SHEAR TRANSFER IN REINFORCED CONCRETE—RECENT RESEARCH

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Shows how concrete strength, shear plane characteristics, reinforcement, and direct stress affect the shear transfer strength of reinforced concrete. Fundamental behavior of test specimens under load is reported, and hypotheses to explain the behavior are developed. It is concluded that shear-friction provisions of ACI 318-71 give a conservative estimate of shear-transfer strength below the stated limit of 800 psi. A design equation to develop higher shear transfer strength is presented.

Test program

Shear transfer across a definite plane must frequently be considered in the design of precast concrete connections[1,2]. A continuing study of the factors affecting shear transfer strength is in progress at the University of Washington. Factors so far included in the study are as follows:

1. The characteristics of the shear plane
2. The characteristics of the reinforcement
3. The concrete strength
4. Direct stresses acting parallel and transverse to the shear plane.

The influence of the first three factors has been studied in tests[3] of monolithically cast "push-off" specimens as seen in Fig. 1(a). Tests[4,5] to study the influence of direct stresses acting parallel and transverse to the shear plane were made on the "pull-off" and modified push-off specimens shown in Figs. 1(b) and 1(c) respectively. In all cases, the shear transfer reinforcement crosses the shear plane at right angles and is securely anchored so that it can develop its yield strength in tension.

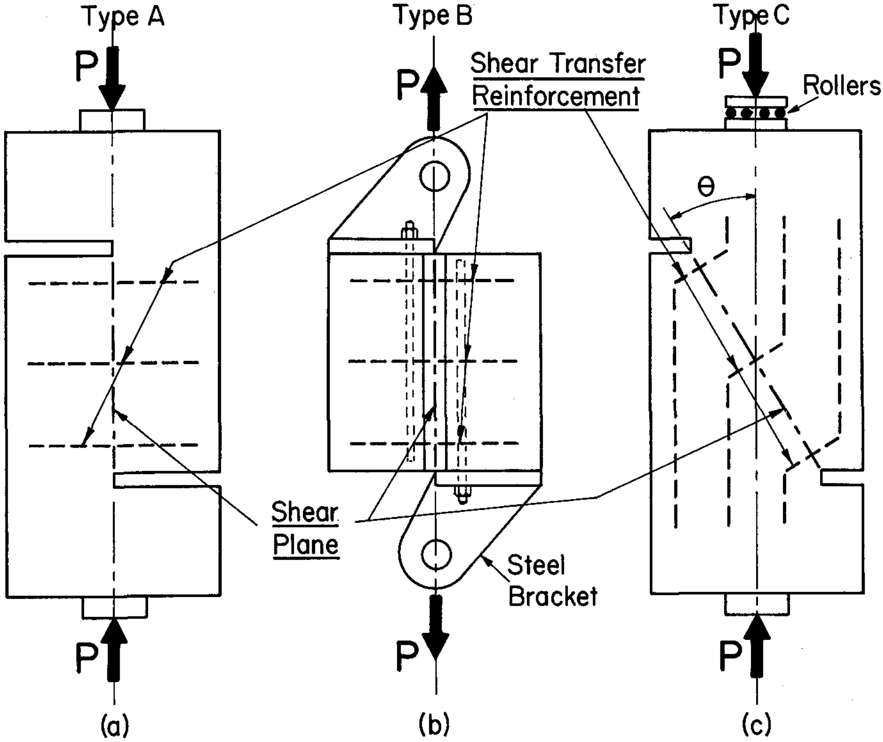

Fig. 1. Shear transfer test specimens: (a) push-off; (b) pull-off; (c) modified push-off

Additional reinforcement is provided away from the shear plane, to prevent failures other than along the shear plane. The length and width of the shear planes were 10 x 5 in., 12 x 4¾ in., and 12 x 6 in. (approx. 25 x 13 cm, 30 x 12 cm, and 30 x 15 cm) in the push-off, pull-off and modified push-off specimens respectively. When loaded concentrically by a force *P*, the shear along the shear plane is equal to *P* in the push-off and pull-off specimens. In the modified push-off specimens, the concentric force *P* produces a shear force $P \cos \theta$ along the shear plane and a compressive normal force $P \sin \theta$ across the shear plane. Six different values of θ were used to give different ratios of shear stress to transverse normal stress. The test program is summarized in Table 1.

The specimens were subjected to monotonic loading to failure. In all cases, slip along the shear plane was measured, and in some instances the lateral separation at the shear plane was also measured. Cracks were marked on the faces of the specimens as they developed. Detailed data for Series 1 to 6 have already been published(3). The data for Series 7 to 10 are summarized in Tables 2 and 3. For convenience, the ultimate shear strengths are expressed as average shear stresses v_u, obtained by dividing the ultimate shear force V_u by the area of the shear plane *bd* (*d* is the length of the shear plane and *b* its width).

Table 1. Test program

Test series	Description	Specimen type	Number of tests
1	Push-off tests of initially uncracked specimens. Reinforcement size constant, spacing varies. $f'_c \approx 4000$ psi, $f_y \approx 50$ ksi.	A	13
2	Push-off tests of initially cracked specimens. Reinforcement size constant, spacing varies. $f'_c \approx 4000$ psi, $f_y \approx 50$ ksi.	A	6
3	Push-off tests of initially cracked specimens. Reinforcement size varies, spacing constant. $f'_c \approx 4000$ psi, $f_y \approx 50$ ksi.	A	5
4	Push-off tests of initially cracked specimens. Higher strength reinforcement, $f_y \approx 66$ ksi. Reinforcement size constant, spacing varies. $f'_c \approx 4000$ psi.	A	5
5	Push-off tests of initially cracked specimens. Low strength concrete, $f'_c \approx 2500$ psi. Reinforcement size constant, spacing varies. $f_y \approx 50$ ksi.	A	5
6	Push-off tests of both initially cracked and uncracked specimens. Dowel action destroyed by short rubber sleeves on reinforcement across shear plane. $f'_c \approx 4000$ psi, $f_y \approx 50$ ksi.	A	4
7	Pull-off tests of initially uncracked specimens. Reinforcement size and spacing varies. $f'_c \approx 5000$ psi, $f_y \approx 50$ ksi.	B	6
8	Pull-off tests of initially cracked specimens. Reinforcement size and spacing varies. $f'_c \approx 5000$ psi, $f_y \approx 50$ ksi.	B	6
9	Modified push-off tests of initially uncracked specimens. Reinforcement size constant, spacing varies. Angle θ varies (0, 15°, 30°, 45°). $f'_c \approx 5500$ psi, $f_y \approx 52$ ksi.	C	6
10	Modified push-off tests of initially cracked specimens. Reinforcement size constant, spacing varies. Angle θ varies (0, 15°, 30°, 45°, 60°, 75°). $f'_c \approx 4000$ and 6000 psi, $f_y \approx 52$ ksi.	C	10

Table 2. Test data, Series 7 and 8

Specimen number*	Reinforcement bar size	Number of stirrups (2 legs each)	Reinforcement yield point, f_y ksi	Concrete strength, f'_c psi	pf_y, psi	v_u, psi
7.1	#3	2	49.5	4850	384	851
7.2	#3	3	49.5	5120	576	908
7.3	#3	4	49.5	5050	768	974
7.4	#2	2	56.0	5410	193	567
7.5	#2	3	56.0	5070	289	609
7.6	#2	5	56.0	5100	481	846
8.1	#3	2	49.5	4850	384	697
8.2	#3	3	49.5	5120	576	888
8.3	#3	4	49.5	5050	768	925
8.4	#2	2	56.0	5410	193	521
8.5	#2	3	56.0	5070	289	572
8.6	#2	5	56.0	5100	481	746

*Specimens of Series 7 were initially uncracked; specimens of Series 8 were cracked along the shear plane before test.

