OPTIMUM PRESTRESS, ANALYSIS, AND ULTIMATE STRENGTH DESIGN OF PRESTRESSED CONCRETE SHEET PILES

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Sheet piles are vertical flat structural elements driven into the ground to support either backside earth or hydrostatic pressure, or external earth or hydrostatic pressure in excavations, through their flexural resistance. Watertightness if required is achieved through designed interlocks, through grouted joints, or through the formation of cells filled with impervious materials. The complete installation of such piles may include soldier beams, backstays and anchorages, or interior soldier beams and transverse struts if these become necessary in moderately deep excavations or in moderately deep underwater constructions.

Prestressed concrete has been much used in sheet piling work because of its ability to withstand tensile stresses during recoil in the driving process, to withstand bending stresses in service whether temporary or permanent, and to withstand environmental conditions better than either timber or steel sheet piling.

Since the larger sheet piling installations involve considerable expense, and they are often selected as a convenient means for temporary or permanent construction, the pertinent issues of optimum prestress, analysis, and ultimate strength design of prestressed concrete sheet piling merit attention. This paper describes thirteen recommended pile cross-sections and establishes criteria for optimum prestress. However details of analysis and design are limited to sheet piles with interior transverse support.

SHEET PILE CROSS-SECTIONS

Fig. 1 shows all of the recommended transverse cross-sections of prestressed concrete sheet piles[1,2], together with a listing of the practical applications and special merits of each.

PRE-SERVICE STRESSES OF SHEET PILES

Sheet pile design must consider pre-service stresses: 1) handling stresses, 2) storage (supporting) stresses, 3) hauling stresses (supporting and dynamic increment), and 4) driving stresses[3,4]. Li and Huang[4] have dealt with the handling, supporting, and hauling stresses; for practical use in engineering offices, they computed and tabulated locations of one, two, three and four

Presents 13 recommended cross-sections for prestressed concrete sheet piles, establishes standards for selecting optimum prestress, and makes a detailed analysis of sheet piling with interior transverse supports to meet optimum criteria. Ultimate strength design equations are developed for both symmetrical and eccentric prestressing.

pickup points to the nearest ¼ in. (6 mm) for pile lengths from 10 to 230 ft. (3 to 71 m) in 5 ft. (1.5 m) increments, and computed maximum moments in ft.-lb., given in terms of uniform dead load in lb. per linear ft.

For each individual project, driving stresses should be carefully appraised, taking account of substrata conditions, driving equipment, and method of driving[3].

OPTIMUM PRESTRESS FOR SHEET PILES

All prestressed concrete piles are subjected during driving to certain dynamic maximum amplitudes of a compressive stress wave under the hammer blow and of a tensile stress wave upon recoil. Besides this common dynamic phase for all piles, sheet piles as a class are subjected, during service, to certain maximum flexural stresses occurring:

1. At the lower point of fixity when they are short or moderate-height cantilevers.
2. In the vertical span with longer spans when both ends are partially restrained.
3. At both lower point of fixity and one or more intermediate supports.

Given the nature of sheet piling as stated above, the guiding principles for optimum prestress are as follows[5,6,7]:

1. During driving, the maximum amplitude of the compressive stress wave plus prestress at that time must not exceed the allowable compressive stress for the age of the concrete in use.

2. A limited eccentricity for prestress with slight camber on the final compression side may be advantageously introduced, but excessive eccentricity and camber would be harmful during driving.

3. To minimize flexural stresses and the optimum magnitude of prestress, supports may be arranged so that resulting stresses at maximum positive moment sections in the vertical span and at maximum negative moment sections over supports are practically equal, taking into account the effect of prestress and induced camber.

