SHEAR DESIGN OF PRESTRESSED CONCRETE STEPPED BEAMS

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It is demonstrated experimentally that the shear force leading to cracking at the re-entrant corner of a stepped beam can be considered as the shear resisted by the concrete in the shear design. Recommendations for the detailing of the step are given.

The prestressed stepped beam connection shown in Fig. 1 provides an economical and efficient means of connecting precast to precast and precast to cast-in-place concrete members. The connection is economical because all the miscellaneous mild steel is eliminated and because the span of the drop-in beam is effectively reduced if the connection can be located at the inflection point of the total span.

The designer has little information for the shear design of the stepped beam end, because this case is not covered by present design codes. The questions which arise in the design of a stepped beam are: (1) what is the shear force at which shear cracks start to form at the re-entrant corner of the step and (2) can this cracking shear be considered to be the contribution of the concrete to the shear strength of the

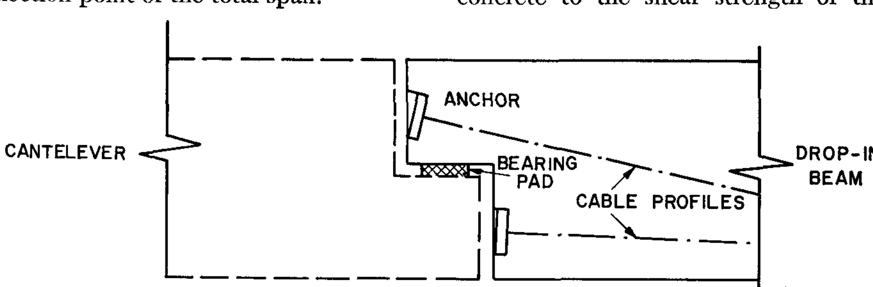


Fig. 1. Prestressed stepped beam connection

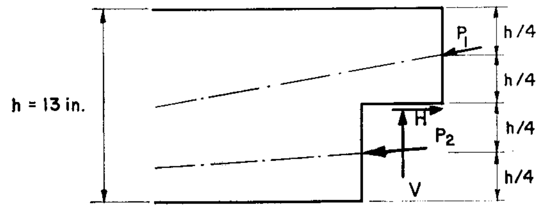


Fig. 2. Forces applied to beam end zone

step? The first question was investigated using the finite element technique and the second question was studied experimentally.

THEORETICAL STUDY

It is generally accepted[1,2] that the total shear resisted by a beam at any cross section is the summation of the shear resisted by the concrete and the shear carried by the shear reinforcement. In prestressed beams with draped tendons the vertical component of the prestressing force is added. The shear resisted by the concrete at a section is assumed to be approximately equal to the shear force at the section at the formation of an inclined crack.

Although the concrete cannot resist any tensile forces after cracking, other mechanisms such as the concrete compression zone, the dowel forces of bars crossing the crack, and the aggregate interlock take over the shear previously resisted by the uncracked concrete. No attempt is made here to discuss the shear resistance of these mechanisms. In zones of prestressed concrete beams where the shear force is high and the moment small, the shear carried by the concrete is equal to the web cracking load which is determined by equating the principal tensile stress at the centroid of the member, to a design tensile strength of the concrete.[1,2]

In a stepped beam this approach is not easily applicable because of the complicated stress configuration at the re-entrant corner.[3] Also, no evidence has been available, to date, that the shear force at the formation of a crack at the re-entrant corner can be considered as the shear force resisted by the concrete. The theoretical part of this study is concerned with the calculation of concrete stresses at the re-entrant corner.

The finite element technique[4] was used to determine the elastic stress distribution in the step. (The computer program with a brief description of input and output data is available from the authors on request.)

Four different loading cases (Fig. 2) were investigated for the beam shown in Fig. 5:

