ANALYSIS OF LATERAL LOAD RESISTING ELEMENTS

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The structure of most high rise concrete buildings is a complex, three-dimensional system of beams, columns, shear walls, and diaphragm slabs. This complex of linear and planar structural elements must be interconnected and disposed in such a way as to insure the stability of the structure when subjected to the extraordinary forces of nature, wind, and earthquake.

LATERAL LOADS

The first step in the lateral analysis process is to calculate the design loads. The specified code, the "Uniform Building Code, 1970 Edition," very specifically establishes the design loads.

Wind load

For non-coastal areas of Georgia, the

Table 1. Computer printout of wind load analysis in x direction using basic wind pressure equal to 25 psf. Total wind load, shear, and overturning moment are listed for each story level.

NAME	HEIGHT (FT)	WIDTH (FT)	LOAD (K)	SHEAR ABOVE (K)	MOMENT (FTK)
ROOF	0.00	96.00	19.28	0.00	0.00
23	10.04	96.00	38.55	19.28	193.54
22	10.04	96.00	38.55	57.83	774.16
21	10.04	96.00	38.55	96.38	1741.85
20	10.04	96.00	38.55	134.94	3096.62
19	10.04	96.00	38.55	173.49	4838.47
18	10.04	96.00	38.55	212.04	6967.40
17	10.04	96.00	38.55	250.60	9483.41
16	10.04	96.00	38.55	289.15	12386.49
15	10.04	96.00	38.55	327.71	15676.65
14	10.04	96.00	38.55	366.26	19353.89
13	10.04	96.00	38.55	404.81	23418.21
12	10.04	96.00	38.55	443.37	27869.61
11	10.04	96.00	38.55	481.92	32708.08
10	10.04	96.00	28.92	520.47	37933.62
9	10.04	96.00	28.92	549.39	43449.47
8	10.04	96.00	28.92	578.30	49255.63
7	10.04	96.00	28.92	607.22	55352.10
6	10.04	96.00	28.92	636.13	61738.88
5	10.04	96.00	24.10	665.05	68415.95
4	10.04	96.00	24.10	689.14	75334.96
3	10.04	96.00	19.28	713.24	82495.89
2	10.04	96.00	24.04	732.52	89850.34
1	15.00	0.00	0.00	756.56	101198.70

Uniform Building Code establishes the basic wind pressure at 25 psf. The design pressures increase for various height zones above the base and these pressures are taken from the Code, Table 23E. The calculation of the total shear and moment at all levels of the structure is simple statics.

Tables 1 and 2 are computer printouts which list the total wind load, shear, and overturning moment at each level throughout the building in each of two normal directions.

Seismic load

The seismic risk map of the UBC places Atlanta in Zone 1. Section 2314 of this Code contains a generally accepted empirical method for the calculation of equivalent static forces to be used for design. Most West Coast engineers designing in Seismic Zones 2 or 3 would make a more elaborate analysis. However, since this building is located in Zone 1, earthquake forces have been calculated empirically in accordance with the Code.

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The base shear formula is:

$$V = ZKCW$$

The zone factor Z, in this case, is ¼

Table 2. Computer printout of wind load analysis in z direction using basic wind pressure equal to 25 psf. Total wind load, shear, and overturning moment are listed for each story level.

NAME	HEIGHT (FT)	WIDTH (FT)	LOAD (K)	SHEAR ABOVE (K)	MOMENT (FTK)
ROOF	0.00	72.00	14.46	0.00	0.00
23	10.04	72.00	28.92	14.46	145.15
22	10.04	72.00	28.92	43.37	580.62
21	10.04	72.00	28.92	72.29	1306.39
20	10.04	72.00	28.92	101.20	2322.47
19	10.04	72.00	28.92	130.12	3628.86
18	10.04	72.00	28.92	159.03	5225.55
17	10.04	72.00	28.92	187.95	7112.55
16	10.04	72.00	28.92	216.86	9289.87
15	10.04	72.00	28.92	245.78	11757.49
14	10.04	72.00	28.92	274.69	14515.41
13	10.04	72.00	28.92	303.61	17563.65
12	10.04	72.00	28.92	332.52	20902.20
11	10.04	72.00	28.92	361.44	24531.05
10	10.04	72.00	21.69	390.35	28450.21
9	10.04	72.00	21.69	412.04	32587.10
8	10.04	72.00	21.69	433.73	36941.72
7	10.04	72.00	21.69	455.41	41514.07
6	10.04	72.00	21.69	477.10	46304.15
5	10.04	72.00	18.07	498.79	51311.96
4	10.04	72.00	18.07	516.86	56501.22
3	10.04	72.00	14.46	534.93	61871.92
2	10.04	72.00	18.03	549.39	67387.78
1	15.00	0.00	0.00	567.42	75899.03

for Seismic Zone 1. A horizontal force factor K of 1.33 is selected because this building has a box structural system. The horizontal force coefficient C is calculated from a formula which includes the fundamental period of the structure and this fundamental period is calculated by an empirical equation in the Code.

Tables 3 and 4 list the resultant total loads, shears, and overturning moments on a story by story basis for each of the two normal directions.

LATERAL ANALYSIS

This 24-story apartment building has a lateral resistant structural system which consists of exterior, perimeter bearing and shear walls with window openings and internally located moment resisting frames consisting of four columns with connecting beams (see Fig. 1).

In order to proceed with a lateral analysis it is necessary to make certain

Table 3. Computer printout of earthquake load analysis in x direction. Total seismic load, shear, and overturning moment are listed for each story level.

FUNDAMENTAL PERIOD OF VIBRATION $T = 1.3902$ (SEC) EMPIRICAL $= \dfrac{0.05 \times 235.9}{\sqrt{72}}$

