DEFLECTION OF DOUBLE TEES AND HOLLOW-CORED SLABS WITH HIGH TENSION ALLOWED IN CONCRETE

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General load-deflection characteristics through ultimate of standard double tee and hollow-cored slab sections are studied.
A numerical analysis is carried out for members designed following ACI 318-71 using tensile stresses $6 \sqrt{f_{c}'}, 12 \sqrt{f_{c}'}$, and $16 \sqrt{f_{c}'}$. The double tee is found to be more efficient in controlling post-cracking deflections, and composite topping adds to the post-cracking stiffness of the double-tee section.
The hollow-cored slabs studied are found to deflect very rapidly after cracking and would develop deflection problems if $12 \sqrt{f_{c}'}$ tension were utilized. Ponding on roofs and time-dependent deflections are not included in this study.
An advantage in the use of prestressed concrete structural elements is the control of deflection under normal service level loads.

Prestressing permits the use of smaller cross-sectional properties by keeping the section uncracked through design level loads. Also, an uncracked section is particularly desirable where exposure to a corrosive environment is a factor.

Many applications, however, do not require an uncracked section, and thus post-cracking deflection behavior becomes the controlling design criterion.

The 1971 ACI Code [Sections 18.4.2(c) and 18.4.3] now allows apparent design tensile stresses of $12 \sqrt{F_e}$ in precompressed tension zones of members. The Code states that greater apparent tensile stresses may be allowed provided that deflection performance in accordance with Section 9.5 of the Code is met.

This possible reduction in initial prestressing forces would result in reduced initial cambers, which in many cases is highly desirable. These new allowable stresses also increase the importance of accurately predicting the post-cracking deflection behavior of prestressed structural elements.

During the fall of 1972 and spring of 1973, a study was undertaken at The University of Texas at Austin to determine general load-deflection characteristics through ultimate of standard prestressed concrete beams. Time-dependent deflections were not included in the analysis of load-deflection response.

It was decided to use data obtained by numerical analysis instead of experimentally derived data in order to evaluate the deflection response of a varied group of beams subjected to applied load.

The design levels of effective prestress were varied to check load-deflection response of members allowing 1971 ACI Code apparent tensile stresses.

It was hoped that any extreme behavior of an element caused by using the higher allowable design stresses would be identified and that general load-deflection characteristics would be shown.
The standard sections chosen for analysis were of two basic types which were expected to indicate extremes in characteristic load-deflection behavior. The two types were the “double-tee” section and the “hollow-cored slab” section.

Each of these sections was evaluated for long and short spans in order to check beams with relatively low and high percentages of steel. Further, each long and short span was stressed to a level of prestress to allow $6\sqrt{f'_c}$, $12\sqrt{f'_c}$, and $16\sqrt{f'_c}$ tensile stress at midspan to determine post-cracking behavior for partially prestressed members.

A check was performed early in the design procedure in an attempt to provide less prestressing force by reducing the amount of prestressing reinforcement. In nearly all cases, ultimate strength requirements prohibited any reduction of reinforcement.

It was decided to utilize the same prestressing reinforcement in each design for the increased apparent tensions and merely to reduce the level of prestress. For the two cross-sectional shapes investigated in this study, designs and analyses for a total of 28 members were included.

The hollow-cored slab sections chosen were the 8 and 12-in. nonproprietary types from the PCI Design Handbook. The shallowest depth hollow-cored slab was not chosen since it was felt that the shallower sections approached the load-deflection behavior of solid slabs.

The 8 and 12-in. sections were chosen to give deflection behavior peculiar to hollow-cored slabs. No topping was used with the long span hollow-cored members since topping is usually not used on these floor members. Here it is very important to keep flat under full dead load, (i.e., to reduce camber).

The long span double-tee sections (8 DT 24 and 8 DT 12) were chosen to indicate behavior at their practical limit as floor members. The practical long span limit for the 24-in. double tee is a 60-ft span parking garage.

