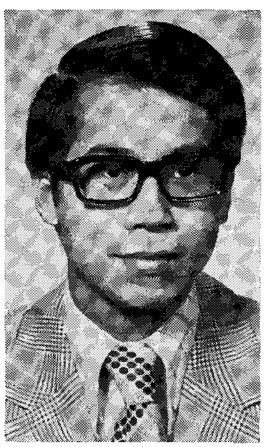
Shear transfer in lightweight reinforced concrete

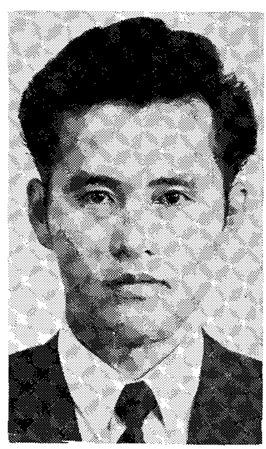
Alan H. Mattock
Professor of Civil Engineering and
Head, Structures and Mechanics Division
University of Washington
Seattle, Washington

W. K. Li*
Structural Engineer
Surveyor, Nenniger and Chenevert, Inc.
Consulting Engineers
Montreal, Quebec

T. C. Wang*
Structural Engineer
Kramer, Chin and Mayo, Inc.
Consulting Engineers
Seattle, Washington


Alan H. Mattock


W. K. Li


T. C. Wang

*Former Graduate Student, Department of Civil Engineering,
University of Washington, Seattle, Washington.

Lightweight concrete is being used increasingly in precast concrete construction. This has resulted in a need for connection design data when lightweight concrete is used. Neither the ACI Building Code, ACI 318-71,[1] nor the *PCI Design Handbook*[2] provides any guidance in this respect.

The shear-friction provisions contained in Section 11.15 of ACI 318-71, which are used extensively in the connection design procedures developed in the *PCI Design Handbook*, have only been validated experimentally for the case of shear transfer in normal weight concrete.

The study reported here was directed toward developing shear transfer design recommendations for use in the design of connections in precast structures made of lightweight concrete.

Synopsis

A study is reported of the single direction shear transfer strength of lightweight aggregate concrete.

Push-off tests were carried out on specimens made from sanded lightweight concrete, two types of all-lightweight concrete, and sand and gravel concrete.

Both initially uncracked specimens and specimens cracked in the shear plane before being subjected to shear, were tested.

It was found that the shear transfer strength of lightweight concrete is less than that of sand and gravel concrete having the same compressive strength.

The shear-friction provisions of Section 11.15 of ACI 318-71 may be used in the design of connections in lightweight concrete providing the value of the coefficients of friction μ, contained in Section 11.15.4, are multiplied by the following factors:

(a) For all-lightweight concrete having a unit weight not less than 92 lb per cu ft, multiply μ by 0.75.

(b) For sanded lightweight concrete having a unit weight not less than 105 lb per cu ft, multiply μ by 0.85.

Experimental Study

The experimental program reported here was designed to study the influence on single direction shear transfer strength and behavior, of the type of aggregate used in making the concrete. The primary variable in the tests was the type of aggregate, four types being used:

1. Naturally occurring gravel and sand.
2. A predominantly coated rounded lightweight aggregate.
3. A predominantly crushed angular lightweight aggregate.
4. A "sanded lightweight" aggregate, in which most of the lightweight fine particles were replaced with normal weight sand.

The test specimens were of the "push-off" type shown in Fig. 1, with a shear plane of 50 sq. in. area. When loaded as indicated by the arrows, shear without moment is produced in the shear plane. The reinforcement crossing the shear plane was in the form of

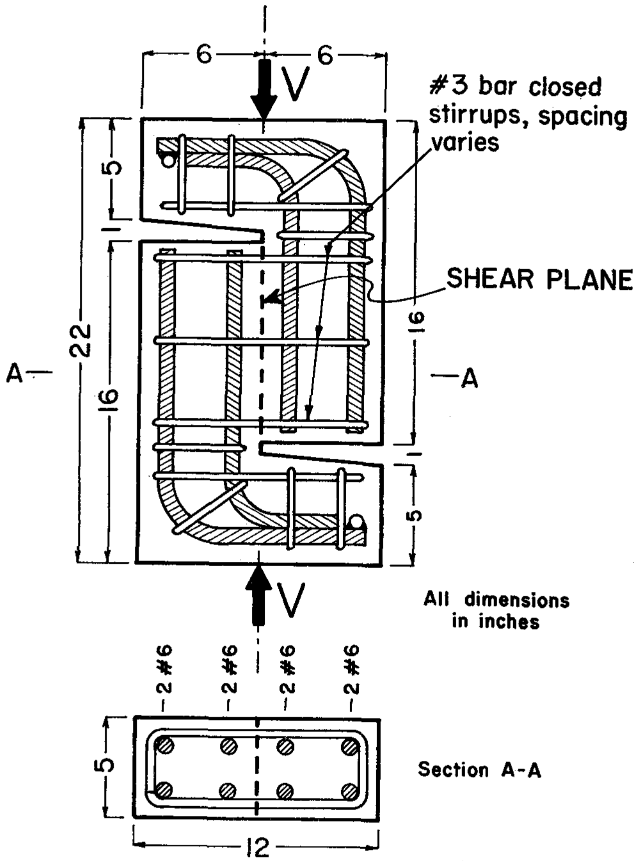


Fig. 1. Push-off specimen.

welded closed stirrups. This was to ensure the effective anchorage of the reinforcement on both sides of the shear plane. The specimens were cast on their sides, so that at the time of casting the shear plane was horizontal.

Ten series of push-off specimens were tested, as indicated in Table 1. The variables between test series were the type of aggregate, the concrete strength, and whether or not a crack existed in the shear plane before the shear transfer test. The variable within each series was the reinforcement parameter, ρf_y. The number of stirrups provided in the specimens within each series is set out in Table 2.

Materials and fabrication

When planning the project, the advice of the Expanded Shale, Clay and Slate Institute was sought as to the types of aggregate and concrete mix designs that should be used in order that the results of the study would have the widest practical applicability.

Current production of rotary kiln expanded lightweight aggregate is predominantly of the coated surface type in the coarse fraction and a blend of crushed and coated surface aggregate in the fine fraction.

For the sake of brevity, lightweight aggregate having predominantly coated

Table 1. Program of push-off tests.

Series	Aggregate Type	Design f'_c at test (psi)	Initial Condition
A	Coated lightweight aggregate and sand	4000	Uncracked
B	Coated lightweight aggregate and sand	4000	Cracked
C	Coated lightweight aggregate and sand	2500	Cracked
D	Coated lightweight aggregate and sand	6000	Cracked
E	Coated lightweight aggregate only	4000	Uncracked
F	Coated lightweight aggregate only	4000	Cracked
G	Crushed lightweight aggregate only	4000	Uncracked
H	Crushed lightweight aggregate only	4000	Cracked
M	Natural gravel and sand	4000	Uncracked
N	Natural gravel and sand	4000	Cracked

Table 2. Specimen numbering and reinforcement details.