$ZKC = \frac{1}{4} \times 1.33 \times 0.045 = 0.015$ $\quad Z = \frac{1}{4}$

$C = \dfrac{0.05}{T^{1/3}} = \dfrac{0.05}{1.39^{1/3}} = 0.045$ $\quad K = 1.33$

J MIN. $= 0.538$

NAME	HEIGHT (FT)	WEIGHT (K)	LOAD (K)	SHEAR ABOVE (K)	MOMENT (FTK)
ROOF	0.00	858.00	35.63	0.00	0.00
23	10.04	858.00	22.02	35.63	337.37
22	10.04	858.00	21.04	57.65	834.86
21	10.04	858.00	20.06	78.69	1457.82
20	10.04	858.00	19.09	98.76	2178.81
19	10.04	858.00	18.11	117.84	2976.71
18	10.04	858.00	17.13	135.95	3835.83
17	10.04	858.00	16.15	153.07	4745.17
16	10.04	858.00	15.17	169.22	5697.65
15	10.04	858.00	14.19	184.39	6689.36
14	10.04	858.00	13.21	198.58	7718.93
13	10.04	858.00	12.23	211.79	8786.89
12	10.04	858.00	11.25	224.02	9895.04
11	10.04	858.00	10.27	235.28	11045.99
10	10.04	858.00	9.29	245.55	12242.58
9	10.04	858.00	8.32	254.85	13487.43
8	10.04	858.00	7.34	263.16	14782.54
7	10.04	858.00	6.36	270.50	16128.93
6	10.04	858.00	5.38	276.86	17526.24
5	10.04	858.00	4.40	282.23	18972.50
4	10.04	858.00	3.42	286.63	20463.85
3	10.04	858.00	2.44	290.05	21994.32
2	10.04	858.00	1.46	292.50	23555.69
1	15.00	0.00	0.00	293.96	25920.99
		19734.00			

initial decisions and/or assumptions. This entire building is put together with precast pieces. Thus, the structural engineer must decide how and where he connects the pieces together. Temperature and other volumetric effects as well as seismic considerations may suggest the incorporation of some flexible or sliding joints. However, since this discussion is limited to the lateral system, it has been assumed that all beams and columns are rigidly connected together and that all wall panels are connected together along all edges.

This is a very complex system. Were we to completely mathematically model the entire building the resultant analytical problem would be huge and very expensive to solve. However, as is often the case, it is the design engineer's task to reduce the analysis problem to a size that can be handled in the time and within the fee available. Nonetheless, when this is done, it is essential to retain an analysis that recognizes the significant aspects of the structural system.

Table 4. Computer printout of earthquake load analysis in z direction. Total seismic load, shear, and overturning moment are listed for each story level.

FUNDAMENTAL PERIOD OF VIBRATION T = 1.2039 (SEC) EMPIRICAL

$ZKC = \frac{1}{4} \times 1.33 \times 0.046 = 0.016$ $Z = \frac{1}{4}$

$C = \frac{0.05}{T^{1/3}} = \frac{0.05}{1.20^{1/3}} = 0.045$ $K = 1.33$

J MIN. = 0.538

NAME	HEIGHT (FT)	WEIGHT (K)	LOAD (K)	SHEAR ABOVE (K)	MOMENT (FTK)
ROOF	0.00	858.00	25.21	0.00	0.00
23	10.04	858.00	24.14	25.21	239.59
22	10.04	858.00	23.07	49.35	672.07
21	10.04	858.00	21.99	72.42	1259.21
20	10.04	858.00	20.92	94.42	1970.34
19	10.04	858.00	19.85	115.34	2781.42
18	10.04	858.00	18.77	135.18	3674.17
17	10.04	858.00	17.70	153.96	4635.23
16	10.04	858.00	16.63	171.66	5655.41
15	10.04	858.00	15.55	188.29	6728.94
14	10.04	858.00	14.48	203.84	7852.77
13	10.04	858.00	13.41	218.33	9025.89
12	10.04	858.00	12.34	231.73	10248.79
11	10.04	858.00	11.26	244.07	11522.84
10	10.04	858.00	10.19	255.33	12849.79
9	10.04	858.00	9.12	265.52	14231.27
8	10.04	858.00	8.04	274.64	15668.40
7	10.04	858.00	6.97	282.68	17161.35
6	10.04	858.00	5.90	289.65	18709.04
5	10.04	858.00	4.82	295.54	20308.78
4	10.04	858.00	3.75	300.37	21956.08
3	10.04	858.00	2.68	304.12	23644.36
2	10.04	858.00	1.60	306.79	25364.83
1	15.00	0.00	0.00	308.40	27968.86
		19734.00			

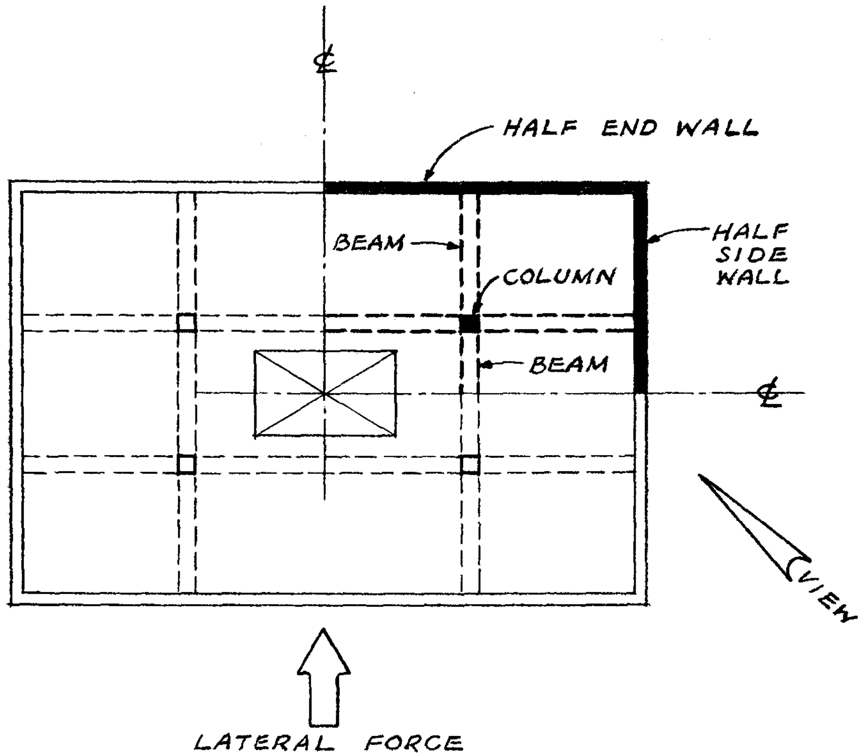


Fig. 1. Schematic plan showing principal structural components of building.

For the subject structure, the most significant features are:

1. The reduction of exterior wall stiffness due to the window openings.
2. The overall box action of the exterior walls.
3. The shear deformation (or shear lag) in both the side and end walls.
4. The interaction between the walls and the internal moment resisting frames.

MATHEMATICAL MODEL

These significant factors have been retained in the following described analysis which has been reduced to a size that has been solved using the computer program STRESS, the "Structural Engineering System Solver." Other computer programs could have been used provided that they include both flexural and shear deformation. The significant steps involved in the development of the mathematical model are as follows:

Step 1

Recognize the symmetry of the building about its center line axis each way. This permits an analysis of one quarter of the building subjected to one quarter of the load (see Fig. 1).

Step 2

Calculate the shear and shear type deformation of the exterior wall with openings and calculate equivalent solid wall properties such that the wall may be approximated analytically as a string of linear members.