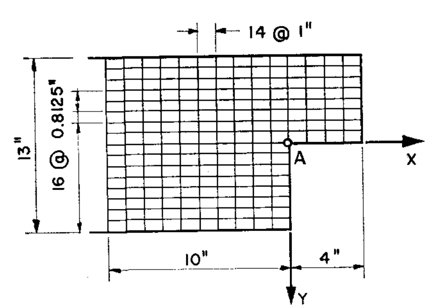


Fig. 3. Fine mesh for finite element analysis

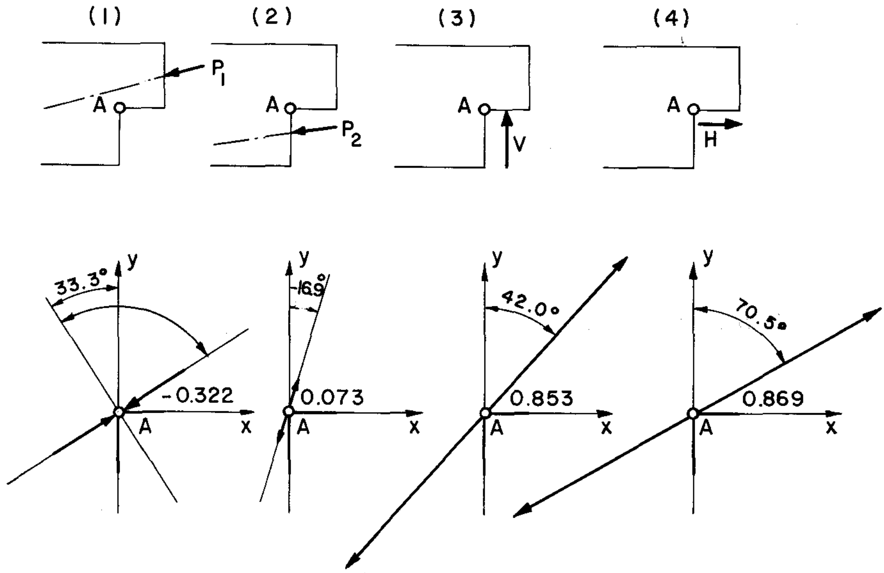


Fig. 4. Principal stresses at re-entrant corner due to unit forces P_1, P_2, V, and H

1. Prestressing force $P_1 = 1$
2. Prestressing force $P_2 = 1$
3. Vertical force $V = 1$
4. Horizontal force $H = 1$

These forces were applied to a beam of 1 in. thickness. The finite element analysis was carried out in two steps: first, the whole shear zone of the beam was analyzed using a coarse mesh of rectangular elements to find the nodal forces at a distance of 10 in. from the step. These nodal forces were then applied to the fine mesh of Fig. 3. The stresses at the re-entrant corner were determined by averaging the stresses at the corners of the three elements which are connected there. The stress components and the principal stresses at the re-entrant corner (Point A) are listed in Table 1 for all four loading cases, and shown in Fig. 4. With the exception of load P_1, all forces cause tensile stresses at Point A. With the stress components due to unit forces known, the cracking loads can be determined if the tensile strength of the concrete is known. Calculation of the cracking load will be

Table 1. Stress Components and Principal Stresses at Re-entrant Corner Due to Unit Forces (Fig. 2).

Stresses	$P_1 = 1.0$	$P_2 = 1.0$	$V = 1.0$	$H = 1.0$
f_x	−0.248	0.023	0.508	0.801
f_y	−0.150	0.068	0.575	0.327
f_{xy}	0.113	−0.015	−0.315	−0.192
f_1	−0.075	0.073	0.858	0.869
f_2	−0.322	0.018	0.225	0.259
θ	33.3	−16.9	−42.0	−70.5

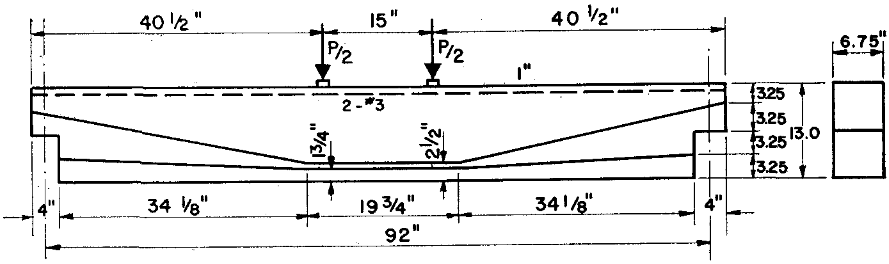


Fig. 5. Dimensions of test beams and reinforcement of Beam W1.

shown after discussion of the experimental investigation.

EXPERIMENTAL STUDY

Five prestressed beams were tested to verify the theoretical studies on crack initiation, and to determine whether the cracking load is equal to the shear force which can be supported by the concrete. The principal test variables were:

1. Support condition.
2. Type and amount of shear reinforcement.

Three beams were supported vertically, the remaining two were supported such that an inclined reaction developed as the beam was loaded. This inclined reaction was introduced to simulate the horizontal tension which can develop at the support due to creep, shrinkage, and temperature if shortening of the drop-in beams is prevented.