Specimen No.	X.0*	X.1	X.2	X.3	X.4	X.5	X.6
Number of #3 Stirrups	0	1	2	3	4	5	6
Reinforcement Area, sq in.	0	0.22	0.44	0.66	0.88	1.10	1.32

* This specimen appears only in Series A, E, G, and M.

"X" is the letter designating the particular series.

rounded aggregate particles, will in this report be referred to as a "coated aggregate." Similarly, aggregate in which the various particle sizes are predominantly obtained by crushing larger-sized particles is referred to as "crushed aggregate." Most structural lightweight aggregate concretes use mainly sand for the fine fraction of the aggregate. Such concretes are referred to as "sanded lightweight" concretes and have a dry density of about 110 lb per cu ft.

In view of the foregoing, it was decided that the results of the study would have the widest applicability if emphasis were placed on tests involving lightweight aggregate concrete made of a "coated aggregate" in "sanded lightweight" mixes.

The first four test series (Series A, B, C, and D), therefore involve this type of concrete. The second group of four test series (Series E, F, G, and H) involved "all-lightweight" concrete, Series E and F using a "coated aggregate" and Series G and H using a "crushed aggregate." The final two series (Series M and N) used a natural gravel concrete as a reference against which to compare the performance of the lightweight concretes.

The natural aggregate used was a glacial outwash gravel obtained from a

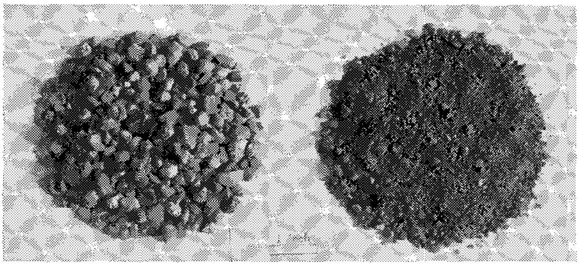


Fig. 2. Sample of coated lightweight aggregate.

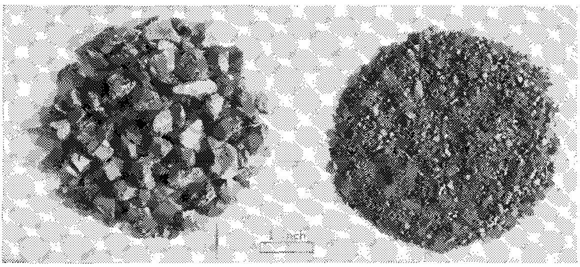


Fig. 3. Sample of crushed lightweight aggregate.

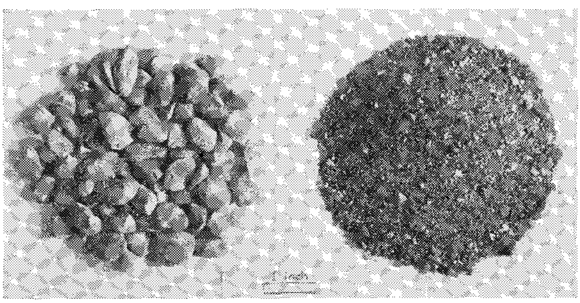


Fig. 4. Sample of natural sand and gravel.

local pit. The gradings of the coarse and fine fractions of each aggregate as determined by sieve analysis, are set out in Table 3. The appearance of the aggregates is seen in Figs. 2, 3, and 4.

Trial batches were made of the lightweight concretes, using proportions suggested by the producers of the aggregates. The mix proportions selected for the various lightweight aggregate concretes are given in Table 4. The mix proportions for the natural gravel concrete were arrived at through the testing of trial batches and they are shown in Table 5. The weights shown in Tables 4 and 5 are of aggregate as supplied, i.e., slightly damp. Exact moisture contents were not measured.

The proportions of the lightweight concrete mixes were chosen so that for a 3-in. slump the design strength would be attained at 28 days, after 7 days moist curing and a further 21 days curing in air. The proportions of the sand and gravel concrete were chosen so that for a 3-in. slump the design strength would be attained at 4 days, after 2 days moist curing and 2 days curing in air. Approximately 6 percent of entrained air was included in all mixes to conform to usual practice with lightweight concrete. Complete details of the procedures followed are reported elsewhere.[4]

Table 3. Grading of aggregates (cumulative percent retained).

Sieve Size \ Aggregate	Coated Lightweight	Crushed Lightweight	Natural Sand and Gravel
Coarse Aggregate			
3/4 in.	0	0	0
1/2 in.	0	19.4	53.8
3/8 in.	2.7	61.6	87.0
No. 4	92.5	96.4	99.9
Fine Aggregate			
No. 4	1.8	0.4	1.8
No. 16	43.1	43.7	26.3
No. 50	81.9	74.2	76.2
No. 100	93.8	86.9	96.9

Table 4. Mix proportions for lightweight concretes (lbs per cu yd)

Type of Concrete	Type I Portland Cement	Lightweight Aggregate		Natural Sand
		Coarse	Fine	
Sanded Lightweight (coated aggregate)				
Series A & B ($f'_c \simeq 4000$ psi)	500	740	227	1369
Series C ($f'_c \simeq 2500$ psi)	400	744	220	1426
Series D ($f'_c \simeq 6000$ psi)	640	784	220	1091
All-Lightweight				
Series E & F (coated aggregate, $f'_c \simeq 4000$ psi)	500	740	1049	--
Series G & H (crushed aggregate, $f'_c \simeq 4000$ psi)	517	825	1284	--

Note: In all mixes, water was provided to produce a 3-in. slump.

Table 5. Mix proportions for natural sand and gravel concrete (lbs per cu yd)

Concrete	Type III Portland Cement	Gravel	Sand
Series M & N ($f'_c \simeq 4000$ psi)	489	1798	1448

Note: - In all mixes, water was provided to produce a 3-in. slump.

Table 6. Data concerning test specimens.