Characteristics of the shear plane. Mast[2] pointed out the need to consider the case where a crack may exist along the shear plane before shear is applied. Such cracks occur for a variety of reasons unrelated to shear, such as tension forces caused by restrained shrinkage or temperature deformations or accidental dropping of a member. Certain shear transfer test specimens were therefore cracked along the shear plane by the application of transverse line loads, before application of shear loading.

Slip was measurable from the beginning of the shear test for the initially cracked specimens. However, no movement occurred in the initially uncracked Type A specimens until diagonal tension cracks became visible at shear stresses of from 500 to 700 psi (35–49 kgf/cm²). These cracks crossed the shear plane at an angle of from 40 to 50 deg. They were each about 2 in. (5 cm) long, spaced 1 to 2 in. (2½ to 5 cm) apart along the shear plane. After these cracks formed, there was a relative longitudinal movement of the two halves of the initially uncracked specimens. This was due to rotation of the short concrete struts formed by the diagonal tension cracks, when the shear transfer reinforcement stretched. It was found that if a crack exists in the shear plane before the application of shear, then the slip at all stages of loading is greater than when such a crack does not exist.

A crack in the shear plane reduces the ultimate shear strength of under-reinforced specimens (Fig. 2). The decrease is greater in the push-off specimens than in the pull-off specimens. The shear strength of the initially cracked specimens is not directly proportional to the amount of reinforcement. Because of the observed weakening effect of a crack in the shear plane, most of the subsequent tests were made on initially cracked specimens, in order to obtain lower bound values of shear strength.

Table 3. Test data, Series 9 and 10

Specimen number[1]	Angle θ, deg.	Number of bars[2]	Reinforcement yield point, f_y, ksi	Concrete strength, f'_c psi	pf_y, psi	σ_{Nx}, psi	$pf_y + \sigma_{Nx}$, psi	v_u, psi	Failure type[3]
9.1	45	10	52.4	5500	800	2460	3260	2460	S
9.2	30	12	52.2	5500	956	1480	2436	2560	S
9.3	15	12	52.3	3940	976	406	1382	1515	S
9.4	0	12	53.7	3940	985	0	985	1389	S
9.5	30	8	51.0	6440	623	1655	2278	2870	S
9.6	30	4	51.0	6440	312	1600	1912	2770	S
10.1	75	6	51.8	3450	475	3220	3695	862	C
10.2	75	6	52.0	4390	476	3920	4396	1049	C
10.3	60	8	51.8	3450	632	2780	3412	1610	C
10.4	60	8	53.0	4390	648	3060	3708	1770	C
10.5	45	10	52.7	4630	805	2265	3070	2265	S
10.6	30	12	52.0	4630	954	1250	2204	2165	S
10.7	15	12	52.4	4020	962	387	1349	1445	S
10.8	0	12	53.7	4020	985	0	985	1115	S
10.9	30	8	51.0	5800	623	1490	2113	2590	S
10.10	30	4	51.0	5800	312	813	1125	1410	S

1. Specimens of Series 9 were initially uncracked; specimens of Series 10 were cracked along the shear plane before test.
2. All reinforcing bars were No. 3's arranged in pairs crossing the shear plane.
3. S = shear; C = compression.

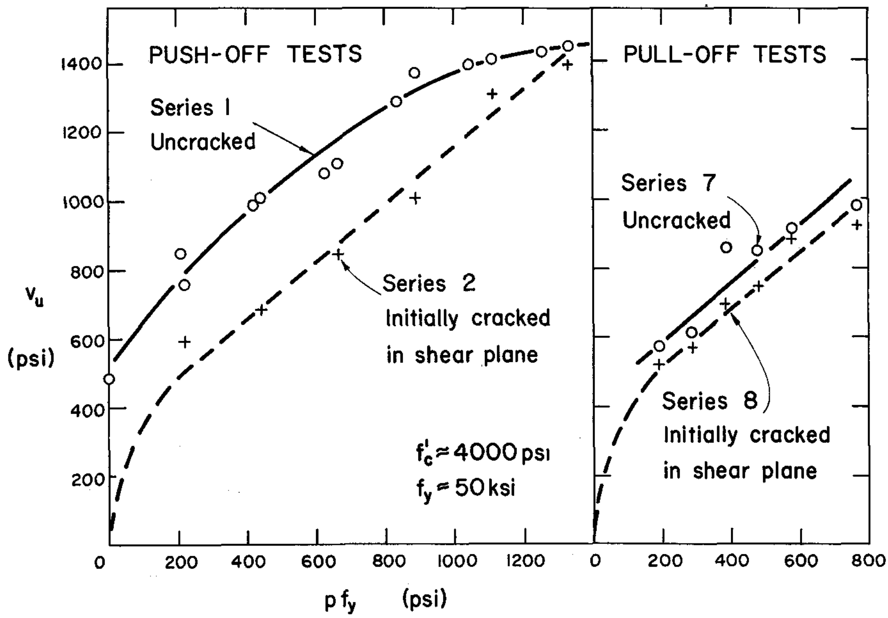

Fig. 2. Variation of shear transfer strength reinforcement parameter pf_y, with and without an initial crack along the shear plane

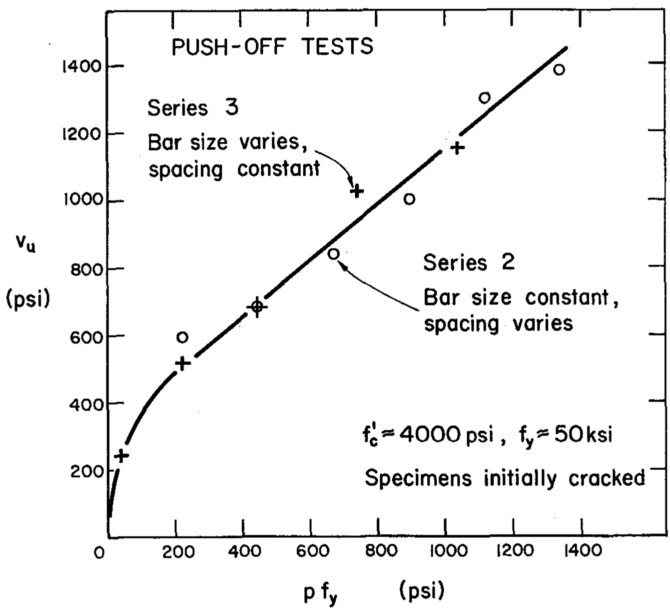

Fig. 3. Effect of stirrup bar size and spacing on the shear transfer strength of initially cracked push-off specimens

Characteristics of the reinforcement. The reinforcement parameter pf_y can be changed by varying either the reinforcement ratio p, the reinforcement yield strength f_y, or both. Also, for a given shear plane the reinforcement ratio can be changed by changing the bar size and/or the bar spacing. In Fig. 3, the results of tests of Series 2 and 3 are compared to determine whether the way in which the reinforcement ratio is changed has any effect on the relationship between ultimate shear strength and the reinforcement parameter pf_y. In Series 2, p was changed by varying the stirrup spacing, the bar size (No. 3) (9.5 mm) being constant. In Series 3, P was changed by varying the bar size between ⅛ in. diam. and No. 5 (3.2 and 15.9 mm) while maintaining a constant spacing of 5 in. (12.7 cm). Fig. 3 shows that the way in which p is changed does not affect the relationship between shear strength and the reinforcement parameter pf_y.