TEST SPECIMENS

All the beams had the same cross

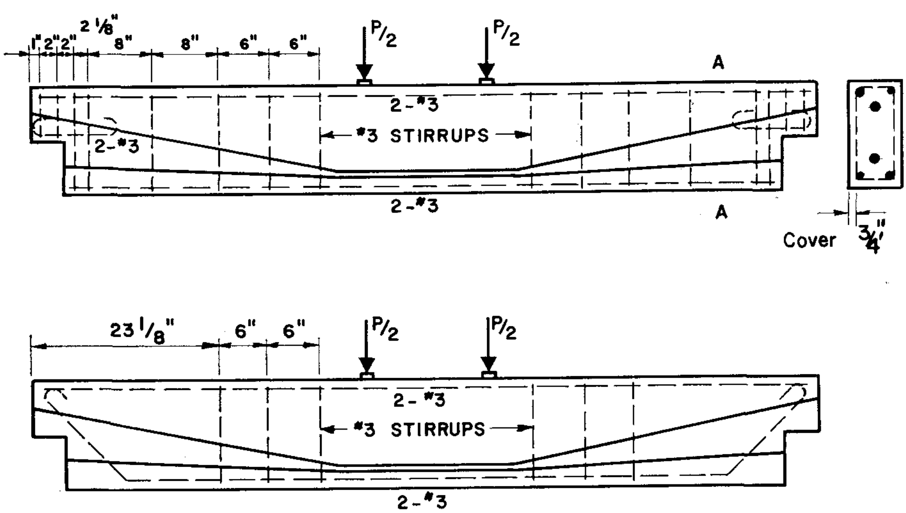


Fig. 6. Reinforcement of beams with shear reinforcement.
Top: Beams W2 and W4. Bottom: Beams W3 and W5.

section, $b/h = 6.75 \times 13.0$ in., an overall length of 96 in. and a span of 92 in. The steps on each end were 4 in. long. All beams were post-tensioned by two 0.6 in. draped deformed bars (Fig. 5). The 1-in. ducts were grouted after stressing of the bars. Beam W1 had no shear reinforcement, Beams W2 and W4 had vertical stirrups only [Fig. 6 (top)], and Beams W3 and W5 had bars bent up at 45 deg at the beam end [Fig. 6 (bottom)]. Beams W1, W2, and W3 were supported vertically.

To introduce vertical and tensile forces in Beams W4 and W5 they were supported on a plane inclined at about 30 deg to the beam axis.

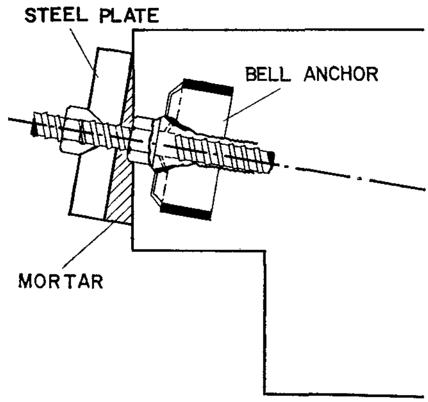
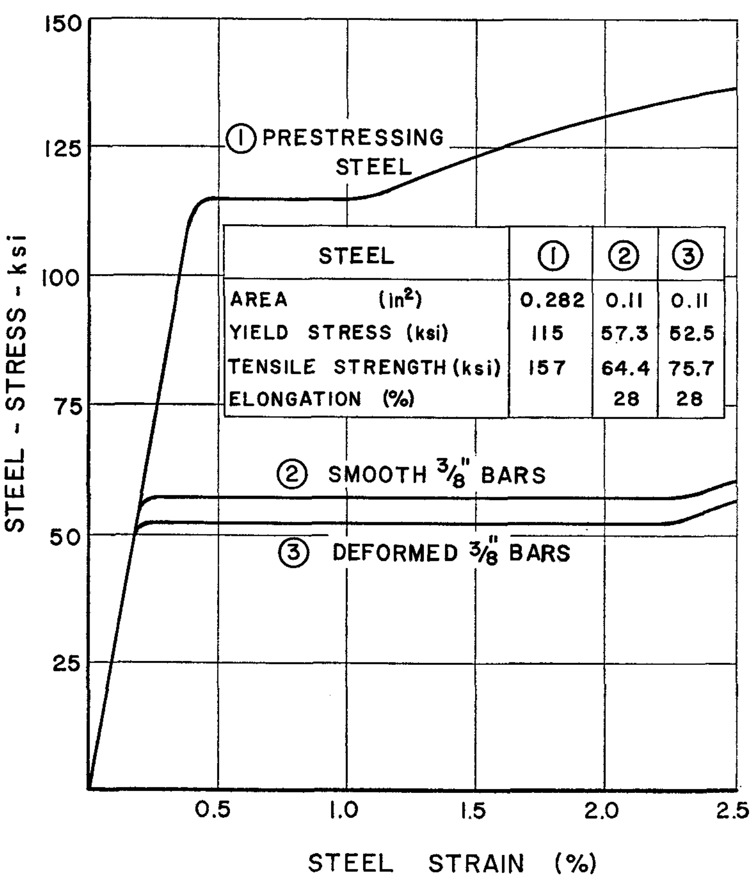


Fig. 7. Bell anchor of prestressing bars. Strengthening of anchor zones using steel plate