4. The precompression should be such that the prestress plus allowable tension at the age of the pile

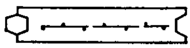
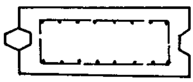
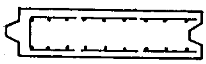
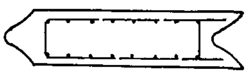
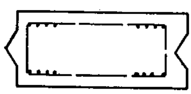
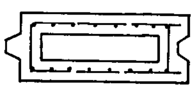
Cross-Section	Practical Applications	Special Merits
 (a)	Widely used for bank protection in Florida, where free cantilever is low	Low bank protection, low retaining wall, and low embankment near right-of-way line
 (b)	Widely used for bank protection in Florida, where free cantilever is high	High bank protection, high retaining wall and high embankment near right-of-way line
 (c)	Similar to (b), but with a tongue-and-groove joint, instead of a grout-filled joint as in (b)	Similar to (b), but dispensing with later grouting
 (d)	Same as (c)	Same as (c), but with the added advantage of less liability to binding during driving
 (e)	Commonly employed on the north coast of Germany	Medium-height bank protection, medium-height retaining wall, and medium-height embankment near right-of-way line
 (f)	Adaptable to combined bearing and sheet piles	When both greater overall thickness and saving in concrete are desired
 (g)	Similar to (f)	More rigid than (f), but more difficult to form

Fig. 1. Prestressed sheet pile cross-sections and applications

being driven is sufficient to counteract the maximum amplitude of the tensile stress wave during recoil.

5. In service, the prestress plus the allowable tensile stress in the concrete shall not be less than the flexural tensile stress caused by lateral pressure either over supports or in the spans.

The following notation is used to express optimum prestress criteria for pretensioned concrete sheet piles:

f_{ca} = allowable compressive stress in concrete under design loads in service

f_{cd} = maximum compressive stress in concrete due to driving at a section of maximum amplitude of the compressive stress wave

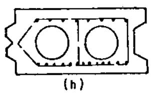
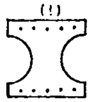
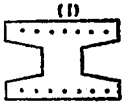
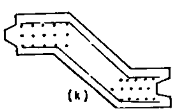
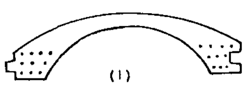
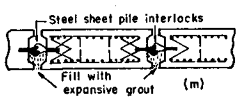
Cross-Section	Practical Applications	Special Merits
 (h)	Used for harbor quay wall of Pensacola, Fla., up to 40.5 m (133 ft.) of water and 18 m (59 ft.) long	Good for quay walls accommodating sea-going ships of moderate draft and subject to heavy bending in one direction; more complicated to form than (g)
 (i)	Adaptable to any application where greater transverse flexural rigidity and saving in concrete are of primary consideration	Good for greater heights and heavy bending; unsymmetrical prestressing may be used; groove may be filled with concrete when watertightness is essential
 (j)	Same as (i)	Same as (i), but easier to form
 (k)	Adaptable to longer overall width, utilizing fold-plate advantage	Good for sheet-piling cell formation in multiple-cell coffer dams
 (l)	Adaptable to longer overall width, utilizing arch action	Similar to (k), but more difficult to form on account of curved extrados and soffit
 (m)	Needed when much leakage is objectionable before grout filling	Conducive to water-tightness during driving and in service

Fig. 1 (continued). Prestressed sheet pile cross-sections and applications

f_{cf}, f_{tf} = respectively flexural compressive or tensile stress in concrete due to most unfavorable combination of external and environmental loads causing bending

f_{ci} = allowable compressive stress at, or shortly after, stress transfer = $0.60\, f'_{ci}$

f'_c = minimum compressive cylinder strength of concrete to be used

f'_{ci} = minimum compressive strength of concrete at stress transfer = $0.80\, f'_c$

f_l = total loss of precompression due to concrete shrinkage, creep, elastic compression, and steel relaxation

f_{op} = optimum compressive prestress in concrete (final effective value)

f_{ta} = allowable tensile stress in concrete corresponding to its age and exposure

f_{tr} = maximum tensile stress (at a section of maximum amplitude of the tensile stress wave) in concrete due to recoil in the process of driving, to be determined through analysis for each set of driving conditions