Specimen No.	Stirrup Yield Point f_y, (ksi)	Reinforcement Parameter ρf_y (psi)	f'_c (1) (psi)	f_{ct} (2) (psi)	Concrete (4) Dry Density (lb./cu. ft)	Ultimate Shear Stress v_u (3), (psi)
Series A						
A0	--	0	4230	402	111	500
A1	47.7	210	3740	336	111	758
A2	53.6	472	4095	367	105	914
A3	53.2	702	3910	349	110	1020
A4	50.9	896	4100	352	108	1100
A5	50.9	1120	3960	351	108	1190
A6	51.8	1368	4250	397	110	1344
Series B						
B1	49.6	218	3740	336	111	450
B2	50.9	448	3360	318	107	652
B3	50.9	672	3910	349	110	840
B4	49.1	864	4100	352	108	940
B5	50.5	1111	3960	351	108	1000
B6	51.8	1368	4250	397	110	1154
Series C						
C1	49.6	218	2330	254	102	364
C2	53.6	472	2330	254	102	514
C3	50.9	672	2000	232	103	526
C4	52.3	921	2050	235	105	560
C5	53.6	1179	2330	269	106	640
C6	49.6	1309	2330	269	106	740
Series D						
D1	51.8	228	5995	376	108	370
D2	52.3	460	5995	376	108	668
D3	52.3	690	5710	379	107	772
D4	52.3	920	5710	379	107	1022
D5	52.3	1151	5600	398	109	1082
D6	51.8	1368	5600	398	109	1220
Series E						
E0	--	0	3960	365	92	560
E1	52.3	230	4150	350	97	780
E2	52.3	460	4030	355	94	872
E3	52.3	690	4065	375	96	960
E4	53.2	936	4040	405	96	1150
E5	50.5	1111	4115	365	98	1200
E6	52.3	1381	4050	365	94	1250

Notes: (1) Concrete compressive strength at time of test, measured on 6 x 12-in. cylinders.

(2) Concrete splitting tensile strength at time of test, measured on 6 x 12-in. cylinders.

(3) $v_u = \frac{V_u}{\phi A_{cr}}$, with $\phi = 1.0$.

(4) Air dried density at time of test.

Table 6. (cont.). Data concerning test specimens

Specimen No.	Stirrup Yield Point f_y, (ksi)	Reinforcement Parameter ρf_y (psi)	f'_c (1) (psi)	f_{ct} (2) (psi)	Concrete (4) Dry Density (lb./cu. ft)	Ultimate Shear Stress v_u (3), (psi)
Series F						
F1	53.2	234	4150	350	97	450
F2	52.3	460	4030	355	94	530
F2A	50.9	448	3970	355	94	620
F3	52.3	690	4065	375	96	734
F3A	51.4	678	3970	355	94	702
F4	50.9	896	4040	405	96	870
F5	51.8	1140	4115	365	98	920
F6	53.2	1404	4050	365	94	982
Series G						
G0	--	0	4030	420	98	530
G1	52.3	230	4145	395	98	820
G2	50.5	444	3880	378	96	846
G3	51.8	684	4100	371	96	1060
G4	53.2	936	4420	406	97	1150
G5	51.8	1140	4005	395	99	1140
G6	51.8	1368	4005	395	99	1190
Series H						
H1	49.8	219	4145	395	98	400
H2	51.8	456	3880	378	96	620
H3	51.8	684	4100	371	96	866
H4	51.8	912	4420	406	97	940
H5	50.5	1111	3950	395	99	990
H6	49.8	1315	4080	385	98	1042
Series M						
M0	--	0	3935	375	145	590
M1	50.9	224	4180	390	145	760
M2	52.7	464	3900	365	145	980
M3	52.3	690	3995	395	148	1110
M4	50.9	896	4150	400	145	1140
M5	52.7	1160	3935	350	146	1280
M6	52.7	1392	4120	395	145	1320
Series N						
N1	50.9	224	4180	390	145	460
N2	52.7	464	3900	365	145	780
N3	52.3	690	3995	395	148	960
N4	50.9	896	4150	400	145	1150
N5	50.9	1120	3935	350	146	1175
N6	50.0	1320	4120	395	145	1190

Notes:

(1) Concrete compressive strength at time of test, measured on 6 x 12-in. cylinders.

(2) Concrete splitting tensile strength at time of test, measured on 6 x 12-in. cylinders.

(3) $v_u = \dfrac{V_u}{\phi A_{cr}}$, with $\phi = 1.0$.

(4) Air dried density at time of test.

Table 7. Average concrete properties at time of test (by series).

Series	Compressive Strength, f'_c (psi)	Splitting Tensile Strength, f_{ct} (psi)	$\frac{f_{ct}}{f'_c}$	$\frac{f_{ct}}{\sqrt{f'_c}}$	Air Dry Density (lb./cu. ft.)
Sanded Lightweight Concretes					
Series A	4040	365	.090	5.74	109
Series B	3890	351	.090	5.63	109
Series C	2230	252	.113	5.34	104
Series D	5770	384	.067	5.06	108
All-Lightweight Concretes					
Series E	4060	370	.091	5.81	95
Series F	4050	365	.090	5.74	95
Series G	4085	395	.097	6.18	97
Series H	4095	390	.095	6.09	97
Sand and Gravel Concrete					
Series M	4020	380	.095	5.99	146
Series N	4045	385	.095	6.05	146

Type I portland cement was used for the lightweight concrete mixes and Type III portland cement was used for the sand and gravel concrete mixes.

The deformed bar reinforcement used conformed to ASTM Specification A615. The #3 bar used for the shear transfer reinforcement had a yield point of approximately 50 ksi. The #6 bar longitudinal reinforcement had a yield point of approximately 60 ksi. Reinforcement details and concrete properties are shown in Tables 6 and 7.

Testing arrangements and procedures

The push-off specimens were tested using a Baldwin hydraulic testing machine to load the specimen along the shear plane as indicated in Fig. 1. Typical arrangements for test are shown in Fig. 5. The specimen stood on the lower platen of the testing machine and was loaded through the spherically seated upper platen of the testing machine, a load cell and a set of parallel plates and rollers.

The rollers ensured that separation of the two halves of the push-off specimen was not restrained by the testing machine. Both slip along the shear plane and separation across it were measured continuously using linear differential transformers attached to reference points embedded in the specimen, as may be seen in Fig. 5.

Mast[3] pointed out the need to consider the case of a crack existing in the shear plane before shear acts. Therefore prior to test, some of the specimens were cracked along the shear plane by applying line loads to their front and rear faces.

These loads were applied through steel wedges with the specimen in a horizontal position. The dilation of the specimen across the shear plane was measured during the cracking operation, using dial gages mounted on a reference frame. The width of the crack produced was approximately 0.01 in..

The specimens were subjected to a continuously increasing load until failure occurred, with short pauses as necessary to mark any cracks which may have occurred. The average length of time taken for a test was about 15 minutes. The ultimate load was defined as the maximum load that could be carried by the specimen.

Specimen behavior

The general behavior of all the initially uncracked specimens was similar. No slip along the shear plane, nor separation across the shear plane occurred in these specimens until the formation of diagonal tension cracks in the region of the shear plane at shear stresses of from 400 to 700 psi.