STEEL	①	②	③
AREA (in²)	0.282	0.11	0.11
YIELD STRESS (ksi)	115	57.3	52.5
TENSILE STRENGTH (ksi)	157	64.4	75.7
ELONGATION (%)		28	28

Fig. 8. Stress-strain diagrams of prestressing and reinforcing bars

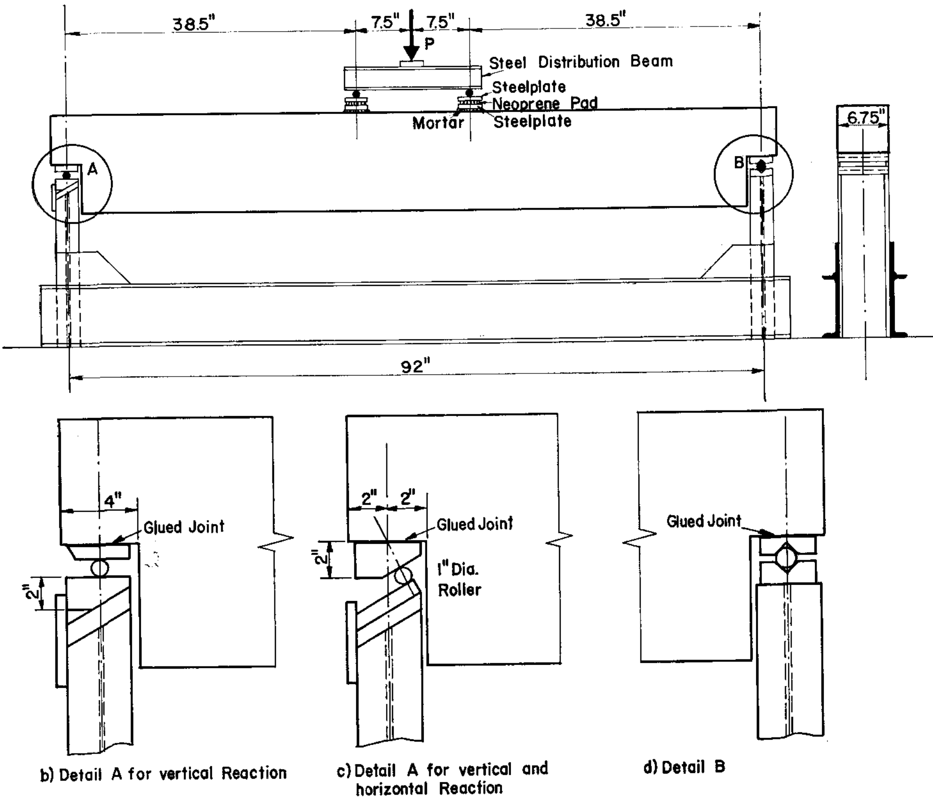


Fig. 9. Test setup

All the beams were cast in one placement using ready-mixed concrete with a ¾ in. maximum aggregate size and a 2-in. slump. The cylinder strength at prestressing (age 21 days) was 5380 psi and at testing (age 42 days) 5800 psi. Stressing of the bars was immediately followed by grouting of the ducts. The grout strength was 5100 psi at 28 days.

The 0.6 in. bars were Dywidag bars with the deformations of the bars serving as a thread for the anchor nuts. Each anchor nut bore against a bell anchor (Fig. 7). The stress-strain relation for the prestressing steel is shown in Fig. 8.

The steel used for the stirrups and the nonprestressed longitudinal reinforcement consisted of plain #3 bars, except the short hooked bars at mid-depth of the beam ends were #3 deformed bars. Their properties, together with their stress-strain curves, are also given in Fig. 8.

LOADING AND MEASUREMENTS

Two symmetrical loads were applied incrementally to failure. For the combined horizontal and vertical reaction, a triangular bearing plate was used (Fig. 9c). Measurements of strains and deflections were taken at the locations indicated in Fig. 10, and the extent of crack propagation was marked at each load increment.

BEHAVIOR OF TEST BEAMS

Up to the formation of the first crack, each of the beams behaved elastically. The first crack in all the beams formed at the re-entrant corner of the step. These cracks propagated nearly horizontally towards the load points. The beams supported vertically, cracked at approximately 17.5 kips (Table 2). The ones with horizontal and vertical reactions cracked under a load of about 13.5 kips. The formation of these horizontal cracks resulted in a sharp increase in the steel stress in the shear reinforcement. The crack patterns of all beams after failure are shown in Fig. 11. After formation of the first crack at 18.0 kips, Beam W1 resisted another 12 kips before failure occurred at 30 kips. Prior to failure the crack had opened about ⅛ in. at its root.