Considering 4 and 5 above, the larger tensile effect of f_{tr} or f_{tf} will usually govern the selection of the optimum prestress for sheet piling. If the tensile stress due to recoil f_{tr} has the larger effect, then

$$f_{op} + f_l + f_{ta} \geqq f_{tr} \quad (1)$$

$$f_{op} + f_l + f_{cd} \leqq f_{ci} \quad (2)$$

In the case where the flexural tensile stress f_{tf} has the larger final effect, then

$$f_{op} + f_{ta} \geqq f_{tf}$$
$$f_{op} + f_{cf} \leqq f_{ca}$$

and their combination will yield the optimum prestress as given by

$$f_{op} = \frac{1}{2}[(f_{ca} - f_{ta}) - (f_{cf} - f_{tf})] \quad (3)$$

However, in general, all the three criteria, Eqs. (1), (2) and (3), must be simultaneously satisfied by the optimum prestress f_{op}, to achieve the minimum concrete section and minimum prestressing strands.

OPTIMUM ANALYSIS OF SHEET PILING

There are three different support conditions for sheet piles: 1) self-supported free cantilevers fixed at some depth below the lower ground surface on the front side, as is usually the case with short sheet piling; 2) fixed at some depth and with top backstays, as in cases of higher sheet piling or of bulkhead sheet piling; and 3) fixed as in (2) with additional interior transverse supports (soldier beams and struts), as for most of the larger and deeper cofferdams. Analysis for support condition 3 is presented below. Since the analysis of self-supported, free-cantilever sheet piling is simple, no further treatment will be given here. Analysis of sheet piling with top backstays has been ably covered by Terzaghi and Tschebotarioff. Their original works[8-13] referenced at the end of this paper are recommended for guidance in analyzing condition 2.

Optimum analysis of sheet piling with interior transverse supports. In this category 3, the best approach is to arrange interior transverse supports to make all maximum positive moments approximately equal to the negative moments over supports[6,7]. Having identical maximum positive and negative moments will permit the minimum concrete section and minimum symmetrical prestressing which are always desirable for piles to be driven into the ground. Fig. 2 shows the concepts of the problem together with the necessary notation.

To start with, the last reaction R_3 at $\frac{1}{3}h'$ is given by

$$R_3 = \frac{1}{2}H^2w - \sum_0^2 R_i \quad (4)$$

in which

$$R_0 = \frac{1}{h_1}\left[\frac{1}{6}h_1^3\,w - M_1\right] \quad (5a)$$

$$R_1 = \frac{1}{h_2}\left[\frac{1}{6}(h_1 + h_2)^3\,w - R_0\,(h_1 + h_2) - M_2\right] \quad (5b)$$

$$R_2 = \frac{1}{h_3'}\left[\frac{1}{2}(h_1 + h_2 + h_3)^2\,w \times \left(\frac{H}{3} + h'\right) - R_0(h_1 + h_2 + h_3')\right.$$