These cracks were initially 2 or 3 in. long and inclined at from 20 to 45 deg to the shear plane. As the load increased, some of the cracks lengthened and additional cracks formed, so that at failure there were in general a larger number of diagonal tension cracks in the more heavily reinforced specimens than in the more lightly reinforced specimens.

At failure some of the cracks propagated parallel to the shear plane, linking with others, and extensive compression spalling occurred in the inclined concrete struts formed by the diagonal tension cracks.

In the initially uncracked specimens there was no slip along the shear plane in the true sense of the word. Relative motion of the two halves of the specimen occurred as a result of the rotation and compression of the inclined concrete struts as the reinforcement crossing the shear plane stretched.

The component of this relative motion parallel to the shear plane is referred to as slip when discussing behavior, for the sake of brevity. The component of the relative motion normal to the shear plane is referred to as separation. The slip and separation data have been reported in detail elsewhere.[4] Typical shear-slip curves for initially uncracked specimens are shown in Fig. 6.

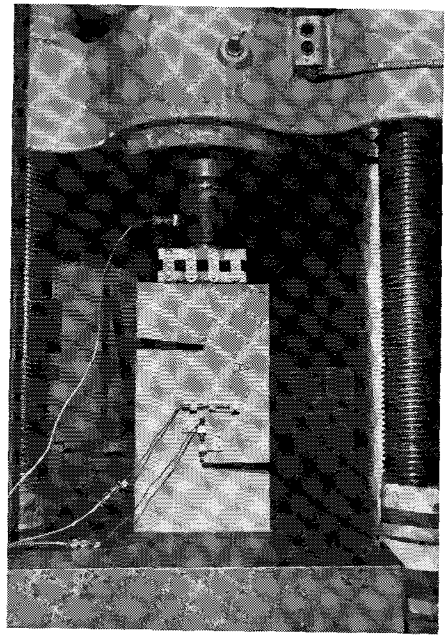


Fig. 5. Arrangements for test of push-off specimen.

The general behavior of all the initially cracked specimens was similar. Slip occurred along the preformed crack in the shear plane from the commencement of loading, and at a progressively increasing rate. In the case of the heavily reinforced sand and gravel concrete specimens, a few diagonal tension cracks occurred across the shear plane at high loads.

This behavior had been observed in previous tests of sand and gravel concrete push-off specimens.[5] However, no such diagonal tension cracks were observed in either the sanded-lightweight or the all-lightweight concrete specimens.

In all the initially cracked specimens, the slip increased at a rapid rate at failure and a small amount of compression spalling of the concrete occurred adjacent to the shear plane crack. Typical shear-slip curves for initially cracked specimens are shown in Fig. 7.

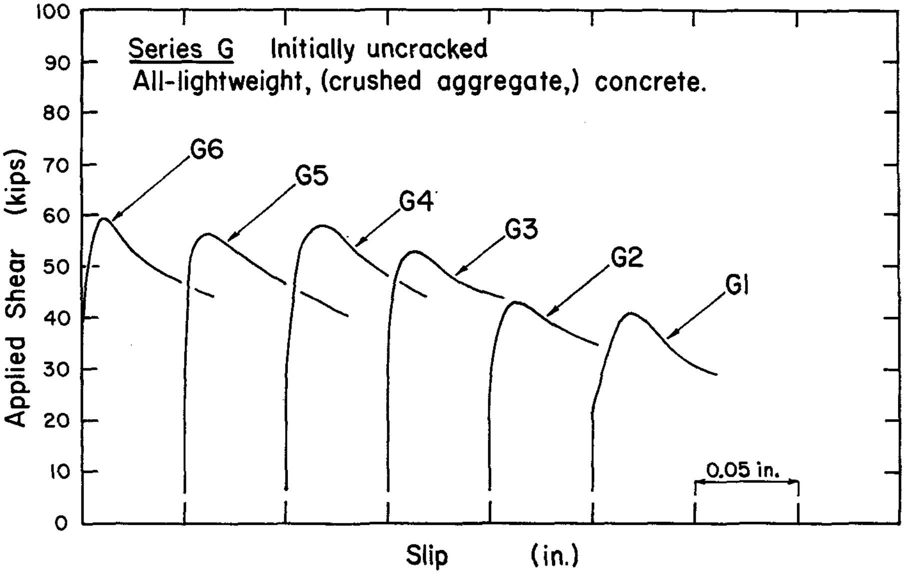


Fig. 6. Typical shear-slip curves for initially uncracked specimens (Series G).

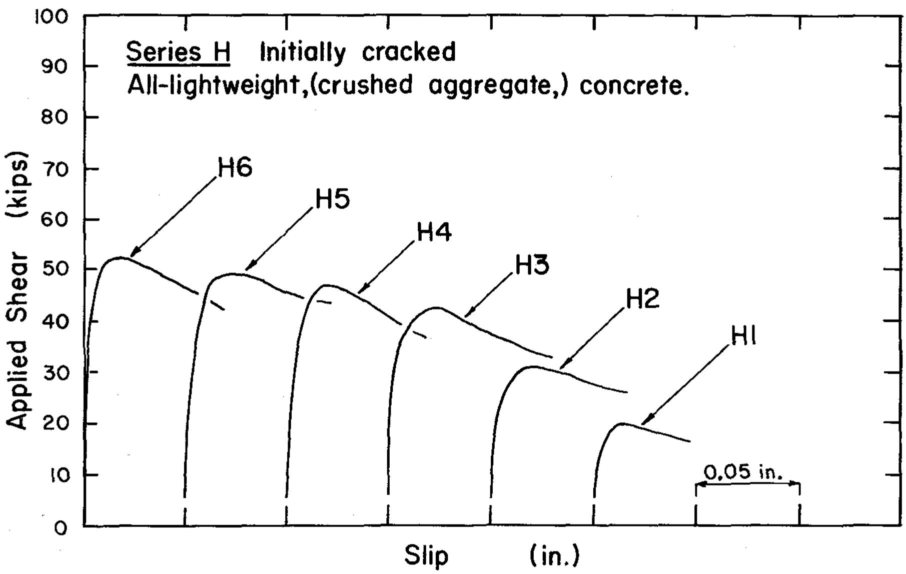


Fig. 7. Typical shear-slip curves for initially cracked specimens (Series H).

It can be seen from the shear-slip curves that the deformation behavior was relatively brittle in all cases. The maximum shear resistance was not maintained as the slip increased beyond the value at which maximum shear resistance was developed.