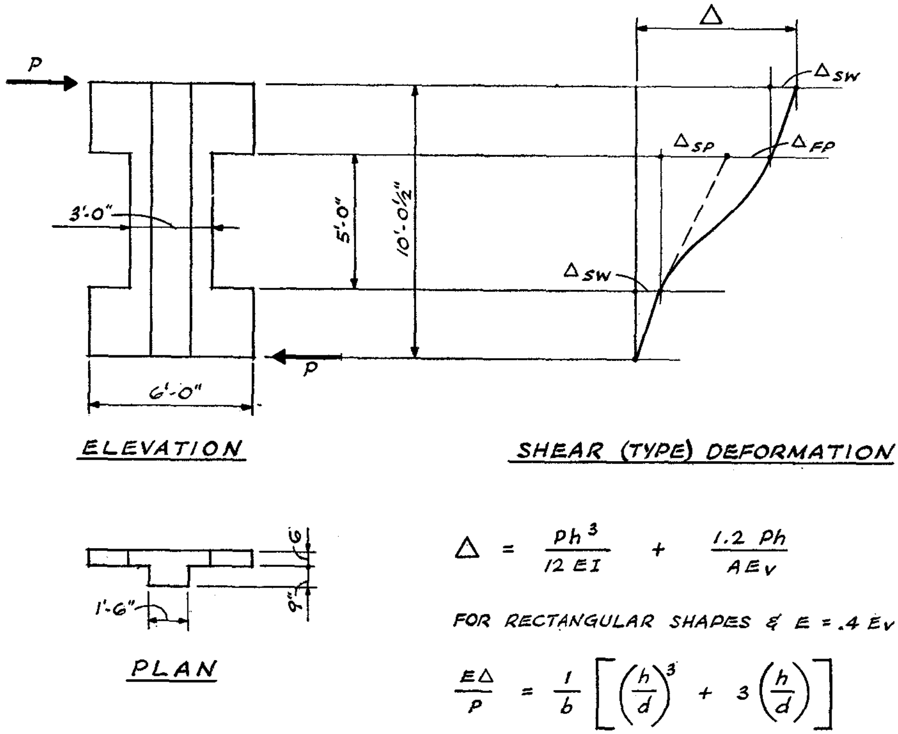
Figs. 2 and 3 illustrate how this has been done. A typical piece of exterior wall one story high and 6 ft wide has been considered. The shear type deformation of this panel consists of the shear deformation of the wall above and below the openings (Δ_{sw}) plus the shear deformation of the pier (Δ_{sp}) plus the flexural deformation of the pier (Δ_{fp}) which is based on the assumption that the piers are fixed against rotation at top and bottom.

The total calculated deflection was set equal to the shear deformation of a solid wall panel 6 ft wide and one story high and an equivalent wall thickness of 2.48 in. was calculated. This thickness was used to calculate the shear area, *AY*, for any portion of the wall used in the STRESS analysis. *AX* (the axial area) and *IZ* (the flexural moment of inertia) were calculated using the average horizontal cross section through the wall including the piers.

Step 3

Model the half sidewall as a string of linear members located at the middle of the wall with stiff stubs projecting out at each floor line to the corners of the building (see Fig. 4). Give these stubs infinitely large flexural properties. Their purpose is to model the full physical dimension of the wall.

Also, model the half end wall as two vertical linear members, one at the corner and one at the middle of the end wall, connected by flexurally rigid struts



$$\Delta = \frac{Ph^3}{12\,EI} + \frac{1.2\,Ph}{AE_v}$$

$$\frac{E\Delta}{P} = \frac{1}{b}\left[\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right]$$

Fig. 2. Formulas for calculating shear type deformation.

AY SHEAR AREA CALCULATION

UPPER WALL $\quad \Delta_{SW} \approx 3\left(\frac{h}{d}\right) = \left(3 \times \frac{2.52}{6}\right) = 1.26$

LOWER WALL $\quad \Delta_{SW} \approx$ do. $= 1.26$

PIER (SHEAR) $\quad \Delta_{SP} \approx 3\left(\frac{h}{d}\right) = \left(3 \times \frac{5.0}{3}\right) = 5.00$

PIER (FLEXURE) $\quad \Delta_{FP} \approx \left(\frac{h}{d}\right)^3 = \left(\frac{5}{3}\right)^3 = 4.63$

$$\frac{E\Delta}{P} = 12.15/b$$

FOR EQUIVALENT SOLID WALL THICKNESS "t"

$$\frac{12.15}{b} = \frac{1}{t} \times 3\left(\frac{h}{d}\right) = \frac{3 \times 10.04}{6t}$$

$t/b = .413 \qquad b = 6''$

$t = 2.48'' = .206'$

$\longrightarrow AY = 2.48'' \times 36\ (12) = 1070\ in.^2$

AX AND IZ CALCULATION

SOLID WALL $\quad \frac{1}{2} \times 6' \times .5 = 1.50$

PIER WALL $\quad \frac{1}{2} \times 3' \times .5 = .75$

PILASTER $\quad .75 \times 1.5 = 1.13$

FOR AVERAGE CROSS SECTION AREA: $t = \frac{3.38}{6'} \times 12 = 6.76''$

$\longrightarrow AX = 6.76 \times 36\ (12) = 2919\ in.^2$

$$\longrightarrow IZ = \frac{2919 \times (72 \times 12)^2}{12} = 181{,}580{,}000\ in.^4$$

Fig. 3. Properties of section for calculating shear type deformation.

at each floor line. For both the side wall stubs and end wall struts, use an equivalent effective shear area, *AY*, thus including the effect of shear deformation.

Step 4

By a judicious selection of joint and member releases in the STRESS analysis, place the side wall, the end wall and the moment resisting frame in the same "PLANE FRAME" (see Fig. 5).

The key release is at the corner where the side wall stubs join the corner member. The only force transfer allowed is a vertical shear.