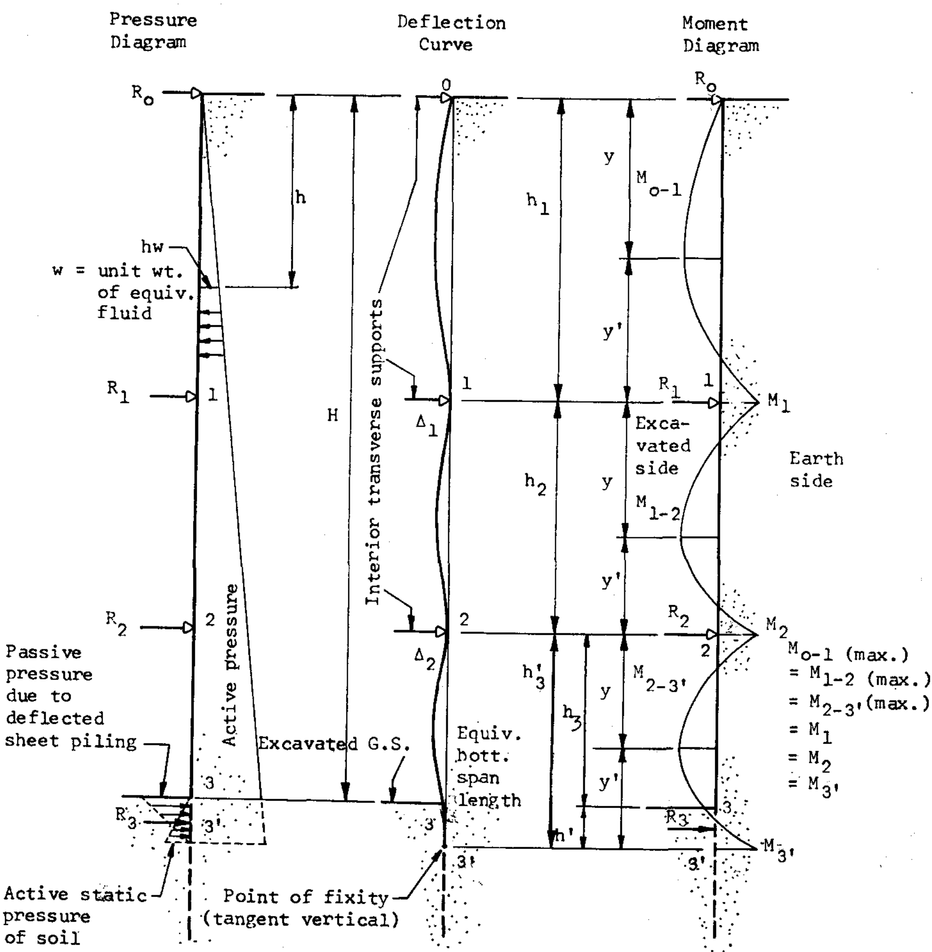


Fig. 2. Diagram of interior transverse support arrangements for equal maximum positive and negative moments

$$- R_1 (h_2 + h_3') - M_{3'}\Big] \qquad (5c)$$

But the premise is that numerically

$$M_{0\text{-}1}\,(\max) = M_{1\text{-}2}\,(\max) = M_{2\text{-}3}\,(\max) = M_1 = M_2 = M_{3'} = M \qquad (6)$$

Knowing $+ M_{max}$ in span 0-1 is at shear = 0, we have

$$R_0 = \frac{1}{2} h_{0\text{-}1}^2 w \qquad (7)$$

$$R_0 h_{0\text{-}1} - \frac{1}{2} h_{0\text{-}1}^2 w \left[\frac{h_{0\text{-}1}}{3}\right] = + \mathrm{M} \qquad (8)$$

Substituting R_0 from Eq. (7) in Eq. (8), there results

$$\frac{1}{2} h_{0\text{-}1}^3 w - \frac{1}{6} h_{0\text{-}1}^3 w = + M$$

$$h_{0\text{-}1} = \sqrt[3]{\frac{3\,M_{max}}{w}}$$

Then Eq. (7) gives

$$R_0 = \frac{1}{2}\,w\left(\frac{3M}{w}\right)^{2/3}$$

$$= \frac{1}{2}\sqrt[3]{9wM^2} \tag{9}$$

We can now solve M in Eq. (6) in terms of h_1 by writing

$$R_0 h_1 - \frac{1}{6}\,h_1^3\,w = -\,M$$

$$\frac{1}{2}\sqrt[3]{9wM^2}\,h_1 - \frac{h_1^3\,w}{6} = -\,M$$

$$M + \frac{1}{2}\,h_1\,(9w)^{1/3}\,M^{2/3} = \frac{wh_1^3}{6}$$

whose solution gives

$$M = 0.1008\,wh_1^3 \tag{10}$$

An examination of Eqs. (5), (6) and (10) shows that R_0, R_1, R_2 and M are in terms of unknowns h_1, h_2, h_3 and h'. Instead of four unknowns, there are really three, because