The initially uncracked specimens behaved in a more brittle fashion than the corresponding initially cracked spe-

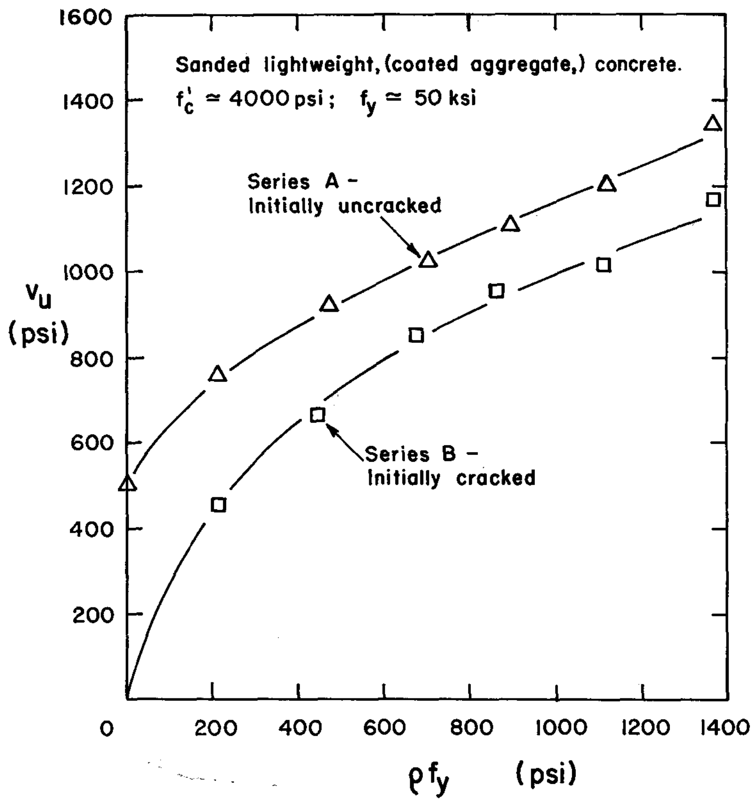


Fig. 8. Effect of a crack in the shear plane on shear transfer strength of sanded lightweight concrete.

cimens, in that the shear resistance after ultimate decreased more rapidly as the slip increased. However, because the ultimate strength of the initially uncracked specimens was greater than that of the corresponding initially cracked specimens, the residual strengths of both types of specimens were about the same for slips of 0.05 in. or more.

Deformation behavior was found to become more brittle as the compressive strength of the concrete increased; and also to be more brittle in the case of the all-lightweight concretes than in the cases of the sanded lightweight concrete and the sand and gravel concrete.

Ultimate shear transfer strength

The values of ultimate shear transfer strength obtained in the tests are shown in Table 6, together with other pertinent data concerning the test specimens. The ultimate shear transfer strength is expressed as a nominal ultimate shear stress:

$$v_u = \frac{\text{Ultimate shear}}{\text{Area of shear plane}}$$

The ultimate shear is defined as the maximum shear carried by the specimen during the test.

Effect of a crack in the shear plane—In Fig. 8 a comparison is made of the ultimate shear transfer strengths of the

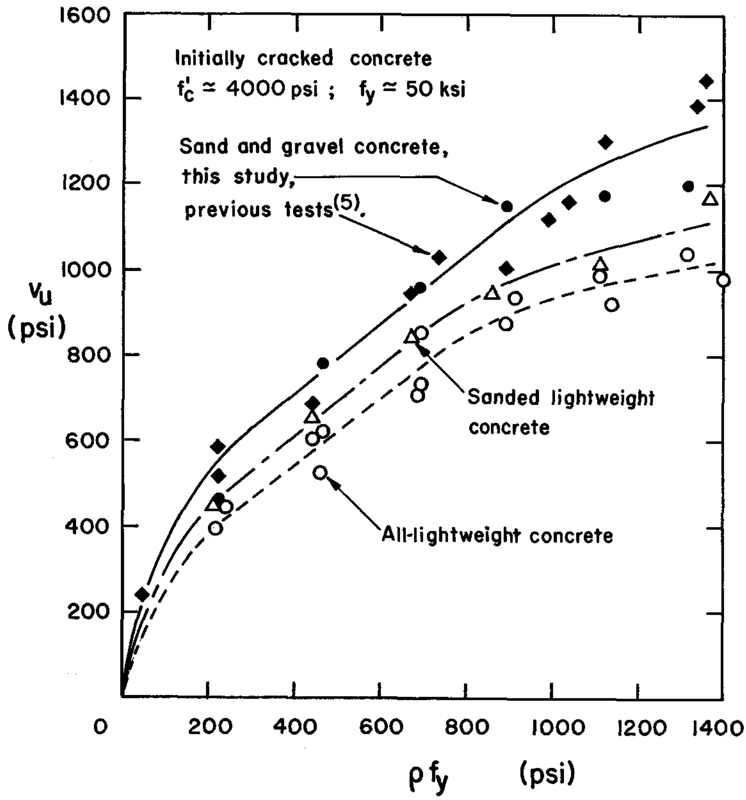


Fig. 9. Effect of aggregate type on shear transfer strength of initially cracked concrete having $f_c' \approx 4000$ psi.

sanded lightweight concrete specimens of Series A and B. The corresponding specimens of these two series were made as nearly identical as possible, except that the specimens of Series A were tested in an initially uncracked condition, while those of Series B were cracked along the shear plane before being tested.

It can be seen that for all values of the reinforcement parameter ρf_y considered, the existence of a crack in the shear plane reduces the shear transfer strength, for a given value of ρf_y, by an almost constant amount.

Similar behavior also occurred in the case of the all-lightweight concrete specimens. However, in the case of the sand and gravel concrete specimens, the difference in strength between the initially uncracked specimens and the initially cracked specimens decreased continually as ρf_y increased.

In previous studies[5] of shear transfer in sand and gravel concrete, similar behavior was observed. However in that case (for concrete with $f_c' \approx 4000$ psi), the strength of the initially cracked concrete became equal to that of the initially uncracked concrete when ρf_y was about 1350 psi.

It was postulated that under the clamping force provided by this large amount of reinforcement, the crack "locked up" and the concrete behaved as if it were initially uncracked. In such a case diagonal tension cracks formed across the shear plane and the failure had the characteristics of a shear transfer failure in initially uncracked concrete.

In the present tests diagonal tension cracks also occurred for this heavy degree of reinforcement, but the strength of the initially cracked concrete fell a little short of that of the initially uncracked concrete. It is posssible that the difference in behavior noted in the two studies is only apparent and was caused by experimental scatter in the strength data.

In the previous study[5] it was postulated that the upper limit to the shear transfer strength of initially cracked concrete of a particular compressive strength resulted from this "locking up" of the crack and subsequent behavior as if initially uncracked. However, this does not appear to be the case for lightweight concrete in which no evidence of such "locking up" behavior was observed.