Beam W2 with vertical stirrups reached the highest failure load ($P_u =$ 43.5 kips). This beam failed by crushing of the concrete in the constant moment zone, after yielding of the prestressed bars indicating flexural failure. The stirrups did not reach the yield point and the shear crack was narrow up till failure.

Beam W3 failed under a load of $P_u =$ 31.6 kips far below the flexural ultimate load. Failure occurred due to bursting of the anchorage zone. The bent-up bars with hooks were obviously inadequate to confine the anchorage zone. The hooks of the plain bars split the concrete anchorage zone so that no resistance to shear was provided by the bent-up bars. As a secondary effect, the shear compression zone near the loading point crushed.

Beam W4 (vertical stirrups, horizontal and vertical forces) showed some cracks in the anchorage zones under P

Table 2. Test Results.

Beam	Web reinforcement	Reactions	Cracking load: Test, kips	Cracking load: Theory, kips	Cracking load: Test / Theory	Ultimate load: Test, kips	Ultimate load: Theory: Flexure, kips	Ultimate load: Theory: Shear,* kips	Ultimate load: Test / Shear theory
W1	—	Vertical	18.0	18.11	0.99	30.0	36.0	27.86	1.08
W2	Vertical stirrups	Vertical	17.8	18.11	0.98	43.5	42.2	43.70	1.00
W3	Inclined bars	Vertical	17.0	18.11	0.94	31.6	42.2	33.20	0.95
W4	Vertical stirrups	Inclined	13.5	13.50	1.02	38.0	40.4	37.54	1.01
W5	Inclined bars	Inclined	13.0	13.50	0.98	40.0	40.4	41.06	0.98

*See Table 3.

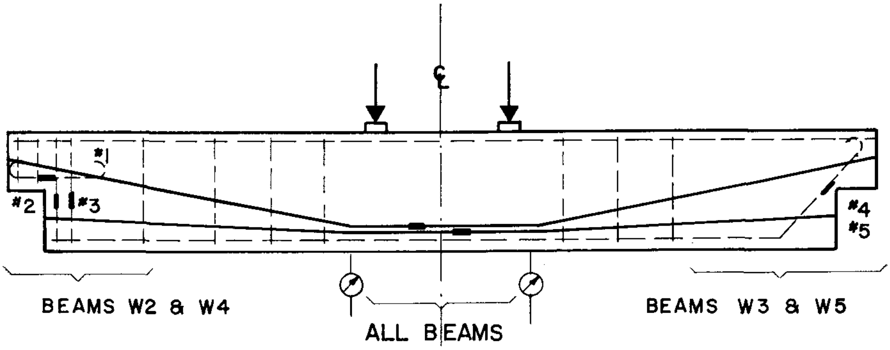


Fig. 10. Position of strain gages and dial gages

= 8 kips. To avoid distress of the anchorage zone, a steel plate was provided at the beam ends (Fig. 7). Failure of Beam W4 occurred due to crushing of the shear compression zone near the load point. The stirrups had not reached yield under the failure load and the width of the shear cracks remained small up till failure. The load reached the theoretical flexural failure load.

Beam W5 (bent-up bars, horizontal and vertical reactions) was also strengthened by steel plates at the ends, after cracks were showing under low loads. This beam collapsed suddenly after the bent-up bars had reached the yield stress.

The ultimate load of $P_u = 40.0$ kips is slightly above the theoretical ultimate flexural load, yet the sudden type of failure (Fig. 11e) clearly indicated a shear failure.

STEEL STRESSES IN WEB REINFORCEMENT

The steel stresses were measured at the locations indicated in Fig. 10. The strain gages were mounted where the shear crack crossed the stirrups so that the measured values can be considered to be the maximum strains occurring in these bars. In Fig. 12 (top), representing the results of Beams W2 and W3 with vertical reactions, the stress increase after formation of the shear crack at $P = 18$ kips is quite pronounced. In Beam W2 the stirrup No. 2 closest to the beam end picks up the highest stress. From the strains recorded in Beams W2 and W3 it is obvious that the shear reinforcement was still far from reaching the yield point of 57.3 ksi at failure. The stirrups in Beams W4 and W5 with vertical and horizontal reactions also had a marked increase in stress after crack formation. The stresses in the vertical stirrups of Beam W4 stayed well below the yield point at failure while yield was reached in the two inclined bars of Beam W5.