In the tests so far discussed the shear transfer reinforcement had a yield strength of about 50 ksi (3500 kgf/cm²). A group of push-off specimens was therefore tested in which the reinforcement had a yield strength of 66 ksi (4640 kgf/cm²), to check whether the relationship between v_u and pf_y was independent of f_y, and also to check whether it was possible to develop the yield strength of this higher strength steel. It was found that for given values of pf_y, the specimens with 66 ksi steel had slightly higher shear strengths than the specimens reinforced with the 50 ksi steel. This appears to indicate that at ultimate strength the higher strength steel stirrups developed a stress

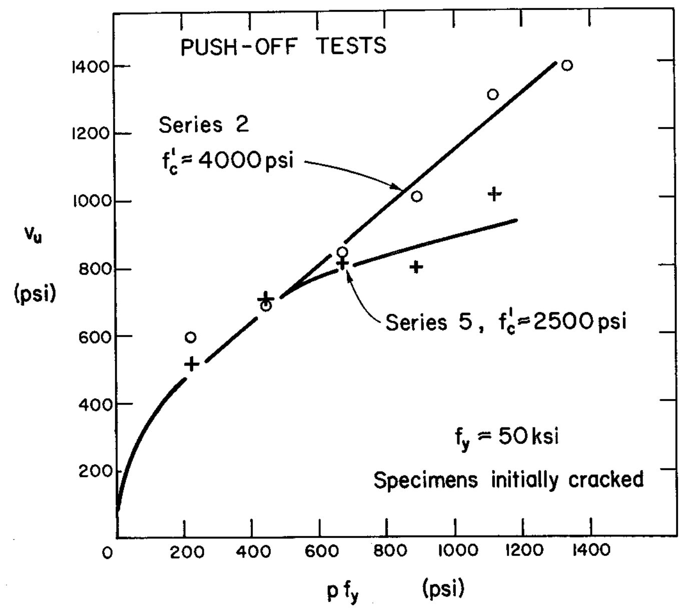

Fig. 4. Effect of concrete strength on the shear transfer strength of initially cracked push-off specimens

greater than their yield point, i.e., strain hardening had occurred. This is quite possible, as the yield plateau of the higher strength reinforcement was considerably shorter than that of the intermediate grade reinforcement. It therefore appears conservative to assume that the relationship between pf_y and v_u is the same for higher strength reinforcement as for intermediate grade reinforcement, provided the yield strength does not exceed 66 ksi.

Concrete strength. The effect of variation in concrete strength on the shear strength of initially cracked push-off specimens is illustrated in Fig. 4. The specimens of Series 2 and 5 were identical in all respects except for concrete strength, Series 2 having 4000 psi (281 kgf/cm^2) concrete and Series 5 having 2500 psi (176 kgf/cm^2) concrete. For values of pf_y below about 600 psi (42 kgf/cm^2) the concrete strength does not appear to affect the shear transfer strength. For higher values of pf_y the shear strength is lower for the lower strength concrete. The concrete strength therefore appears to set an upper limit value of pf_y, below which the relationship between v_u and pf_y established for 4000 psi concrete would hold for any strength of concrete equal to or greater than the strength being considered, and above which the shear strength increases at a lesser rate for the concrete strength being considered. This change in behavior is discussed later.

Direct stress parallel to the shear plane. In an earlier report[3] a method was proposed for the calculation of the shear transfer strength of initially uncracked concrete. This was based on the average shear and normal stresses acting on a concrete element in the shear plane, and made use of the failure envelope for concrete proposed by Zia[6]. This approach predicted the relationship between v_u and pf_y very closely for the tests of initially uncracked push-off specimens reported here, and also for tests of larger initially uncracked composite push-off specimens reported by Anderson[7]. In the push-off test, direct compressive stresses exist parallel to the shear plane, and these were taken into account in the calculation.

Using this method of calculation, an analytical study was made of the influence on shear transfer strength of direct stress parallel to the shear plane. From these calculations it appeared that if a direct tension stress existed parallel to the shear plane, then the shear transfer strength would increase more slowly, as pf_y was increased, than in the push-off test where a direct compressive stress exists parallel to the shear plane. This conclusion was disturbing from the designer's point of view, since in many practical situations there is a direct tensile stress parallel to the shear plane. It was therefore decided to study this problem with pull-off tests using specimens of the type shown in Fig. 1(b). The shear is applied to the shear plane by a concentric tension force acting on the specimen through steel brackets bolted to longitudinal reinforcing bars embedded in the specimen on either side of the shear plane. In these specimens a direct tension stress exists parallel to the shear plane, the average intensity of which is about half the intensity of the applied shear stress. In the push-off specimens tested previously, a direct compressive stress existed parallel to the shear plane, the average intensity of which was equal to that of the applied shear stress.

The ultimate shear strengths of the pull-off and push-off specimens are compared in Fig. 5. For initially uncracked specimens, the pull-off tests gave lower shear strengths than the push-off tests, indicating that a direct tension stress parallel to the shear plane

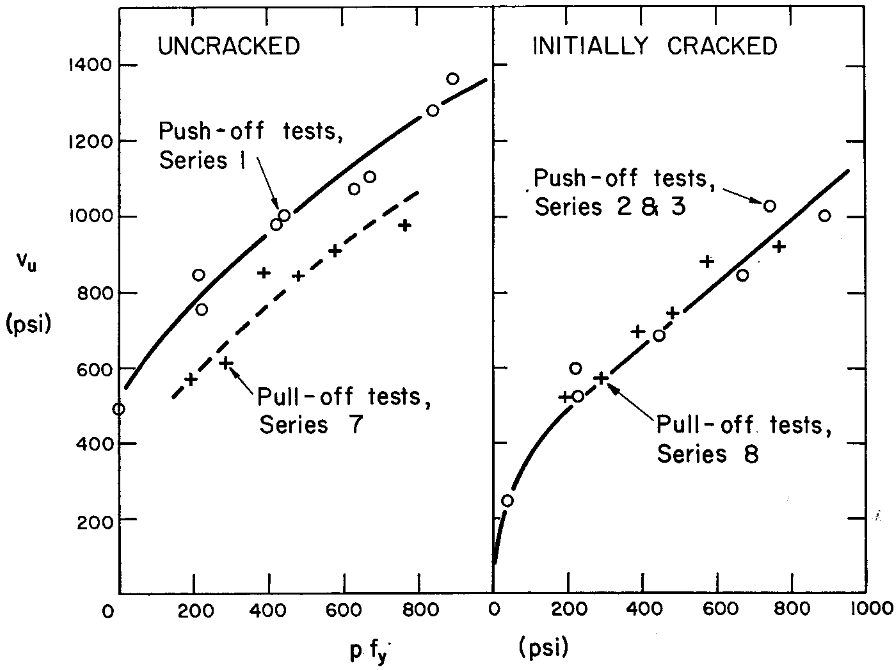

Fig. 5. Effect on shear transfer strength of direct stress acting parallel to the shear plane

is detrimental to shear transfer strength in initially uncracked concrete. However, the reduction in shear strength appears to be due to a reduction in the cohesion contribution of the concrete, and the rate of increase in v_u with increase in pf_y is approximately the same in both the pull-off and push-off tests. This indicates that the method of calculation proposed earlier(3) is faulty and cannot be extrapolated to the case of the pull-off test.