In this application, topping is normally used, and thus the section was designed with a 2-in. composite topping. The 12-in. double-tee long span was designed for a 36-ft span apartment load and was also designed with a 2-in. composite topping.

For the short span double tee and hollow-cored slabs, the area of steel was reduced to a practical minimum and the maximum span allowed for the member with the prestress provided by this area of steel was then used. Roof loads were used for these members.

The choice of the check points for allowable apparent tensions was based mainly on ACI Code requirements. For the long span double tees, the ACI Code under some circumstances allows both $6\sqrt{f'_c}$ and $12\sqrt{f'_c}$ stresses, if deflections are within the allowable (ACI Code, Section 9.5).

An apparent tension of $16\sqrt{f'_c}$ was checked also to determine if the member underwent a radical change in behavior at full live load for apparent tensions beyond $12\sqrt{f'_c}$.

In order to check cases close to normal design practice, the hollow-cored slabs were designed for four basic levels of prestress based on varying apparent tensions at midspan. The four governing apparent tensions are:

(a) $6\sqrt{f'_c}$ apparent tension at mid-span under full live load.
(b) $12\sqrt{f'_c}$ apparent tension at mid-span under full live load.
(c) All strands fully stressed.
(d) Zero psi tension at midspan under full dead load.
The short-span slabs were designed for zero prestress instead of $12 \sqrt{f'_c}$ apparent tensions. This was done because zero prestress existed before allowable apparent tensions of $12 \sqrt{f'_c}$ were reached. Thus, zero prestress was the governing apparent tension.

Normal weight concrete was used (150 pcf) and 5000 psi final strength was chosen. Losses were assumed to be 35 ksi and thus no consideration was given to the initial strength of concrete.

All analysis was performed assuming final $f'_c$ and final effective prestress forces after losses. The 2-in. composite topping for the double tees was assumed to have a final in-place strength of 3000 psi.

**METHOD OF ANALYSIS**

The *Commentary* to the ACI Code suggests two methods for evaluating post-cracking deflection behavior of reinforced concrete beams.

One method utilizes $I_e$ (effective moment of inertia) to predict behavior of partially prestressed members.

The other method idealizes the load-deflection response into an assumed bilinear curve, the first portion being a straight line from initial load up to a load which causes cracking, and the second portion of the curve being a flatter curve proceeding from the point of cracking.

The elastic portion of the load-deflection curve is determined through the use of uncracked section properties.

Another method of analysis as proposed by Burns\(^3\) in "Moment Curvature Relationships for Partially Prestressed Concrete Beams," is a numerical iterative method of balancing internal strains and forces to obtain a moment-curvature history of the element. The deflection is then obtained by use of moment area principles.

This method, while being more exact, is sometimes not practical for hand calculations, especially when the beam has several varying cross sections throughout its length. However, the method does lend itself to computer usage, and the load-deflection analysis was performed through use of such a computer program.

The analysis to find the load-deflection response of the various sections from zero load to ultimate was performed by the use of an existing computer program at The University of Texas at Austin.

In May 1969, Chang developed a computer program PCCBM (Prestressed Composite Concrete Beam) as part of an unpublished MS thesis for The University of Texas at Austin, based on the analytical method proposed by Burns.

PCCBM, through an iterative method of imposing strains and balancing internal forces, solves for the flexibility of the member. Steps in the program are outlined as follows:

All relevant data are input to describe the loaded and restrained beam. Next, the moment-curvature relation is generated for each cross section type, and a recursive technique is applied to determine the deflected shape, assuming the beam is temporarily elastic. From this deflected shape, the beam curvature at each station is computed and a new flexural stiffness is determined based on the actual moment-curvature relation for that section. This primary iteration loop is continued until the deflection previously calculated differs from the new deflection within a specified tolerance.

The relevant input data include stress-strain relations for concrete and steel. The program assumes a stress-strain relationship for concrete based on Hognestad's equation.