EVALUATION OF TEST RESULTS

Cracking

The stresses resulting from unit loads (Table 1) may be used to predict the cracking load. Equating the principal tensile stress f_1 resulting from the various forces, to the tensile strength of the concrete f'_t, we get:

$$f_1 = f'_t = \tfrac{1}{2}(\Sigma f_x + \Sigma f_y) + \sqrt{\tfrac{1}{4}(\Sigma f_x - \Sigma f_y)^2 + \Sigma f^2_{xy}} \quad (1)$$

Rearranging leads to:

$$f'^2_t - f'_t(\Sigma f_x + \Sigma f_y) + (\Sigma f_x)(\Sigma f_y) - (\Sigma f_{xy})^2 = 0 \quad (2)$$

With the prestressing forces $P_1 = P_2 = 3.87$ kips per in. of beam width and putting the tensile strengths of the concrete $f'_t = 6\sqrt{f'_c}$, we can solve Eq. (2) for the shear force leading to cracking.

For the beams supported vertically ($H = 0$) this procedure results in $V = 1.22$ kips per in., which is equivalent to a cracking load $P_{cr} = 18.11$ kips.

For the beams with vertical and horizontal reactions ($H = 0.59V$) solution of Eq. (2) results in a cracking load

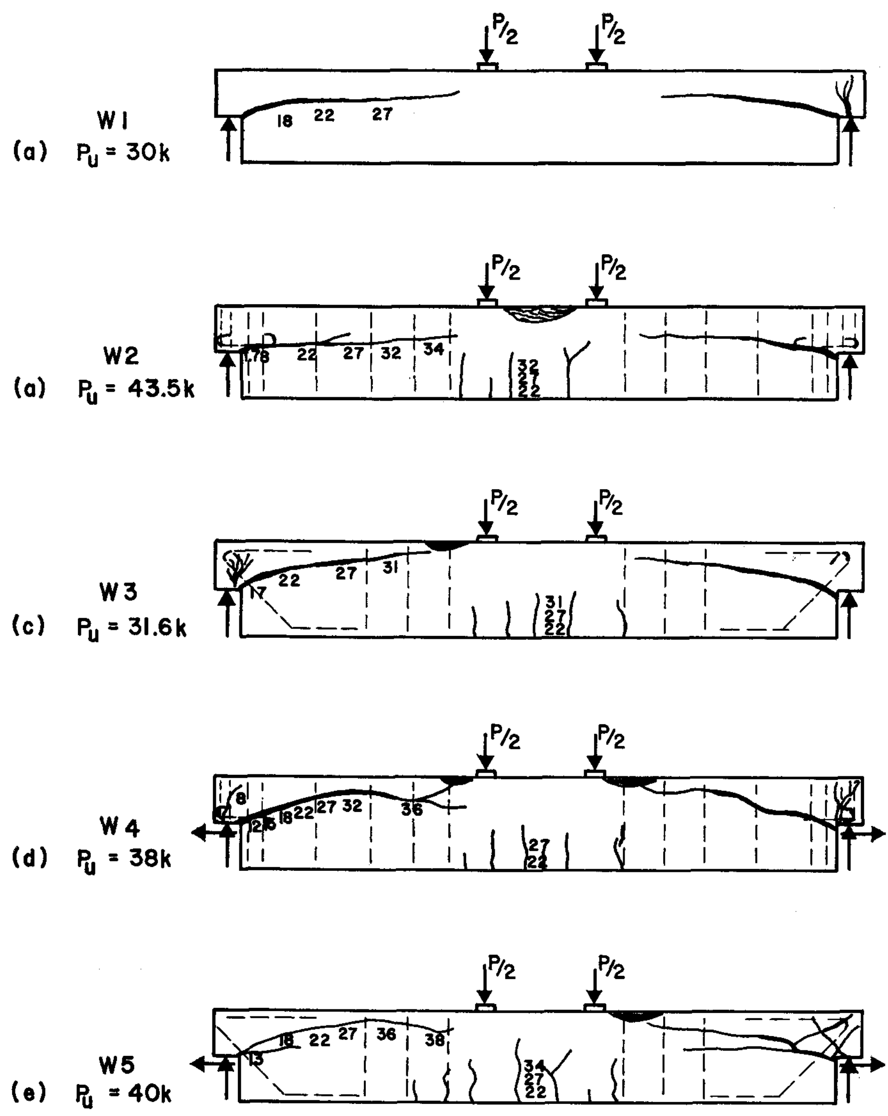


Fig. 11. Crack patterns at failure

