculated using:

$$A_t = \frac{N_u}{\phi f_y}$$

5. A_f and $(2/3)A_{vf}$ are compared. The larger quantity is added to A_t to yield A_s.

6. $A_s(\min)$ is calculated using:

$$A_s(\min) = 0.04\,bdf_c'/f_y$$

The values of A_s and $A_s(\min)$ are compared and the larger quantity is displayed as A_s.

7. A_h is calculated using:

$$A_h = (A_s - A_t)/2$$

Table B1. Operating instructions for HP-65 corbel reinforcement design programs.

STEP	INSTRUCTIONS	INPUT DATA	KEYS		OUTPUT DATA
1.	Insert card 1				
2.	Initialize		f f RTN	REG STK	
3.	Enter data	V_u(kips)	STO	1	V_u
		N_u(kips)	STO	2	N_u
		a (in.)	STO	3	a
		h (in.)	STO	4	h
		(h - d)(in.)	STO	5	(h - d)
		b (in.)	STO	6	b
		f_y (ksi)	STO	7	f_y
		f'_c (psi	STO	8	f'_c
4.	Commence calculations.		A		$(\approx\beta_1 f'_c)$
5.	Insert card 2 if ACI 318-71 shear friction is to be used. Insert card 3 if modified shear friction is to be used.				
6.	Calculate A_f (If A_f/bd>0.75 ρ_b, 0.00 shows. Select new corbel size.)		A		A_f(in.^{2})
7.	Calculate (2/3) A_{vf}		B		
7(a)	If shear friction is used,- Enter μ = 1.40 for normal weight conc. = 1.05 for all-lightweight conc. = 1.15 for sanded lightweight	μ	R/S		(2/3)A_{vf}(in.^{2})
7(b)	If modified shear friction is used,- Enter K = 0.50 for normal weight conc. = 0.25 for all-lightweight conc. = 0.31 for sanded lightweight	K	R/S		(2/3)A_{vf}(in.^{2})
8.	Calculate A_t		C		A_t(in.^{2})
9.	Calculate A_s		D		A_s(in.^{2})
10.	Calculate A_h		E		A_h(in.^{2})

Table B2. HP-65 calculator program for reinforced concrete corbel (Card No. 1).

KEY ENTRY	CODE SHOWN	COMMENTS	KEY ENTRY	CODE SHOWN	COMMENTS
LBL	23	Calculate A_f	f^{-1}	32	
A	11		INT	83	j_1 in x, j in y
.	83		STO	33	
9	09		-	51	
5	05	0.95	8	08	f'_c only in R8
STO	33		g x≷y	35 07	j in x, j_1 in y
+	61		STO	33	
8	08	Add 0.95 to R8	+	61	
LBL	23		8	08	$f'_c + j$ in R8
1	01		-	51	$j_1 - j$
RCL 1	34 01	V_u	g	35	
RCL 3	34 03	a	ABS	06	$\lvert j_1 - j \rvert$
x	71	$V_u a$	.	83	
RCL 2	34 02	N_u	0	00	
RCL 5	34 05	(h - d)	0	00	
x	71	$N_u(h - d)$	5	05	.005
+	61	$[V_u a + N_u(h - d)]$	g x≤y	35 22	If $.005 \leq \lvert j_1 - j \rvert$
.	83		GTO	22	go to 1
9	09	0.9	1	01	
÷	81	$[\quad]/0.9$	g R↓	35 08	$\lvert j_1 - j \rvert$ in x
RCL 7	34 07	f_y	g R↓	35 08	A_f in x
÷	81	$[\quad]/(0.9 f_y)$	RCL 8	34 08	$f'_c + j$
RCL 4	34 04	h	1	01	
RCL 5	34 05	(h - d)	0	00	
-	51	d	0	00	
÷	81	$[\quad]/(0.9 f_y d)$	0	00	1000
RCL 8	34 08	$f'_c + j$	÷	81	$\simeq f'_c/1000$
f^{-1}	32		4	04	4
INT	83	j	-	51	$(f'_c/1000 - 4)$
÷	81	A_f	0	00	0
ENT ↑	41		g x>y	35 24	If $0 > (f'_c/1000 - 4)$
ENT ↑	41	A_f in x, y & z	GTO	22	go to 4
RCL 7	34 07	f_y	4	04	
x	71	$A_f f_y$	g x≷y	35 07	$(f'_c/1000 - 4)$ in x
RCL 8	34 08	$(f'_c + j) \simeq f'_c$	LBL	23	
÷	81	$A_f f_y/f'_c$ (k/psi)	4	04	
5	05		CHS	42	$-(f'_c - 4)$ or 0
8	08		.	83	
8	08	588 = (1000/1.7)	0	00	
x	71	$A_f f_y/(1.7 f'_c)$	5	05	0.05
RCL 6	34 06	b	x	71	$-.05(f'_c/1000 - 4)$ or 0
÷	81	$A_f f_y/(1.7 f'_c b)$	.	83	
RCL 4	34 04	h	8	08	
RCL 5	34 05	(h - d)	5	05	0.85
-	51	d	+	61	$\beta_1 = .05(f'_c/1000 - 4)$
÷	81	$[A_f f_y/(1.7 f'_c bd)]$	g x≷y	35 07	but $\nless 0.85$
1	01	1	g R↓	35 08	β_1 in x, A_f in y
g x≷y	35 07		RCL 8	34 08	$f'_c + j$
-	51	$j = 1 - [\quad]$	x	71	$\beta_1 f'_c$
RCL 8	34 08	$f'_c + j_1$	RTN	24	Stop