The steel data are input as a series of data points describing the stress-strain curve. Values of stress and strain between the input points are interpolated linearly. A normal stress-strain curve for 7-wire strand with 270-ksi ultimate
strength was input for all sections analyzed.

The program is very general and computes load-deflection response of composite as well as noncomposite sections. Figs. 1, 2, and 3 illustrate how the cross-sectional shapes were input by a series of strips approximating the section dimensions.

The above approximation is very close and PCI Design Handbook values for section properties were often obtained within five significant figures from the approximated section input.

Chang verified his program in 1969 by comparing his computed results with experimental beam results which were previously published. His computed data points were very close to the experimentally derived data and also very close to a hand calculated set of values. Detailed information and program de-

Fig. 1. Cross-sectional shapes approximated by strips.
LOAD-DEFLECTION RESPONSE FOR VARIOUS SECTIONS

Double-tee sections

As previously mentioned, a 12-in. double tee (PCI Design Handbook SDT12) was used to illustrate the behavior for a shallow double-tee section. The span was varied to show response for both highly reinforced and lightly reinforced shapes.

A span of 23 ft was chosen for the short span condition based on minimum strength considerations for a minimum of steel. Light roof design loads were assumed.
Dead load (8DT12) ............. 37 psf  
Roofing and misc. ............. 10 psf  
Live load snow ............. 30 psf

The prestressing force was varied to permit $6\sqrt{f_c}$ (424 psi), $12\sqrt{f_c}$ (848 psi), and $16\sqrt{f_c}$ (1131 psi) apparent tension in the bottom beam fiber under full design load. The prestressing calculations were made by hand calculations.

The load-deflection data resulting from the computer analysis are plotted to emphasize both immediate post-cracking behavior and ultimate deflection behavior.

Load-deflection response is plotted in Fig. 4 to emphasize immediate behavior. Fig. 5 illustrates the overall load-deflection behavior.

Fig. 4 shows that the section behaves elastically up to cracking for all three levels of prestress. The slopes (stiffnesses) are virtually the same except for different origins due to the variation in prestress moment.

It must be remembered that the analysis is performed assuming final prestress forces and thus camber at dead load would be slightly greater before losses than the values shown here.

The various deflection limits are plotted on Fig. 4 to illustrate how well the section meets the ACI Code requirements using the various levels of prestress. For a roof which does not damage attachments due to deflections, $L/240$ is allowed.

With $12\sqrt{f_c}$ tension allowed, at full live load, the deflection is about $L/185$. Thus, allowing apparent tension of about $10\sqrt{f_c}$ would meet Code requirements. Cracking load for allowing apparent tension of $6\sqrt{f_c}$ is 83.5 psf.

Therefore, designing the beam to
crack (at $7.5 \sqrt{f_c}$) at design load (77 psf) would be acceptable, since the section does not deflect excessively even at higher apparent tensions.

Fig. 5 shows that for much lower levels of prestress ($16 \sqrt{f_c}$) the post-cracking stiffness tends to remain much flatter for the immediate post-cracking load interval than for the more highly stressed conditions.

This characteristic appears throughout the study and is due to the fact that the section must rotate more at low prestress levels in order to develop the stress in the high strength strands and balance internal forces.

The relative stiffnesses of the section comparing cracked flexibility at cracking load plus 30 psf to uncracked flexibility are (for midspan cracked sections):

$$6 \sqrt{f_c} = \frac{9.512 \times 10^8}{1.243 \times 10^{10}} = 0.0765$$

$$12 \sqrt{f_c} = \frac{1.006 \times 10^9}{1.257 \times 10^{10}} = 0.080$$

Thus, the general characteristic of rate of change of stiffness after cracking is not radically different between $6 \sqrt{f_c}$ and $12 \sqrt{f_c}$ design levels of prestress.

In general, the lightly reinforced double tee behaves with the responses shown in Fig. 4 after cracking, and apparent tension of less than $12 \sqrt{f_c}$ appears to be a practical limit due to deflection limitation.