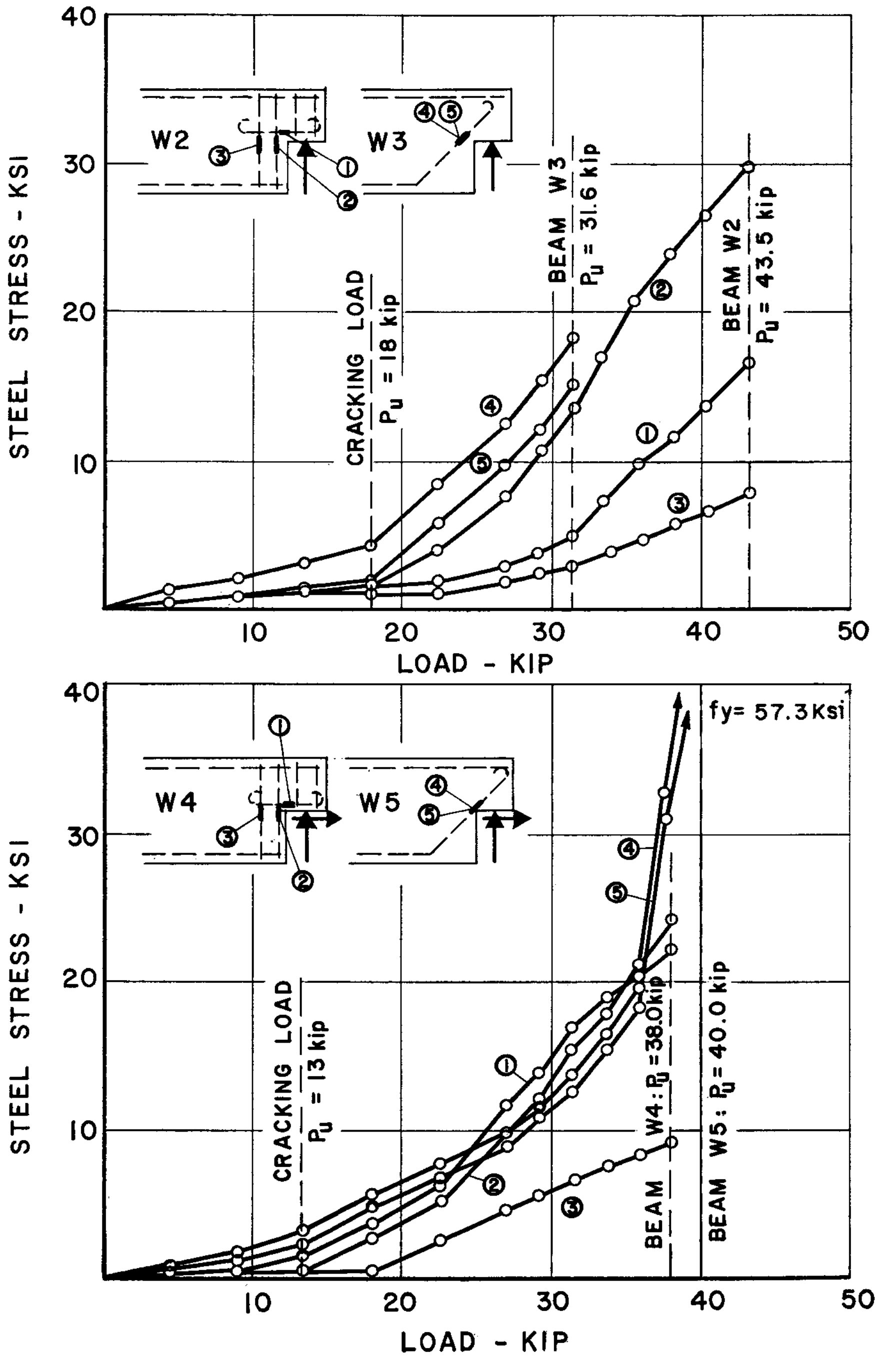


Fig. 12. Stresses in web reinforcement versus load

$P_{cr} = 13.50$ kips. These results compare well with the experimental cracking loads (see Table 2). The good agreement indicates that the finite element method with the mesh selected was appropriate for accurately predicting the cracking load at the re-entrant corner of the step.

Ultimate load

If we assume that the shear force existing when cracking occurs at the re-entrant corner, is equal to the shear force which continues to be resisted by the concrete, then the ultimate load can be calculated by adding to this force the vertical component of the tendon and the shear carried by the web reinforcement. This is done in Table 3 and the results are compared with the experimental failure loads. The shear force due to prestressing is the vertical component of the effective prestressing force. With $P_1 = 26.1$ kips at an angle of 10.7 deg, the vertical component is $V_P = P_1 \sin(10.7°) = 4.87$ kips. The shear resisted by the shear reinforcement is calculated from the stresses shown in Fig. 12.