For specimens cracked along the shear plane before being loaded in shear, the shear strengths of the push-off and the pull-off specmiens are essentially the same for any given value of pf_y. This is important practically, since it indicates that direct stresses parallel to the shear plane may be ignored in design for shear transfer, if the design is based on the relationship between v_u and pf_y obtained in tests of initially cracked specimens.

Direct stress transverse to the shear plane. The effect of compressive stresses acting transverse to the shear plane was studied in Series 9 and 10. Modified push-off specimens were used, as shown in Fig. 1(c). The depth of the block-outs in the specimens was adjusted so that the length of the shear plane joining their ends remained constant as the angle θ varied. A system of rollers on the top of the specimen permitted separations to develop, even for relatively large applied loads. The spec-

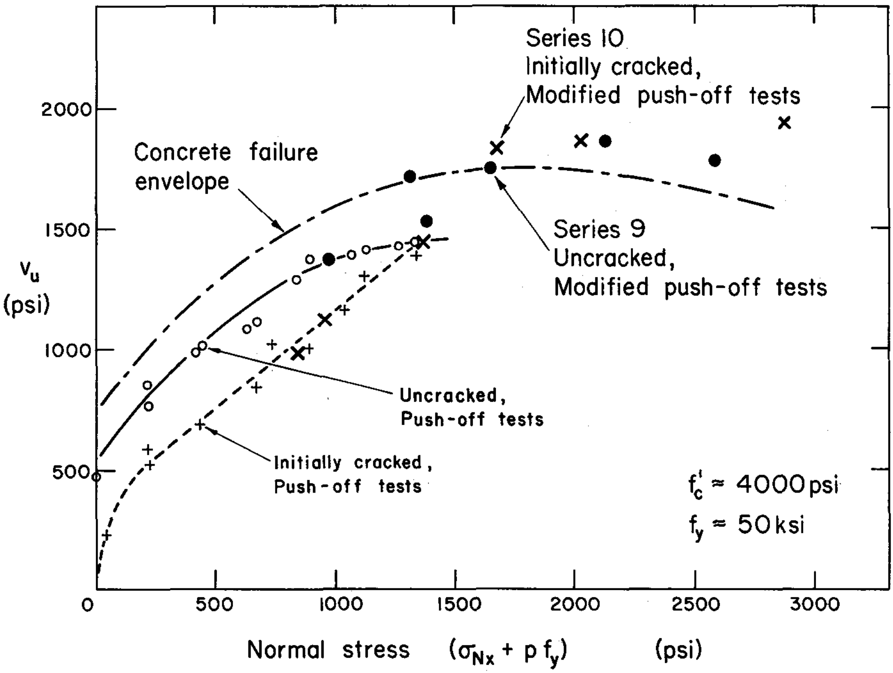

Fig. 6. Effect on shear transfer strength of direct stress acting transverse to the shear plane

imens of Series 10 were initially cracked along the shear plane, while those of Series 9 were initially uncracked.

Failures were characterized by a shearing action along the shear plane when angle θ was 45 deg. or less, and by a crushing failure across the plane for θ of 60 or 75 deg. The deformations of the initially uncracked specimens were extremely small until diagonal tension cracks developed across the shear plane at about 60 to 70 percent of the ultimate strength. As in the push-off specimens, these cracks formed at an angle of about 45 deg. to the shear plane. They were about 2 in. (5 cm) long, and between 1 and 2 in. (2½ to 5 cm) apart. In specimens with angle θ of 30 deg. or less, failure occurred with a continuous crack propagating through the diagonal tension cracks, along the shear plane. Deformations developed rapidly after diagonal tension cracking, at a rate which increased continuously with increasing load, but decreased as θ increased. The slips at failure were in excess of 0.03 in. (0.76 mm) and the separations were large enough to indicate yielding of the reinforcement when θ was 30 deg. or less. For the specimens with θ equal to 45 deg., separations did not develop rapidly until immediately prior to failure. For the specimens with θ of 30 deg. and having differing values of pf_y, the load-slip relationships were not influenced by the value of pf_y until immediately prior to failure.

Significant deformations of the precracked specimens occurred from the commencement of loading. The initial stiffnesses were almost identical for θ ranging from 45 to 75 deg. When θ was between 0 and 45 deg., the initial stiffness increased with both θ and the value of pf_y. When shearing failures occurred, the ultimate slips were similar to those observed in initially uncracked specimens. Separations began to develop rapidly at three-quarters of the ultimate load, for θ between 0 and 30 deg. For θ equal to 45 deg., separations did not develop until immediately prior to collapse, while for angles θ of 60 and 75 deg. only contractions occurred. Separations at ultimate were as large as 0.06 in. (1.52 mm).

The ultimate shear strengths of the modified push-off specimens which had shearing type failures are compared in Fig. 6 with results from the push-off tests of Series 1, 2 and 3. In this figure the data from Series 9 and 10 are normalized to a concrete strength f'_c of 4100 psi (288 kgf/cm²), the average concrete strength of the specimens in Series 1 and 2. The values of applied normal stress σ_{Nx} and of v_u were multiplied by the ratio $4100/f'_c$. The total normal compressive stress across the shear plane is assumed to be equal to $\sigma_{Nx} + pf_y$. Also shown in Fig. 6 is a failure envelope for concrete with a cylinder strength of 4100 psi. The intrinsic shape of this failure envelope was obtained from biaxial tests of concrete reported by Kupfer, Hilsdorf and Rusch[8]. The assumption that σ_{Nx} may be added to pf_y when estimating v_u can be seen to be conservative for all values of σ_{Nx}. Furthermore, under certain conditions, the shear strength can be as large as the intrinsic strength of the concrete. This occurred when $\sigma_{Nx} + pf_y$ was greater than $0.3f'_c$ and the ratio of σ_{Nx} to pf_y was simultaneously greater than 1.3. (An initially cracked specimen having σ_{Nx}/pf_y equal to 2.6, but with $\sigma_{Nx} + pf_y$ of only $0.2f'_c$ developed a strength almost identical with that of a simple push-off specimen having pf_y equal to $0.2f'_c$.)

Further investigations are needed to define completely the effect on shear transfer strength of the ratio of σ_{Nx} to pf_y, of direct tensile stresses acting transverse to the shear plane, and of applying the shearing force after the direct stress has been increased to its maximum value.

Hypotheses for behavior

Shear transfer behavior of initially uncracked concrete with reinforcement normal to the shear plane. External loads are assumed to cause a shear stress v along the shear plane and direct stresses σ_{Ny} and σ_{Nx} parallel to and normal to the shear plane, respectively. As loading begins the concrete is uncracked; the transverse reinforcement A_{vf} is unstressed and therefore does not contribute an additional direct stress across the shear plane.