Limiting apparent tension to $10 \sqrt{f_c}$ or perhaps $7.5 \sqrt{f_c}$ (cracking) would probably be a good general rule for lightly reinforced 12-in. double tees based on the analysis of this section.

A span of 36 ft and apartment live loads were selected for design of the long span unshored composite section. The loads are as follows:

Dead load: 8 DT 12 ........ 37 psf
2 in. comp. conc. .... 25 psf
Partitions ........ 10 psf
Live load: Apartment .... 40 psf

This composite section (8DT 12 + 2) was also designed for varying levels of
prestress allowing $6\sqrt{f'_c}$ (424 psi), $12\sqrt{f'_c}$ (484 psi), and $16\sqrt{f'_c}$ (1131 psi) apparent tension at midspan under service load.

Due to the additional stiffness from the composite topping, the composite section remained uncracked for higher load levels than would be the case if noncomposite. Figs. 6 and 7 give the immediate post-cracking deflection behavior and the behavior through ultimate.

Figs. 6 and 7 illustrate that this composite section behaves as a stiff section after cracking due to the large area of steel provided for the long span. This is also due to the fact that the moment arm for the prestressing force couple is increased due to the composite topping.

As shown by Fig. 7, the section is not radically affected by changes in prestress level. Deflection at $6\sqrt{f'_c}$ apparent tension is $L/429$ at design level loads, while deflection at $16\sqrt{f'_c}$ apparent tension is $L/348$.

This amount of deflection is allowable for most applications.

The ratio of cracked (at service load) to uncracked stiffness for the $16\sqrt{f'_c}$ design is:

$$\frac{7.48 \times 10^9}{1.233 \times 10^{10}} = 0.606$$

The short span deep double-tee section consists of a 24-in. noncomposite section spanning 42 ft with roof loads. The loads used in design are:

- Dead load: 8 DT 24 ..........52 psf
- Rfg. and misc. .....10 psf
- Live load: Snow ..........30 psf

Again, the levels of prestress were varied to compare behavior.

Fig. 8 reveals that $12\sqrt{f'_c}$ apparent tension would be a limiting value for this lightly reinforced 24-in. double tee. The values of deflection exceed
allowables at $16\sqrt{f_c} (L/153)$.

Possibly, values of apparent allowable tension of slightly higher than $12\sqrt{f_c}$ could be used but control of materials and prestress losses would have to be very close.

Fig. 9 gives the complete load-deflection response to ultimate for the 24-in. double tee with a 42-ft span.

For the $12\sqrt{f_c}$ design, the cracked stiffness at design level load divided by the uncracked stiffness is:

\[
\frac{8.54 \times 10^9}{9.118 \times 10^{10}} = 0.09
\]

Thus, it is evident the cracked stiffness for midspan sections is much less than uncracked (about one-tenth) and care must be taken when cracking loads are lowered below about 75 psf.

This load is the cracking load for the $12\sqrt{f_c}$ designs. Fig. 9 illustrates the extreme ductility of the section through ultimate.

The long span deep section example consisted of a 60-ft span 24-in. composite double tee. The loads are:

- Dead load: 8 DT 24 .............. 52 psf
  2 in. comp. concrete 25 psf
  Misc. dead load . . . . 10 psf
- Live load: Parking (reduced) . . . 40 psf

Figs. 10 and 11 show the stiff post-cracking stiffness of this composite, highly reinforced section.

Designing with $16\sqrt{f_c}$ apparent tension or greater could be allowed because of the small change in stiffness caused by prestress reduction. The deflection at design level load allowing $16\sqrt{f_c}$ allowable tension is $L/240$.

The ratio of post-cracking stiffness to uncracked stiffness is approximately 0.5.
Fig. 8. Load-deflection curves for 42-ft span, 24-in. deep double tees.

Fig. 9. Complete load-deflection curves for 42-ft span, 24-in. deep double tees.
for a range of load values well beyond design load.
Thus, the deep, highly reinforced composite double tees exhibited the stiffest post-cracking deflection behavior of the double tees included in this study.