$$H = h_1 + h_2 + h_3 \tag{11a}$$

and $$h_3' = h_3 + h' \tag{11b}$$

We need to find three compatible equations to determine, say h_1, h_2 and h', knowing that $h_3 = H - (h_1 + h_2)$.

Writing total internal flexural strain energy from top to bottom, we get

$$U = \frac{1}{2EI}\left[\int_0^1 M_y\,dy + \int_1^2 M_y\,dy + \int_2^{3'} M_y\,dy\right] \tag{12}$$

Again, letting y' increase from bottom to top, beginning from 3′, 2 and 1, we have

$$U' = \frac{1}{2EI}\left[\int_{3'}^2 M_y\,dy' + \int_2^1 M_y\,dy' + \int_1^0 M_y\,dy'\right] \tag{13}$$

Realizing that for compatibility of continuity, there is only one tangent over both supports 1 and 2, and the rotation at $3' = 0$, we write:

$$\left[\frac{\partial U}{\partial M_1}\quad \frac{\partial U}{\partial M_2}\quad \frac{\partial U}{\partial M_3}\quad \Delta_1\quad \Delta_2\right] =$$

$$\left[-\frac{\partial U'}{\partial M_1}\quad -\frac{\partial U'}{\partial M_2}\quad 0\quad \frac{\partial U}{\partial R_1}\quad \frac{\partial U}{\partial R_2}\right] \tag{14}$$

which are the five simultaneous equations that will determine h_1, h_2, h', Δ_1 and Δ_2 satisfying both compatibility and equilibrium.

In order to discern M_i and R_i for partial differentiation, they must be distinguished in the equations before partial differentiation, and their values from Eqs. (5) and (10) substituted after partial differentiation, to proceed with the necessary numerical solution.

The solution of the first three of Eqs. (14) will provide h values for the solution of Δ_1 and Δ_2 in the last two of Eqs. (14). If the solution of Δ_i is positive, it is upward; if negative, it is downward.

The general mathematical set-up given here can be reduced for fewer tiers or extended for more tiers of interior transverse supports. In actual cases, the numerical combination of values will simplify the process, which is better than the derivation of complicated algebraic expressions for each h and Δ.

OPTIMUM DESIGN OF PRESTRESSED CONCRETE SHEET PILES

For optimum design of prestressed concrete sheet piles, we emphasize optimum permissive eccentricity, optimum load factors, and ultimate strength design for flexure. Allowable design stresses are first given[14,15].

Allowable design stresses. The following allowable stresses are adapted with the author's extensions from a draft of Section 2, Design, of the report of the PCI Committee on Prestressed Concrete Piling[14].

Compression in concrete due to bending:

(a) For sheet piles used with building work, or where computations are reliable to within 5 percent of actual design governing conditions . . $0.45f'_c$

(b) For sheet piles used with bridge work, or where the reliability of computations is not certain to be within 5 percent of actual design governing conditions . . . . $0.40f'_c$

(c) For sheet piles at, or shortly after, stress transfer (where f'_{ci} = minimum compressive strength of concrete at stress transfer $\leqq 0.80f'_c$) . . . . . . $0.60f'_{ci}$

Tension in concrete due to bending:

(a) For sheet piles exposed to corrosive environment or to freezing temperatures 0

(b) For sheet piles under design loads in service, not exposed to corrosive environment or to freezing temperatures . . . $3\sqrt{f'_c}$