Several short diagonal tension cracks will occur along the length of the shear plane and inclined to it at an angle α when, under increasing shear, the principal tensile stress in the concrete becomes equal to the tensile strength of the concrete. The angle α will depend upon the particular combination of v, σ_{Nx} and σ_{Ny} existing at the time of cracking. In push-off tests without additional externally applied direct stress σ_{Nx}, α is usually about 45 deg.

When the shear load is further increased a truss action develops, as shown in Fig. 7(a). Diagonal struts of concrete are formed by the short, parallel diagonal tension cracks. When the shear acts on the truss, the struts tend to

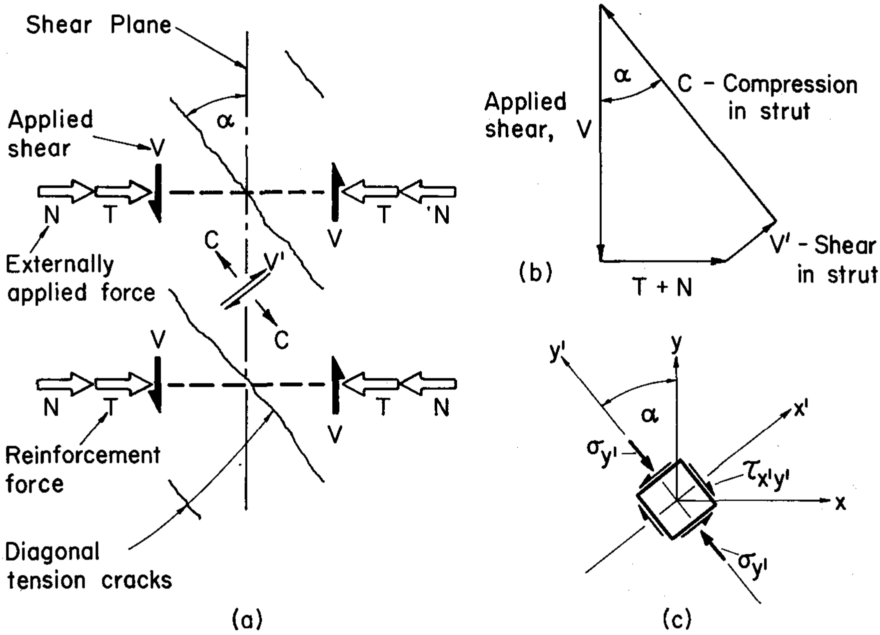

Fig. 7. Shear transfer in initially uncracked concrete

rotate and so stress the transverse reinforcement. Because the diagonal struts are continuous with the concrete on both sides of the shear plane, there will be both compression and transverse shear in the strut. The applied shear is therefore resisted by the components of the strut compression and shear forces acting parallel to the shear plane, as shown in Fig. 7(b).

The reinforcement crossing the shear plane will eventually develop its yield strength $A_{vf}f_y$, provided a failure of the concrete does not occur first. Failure will finally occur when the concrete struts fail under the combined action of compression and shear in the struts, while the reinforcement continues to develop its yield strength.

Consider an element of concrete lying in the shear plane, at the middle of the thickness of a strut. With reference to coordinates x' and y', the stresses acting on the element will be as shown in Fig. 7(c). They comprise a compression $\sigma_{y'}$ acting parallel to the direction of the diagonal tension cracks, and shear stresses $\tau_{x'y'}$ oriented as shown. Because the faces of the strut formed by the diagonal tension cracks are unloaded free surfaces, $\sigma_{x'}$ is zero. The pairs of values of $\sigma_{y'}$ and $\tau_{x'y'}$ at failure of the concrete can be obtained from the failure envelope for the concrete using the geometrical construction shown in Fig. 8. A succession of Mohr circles is drawn tangent to the failure envelope. The intersection of any particular circle and the τ axis will define the point ($\sigma_{x'}$, $\tau_{x'y'}$), since $\sigma_{x'}$ is zero. The diametrically opposite point on the circle must therefore be the point ($\sigma_{y'}$, $\tau_{x'y'}$), where $\sigma_{y'}$ and $\tau_{x'y'}$ are a pair of stresses corresponding to failure of the concrete.

The state of stress in the element on the shear plane can also be expressed as σ_x, σ_y and τ_{xy} with respect to the axes x and y, normal and parallel to the shear plane, respectively. These stresses can be stated in terms of $\sigma_{y'}$ and $\tau_{x'y'}$ as follows:

$$\sigma_x = \sigma_{y'} \sin^2 \alpha - 2\tau_{x'y'} \sin \alpha \cos \alpha \quad (1)$$

$$\sigma_y = \sigma_{y'} \cos^2 \alpha + 2\tau_{x'y'} \sin \alpha \cos \alpha \quad (2)$$

$$\tau_{xy} = -\sigma_{y'} \sin \alpha \cos \alpha + \tau_{x'y'} (\cos^2 \alpha - \sin^2 \alpha) \quad (3)$$

If $\alpha = 45$ deg., then

$$\sigma_x = \frac{\sigma_{y'}}{2} - \tau_{x'y'} \quad (1a)$$

$$\sigma_y = \frac{\sigma_{y'}}{2} + \tau_{x'y'} \quad (2a)$$

$$\tau_{xy} = -\frac{\sigma_{y'}}{2} \quad (3a)$$

Since pairs of values of $\sigma_{y'}$ and $\tau_{x'y'}$ corresponding to failure of the concrete can be obtained as shown in Fig. 8, it is possible to calculate values of σ_x, σ_y and τ_{xy} which correspond to failure of the concrete.

Now at failure, σ_x is the direct stress acting across the shear plane as a result of the shear transfer reinforcement being stressed to yield, plus any externally applied direct stress σ_{Nx} act-

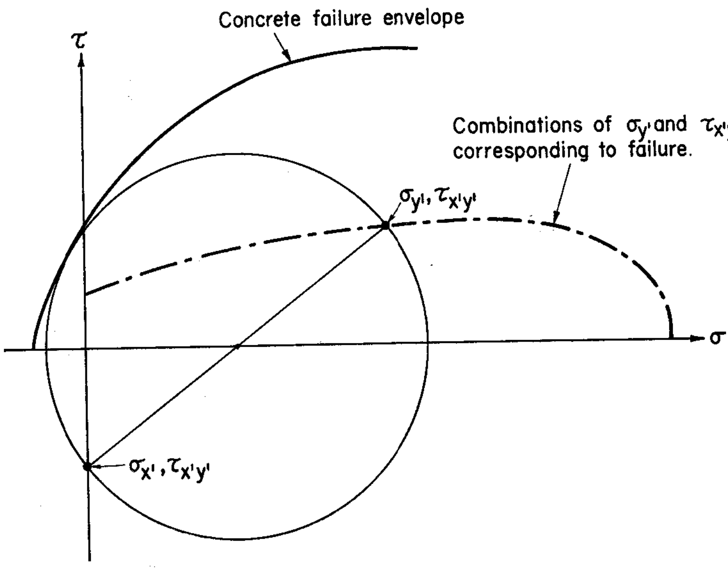

Fig. 8. Derivation of combinations of $\sigma_{y'}$ and $\tau_{x'y'}$ which cause failure of the concrete