**Hollow-cored slab section**

The short-span, shallow hollow-cored slab chosen was the 4 ft x 8 in. proprietary section given in the PCI Design Handbook.

This section was used as a 24-ft roof span with the following loads:
- Dead load: 4 ft HC 8 in. .... 56 psf
- Rfg. and misc. .... 10 psf
- Live load: Snow ........ 30 psf

Load-deflection values were obtained for levels of prestress permitting tensile stresses at midspan of $6\sqrt{f'c}$ at full live load and zero psi at full dead load. The load-deflection response is plotted in Figs. 12 and 13. One fact obvious from Fig. 12 is that once the slab has cracked the stiffness is very much reduced.

This is due to the large rotation necessary after cracking to strain the prestressing strands to the level of stress necessary for internal force equilibrium. The shallow depth of the section reduces the moment arm of the internal couple and thus large rotations are necessary for force equilibrium after the loss of tensile force from the concrete after cracking.

Fig. 13 indicates that the cracked stiffness increases slightly after the ini-
**Fig. 11.** Complete load-deflection curves for 60-ft span, 24-in. deep double tees with 2-in. topping.

**Fig. 12.** Load-deflection curves for 24-ft span, 8-in. hollow-cored slabs.
tial radical reduction at cracking, but this increase comes at a deflection level well beyond Code allowables.

The cracked to uncracked stiffness ratio is (within allowable deflection limits):

Zero prestress \[
\frac{3.487 \times 10^8}{7.141 \times 10^6} = 0.048
\]

\[6 \sqrt{f'_c} \frac{4.588 \times 10^8}{7.141 \times 10^6} = 0.064\]

This very radical change in stiffness at cracking would seem to make cracking prior to service load levels undesirable.

Cracking at a very small load increment prior to design (service) load level could cause deflections in excess of Code allowables.

The long span shallow-cored slab choice was a 30-ft span with a floor loading. The loading is:

Dead load: 4 ft HC 8 in. ....... 56 psf
Partition ............ 10 psf
Live load: Apartment ........... 40 psf

Four levels of prestress designs were checked:

a. All strands fully stressed.
b. Zero psi tensile stress at full dead load at midspan.
c. \(6\sqrt{f'_c}\) tensile stress at full live load at midspan.
d. \(12\sqrt{f'_c}\) tensile stress at full live load at midspan.

The load-deflection response of the long span section was very similar to the short span behavior.

Figs. 14 and 15 show that the stiffness of the section is drastically reduced immediately after cracking.

However, due to the high percentage of reinforcing steel, the section began to stiffen sooner than the more lightly reinforced 8-in. slab, as indicated in Fig. 15.

Again, it appears that after cracking before design level loads by only a small load increment above the cracking load could cause excessive deflections.

The small moment arm depth for the internal force couple and the loss
Fig. 14. Load-deflection curves for 30-ft span, 8-in. deep hollow-cored slabs.

Fig. 15. Complete load-deflection curves for 30-ft span, 8-in. deep hollow-cored slabs.
Fig. 16. Load-deflection curves for 32-ft span, 12-in. deep hollow-cored slabs.

of more than half the tensile concrete area after cracking accounts for the greatly reduced stiffness. Limiting cracking \(7.5 \sqrt{f_{c}}\) tensile stress) to design level loads seems a practical limit.

The post-cracking to uncracked stiffness ratio for \(12 \sqrt{f_{c}}\) is:

\[
\frac{8.02 \times 10^8}{7.102 \times 10^9} = 0.11
\]

The lightly reinforced, deep slab section is a 12-in. hollow-cored slab spanning 32 ft. The loading is a typical apartment load level.

Dead load: 4 ft HC 12 in. ...68 psf
Rfg. and misc. ....10 psf
Live load: Snow ...............30 psf
Prestress levels investigated are:
All strands fully stressed.

Zero psi at full dead load.
6 \(\sqrt{f_{c}}\) at full live load.
12 \(\sqrt{f_{c}}\) at full live load.