Figs. 6 and 7 are computer plots of the resultant "PLANE FRAME" models. Solutions were obtained in each of

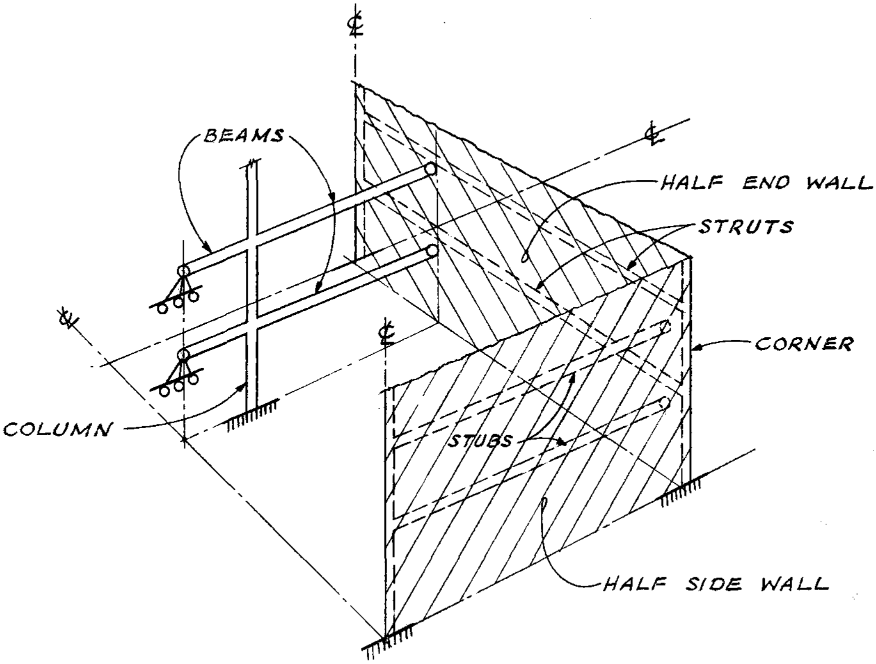


Fig. 4. Modeling of structural frame.

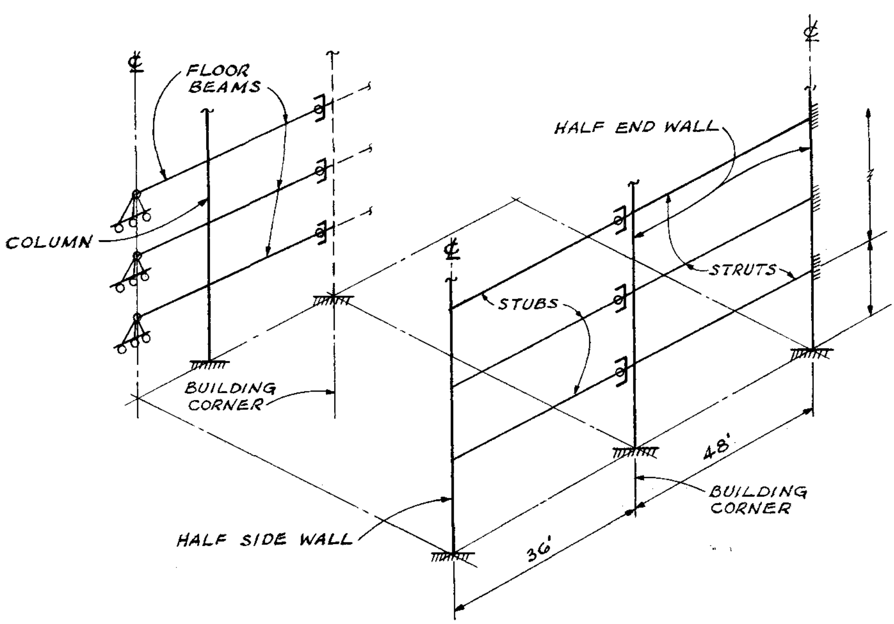


Fig. 5. Mathematical model of structural frame.

Table 5. Sample computer output of member forces (axial, shear, and moment) in foot and kip units due to wind load (Loading 1).

MEMBER	JOINT	AXIAL	SHEAR	MOMENT
1	1	0.000	186.064	11771.14
1	5	0.000	-186.064	-8980.17
2	5	0.000	180.293	9299.53
2	9	0.000	-180.293	-7489.03
3	9	0.000	175.166	7959.12
3	13	0.000	-175.166	-6200.10
4	13	0.000	169.117	6782.63
4	17	0.000	-169.117	-5084.35
5	17	0.000	163.068	5748.02
5	21	0.000	-163.068	-4110.49
6	21	0.000	155.893	4829.74
6	25	0.000	-155.893	-3264.26
7	25	0.000	148.735	4018.21
7	29	0.000	-148.735	-2524.61
8	29	0.000	141.595	3296.16
8	33	0.000	-141.595	-1874.26
9	33	0.000	134.492	2649.44
9	37	0.000	-134.492	-1298.87
10	37	0.000	127.156	2066.15
10	41	0.000	-127.156	-789.24
11	41	0.000	118.513	1538.79
11	45	0.000	-118.513	-348.68
12	45	0.000	108.832	1072.53
12	49	0.000	-108.832	20.85
13	49	0.000	99.342	671.52
13	53	0.000	-99.342	326.06
14	53	0.000	89.854	329.21
14	57	0.000	-89.854	573.09
15	57	0.000	80.370	42.36
15	61	0.000	-80.370	764.71
16	61	0.000	70.773	-191.11
16	65	0.000	-70.773	902.52
17	65	0.000	61.854	-371.61
17	69	0.000	-61.854	992.75
18	69	0.000	52.246	-504.19
18	73	0.000	-52.246	1028.86
19	73	0.000	42.713	-581.34
19	77	0.000	-42.713	1010.26
20	77	0.000	33.179	-601.14
20	81	0.000	-33.179	934.32
21	81	0.000	23.643	-559.55

the two normal directions for both wind load and earthquake load. The total running time on an IBM 1130 Computer was 102 minutes.

The results of this analysis are the member forces (axial, shear, and moment) in all 184 members as well as all reactions and joint displacements. Table 5 is the first page of the computer printout.