REGISTERS	
R_1	V_u (kip)
R_2	N_u (kip)
R_3	a (in.)
R_4	h (in.)
R_5	(h - d) (in.)
R_6	b (in.)
R_7	f_y (ksi)
R_8	f'_c (psi) +j
R_9	

LABELS	
A	A_f
B	
C	
D	
E	
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

FLAGS	
1	
2	

Table B3. HP-65 calculator program for reinforced concrete corbel (Card No. 2).

KEY ENTRY	CODE SHOWN	COMMENTS
LBL	23	Calculate A_f
A	11	
1	01	
8	08	18
÷	81	$\beta_1 f'_c/18$
RCL 7	34 07	f_y
÷	81	$\beta_1 f'_c/18f_y$
RCL 7	34 07	f_y
8	08	
7	07	87
+	61	87 + f_y
÷	81	$\beta_1 f'_c/[18f_y(87+f_y)]$
RCL 4	34 04	h
RCL 5	34 05	(h - d)
-	51	d
RCL 6	34 06	b
x	71	bd
STO 6	33 06	bd in R6
x	71	A_f(max)
g x>y	35 24	If A_f(max)>A_f
g R↓	35 08	
RTN	24	Stop, read A_f
0	00	0
RTN	24	Read 0.00 if A_f>A_f(max)
LBL	23	Calculate 2/3 A_{vf}
B	12	
STO 5	33 05	A_f in R5
RCL 1	34 01	V_u
.	83	
8	08	
5	05	0.85
÷	81	$V_u/0.85$
RCL 7	34 07	f_y
÷	81	$V_u/(0.85f_y)$
R/S	84	Stop and enter μ
÷	81	A_{vf}
2	02	2
x	71	$2A_{vf}$
3	03	3
÷	81	$(2/3)A_{vf}$
RTN	24	Stop, read $(2/3)A_{vf}$
LBL	23	Calculate A_t
C	13	
RCL 5	34 05	A_f
g x>y	35 24	If $A_f>(2/3)A_{vf}$
GTO	22	go to 2
2	02	
g R↓	35 08	$(2/3)A_{vf}$
LBL	23	
2	02	Greater of A_f & $2/3A_{vf}$

KEY ENTRY	CODE SHOWN	COMMENTS
RCL 2	34 02	N_u
.	83	
8	08	
5	05	0.85
÷	81	$N_u/0.85$
RCL 7	34 07	f_y
÷	81	A_t
STO 5	33 05	A_t in R5
RTN	24	Stop, read A_t
LBL	23	Calculate A_s
D	14	
+	61	A_s
RCL 6	34 06	bd
RCL 7	34 07	f_y (ksi)
÷	81	bd/f_y
2	02	
5	05	
EEX	43	
3	03	25000
÷	81	$0.04bd/(1000f_y)$
RCL 8	34 08	f'_c (psi)
x	71	A_s(min)
g x>y	35 24	If A_s(min)>A_s
GTO	22	go to 7
7	07	
g R↓	35 08	A_s
LBL	23	
7	07	A_s(min)
RTN	24	Read greater of A_s&A_s(min)
LBL	23	Calculate A_h
E	15	
RCL 5	34 05	A_t
-	51	$A_s - A_t$
2	02	2
÷	81	A_h
RTN	24	Stop, read A_h
g NOP	35 01	
g NOP	35 01	
g NOP	35 01	
g NOP	35 01	
g NOP	35 01	
g NOP	35 01	
g NOP	35 01	
g NOP	35 01	
g NOP	35 01	
g NOP	35 01	
g NOP	35 01	
g NOP	35 01	
g NOP	35 01	
g NOP	35 01	

REGISTERS	
R_1	V_u (kip)
R_2	N_u (kip)
R_3	a (in.)
R_4	h (in.)
R_5	(h - d) A_f A_t
R_6	b (in.) bd
R_7	f_y (ksi)
R_8	f'_c (psi) +j
R_9	

LABELS	
A	A_f
B	$(2/3)A_{vf}$
C	A_t
D	A_s
E	A_h
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

FLAGS	
1	
2	

Table B4. HP-65 calculator program for reinforced concrete corbel (Card No. 3).