(c) For sheet piles under transient loads . . . . $6\sqrt{f'_c}$

(d) For sheet piles at, or shortly after, stress transfer (where f'_c is the cylinder strength at the age of stress transfer) . . . . $3\sqrt{0.80f'_c}$

Range of effective prestress in concrete to meet driving, handling, and hauling requirements:

(a) For sheet piles 40 ft. (12m) or shorter . . . .
700-400 psi (49-28 kgf/cm^2)

(b) For sheet piles longer than 40 ft. (12m). . . . .
700-1250 psi (49-88 kgf/cm^2)

Effective prestress may be increased or decreased as required for driving, handling, or hauling. In general, effective prestress should not exceed $0.25f'_c$ for sheet piles unless the concrete in use has a strength higher than 5000 psi (350 kgf/cm^2).

Shear, bond, and torsion:

(a) Use ACI 318 latest allowable stresses for buildings.

(b) Use AASHO latest allowable stresses for highway bridges.

Steel stresses:

(a) Effective prestress . . . . . $0.60f'_s$ or $0.80f_{sy}$ (whichever is smaller)

(b) Pretensioning tendons immediately after transfer, or post-tensioning tendons immediately after anchoring . . . . . $0.70f'_s$

(c) Tension due to transient loads . . . . $0.66f'_s$

(d) Tension due to temporary jacking force (never greater than the maximum value recommended by the manufacturer of the steel) . . . . . . $0.80f'_s$

(e) Unstressed prestressing steel . . . . 30,000 psi (max) (2100 kgf/cm^2)

Optimum concrete strength. Li and Kuo have demonstrated[16] under the 1969 price structure in the U.S.A. that:

1. Unit cost of typical prestressed concrete flexural members with 9000 psi (630 kgf/cm^2) concrete reaches a practical minimum for both WSD and USD. While lower concrete strengths tend to increase the cost, the reduction in cost beyond 8000

psi (560 kgf/cm^2) concrete becomes insignificant.

2. It is more economical to use high-strength concrete in prestressed concrete piles. In ultimate strength design, while the unit cost would approach a minimum beyond 9000 psi concrete, the economic gain beyond this strength becomes insignificant.

Both of the above findings should apply to prestressed concrete sheet piles, provided the quality of local materials and workmanship permit the production of 8000 to 9000 psi concrete.

Optimum permissive eccentricity. Where it is impossible to balance the maximum positive and negative moments, it is best to introduce eccentric prestressing with eccentricity on the side of the larger tensile fiber stress.

How large an eccentricity is permissible? Any amount less than one-sixth of the depth of the concrete section would be permissible if the pile could be installed without driving. For a sheet pile to be driven into the ground, the eccentricity must be such that the smaller precompression on one face will not be less than that required to resist the maximum tensile stress wave amplitude during recoil in driving.

A portion of the allowable tension in the concrete may be taken into account wherever justified by engineering judgment in cases of very high-strength concrete. Under no circumstances, even for transient loading such as driving, should the allowable tension in the concrete exceed 0.4 of the modulus of rupture.

Load factors. Sheet piles are primarily flexural elements. The following minimum load factors(2,14,15) are recommended for ultimate strength design:

Normal service loading 2

Seismic loading; wind loading; waves and currents; ice 1½

Handling and hauling (after adding at least 50 percent of the dead load as impact, allowing tension in concrete not more than 0.4 of its modulus of rupture) 1½

Provision for crack resistance during handling 1¼

These minimum load factors should be reasonably increased when there is doubt as to the accuracy of the evaluation of loads or the analysis of design stresses.

ULTIMATE STRENGTH DESIGN FOR FLEXURE

Since working stress design of prestressed concrete sheet piles in flexure is not different from the design of other flexural members, we shall limit our treatment to ultimate strength design. There are two cases —symmetrical prestressing and eccentric prestressing. In either case, there must be enough precompression on the tension side to resist tensile stress during driving and in service, and enough compression capacity in the concrete on its compression side to give the above minimum load factors both during driving and in service.