Figs. 8 and 9 are plots of the wall moment, shear, and deflection diagrams

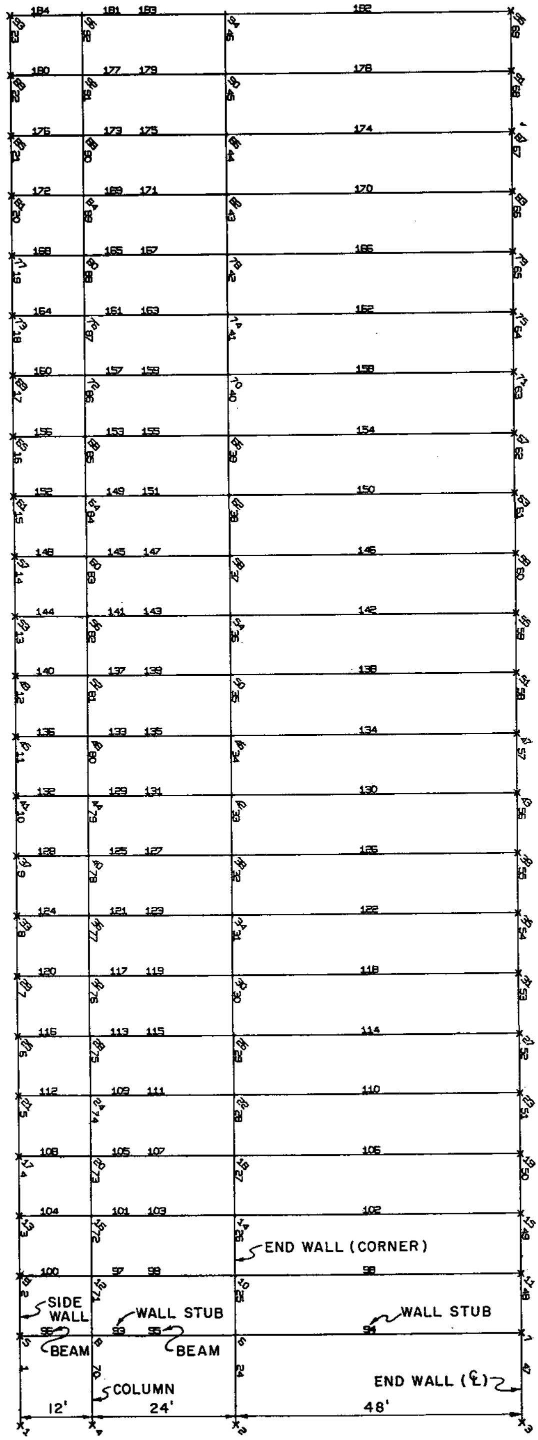


Fig. 6. Plot of frame in x direction.

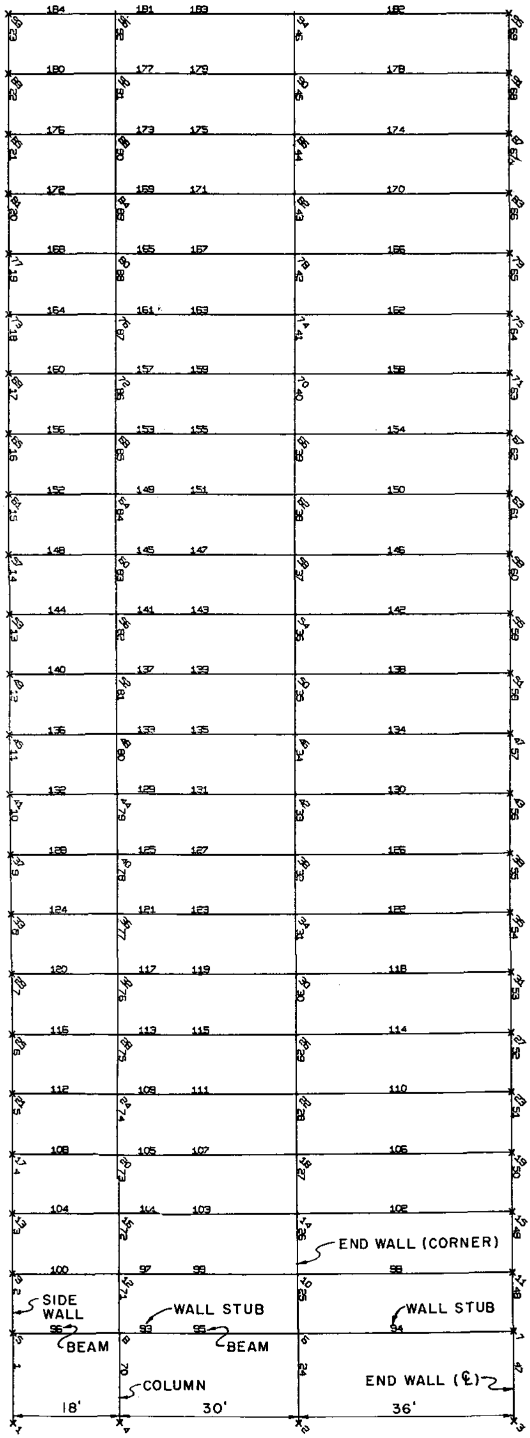


Fig. 7. Plot of frame in z direction.

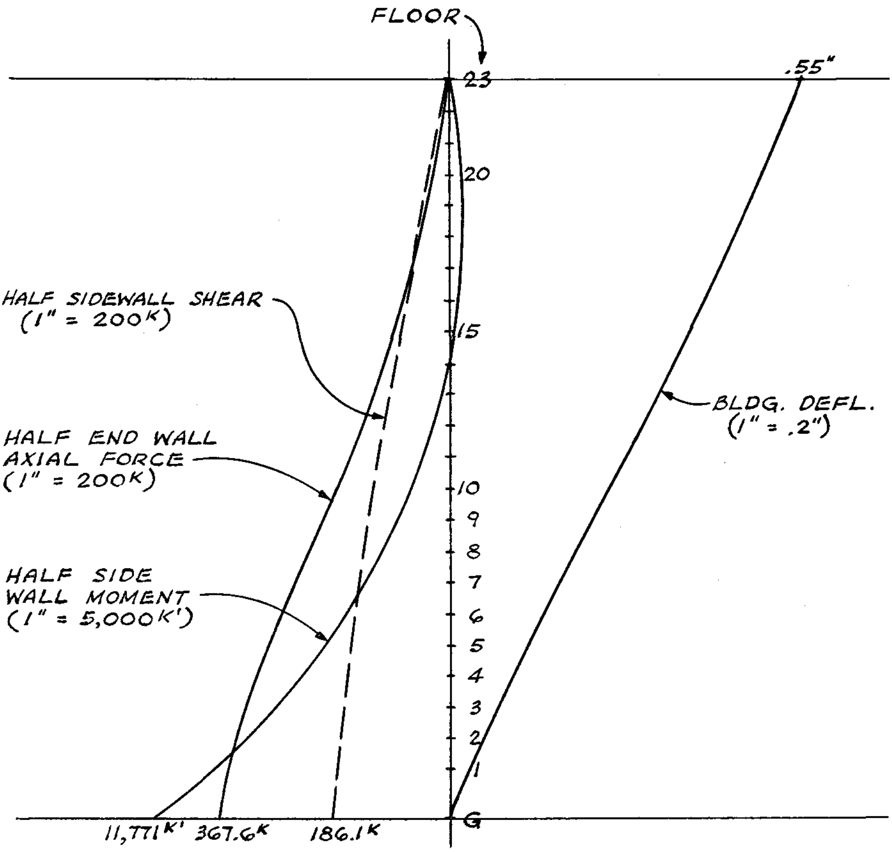


Fig. 8. Wall moment, shear, and deflection. Wind load is in x direction.

in each of the two directions. Notice that the total overturning moment resisted by the wall system consists of the sum of the moment in the side wall plus the couple due to the axial forces in the end wall.