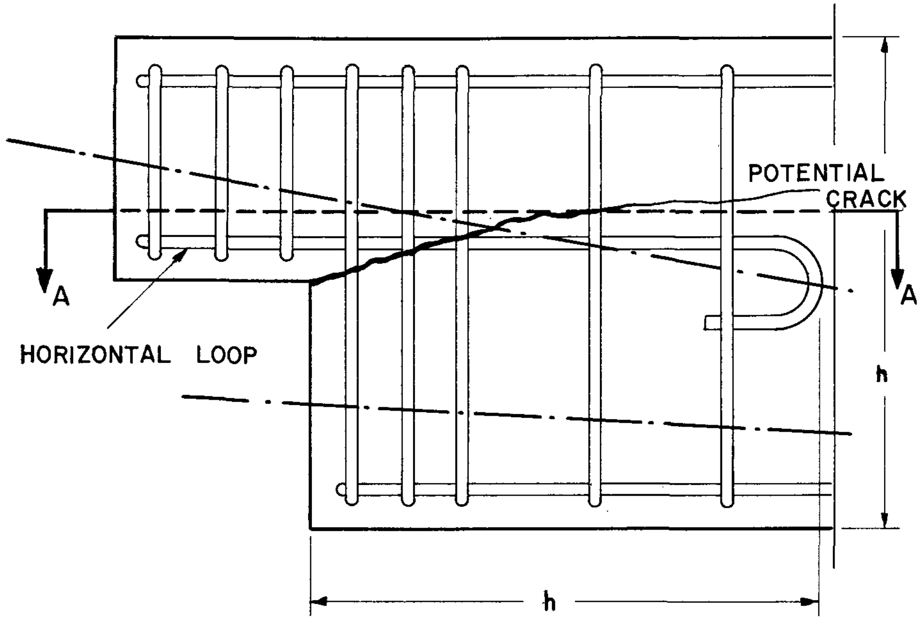
Comparison of the actual failure load with the shear force resulting from the shear components discussed above is shown in Column 7 of Table 3. The agreement is excellent.

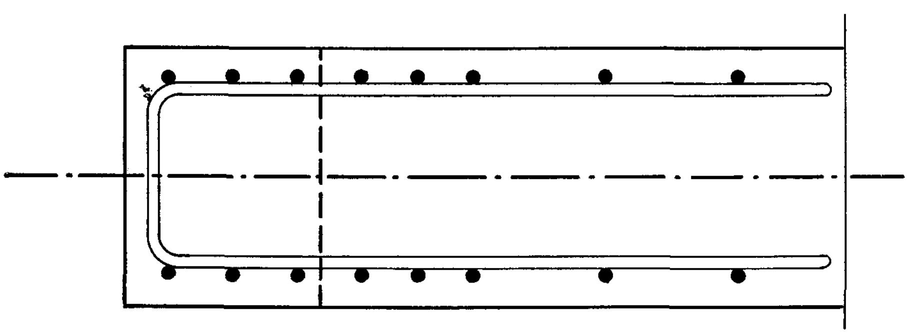
CONCLUSIONS

The shear force which causes shear cracks to form at the re-entrant corner of a stepped beam can be accurately predicted by a finite element analysis using a tensile strength for the concrete equal to $6\sqrt{f'_c}$. This cracking load can be taken as the shear force which is resisted by the concrete. The shear force which is in excess of the cracking load is resisted by the inclined tendon and the shear reinforcement crossing the crack.

Table 3. Shear Force Components at Ultimate and Comparison of Ultimate Shear Force With Test Results.

1	2 V_c	3 V_p	4 V_s	5 $V_u = V_c + V_p + V_s$	6 V_u	7
Beam	Cracking shear force,	Shear force due to prestressing,	Shear force in web reinforcement,	Total shear force,	Experiment	Experiment / Total shear force
	kips	kips	kips	kips	kips	
W1	9.06	4.87	—	13.93	15.00	1.08
W2	9.06	4.87	7.92	21.85	21.75	1.00
W3	9.06	4.87	2.67	16.60	15.80	0.95
W4	6.75	4.87	7.15	18.77	19.00	1.01
W5	6.75	4.87	8.91	20.53	20.00	0.98





SECTION A - A

Fig. 13. Recommended design details

Vertical and inclined shear reinforcement seem to be equally efficient in resisting shear.

As a result of the cracking of the anchorage zone under low load it must be concluded that special attention must be devoted to the choice of anchorage used for the tendons, particularly if the step is short.

RECOMMENDATIONS FOR THE SHEAR DESIGN OF THE END ZONE OF STEPPED BEAMS

For practical design the tensile stress resulting in cracking at the re-entrant corner should be assumed in accordance with present design codes to be equal to $4\sqrt{f'_c}$.

Accurate analytical methods must be used to predict the cracking shear. The vertical component of the prestressing force should be included. The shear force at ultimate which is not resisted by the concrete and the inclined prestressing steel must be supported by shear reinforcement, preferably vertical stirrups placed as close as possible to the beam end.

Careful detailing of the end zones is required to avoid spalling around the tendon anchors. If the step is short, tendons with steel anchor plates should be used to prevent bursting of the anchor zone. Horizontal bars at middepth must extend at least a distance equal to the beam depth beyond the re-entrant corner and should be provided with downward hooks to ensure anchorage below the plane of a potential shear crack (Fig. 13).

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