To facilitate the development of ultimate strength design equations, the following notation is defined.

A_{sc} = area of prestressing steel in the compression zone

A_{st} = area of prestressing steel in the tension zone

b = width of the section

C = total compressive force in the concrete

C_u = total compressive force in the concrete at the ultimate stage

c = depth of neutral axis from the extreme compression fiber

D = total depth of the section

d = depth of the section from the extreme compression fiber to the centroid of the prestressing steel in the tension zone

d_c = distance of the centroid of the prestressing steel in the compression zone from the extreme compression fiber

d_t = distance of the centroid of the prestressing steel in the tension zone from the extreme tension fiber

E_s = Young's modulus for steel

e = eccentricity of the load or the prestressing force from the centroid of concrete section

ϵ_{se} = average strain in steel due to prestress after losses

ϵ_{su} = average strain in steel at failure of member

ϵ_u = ultimate strain in concrete

f_c = compressive stress in concrete at any point across the section

f_{pe} = effective prestress after losses

f_{sc} = tensile stress in the prestressing steel in the compression zone

f_{su} = average stress in tension steel at failure of member

f'_c = cylinder compressive strength of concrete

f'_s = ultimate tensile strength of prestressing steel

ϕ = capacity reduction factor

I = moment of inertia of the section

k_1 = ratio of average compressive stress to the practical limit of compressive stress at imminent failure of concrete in the compressive stress block (taken as 0.85 for a rectangular section)

k_2 = ratio of the depth of the resultant compressive stress in the concrete to the depth of the compression zone (taken as 0.425 for rectangular sections)

k_3 = reduction factor of f'_c to give the practical limit of compressive stress at imminent failure (normally taken as 0.85)

L = effective length of the pile

M_u = ultimate moment capacity of the pile under flexure alone (ultimate flexural capacity of uniformly prestressed pile)

o = axis through the centroid of the pile

p = ratio of prestressing steel

q = the ratio of $A_{st} f'_s / \mathrm{bd} f'_c$

r = radius of gyration

T_c = tensile force in the prestressing steel in the compression zone taken to act through the centroid of the steel in the compression zone

T_t = tensile force in the prestressing steel in the tension zone at ultimate load taken to act through its centroid

y_s = the distance from the centroid of the steel in the compression zone to the neutral axis

y = the distance from the neutral axis to a differential elementary area dA

Case A—Symmetrical prestressing for bending. Whenever the maximum positive span moments and negative moments over interior transverse supports can be made equal or approximately equal, or where the sheet piling is very short or subject to rather low bending

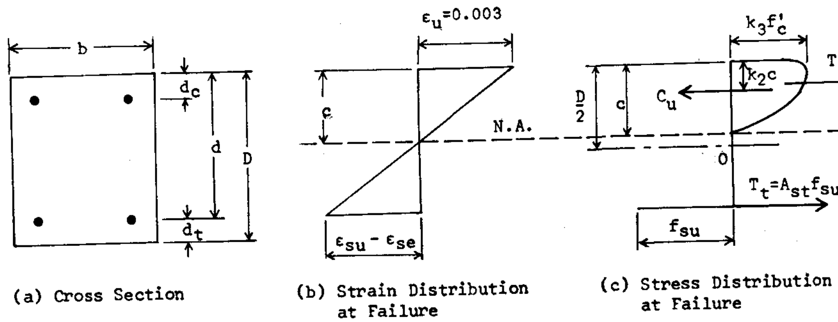


Fig. 3. Stress and strain conditions at ultimate load

moment, symmetrical prestressing to resist bending in sheet piles is highly recommended, for it is the most favorable prestressing under driving conditions.

If the tendons in a uniformly prestressed concrete pile are bonded, as is usually the case in pretensioned piles, there is no shift in tendon location relative to the concrete when the pile bends, and the tendons will not tend to buckle the pile. Unbonded tendons shift location relative to the concrete when the pile bends, and the tendons may tend to buckle the pile. Therefore, in post-tensioned piles, the tendons should always be bonded to the concrete by grouting immediately after prestress is transferred.