Figs. 10 and 11 are plots of moment diagrams for the moment resisting frame. As might be expected, considering the relative stiffnesses of the elements, the moments in the columns and beams are relatively small.

Figs. 12 and 13 illustrate the stresses applied to the foundations. Note the axial force, moment, and shear at the base of the column, the shear and overturning moment at the base of the side wall and the axial force at the base of the end wall.

Fig. 14 illustrates the distribution of the vertical shear transfer at the corners.

RESULTS OF ANALYSIS

The results of the analysis can be summarized as follows:

1. The moments, shears, and axial forces in the column and beam framing have been obtained.

2. The total shear and moment in the side wall has been obtained, but

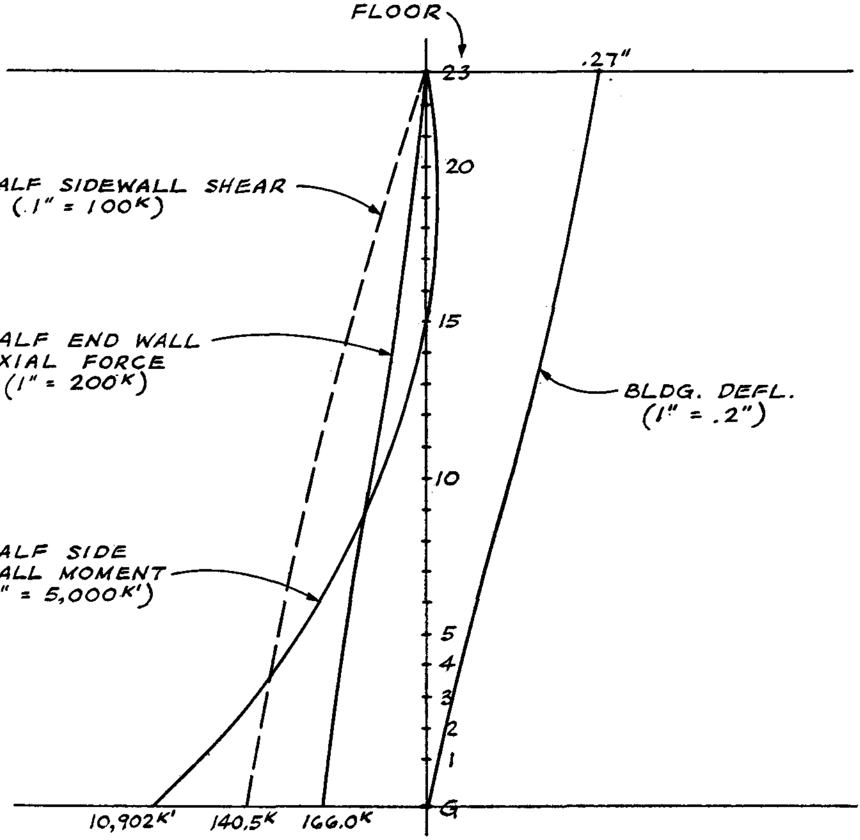


Fig. 9. Wall moment, shear, and deflection. Wind load is in z direction.

(please note) *not* the distribution of the shear or the flexural fiber stress in the side wall.

3. The total axial force in the end wall has been obtained but *not* its distribution.

Note that the above analysis does have its limitations. However, based on other more sophisticated analyses of similar buildings, we know that the fiber stress distribution tends to concentrate at the corners and it is possible to draw an approximate stress distribution in the walls as noted in Figs. 12 and 13.

Since our normal structural design method is one of making elastic analyses and then proportioning by the ultimate strength design method, it seems that the results obtained should be adequate for design. Other significant data obtained are the interacting shears between the side wall and end wall, and the building deflections.

CONCLUDING REMARKS

In conclusion it might be of interest to compare the results of this analysis to several more conventional analyses

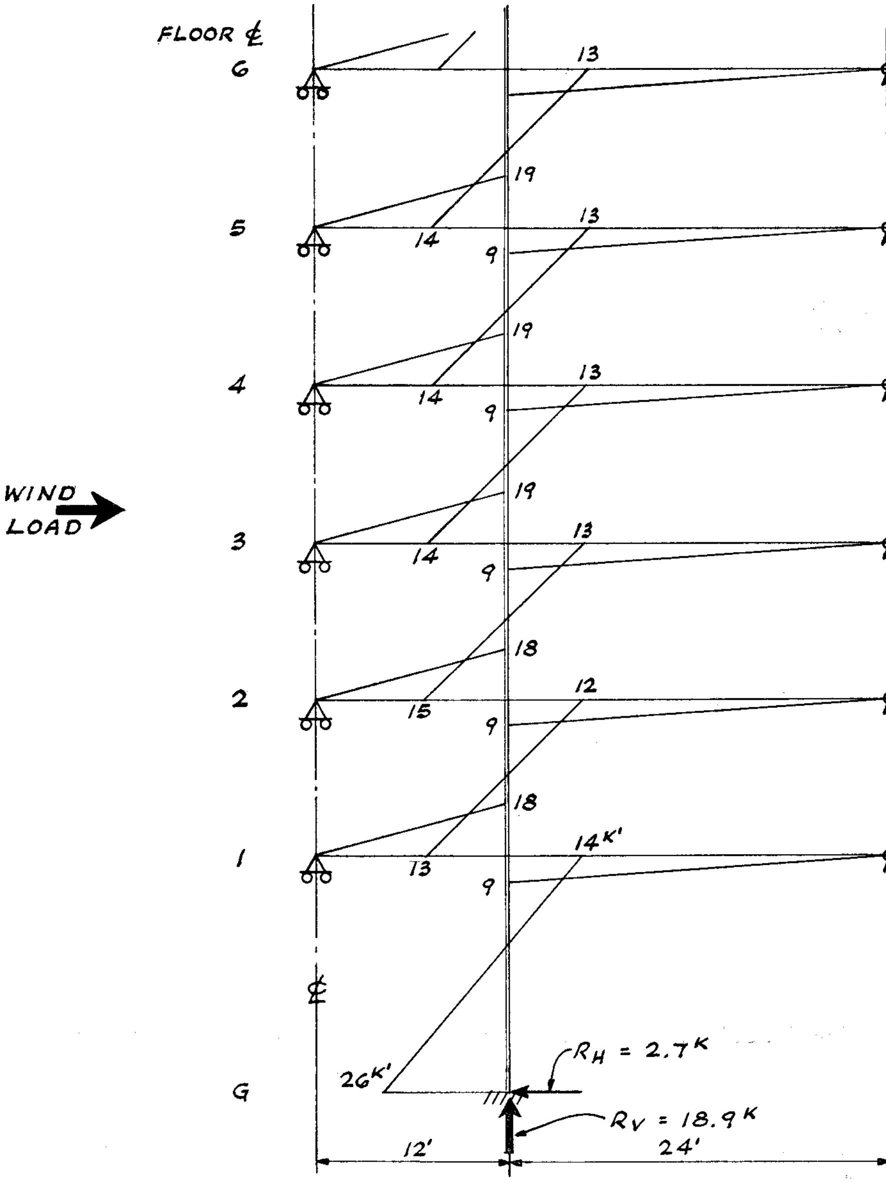


Fig. 10. Column and beam moments (ft-kips). Wind load is in x direction.