No diagonal tension cracks occurred in any of the initially cracked lightweight concrete and the shear transfer strength of the initially cracked lightweight concrete did not approach that of the initially uncracked lightweight concrete at high values of ρf_y. The ceiling value of shear transfer strength for lightweight concrete must therefore result from some other aspect of shear transfer behavior as yet unidentified.

Effect of aggregate type on shear transfer strength—In Fig. 9 a comparison is made of the shear transfer strength of initially cracked concrete of three kinds, all having compressive strengths of close to 4000 psi.

It can be seen that the shear transfer strength of the sand and gravel concrete is consistently greater than that of the lightweight concretes, for the same amount of reinforcement, and that the sanded lightweight concrete has a greater shear transfer strength than all-lightweight concrete.

The differences in shear transfer strength do not correlate with the differences in concrete splitting tensile strength. The shear transfer strength of lightweight concrete in design should not therefore be related to the splitting tensile strength of the concrete.

The difference in the shear stresses which can be carried by lightweight and normal weight concretes of the same compressive strength is usually attributed to differences in the tensile strength of the concretes. However this cannot be the case in this instance, since the average splitting tensile strengths of the 4000 psi concretes were $6.02\sqrt{f'_c}$ for the sanded and gravel concrete, $5.69\sqrt{f'_c}$ for the sanded lightweight concrete, $5.78\sqrt{f'_c}$ for the coated all-lightweight aggregate concrete and $6.14\sqrt{f'_c}$ for the crushed lightweight aggregate concrete. (see Table 7).

The difference in shear transfer strength between lightweight concretes and sand and gravel concretes of the same compressive strength is probably due to differences in roughness of the crack faces in contact. Some of the push-off specimens were cut open after test and it could be seen that the crack faces in the sand and gravel concrete were rougher than those in the lightweight concrete.

In sand and gravel concrete, the bond strength between the mortar and the aggregate particles is smaller than the tensile strength of the aggregate particles. Cracks therefore generally propagate around the aggregate particles, producing a rough surface.

In lightweight concrete the bond strength between the mortar and the aggregate particles is apparently greater than the tensile strength of the aggregate particles. In this case therefore, cracks propagate through the aggregate

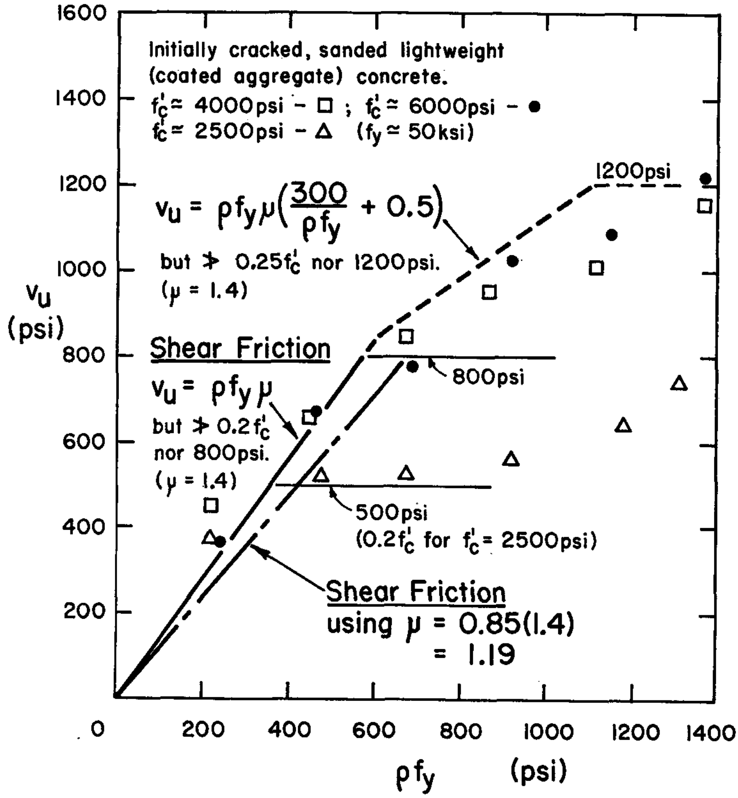


Fig. 10. Comparison of shear transfer strength of initially cracked, sanded lightweight concrete, with strength predicted by current design equations.

and a smoother crack face results. This difference in cracking behavior was readily apparent in both the push-off specimens that were cut open after test, and in the appearance of the two halves of cylinders subject to the splitting tensile strength test.

The difference in shear transfer between the sanded lightweight concretes and the all-lightweight concretes of similar compressive strength was probably due to similar causes. In the sanded lightweight concrete the cracks propagated round the sand particles, producing a rougher surface than in the all-lightweight concrete.

The roughness produced by this effect is only minor. However it is considered that it could influence shear transfer behavior, since the slips and separations at ultimate measured in the tests are only of the order of the size of the sand grains.

It can also be seen in Fig. 9 that while the rate of increase of shear transfer strength with increase in ρf_y is about the same for all three concretes for moderate values of ρf_y, the shear transfer strength of the lightweight concretes commences to increase at a lesser rate at a lower value of ρf_y than does the sand and gravel concrete.

It apears that for a given strength, the absolute maximum shear transfer strength obtainable is less in the case of lightweight concretes than in the case of sand and gravel concrete. This may be due to the resistance to abrasion of the lightweight concrete crack faces being less than that of the sand and gravel concrete crack faces.

"Skid" marks could be seen on the crack faces of the more heavily reinforced lightweight concrete push-off specimens cut open after test. Similar damage to the crack faces was not observed in the case of sand and gravel specimens.

This may be the reason why the "locking up effect noted in heavily reinforced sand and gravel concrete push-off specimens did not occur in heavily reinforced lightweight concrete specimens. Failure of the lightweight concrete crack faces through extensive shearing off of local roughness apparently occurs before the crack faces can become locked together.

Calculation of shear transfer strength in design—Currently, the calculation of shear transfer strength in design is usually based on the provisions of Section 11.15—Shear-friction, of ACI 318-71.[1] According to these provisions, shear transfer strength is given by:

$$v_u = \phi A_{vf} f_y \mu \tag{1}$$

which may also be expressed as an ultimate shear stress:

$$v_u = \frac{V_u}{\phi A_{cr}} = \mu \rho f_y \tag{2}$$

Section 11.15 also specifies that v_u shall not exceed $0.2f'_c$ nor 800 psi.

The value of the coefficient of friction μ is to be taken as 1.4 for concrete cast monolithically, 1.0 for concrete placed against hardened concrete, and 0.7 for concrete placed against as-rolled structural steel.