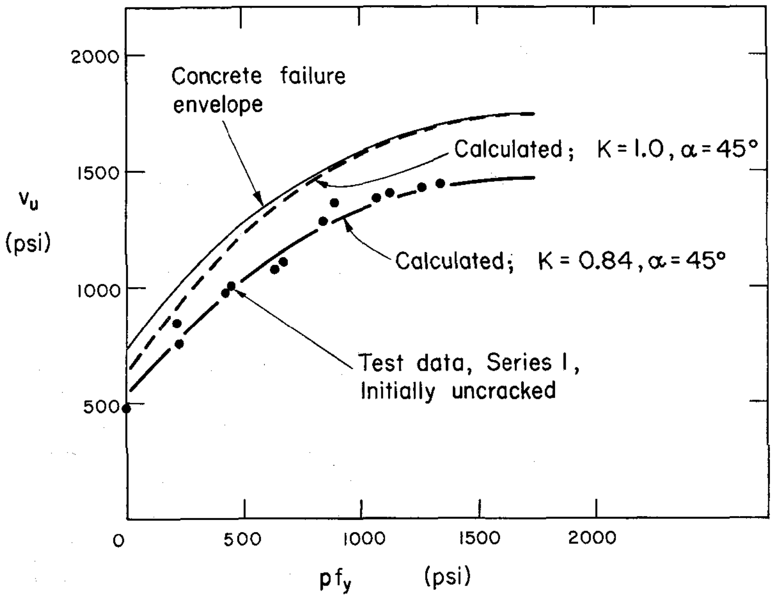

Fig. 9. Comparison of calculated and test shear transfer strengths of initially uncracked push-off specimens

ing across the shear plane at failure, i.e.,

$$\sigma_x = \frac{A_{vf}f_y}{bd} + \sigma_{Nx} = pf_y + \sigma_{Nx} \quad (4)$$

Where A_{vf} is the total cross-sectional area of shear transfer reinforcement, and f_y its yield strength. Also, τ_{xy} is the shear stress in the shear plane, at the center of a strut. We may then write,

$$v_u = \frac{V_u}{bd} = K\tau_{xy} \quad (5)$$

Hence if $\alpha = 45$ deg., then $v_u = -K\sigma_{y'}/2$ and $pf_y + \sigma_{Nx} = (\sigma_{y'}/2 - \tau_{x'y'})$. The value of the coefficient K would be 1.0 if the shear stresses were uniformly distributed across the strut, and could be as low as 0.67 if the shear stress distribution across the strut were parabolic. When the external normal stress σ_{Nx} is zero or small, the concrete struts must rotate slightly in order to strain the transverse reinforcement. This causes the ends of the diagonal tension cracks to propagate parallel to the shear plane for a short distance. As a result, the shear stress in the strut is increased locally to a value higher than the average shear stress v_u based on the total area of the shear plane. Failure then tends to shift from the shear plane to a parallel plane containing the ends of the cracks, and occurs when the locally higher shear stress reaches a critical value. The diagonal struts at each end of the shear plane may be incomplete, depending on the exact location of the diagonal tension cracks. In an extreme case this could reduce the total effective cross-section resisting shear by an amount equal to the width of the shear plane multiplied by the projected length on the shear plane of a single diagonal tension crack. Both of the foregoing types of behavior can be expected to result in K becoming less than 1.0.

Using Eqs. (1) and (3) it is therefore possible to calculate pairs of values of $(pf_y + \sigma_{Nx})$ and v_u corresponding to shear transfer failure, provided some value can be assigned to the coefficient K. The actual distribution of shear stress across each strut will probably be intermediate between the uniform and parabolic distributions. In Fig. 9 the calculated relationship between v_u and pf_y, corresponding to an average value of 0.84 for K and the assumption that angle α is 45 deg., is seen to be in reasonably close agreement with results obtained in push-off tests of initially uncracked specimens in which σ_{Nx} was zero. Also shown are the relationship between v_u and pf_y corresponding to a value of 1.0 for K, and the failure envelope used in the calculations. The intrinsic shape of the failure envelope was obtained from biaxial tests of concrete reported by Kupfer, Hilsdorf and Rusch[8]. Use was made of data reported for a concrete having a ratio of tensile to compressive strength corresponding to that of the concrete used in the push-off specimens, $(f_t/f'_c = 1/12.33)$.

The results of the modified push-off tests in which the ratio of σ_{Nx} to pf_y is greater than 1.3 agree reasonably well with calculations assuming $K = 1.0$. In this case the ends of the cracks do not propagate parallel to the shear plane, and therefore the total cross section of the shear plane acts to resist the applied shear.

In the pull-off specimens a direct tensile stress existed parallel to the shear plane at the time of diagonal tension cracking. The intensity of the direct tensile stress was half the intensity of the shear stress. The diagonal tension crack angle α which corresponds to this is 52 deg. This increase in the crack angle from 45 to 52 deg. would lead to a reduction in calculated strength of about 10 percent, other characteristics remaining the same. The actual reduction found in the pull-off tests was greater than this, indicating that the K value is less in the pull-off specimen than in the push-off specimen. The extension of the ends of the diagonal tension cracks parallel to the shear plane was more marked in the pull-off tests than in the push-off tests. For a given applied shear, this would increase the local intensity of shear stress in the pull-off specimen. This increase would result in failure at a lower average shear stress, corresponding to a reduction in the value of K.

Shear transfer behavior of initially cracked concrete with reinforcement normal to the shear plane. When an initially cracked specimen is loaded in shear, slip will occur along the shear plane. The faces of the crack are rough and hence when slip occurs, the crack faces are forced to separate. This separation causes tension strains in the reinforcement crossing the shear plane. The tension force so induced in the reinforcement is balanced by an equal compression force acting across the crack. This compression force produces a frictional resistance to sliding between the faces of the crack, thus opposing the applied shear. The relative movement of the concrete on opposite sides of the crack also subjects the individual reinforcing bars to a shearing action. The resistance of the bars to this shearing action, sometimes referred to as dowel action, also contributes to the shearing resistance.

In an under-reinforced shear plane, the separation of the crack faces is eventually sufficient to strain the reinforcement to its yield point. At ultimate strength therefore, the compression force across the crack is equal to the yield strength of the reinforcement $A_{vf}f_y$. The frictional resistance to shear along the crack is then equal to this force multiplied by the coefficient of friction for concrete. In addition to the frictional resistance to shear, there is also shear resistance due to the dowel action of the reinforcement crossing the crack in the shear plane, and the resistance to shearing off of asperities projecting from the faces of the crack. It is hypothesized that the frictional resistance to sliding and the reinforcement dowel effect are the principal contributors to shear resistance. This view is supported by the fact that for values of pf_y greater than 200 psi (14 kgf/cm^2), the slope of the curve relating v_u and pf_y is equal to the coefficient of friction between formed concrete surfaces measured by Gaston and Kriz[9]. Further, when the dowel action was destroyed in two initially cracked push-off specimens (Series 6), the shear strength dropped almost to that which could be provided by friction alone. In these specimens the reinforcement became kinked at ultimate, and hence a component of the reinforcement force acted along the shear plane. It is thought that the excess strength of these specimens above the frictional resistance was due to this kinking effect.