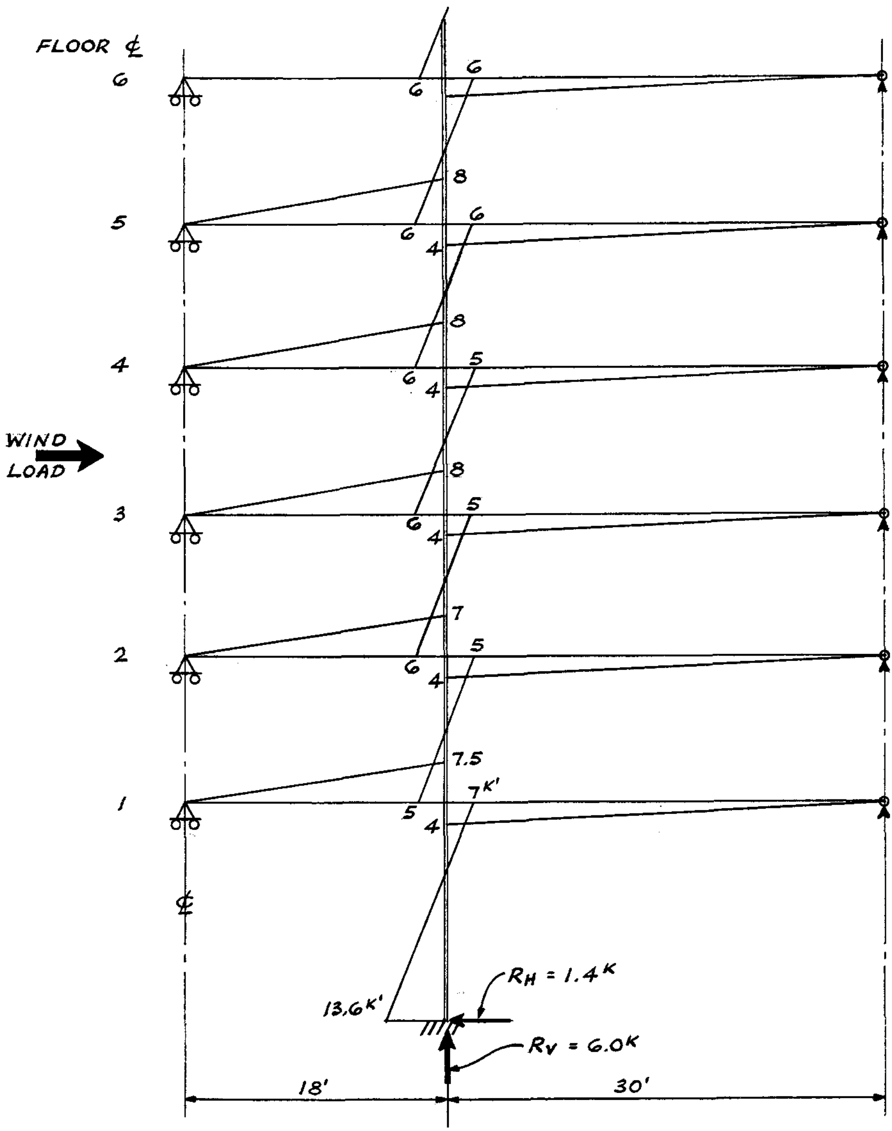


Fig. 11. Column and beam moments (ft-kips). Wind load is in z direction.

based on very simple assumptions. One possible assumption is that all overturning moment is resisted entirely by the side wall.

In this case the moment of inertia and the effective area of the side wall may be calculated as follows:

$$I = 6.76 \times (72 \times 12)^3/12$$
$$= 363{,}319{,}600 \text{ in.}^4$$

$$A = 2.48 \times (72 \times 12)$$
$$= 2142 \text{ sq in.}$$

The deflection may be calculated as the sum of the shear and flexural de-

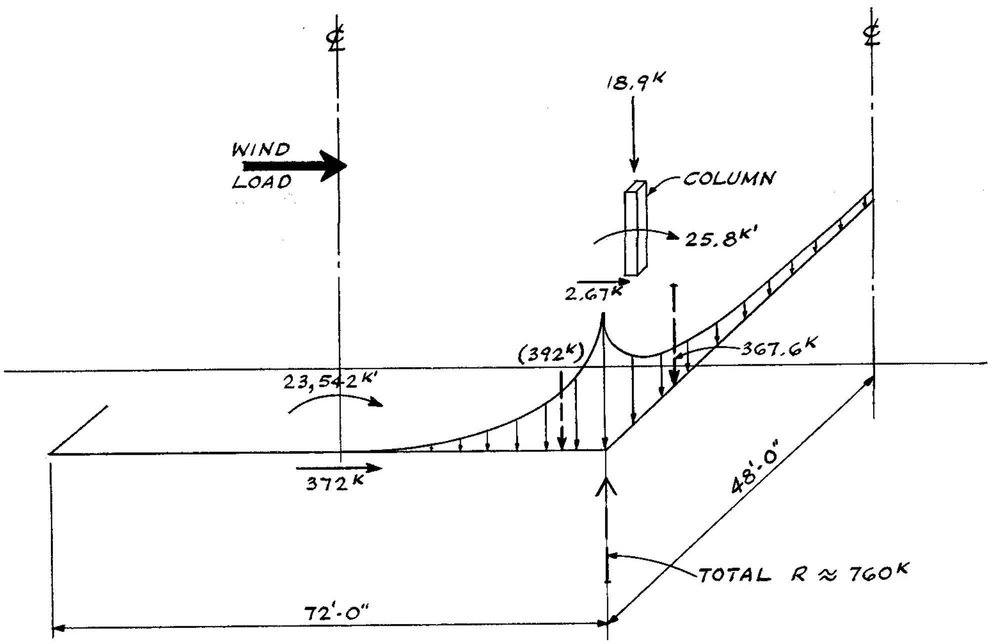


Fig. 12. Stresses applied to foundations in transverse x direction.