KEY ENTRY	CODE SHOWN	COMMENTS	KEY ENTRY	CODE SHOWN	COMMENTS
LBL	23	Calculate A_f	6	06	
A	11		2	02	2
1	01		x	71	$2A_{vf}$ or $2A_{vf}(min)$
8	08	18	3	03	3
÷	81	$\beta_1 f'_c/18$	÷	81	$(2/3)A_{vf}$ or $(2/3)A_{vf}(min)$
RCL 7	34 07	f_y	RTN	24	Read greater of above
÷	81	$\beta_1 f'_c/18f_y$	LBL	23	Calculate A_t
RCL 7	34 07	f_y	C	13	
8	08		RCL 5	34 05	A_f
7	07	87	g x>y	35 24	If $A_f>(2/3)A_{vf}$
+	61	$87 + f_y$	GTO	22	go to 2
÷	81	$\beta_1 f'_c/[18f_y(87+f_y)]$	2	02	
RCL 4	34 04	h	g R↓	35 08	$(2/3)A_{vf}$
RCL 5	34 05	(h - d)	LBL	23	
−	51	d	2	02	Greater of A_f & $(2/3)A_{vf}$
RCL 6	34 06	b	RCL 2	34 02	N_u
x	71	bd	.	83	
STO 6	33 06	bd in R6	8	08	
x	71	$A_f(max)$	5	05	0.85
g x>y	35 24	If $A_f(max)>A_f$	÷	81	$N_u/0.85$
g R↓	35 08		RCL 7	34 07	f_y
RTN	24	Stop, read A_f	÷	81	A_t
0	00	0	STO 7	33 07	A_t in R7
RTN	24	Read 0.00 if $A_f>A_f(max)$	RTN	24	Stop, read A_t
LBL	23	Calculate $(2/3)A_{vf}$	LBL	23	Calculate A_s
B	12		D	14	
STO 5	33 05	A_f in R5	+	61	A_s
RCL 1	34 01	V_u	RCL 6	34 06	$0.2bd/f_y$
.	83		5	05	
6	06		EEX	43	
8	08	0.68	3	03	5000
÷	81	$V_u/0.68$	÷	81	$0.04bd/(1000f_y)$
RCL 6	34 06	bd	RCL 8	34 08	f'_c
R/S	84	Stop to enter K	x	71	$A_s(min)$
x	71	Kbd	g x>y	35 24	If $A_s(min)>A_s$
−	51	$(V_u/0.68 - Kbd)$	GTO	22	go to 7
RCL 7	34 07	f_y	7	07	
÷	81	A_{vf}	g R↓	35 08	A_s
RCL 6	34 06	bd	LBL	23	
.	83		7	07	$A_s(min)$
2	02	0.2	RTN	24	Read greater of A_s & $A_s(min)$
x	71	0.2bd	LBL	23	Calculate A_h
RCL 7	34 07	f_y	E	15	
÷	81	$A_{vf}(min)$	RCL 7	34 07	A_t
STO 6	33 06	$0.2bd/f_y$ in R6	−	51	$A_s - A_t$
g x>y	35 24	If $A_{vf}(min)>A_{vf}$	2	02	2
GTO	22	go to 6	÷	81	A_h
6	06		RTN	24	Stop, read A_h
g R↓	35 08	A_{vf}	g NOP	35 01	
LBL	23		g NOP	35 01	

REGISTERS	
R_1	V_u (kip)
R_2	N_u (kip)
R_3	a (in.)
R_4	h (in.)
R_5	(h - d) A_f
R_6	b (in.) bd $0.2bd/f_y$
R_7	f_y (ksi) A_t
R_8	f'_c (psi) ±j
R_9	

LABELS	
A	A_f
B	$(2/3)A_{vf}$
C	A_t
D	A_s
E	A_h
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

FLAGS	
1	
2	

APPENDIX C—NOTATION

a = shear span, distance between vertical load and face of column, in.

A_f = area of reinforcement required to resist flexure, sq in.

A_h = total area of shear reinforcement parallel to main tension reinforcement, sq in.

A_s = total area of main tension reinforcement, sq in.

A_t = area of reinforcement required to resist horizontal force N_u, sq in.

A_{vf} = area of shear transfer reinforcement required to resist shear V_u, sq in.

b = width of corbel, in.

d = distance from extreme compression fiber to centroid of tension reinforcement, measured at column-corbel interface, in.

f_b = bearing stress, psi

f_c' = specified compressive strength of concrete (measured on 6 x 12-in. cylinders, psi)

f_y = specified yield strength of reinforcement, ksi

h = total depth of corbel, measured at corbel-column interface, in.

K = coefficient in modified shear-friction design equation

M_u = design load moment acting on corbel-column interface, in.-kips

N_u = design tensile force on corbel, acting simultaneously with V_u, kips

v_u = nominal design shear stress, psi $= \dfrac{V_u}{\phi bd}$

V_u = design shear force acting on corbel, kips

w = width of bearing plate, in.

β_1 = ratio of depth of equivalent rectangular stress block to total depth of flexural compression zone (defined in Section 10.2.7 of ACI 318-71)

μ = coefficient of friction used in shear-friction calculations

$\rho = \dfrac{A_s}{bd}$

ρ_b = flexural reinforcement ratio corresponding to balanced flexural conditions

$\rho_f = \dfrac{A_f}{bd}$

$\rho_h = \dfrac{A_h}{bd}$

$\rho_v = \dfrac{A_{vf}}{bd}$

ϕ = capacity reduction factor

Discussion of this paper is invited.
Please forward your comments to
PCI Headquarters by October 1, 1976.