Design application of the “PCI Manual on Design of Connections for Precast Prestressed Concrete” regarding embedded steel shapes from one side only have indicated the need for additional data. This paper presents design relationships for determining the increased capacity of embedded steel shape connections when strengthened by reinforcement. Practical design considerations are given on general connection design. In addition, some discussion is devoted to the philosophy of ultimate connection load factors. A fully worked numerical example is included to supplement the proposed design method.

Application of relationships for one-sided embedded structural steel connections, as defined by Section 2.4.4 of the PCI Connections Manual,® has indicated the need for additional design data.

Eq. (2-9) of the Connections Manual calculates \( V_u' \) for embedded shapes as controlled by the concrete strength. This design relationship approximates, conservatively, the complex bearing conditions occurring at ultimate. Fig. 1 illustrates the basic approximations used in developing Eq. (2-9). The condition shown by Fig. 1 is for the case where the column, or the concrete, properly reinforced above the embedded shape, extends above the embedded shape, and additional embedded shape auxiliary reinforcement is not present.
As stated in Section 2.4.4 of the Connections Manual, the ultimate concrete capacity $V_u'$ may be increased by either increasing $b$, the width of the embedded shape, or by the addition of reinforcement. The changing of $b$ to some greater value by adding additional steel width is straightforward and clear in application. Increasing $V_u'$ by addition of reinforcement to complement $C_f$ is not as clear or straightforward.

It may be necessary to add reinforcement, depending upon the design conditions concerning $b$, $f'_c$, $l_e$ or $l_e$, as shown by Fig. 2. The $A_{s'}$ and $A_s$ reinforcement can be either above, below or a combination of both as indicated by Fig. 2. Good design practice would

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Fig. 1. Concrete force system at ultimate.
locate $A'_s$ below and $A_s$ above, when possible, to insure that the welds and bars are in compression.

The increase in $V_u'$ due to the addition of $A'_s$ is dependent upon whether the $A'_s$ reinforcement yields. Further, for the typical condition shown by Fig. 1, where the column (having adequate column reinforcement) extends at least the required distance above the embedded shape, the addition of $A'_s$ may also require the addition of $A_s$.

Note that the required distance above the embedded shape is that distance required to satisfy the development length of column tension reinforcing bars resisting the ultimate bending moment induced by the eccentric load of the steel shape.

Fig. 3 sets out approximate simplifying strain assumptions for determining whether the $A'_s$ reinforcement yields. It should be noted that locating the ultimate neutral strain axis at the bottom of the rectangular stress block is conservative. The strain conditions of Fig. 3 approximates the so-called balance point relative to column $P_u-M_u$ ultimate interaction diagrams.

Rational design relationships can be set out for the general condition where reinforcement is welded to the embedded shape. $V_u'$ capacity at ultimate can be expressed as the sum of $V_D$ and $V_R$. $V_D$ is that ultimate strength developed by the concrete, and $V_R$ is that portion of the total ultimate strength developed by the additional reinforce-
ment attached to the embedded shape.

The ultimate strength of the concrete $V_C$ is expressed by Eq. (2-9a) where the conditions at ultimate are as shown by Fig. 1.

$$V_C = \phi f_{c'} b l_c / [3 + 4l_c/l_e] \quad (2-9a)$$

By statics, and rearranging Eq. (2-9a), it can be shown that $C_F$ and $C_B$ of Fig. 1 can be expressed as:

$$C_F = \phi f_{c'} b l_c / 3$$
$$C_B = \phi f_{c'} b [4l_c/3] / [3 + 4l_c/l_e]$$

Addition of compression reinforcement $A_s'$, in that concrete face closest to the applied $V_w$ as shown by Figs. 2 and 3, increases the connection capacity by $V_B$. The increased $V_B$ capacity can be expressed by Eq. (2-9b):

$$V_B = 3A_s' f_s' / [3 + 4l_c/l_e] \quad (2-9b)$$

The expression for Eq. (2-9b) is derived by employing the simplifying strain assumptions of Fig. 3 and simple statics by taking sum of moments about $C_B$ of Fig. 1. $A_s'$ and $f_s'$ both refer to compression reinforcement where $f_s'$ is equal to or less than $f_y$, and:

$$f_s' = 87,000 [1 - 3d'/l_e] \leq f_y$$

![Diagram](image)