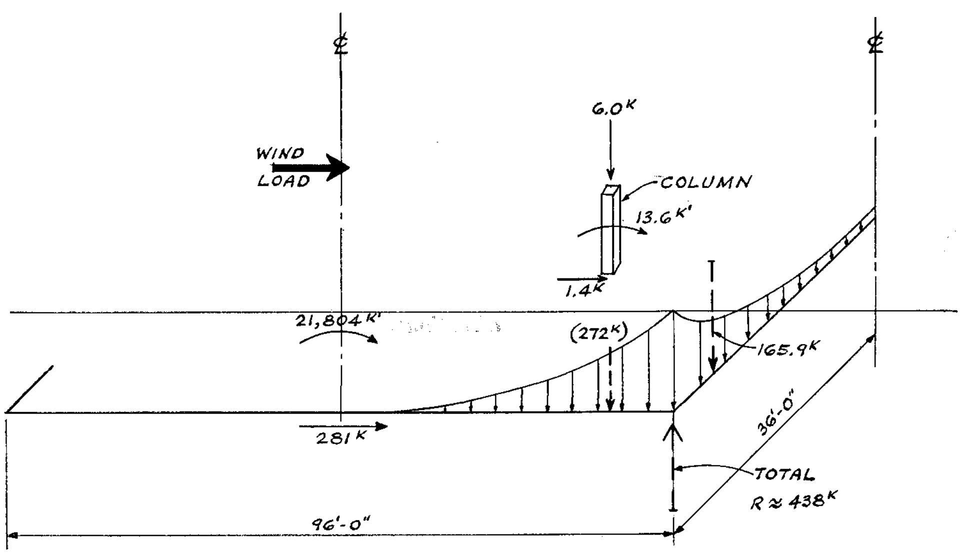


Fig. 13. Stresses applied to foundations in longitudinal z direction.

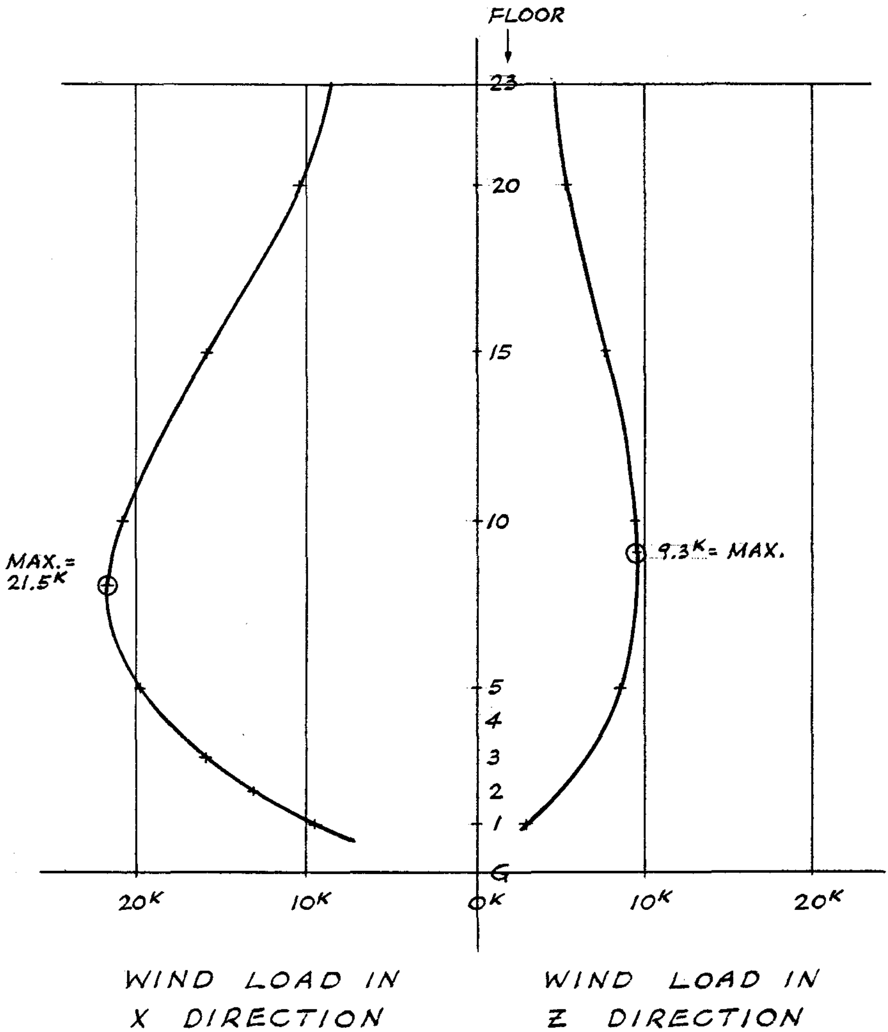
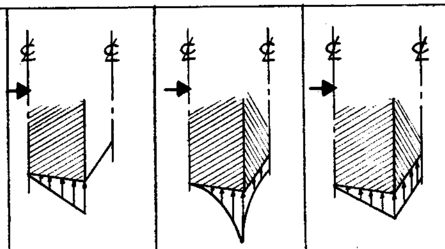


Fig. 14. Shear per story in corner of building due to wind load in x and z directions.



FORCES AND DEFLECTION WIND LOAD IN X DIRECTION	SIDE WALL ONLY	COMPUTER SOLUTION	FLEXURAL TUBE
HALF SIDEWALL SHEAR (K)	189^K	186^K	189^K
HALF SIDEWALL MOMENT (K-FT)	$25,300^{K'}$	$11,771^{K'}$	$5,038^{K'}$
HALF END WALL AXIAL FORCE (K)	0	368^K	563^K
BUILDING DEFLECTION (IN)	1.58"	.55"	.49"

Fig. 15. Forces and deflections due to wind load in x direction.

flections. This method errs in totally neglecting the flange effect of the end walls.

Another assumption could be that the full, tubular section is developed in which case the moment of inertia of half the building may be calculated as an equivalent rectangular tube:

$$6.67 \times (72 \times 12)^3/12 = 363{,}319{,}600 \text{ in.}^4$$

$$2 \times 6.67 \times (48 \times 12) \times (36 \times 12)^2 = 1{,}451{,}138{,}200 \text{ in.}^4$$

$$IZ = 1{,}814{,}507{,}800 \text{ in.}^4$$

Again, the deflection is the sum of the shear and flexural deflections. This assumption errs in not considering the shear deformations of the walls and their effect on stress distributions.

Fig. 15 tabulates the side wall moment and shear, the end wall axial force and the maximum building deflection for these two simplifying assumptions together with the computer results. It can be seen that the simplifying assumptions may lead to substantially different results.

Discussion of this paper is invited.
Please forward your discussion to PCI Headquarters by April 1, 1974, to permit publication in the May-June 1974 PCI JOURNAL.