Figs. 16 and 17 indicate that this section deflects very rapidly after cracking. Fig. 16 reveals that cracking could occur at a small level of load below design level and still fall within allowable ACI Code limits.

However, this would not allow for much of a margin of error and, again, it seems that keeping cracking at a load level above design load is prudent. Slightly more than 6 \(\sqrt{f_{c}}\) tensile stress could be allowed as shown by Fig. 16.

The long span, deep slab is a 44-ft span, 12-in. hollow-cored slab. Apartment loading is specified.
Fig. 17. Complete load-deflection curves for 32-ft span, 12-in. deep hollow-cored slabs.

Fig. 18. Load-deflection curves for 44-ft span, 12-in. deep hollow-cored slabs.
Dead load: 4 ft HC 12 in. ... 68 psf
Partitions ................ 10 psf
Live load: Apartment ............ 40 psf

Prestress levels based on tensile stresses in extreme fibers at midspan of $6\sqrt{f_c}$ and $12\sqrt{f_c}$ at full live load as well as for fully stressed strands and zero psi at full dead load are used in the analysis.

Figs. 18 and 19 show that although this deep section undergoes radical changes in stiffness at cracking, it displays the stiffest post-cracking behavior of all the slabs analyzed in this study.

It appears that cracking could be allowed 10 to 15 psf below design level and still possibly limit deflections to within ACI Code allowables (L/240).

Use of $12\sqrt{f_c}$ tensile stress in determining prestress level causes deflections at midspan well past ACI deflection limits.

Although this highly reinforced slab displays the stiffest post-cracking behavior of the slabs investigated, it still seems prudent to limit cracking to levels of load above service load levels.

The approximate ratio of cracked stiffness to uncracked stiffness in the allowable load range for $12\sqrt{f_c}$ is:

$$\frac{4.48 \times 10^9}{2.11 \times 10^{10}} = 0.212$$

**SUMMARY AND CONCLUSIONS**

In summary, the double tee is seen to be more efficient in controlling post-cracking deflections than the hollow-cored slab.

Composite topping is also seen to add to the post-cracking stiffness of the double-tee element.

Hollow-cored slabs deflect very rapidly after cracking due to both the
small internal force couple moment arm and to the excessive loss of stiffness after cracking.

It appeared that hollow-cored slab sections were more radically affected from use of $12 \sqrt{f_c}$ apparent tensions than the double tee sections of the same depth.

The radical behavior of the noncomposite hollow-cored slabs points out the importance of an accurate post-cracking deflection behavior analysis.

Further experimental data to verify the computer results are desirable, as well as an accurate hand calculation method for determining post-cracking behavior.

Overall, the double tees met ACI Code requirements for deflection allowing $12 \sqrt{f_c}$ tensile stresses in extreme fibers at midspan under full live load.

Careful attention must be given to time-dependent deflection, especially for roof structures where ponding of water might occur.

The hollow-cored slabs changed stiffness quite radically after cracking as shown in the previous load-deflection graphs.

From the sections analyzed, it is concluded that hollow-cored slabs must be prevented from cracking at less than service load levels in order to meet ACI deflection requirements.

Some indication as to the comparative effectiveness of the various structural shapes has been discovered by checking how well each section meets ACI Code requirements.

Some general observations are as follows:

1. Sections with higher percentages of prestressing steel displayed stiffer post-cracking behavior than the more lightly reinforced sections.

2. Double-tee sections, in general, displayed stiffer post-cracking behavior than the hollow-cored slab sections and could be designed with higher apparent tensions at midspan.

3. Lower levels of prestress tended to cause a greater reduction in stiffness immediately after cracking than higher levels of prestress for the same section.

4. Composite sections remained relatively stiff after cracking in the general vicinity of design level loads. The ratios of cracked to uncracked stiffnesses were approximately 0.6 to 0.5.

5. After an immediate loss of stiffness at cracking, the hollow-cored section gain in stiffness usually occurred at deflection levels well beyond ACI allowable limits.

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