**Fig. 4** Effective width "b" for double flanged embedded shapes.
The combined ultimate capacity of the concrete and $A_s'$ reinforcement can be obtained by the sum of Eqs. (2-9a) and (2-9b) as expressed by Eq. (2-9c).

$$V_u' = V_e + V_R = [\phi f_{c'} b l_e + 3A_s' f_s'] / [3 + 4l_v/l_e]$$  \(2-9c\)

Depending on the amount of $A_s'$ reinforcement, it may be necessary to also add "hold-down" reinforcement as shown by Fig. 2.

Conservatively assuming if the static $\Sigma F_y$ for $V_u'$, $A_s' f_s'$, and $C_R$ exceeds $C_r$ that $A_s$ "hold-down" reinforcement is required, then $A_s$ can be determined from Eq. (2-9d).

$$A_s = (1/f_y)[\phi f_{c'} b l_e/3 + A_s' f_s' - V_u']$$  \(2-9d\)

Nonprestressed tension reinforcement $A_s$ is only required when $A_s' f_s'$ exceeds the value expressed by Eq. (2-9e).

$$A_s' f_s' > \phi f_{c'} b [(4l_v/3) / (3 + 4l_v/l_e) - l_e/3] + V_u'$$  \(2-9e\)

Alternately, if an insufficient height of or nonreinforced concrete extends above the embedded shape, it would be mandatory that $A_s$ reinforcement be provided by Eq. (2-9d).

OTHER DESIGN CONSIDERATIONS

Application of the various Eqs. (2-9) require a review of engineering judgement items such as $b$, $l_v$, and ultimate load factors. Moreover, when considering these engineering judgement items, a realistic review of how material or construction tolerances vary, behavior or members being connected in terms of deformation at ultimate, and understanding of all possible loading conditions must enter into final design decisions.

Effective width $b$

The effective width $b$ can be strongly influenced by the type of embedded shape and other details used with the embedded shape. For example, when using embedded wide flanges, I-sections or channels having holes in the web and headed studs attached to the web, it is possible to consider a $b$ greater than just the width of the steel section.

Fig. 4 illustrates the importance of holes in the web of a steel shape and headed stud anchors attached to the web. Holes in the web, located as required, (greater than 1 in. diameter) insure good concrete consolidation between flanges. Likewise, headed stud anchors insure bearing confinement of the concrete between the flanges as well as distribution of bearing stresses. The size and amount of headed studs can be determined by using shear friction principles and a $\mu = 1.7$ as set out in Section 2.2 of the PCI Connections Manual.

The selection of a design $b$ greater than the actual steel section width should have a maximum limit. It appears rational to set a conservative limitation as shown by Fig. 4. Additionally, good design practice would require the use of three or four closely spaced ties just above and below the embedded shape when using a greater $b$.

Single flanged embedded shapes or steel bars should use the actual width of the steel section for $b$. An example of a single flange section would be a structural "T".

Design $l_v$

The determination of the design $l_v$ is important relative to the ultimate $V_u'$ capacity. Many factors such as production, final erected positions of connected members, rotations of supported members at ultimate relative to shifting load positions and $T_u$ forces all can significantly influence the ultimate location of $l_v$. It is far better to employ a design $l_v$ greater than will actually occur than a smaller value which would result in a decreased ultimate capacity.

Embedded shapes must have a po-
sitioning tolerance during manufacture, and can have bearing deviations. Also, if bearing pads are employed in the design, it is possible that they may be placed within ±1 in. of the planned position. Considering the case where the embedded shape projects from a column, the column itself is subject to location tolerances, and thus can shift the location of load application. Finally, rotations of beams or other supported members at their ultimate load can shift the point of load position.

Fig. 5 illustrates the various above factors influencing the location of the applied ultimate load. The key concept, in sound and realistic connection design, is review of factors affecting $l_v$.
and designing for the controlling tolerance condition.