The concrete strength does not appear to affect the shear transfer strength of an initially cracked under-reinforced specimen. This is consistent with the shear strength being primarily developed by friction, since the coefficient of friction is independent of the concrete strength. The behavior hypothesis also explains why the shear transfer strength of initially cracked pull-off and push-off specimens are the same for the same value of the reinforcement parameter pf_y. Direct stresses parallel to the shear plane will not affect either the frictional resistance to sliding along the shear plane, or the dowel effect. Hence a change in this longitudinal direct stress from tension to compression does not affect the shear transfer strength in this case.

In a heavily reinforced shear plane, or one subject to a substantial externally applied normal compressive stress, it is possible for the theoretical shear resistance due to friction and dowel effects to become greater than the shear which would cause failure in an initially uncracked specimen having the same physical characteristics. In such a case, the crack in the shear plane "locks up" and the behavior and ultimate strength then become the same as for an initially uncracked specimen. When this occurs, the shear strength becomes dependent upon the concrete strength, whereas before it was independent. This change in behavior corresponds to the change in slope of the v_u/pf_y curve for 2500 psi (176 kgf/cm^2) concrete in Fig. 4. In Fig. 2 it can also be seen that at the highest values of pf_y, the strengths of both initially cracked and initially uncracked specimens are the same.

In a moderately to heavily reinforced shear plane, diagonal tension cracks may form at angle α to the shear plane, but failure still occurs by sliding along the crack in the shear plane at an ultimate shear strength less than that of the corresponding initially uncracked specimen.

Shear transfer in design

Section 11.15 of the *ACI Building Code*, ACI 318-71[10], allows design for shear transfer to be based on the "shear-friction" hypothesis proposed by Birkeland[1] and Mast[2]. This is a simplification for design purposes of the

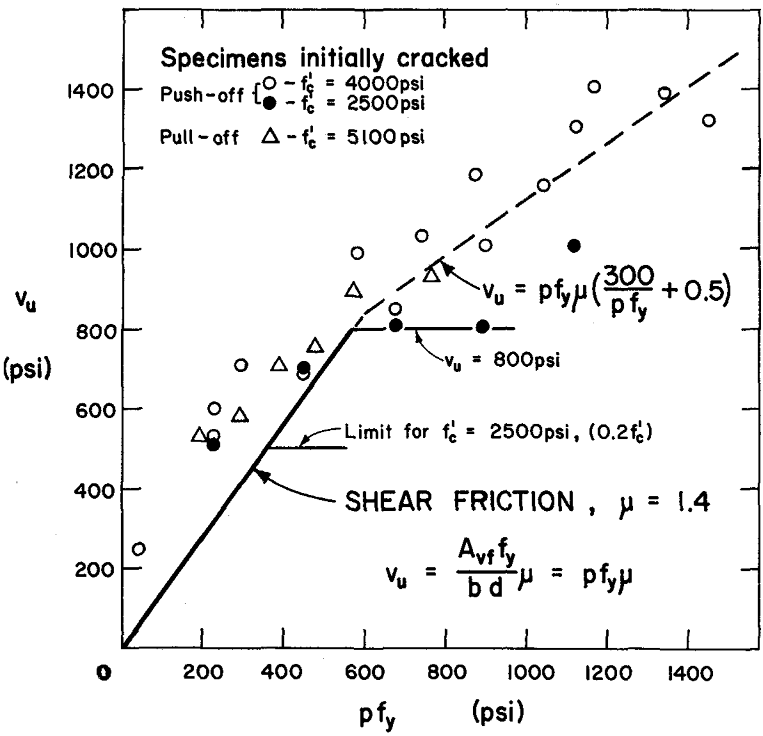

Fig. 10. Comparison of shear transfer strength calculated using the shear friction provisions of ACI 318-71 with measured strengths of initially cracked push-off and pull-off specimens

hypothesis for the behavior of initially cracked concrete described above. In the shear-friction approach, it is assumed that for some unspecified reason a crack exists in the shear plane. The shear resistance is then assumed to be developed entirely by the frictional resistance to sliding of one crack face over the other, when acted upon by a normal force equal to the yield strength of the reinforcement crossing the shear plane. A fictitiously high value of the coefficient of friction μ is used to compensate for neglect of dowel action and other factors. For a crack in monolithic concrete, μ is taken as 1.4. For conservative calculation of strength, the shear transfer strength is limited to $0.2f'_c$ or 800 psi (56 kgf/cm²) whichever is the less. The shear-friction equation may be written as

$$v_u = \frac{A_{vf}f_y}{bd}\,\mu = pf_y\mu \qquad (6)$$

but not more than $0.2f'_c$ or 800 psi.

In Fig. 10 the shear transfer strength calculated according to Eq. (6) is indicated by unbroken lines and is com-

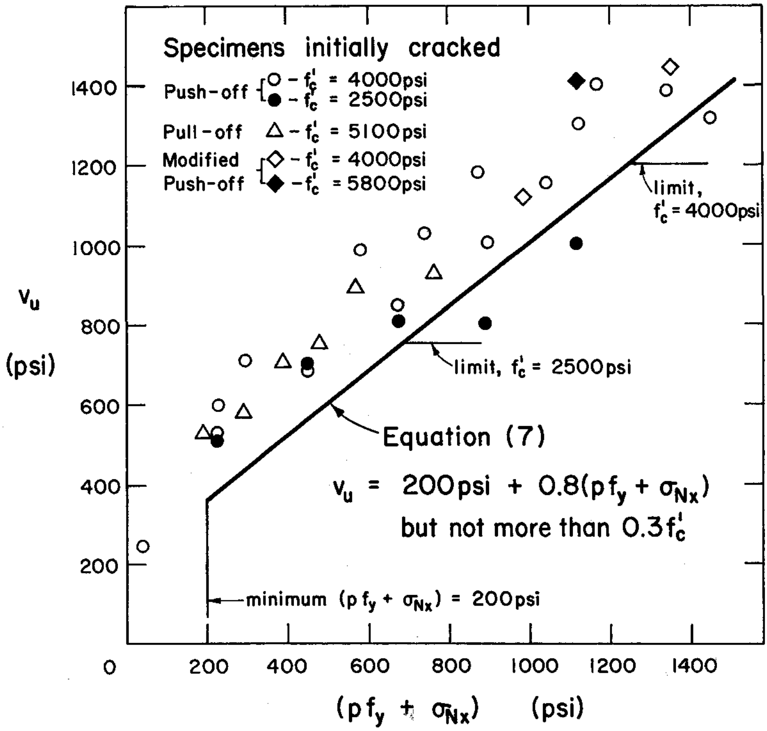

Fig. 11. Comparison of shear transfer strength calculated using Eq. (7) with the measured strengths of initially cracked push-off, pull-off and modified push-off specimens

pared with the measured strength of all the initially cracked push-off and pull-off specimens tested in the program reported here. It can be seen that the provisions of ACI 318-71 yield a conservative estimate of the shear transfer strength of concrete cracked along the shear plane. However, it is also clear that shear stresses considerably in excess of the arbitrary upper limit of 800 psi (56 kgf/cm^2) can be developed if appropriate reinforcement is provided and the concrete strength is high enough. Section 6.1.9 of the *PCI Design Handbook*[11] suggests that this reinforcement may be designed using Eq. (6), providing that μ is multiplied by $(300/pf_y + 0.50)$ when pf_y exceeds 600 psi (42 kgf/cm^2). This proposal is indicated by the broken line in Fig. 10. It can be seen that this proposal is in accord with the trend of the experimental data for concrete strengths greater than 2500 psi (176 kgf/cm^2) and for shear strengths less than about 1300 psi (91 kgf/cm^2). A deficiency in this proposal is that no upper limit on either pf_y or v_u is specified.