In Section 6.1.9 of the *PCI Design Handbook*,[2] it is proposed that for values of ρf_y greater than 600 psi, the shear-friction equations can continue to be used provided that the coefficient μ is multiplied by

$$\left(\frac{300}{\rho f_y} + 0.5\right)$$

That is, Eq. (2) then becomes:

$$v_u = \rho f_y \, \mu \left(\frac{300}{\rho f_y} + 0.5\right) \tag{3}$$

Eq. (3) will be referred to as the PCI equation. Initially, no upper limit was specified for v_u calculated using the PCI equation, but subsequently an upper limit of the lesser of $0.25f'_c$ or 1200 psi was proposed for v_u.

In Figs. 10 and 11 comparisons are made between measured shear transfer strengths in initially cracked lightweight concretes and the strengths predicted by Eq. (2) and (3).

(Note that in making these comparisons, the value of the capacity reduction factor ϕ is taken as 1.0, since the material strengths and specimen dimensions are accurately known.)

It is seen that the PCI equation is unconservative for both sanded lightweight and all-lightweight concretes and should therefore not be used in the design of connections between members of lightweight concrete.

In Fig. 10 it can be seen that the shear-friction equation, using $\mu = 1.4$, becomes unconservative for sanded lightweight concrete when ρf_y exceeds about 450 psi. It is therefore proposed that the multiplying factor for sanded lightweight concrete, 0.85, contained in Section 11.3.2 of ACI 318-71 should also be applied to the coefficients of friction μ contained in Section 11.15.4.

For a crack in monolithic sanded lightweight concrete, μ then becomes 1.19. A line corresponding to use of this value of μ in shear-friction calculations has been drawn in Fig. 10 and it is seen to be reasonably conservative for sanded lightweight concrete.

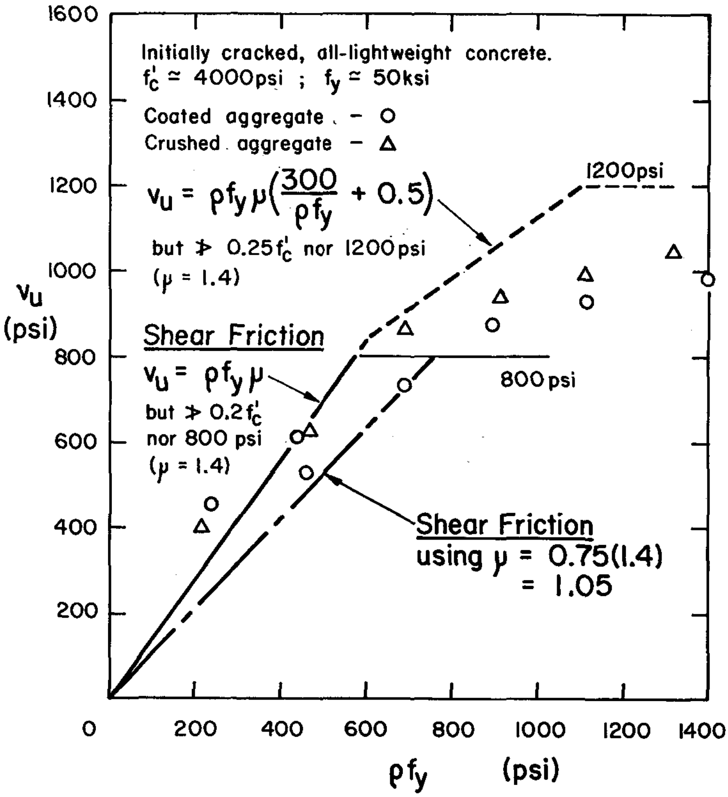


Fig. 11. Comparison of shear transfer strength of initially cracked, all-lightweight concrete, with strength predicted by current design equations.

It can be seen in Fig. 11 that the shear-friction equation, using $\mu = 1.4$ for a crack in monolithic concrete (as specified in Section 11.15.4 of ACI 318-71), can be considerably unconservative for all-lightweight concrete.

It is therefore proposed that the multiplying factor for all-lightweight concrete, 0.75, contained in Section 11.3.2 of ACI 318-71 should also be applied to the coefficients in friction μ contained in Section 11.15.4. For a crack in monolithic all-lightweight concrete, μ then becomes 1.05.

A line corresponding to use of this value of μ in shear-friction calculations has been drawn on Fig. 11 and it is seen to be reasonably conservative for all-lightweight concrete.

It can be seen from Figs. 10 and 11 that the limiting value for v_u contained in Section 11.15.3 of ACI 318-71 (0.2 f'_c but not more than 800 psi) is also appropriate for both sanded lightweight and all-lightweight concretes, provided that the reduced values of μ proposed above are used.

Subsequent to the publication of ACI 318-71 and the *PCI Design Handbook*, Mattock proposed[6] the following equation for the shear transfer strength of sand and gravel concrete:

$$v_u = 0.8\,\rho f_y + 400 \text{ psi} \qquad (4)$$

but not more than $0.3\,f_c'$.

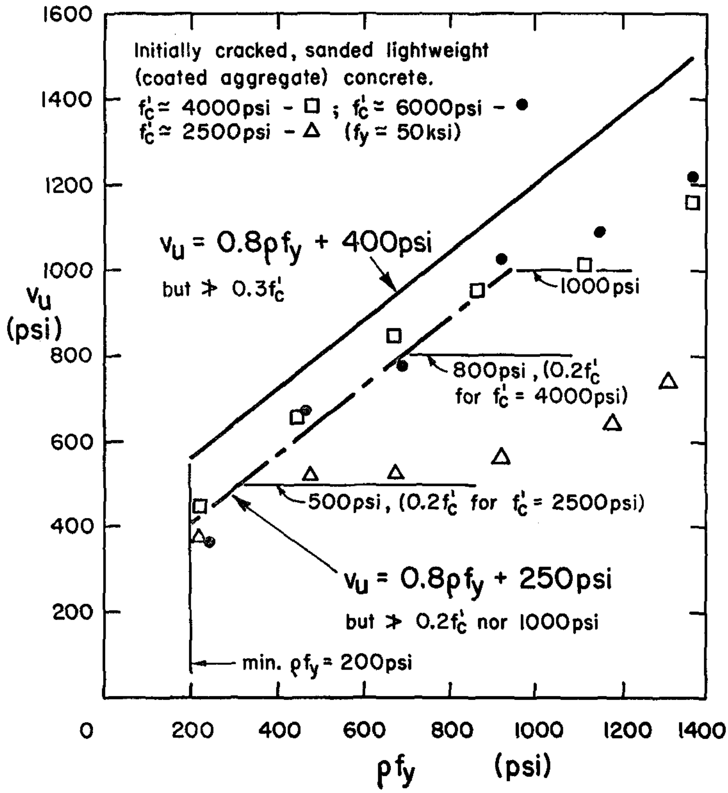


Fig. 12. Comparison of shear transfer strength of initially cracked, sanded lightweight concrete, with strength predicted by Eqs. (4) and (5).