**Load factors**

Section 2.1.3 of the *PCI Connections Manual* regarding ultimate load factors recommends that ACI (318-71) load factors for connections be increased above those used by connected elements, and suggests using a 4/3 factor. This implies for most precast connection design, that an ultimate load factor of $\frac{4}{3} \times 1.5 = 2$ be used where typically:

$$1.4D + 1.7L = 1.5(D + L)$$

At the very best, considering the conditions shown by Fig. 5, as well as workmanship, fabrication, volume change forces, and other possible tolerance conditions, a load factor of 2 appears to be the very minimum. Further, if the connection design is sensitive to $l_v$ variations, and considering the fact that it is not possible to ascertain all the secondary factors of Fig. 5, it would appear that ultimate connection load factors of 2.5 might be employed.

Another reason for using connection ultimate load factors of 2 or greater is a general philosophy relative to connection design. Flexural members, such as beams, exhibit large deflections as they approach ultimate conditions. Connections, unfortunately, are generally hidden from view, and do not indicate significant readily observable deformations at ultimate. Thus, to insure that connections perform as required, it appears reasonable that the connection should have a greater load factor than those members being supported by the connection, and hence load factors of 2 or greater.

**CONCLUDING REMARKS**

The design expressions suggested for one-sided embedded shapes combined with reinforcement approximate an extremely complex condition. It is believed that the assumptions employed for the additional capacity due to reinforcement are conservative. A more realistic evaluation of ultimate capacities appears to be dependent upon a comprehensive research program.

Basic connection concepts regarding load positions and load factors are more critical to connection design than the engineering relationships used. It appears most rational to require that all precast concrete connections be designed for a load factor of at least 2, and many connections might employ load factors of 2.5, or greater.

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**DESIGN EXAMPLE**

The following design example is the same as Problem 7—Embedded Steel Haunch, taken from the *PCI Connections Manual*, p. 58.

The initial part of the solution is identical to that presented in the Manual. However, the latter part of the solution shows how an increase in the concrete capacity can be gained using reinforcement.

**Data**

Assume that a S10 x 35 steel section (A36 structural steel) is embedded into one face of a 20 in. square precast concrete column and projecting 6 in. Let $f'_c = 5000$ psi.

**Required**

Determine the ultimate capacity of the steel and concrete and the increase in concrete capacity with reinforcement.

**Solution**

Ultimate capacity of steel

$$l_v = 20 - 2 = 18 \text{ in.}$$

$$l_v = (2/3) 5 + 1 + (18/6) = 7.33 \text{ in.}$$

(with 1 in. clear between beam and column)
Moment capacity:

The section modulus Z for an S10 x 35 steel section is 35.4 in.³

\[ V_u = f_y Z \]
\[ = 36 \times 35.4 / 7.33 = 173 \text{ kips} \]

Shear capacity (AISC):

\[ V_u = 0.55 f_{y,td} \]
\[ = 0.55(36)0.594(10) = 117.6 \text{ kips} \]

Ultimate capacity of concrete

Use Eq. (2-9a)

Flange width of S10 x 35 = 4.94 in.

\[ l_v / l_e = 0.41 \]

\[ V_u = \frac{(0.85)(5000)(4.94)}{1000[3 + 4(0.41)]} \]
\[ = 81.4 \text{ kips (controls)} \]

Increase concrete capacity with reinforcement

Steel shape capacity = 117.6 kips

Concrete capacity, \( V_c = 81.4 \text{ kips} \)

Therefore, the capacity to be carried by \( A_s' \) is

\[ A_s' = 117.6 - 81.4 = 36.2 \text{ kips} \]

Solve Eq. (2-9b) for \( A_s' f_s' \) with \( V_R = 36.2 \text{ kips} \):

\[ 36,200 = 3 A_s' f_s' [3 + 4(7.33)/18] \]

\[ A_s' f_s' = 55,855 \text{ lb (or 55.9 kips)} \]

Assuming \( d' = 2.5 \text{ in.} \) and \( f_y = 40 \text{ ksi} \), find \( f_s' \):

\[ f_s' = 87,000 \frac{[1 - 3d'/l_c]}{18} \]
\[ = 87,000 \frac{[1 - 3(2.5)/18]}{18} \]
\[ = 50,750 \text{ psi} \]

This steel stress is greater than \( f_y \) of 40 ksi and hence use \( f_y \). Therefore, \( A_s' = 55.9 / 40 = 1.40 \text{ sq in.} \)

Select two No. 8 bars.