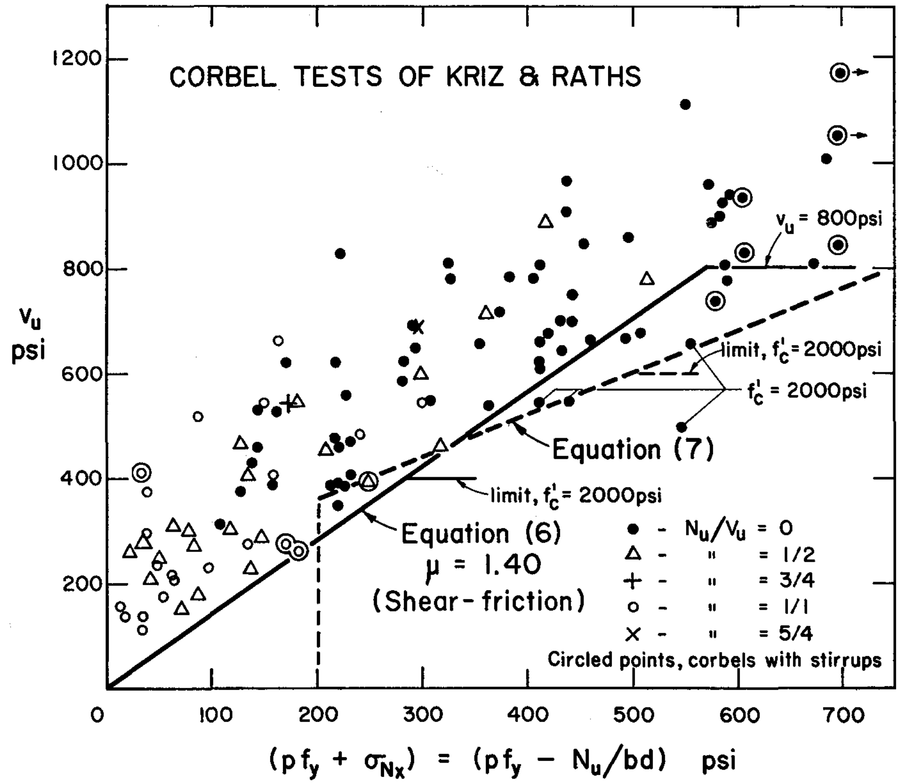

Fig. 12. Comparison of the shear strength of corbels calculated using Eqs. (6) and (7) with the measured strengths of tested corbels

An alternate approach, simpler to apply in design, would be the use of the following equation for shear transfer across a crack in monolithic concrete

$$v_u = 200 \text{ psi} + 0.8\,(pf_y + \sigma_{Nx}) \qquad (7)$$

with the restrictions that v_u shall be not more than $0.3f'_c$ and $(pf_y + \sigma_{Nx})$ shall be not less than 200 psi (14 kgf/cm²). σ_{Nx} is the externally applied direct stress acting across the shear plane, taken as positive for a compressive stress and negative for a tensile stress. In Fig. 11, Eq. (7) is compared with the measured strength of all the initially cracked specimens tested in the program reported here and is seen to be a lower bound to the data. Eq. (7) is slightly less conservative than Eq. (6) for $(pf_y + \sigma_{Nx})$ less than 333 psi (23 kgf/cm²) and slightly more conservative for values of $(pf_y + \sigma_{Nx})$ between 333 and 572 psi (23 and 40 kgf/cm²), the limit of applicability of Eq. (6) according to ACI 318-71.

The results reported here validate Eq. (7) for values of $(pf_y + \sigma_{Nx})$ up to 1400 psi (99 kgf/cm²), when σ_{Nx} is zero or compressive. The results reported by Kriz and Raths[12] for corbels subjected to shear and to tension forces in the direction of the reinforcement, indicate that there are a wide variety of conditions for which Eq. (7) is also valid for values of σ_{Nx} which are tensile. In Fig. 12 Eqs. (6) and (7) are compared with data from Kriz' and Raths' corbel tests. In plotting Fig. 12, v_u was taken as the nominal shear stress at yield of the tension reinforcement, or at ultimate strength of the corbel if the yield of the tension reinforcement did not occur. In accordance with Kriz' and Raths' findings and as required by Section 11.14 of ACI 318-71, the reinforcement ratio p was taken as $(A_s + A_h)/bd$ when shear only acted on the corbel and as A_s/bd when both shear V_u and tension N_u acted on a corbel. In this latter case, $\sigma_{Nx} = N_u/bd$. It can be seen that, providing the limitations placed on them are observed, both Eqs. (6) and (7) yield conservative estimates of the ultimate strength of corbels. (The corbel tests considered included specimens for which the ratio of the tensile stress σ_{Nx} to the shear stress v_u varied from 0 to 1.25, and for which the ratio of moment acting on the corbel to the shear times the effective depth at the column face (a/d) varied from 0.11 to 0.62. The maximum value of $(pf_y - N_u/bd)$ was 514 psi (36 kgf/cm²), and the maximum value of pf_y considered was 700 psi (49 kgf/cm²).

Conclusions

Concerning design.

1. Within their range of applicability, the shear-friction provisions of ACI 318-71 yield a conservative estimate of the shear transfer strength of reinforced concrete whether or not a crack exists in the shear plane.

2. Higher shear transfer strengths than the upper limit of 800 psi (56 kgf/cm²) specified in ACI 318-71 can be developed if appropriate reinforcement is provided and the concrete strength is adequate. Such reinforcement may be proportioned using Eq. (7).

Concerning fundamental behavior.

1. A pre-existing crack along the shear plane will both reduce the ultimate shear transfer strength and increase the slip at all levels of load.

2. Changes in strength, size, and spacing of reinforcement affect the shear transfer strength only insofar as they change the value of the reinforcement parameter pf_y for $f_y \leqq 66$ ksi (4640 kgf/cm²).

3. In initially cracked concrete, the concrete strength sets an upper limit value for pf_y below which the relationship between v_u and pf_y is independent of concrete strength. Above this value of pf_y, the shear transfer strength increases at a much reduced rate for lower strength concrete and is equal to that of similarly reinforced, initially uncracked concrete.

4. Direct tension stresses parallel to the shear plane reduce the shear transfer strength of initially uncracked concrete, but do not reduce the shear transfer strength of concrete initially cracked in the shear plane.

5. An externally applied compressive stress acting transversely to the shear plane is additive to pf_y in calculations of the ultimate shear transfer strength of both initially cracked and uncracked concrete.

6. The shear transfer strength of initially uncracked concrete is developed by a truss action after diagonal tension cracking. Failure occurs when the inclined concrete struts fail under a combination of shear and axial force.

7. The shear transfer strength of initially cracked concrete with moderate amounts of reinforcement is developed primarily by frictional resistance to sliding between the faces of the crack and by dowel action of the reinforcement crossing the crack. When large amounts of reinforcement, or sufficient externally applied compression stresses normal to the shear plane are provided, then the crack in the shear plane "locks up" and shear transfer strength is developed as in initially uncracked concrete.

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