Also, ρf_y was to be not less than 200 psi.

In Figs. 12 and 13, the shear transfer strength predicted by Eq. (4) is compared with the measured shear transfer strength of sanded lightweight and all-lightweight concrete, respectively.

It can be seen that Eq. (4) is non-conservative in both cases and therefore it should not be used in the design of connections between members made of lightweight concrete. However, it can also be seen that for moderate values of ρf_y, v_u increases with ρf_y at the same rate as predicted by Eq. (4).

It is therefore proposed that the following equations for shear transfer strength be used for lightweight concrete:

(a) **For sanded lightweight concrete:**

$$v_u = 0.8\rho f_y + 250 \text{ psi} \qquad (5)$$

but not more than $0.2f'_c$ nor 1000 psi.

(b) **For all-lightweight concrete:**

$$v_u = 0.8\,\rho f_y + 200 \text{ psi} \qquad (6)$$

but not more than $0.2f'_c$ nor 800 psi.

Lines representing Eqs. (5) and (6) have been drawn on Figs. 12 and 13, respectively, and it can be seen that these equations provide a reasonably conservative estimate of the shear transfer strength of the two types of lightweight concrete.

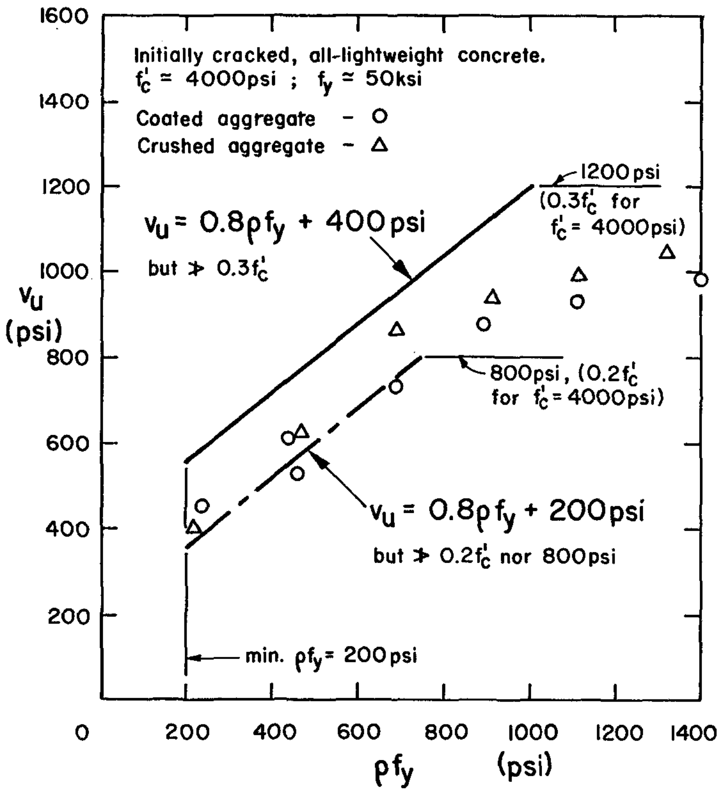


Fig. 13. Comparison of shear transfer strength of initially cracked, all-lightweight concrete with strengths predicted by Eqs. (4) and (6).

Conclusions for Design

On the basis of the study reported here, the following conclusions are drawn concerning shear transfer in lightweight aggregate concrete:

1. The shear transfer strength of lightweight concrete is less than that of sand and gravel concrete of the same compressive strength.

2. The shear transfer strength of lightweight concrete is not significantly affected by the type of lightweight aggregate, i.e., by whether the aggregate is a "coated aggregate" or "crushed aggregate."

3. The shear-friction provisions of Section 11.15 of ACI 318-71 may be used to calculate shear transfer strength in the design of connections in lightweight concrete, *providing* the value of the coefficients of friction μ, contained in Section 11.15.4, are multiplied by the following factors:

(a) For *all-lightweight* concrete having a unit weight not less than 92 lb per cu ft, multiply μ by 0.75.

(b) For *sanded lightweight* concrete having a unit weight not less than 105 lb per cu ft, multiply μ by 0.85.

4. The provisions of Section 6.1.9 of the *PCI Design Handbook* are not applicable to lightweight concrete.

5. The shear transfer strength of *sanded lightweight* concrete having a unit weight not less than 105 lb per cu ft can be calculated using the following equation:

$$v_u = 0.8\,\rho f_y + 250 \text{ psi}$$

but not more than $0.2f'_c$ nor 1000 psi and ρf_y to be not less than 200 psi.

6. The shear transfer strength of *all-lightweight* concrete having a unit weight not less than 92 lb per cu ft can be calculated using the following equation:

$$v_u = 0.8\,\rho f_y + 200 \text{ psi}$$

but not more than $0.2f'_c$ nor 800 psi and ρf_y to be not less than 200 psi.

Notation

A_{cr} = area of shear plane

A_{vf} = total area of shear transfer reinforcement

f'_c = compressive strength of concrete measured on 6 × 12-in. cylinders.

f_{ct} = splitting tensile strength of concrete measured on 6 × 12-in. cylinders

v_u = nominal ultimate shear stress $[v_u = V_u/(\phi\, A_{cr})]$

V_u = ultimate shear force

μ = coefficient of friction used in the shear-friction hypothesis

ρ = A_{vf}/A_{cr}

ϕ = capacity reduction factor

Discussion of this paper is invited. Please forward your discussion to PCI Headquarters by June 1, 1976.

References

1. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-71)," American Concrete Institute, Detroit, 1971.
2. *PCI Design Handbook,* Prestressed Concrete Institute, 1971.
3. Mast, R. F., "Auxiliary Reinforcement in Concrete Connections," *Journal of the Structural Division,* American Society of Civil Engineers, V. 94, ST 6, June 1968, pp. 1485-1504.
4. Mattock, A. H., "Effect of Aggregate Type on Single Direction Shear Transfer Strength in Monolithic Concrete," Report SM74-2, Department of Civil Engineering, University of Washington, Seattle, Washington, August 1974.
5. Mattock, A. H., and Hawkins, N. M., "Research on Shear Transfer in Reinforced Concrete," PCI JOURNAL, V. 17, No. 2, March-April, 1972, pp. 55-75.
6. Mattock, A. H., "Shear Transfer in Concrete Having Reinforcement at an Angle to the Shear Plane," Publication SP-42, *Shear in Reinforced Concrete,* American Concrete Institute, 1974, pp. 17-42.

Acknowledgment

This study was carried out in the Structural Research Laboratory of the University of Washington. It was supported by the National Science Foundation, through Grant No. GK-33842X. Lightweight aggregate was supplied by members of the Expanded Shale, Clay and Slate Institute.