\( l_v \) per weld for one-half of the bar perimeter = \( \pi D / 2 = 1.57 \text{ in.} \)

For weld \( t_v = 1/2 \text{ in.} \), use 0.707(1/2) = 3/8 in.

From Design Aid B-18, \(^*\) \( T_w = 6.6 \text{ kips} \)
per in.

\[ l_v = \frac{A_s f_y}{6.6} = \frac{0.79(40)}{6.6} = 4.79 \text{ in.} \]

Check if nonprestressed tension reinforcement \( A_s \) is required by Eq. (2-9e):

\[ A_s' f_s' > 0.85(5000/1000)4.94 \frac{[4(7.33)/3]}{[3 + 4(7.33)/18]} - 18/3 \]
\[ + 117.6 \]

\[ = 73.3 \text{ kips} \]

If \( A_s' f_s' > 44.3 + 117.6 = 73.3 \text{ kips} \),

\( A_s \) is required by Eq. (2-9d):

\[ A_s' f_s' = 2(0.79)(40) = 63.2 \text{ kips} \]

This value is less than 73.3 kips, and thus no \( A_s \) reinforcement is required.

Check \( V'' \) capacity by Eq. (2-9c), using two No. 8 bars.

\[ V'' = \left[ \frac{0.85(5000)(4.94)(18)}{3 + 4(7.33/18)} \right] \]
\[ + 117.6 \]
\[ = 122,600 \text{ lb} \]

Number of welds per bar = 4.79/1.57 = 3.05

Therefore, use four welds (see sketch).

From Design Aid B-19\(^*\), for No. 8 bars, length below embedded shape = 12 in.

\(^*\)See PCI Connections Manual.
Total No. 8 bar length = \( 2 + 10 + 12 = 24 \text{ in.} \)

Other ways of increasing concrete capacity are as follows:
1. Use deformed stud bar anchors.
2. Use angles to increase width \( b \).
3. Use headed studs to web in combination with holes in web allowing \( b' = \frac{h}{2} = \frac{10}{2} = 5 \text{ in.} \), exceeds \( \frac{w}{2} = \frac{4.94}{2} = 2.47 \text{ in.} \).
   \[ b = 4.94 + 2(2.47) = 9.88 \text{ in.} \]
   This width of embedded steel is satisfactory since it is less than the 16 in. column tie dimension.

**NOTATION**

- \( A_s = \text{tension area of reinforcement located \( l_v + \frac{3}{4}l_e \) from applied } V_u \text{ load, sq in.} \)
- \( A_s' = \text{compression area of reinforcement located \( l_v + \frac{l_e}{6} \) from applied } V_u \text{ load, sq in.} \)
- \( b = \text{effective width of embedded structural shape, in.} \)
- \( C_T = \text{ultimate force developed by concrete having its centroid located \( l_v + \frac{l_e}{6} \) from the applied } V_u \text{ load, lb} \)
- \( C_T = \text{ultimate force developed by concrete having its centroid located \( l_v + \frac{3}{4}l_e \) from applied } V_u \text{ load, lb} \)
- \( d' = \text{location of } A_s' \text{ from concrete face closest to applied } V_u \text{ load, in.} \)
- \( f_c' = \text{concrete compression strength, psi} \)
- \( f_s' = \text{stress in } A_s' \text{ reinforcement, psi} \)
- \( f_y = \text{yield stress of } A_s \text{ reinforcement, psi} \)
- \( l_e = \text{embedded length of structural shape, in.} \)
- \( l_v = \text{shear span of applied } V_u \text{ load, in.} \)
- \( V_c = \text{ultimate capacity as controlled by concrete of embedded shape, lb} \)
- \( V_R = \text{ultimate capacity due to } A_s' \text{ reinforcement only, lb} \)
- \( V_u' = \text{applied ultimate load, lb} \)
- \( V_u = \text{ultimate capacity of embedded steel shape, lb} \)
- \( \phi = \text{capacity reduction factor, 0.85} \)

Discussion of this paper is invited. Please forward your discussion to PCI Headquarters by October 1, 1974, to permit publication in the November-December 1974 PCI JOURNAL.