The ultimate flexural capacity of uniformly prestressed concrete sheet piles*, in which the tendons are fully bonded to the surrounding concrete, is given by the following equation:

$$M_u = 0.85\, bck_1 f'_c \left[\frac{D}{2} - k_2 c\right] + A_{st} f_{su}\left[\frac{D}{2} - d_t\right] - A_{sc}\left[f_{pe} - \frac{0.003\, E_s\, y_s}{c}\right] \times \left[\frac{D}{2} - d_c\right] \quad (15)$$

In deriving the above equation, the stress and strain conditions at ultimate failure are assumed as shown in Fig. 3, and the forces C_u, T_c and T_t are given by the following equations:

$$C_u = 0.85\, k_1 f'_c\, cb \quad (16)$$

$$T_c = A_{sc}\left[f_{pe} - \frac{0.003\, E_s\, y_s}{c}\right] \quad (17)$$

$$T_t = A_{st} f_{su} \quad (18)$$

$$0.85\, bck_1 f'_c = A_{st} A_{su} + A_{sc}\left[f_{pe} - \frac{0.003\, E_s\, y_s}{c}\right] \quad (19)$$

The proper value of f_{su} should be used in the above equations for an accurate estimate of the ultimate moment capacity of the pile. If the full stress-strain curve for the steel is available, the following rigorous method based on the compatibility

**Capacity reduction factor must be applied to obtain design value as explained on p. 72.*

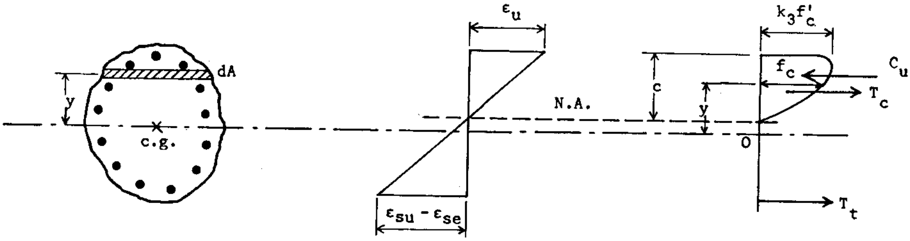


Fig. 4. Strain and stress distribution in a uniformly prestressed concrete pile of any transverse configuration

of strains can be followed for calculating f_{su}:

1. Calculate the effective prestress in the tensioned steel, and hence the effective strain ϵ_{se} due to prestress;
2. Assume a trial value for c, find $(\epsilon_{su} - \epsilon_{se})$ from the strain diagram in Fig. 3b, and then ϵ_{su};
3. From the stress-strain curve for the steel, find the stress, f_{su}, corresponding to a strain of ϵ_{su};
4. Evaluate C_u, T_c and T_t; if they satisfy Eq. (19), then the assumed value of c is correct; if not, assume a progressively better value for c and repeat steps 2 to 4 until Eq. (19) is practically satisfied;
5. Find M_u from Eq. (15).

The above method is laborious. Alternatively, the stress f_{su} for all pretensioned and post-tensioned piles with bonded tendons could be obtained from the following expression suggested by the ACI Building Code 318-71[17]

$$f_{su} = f'_s (1 - 0.5\, p\, f'_s / f'_c) \qquad (20)$$

For pretensioned piles, the steel stress f_{su} will nearly always reach the ultimate strength f'_s, the only exception being the case of high values of the ratio $q = A_{st} f'_s / bd f'_c$. This ratio, which is sometimes called the effective proportion of steel, has a much greater influence on the maximum steel stress in fully bonded post-tensioned concrete, and at high values of q the maximum stress may be as low as $0.75 f'_s$, which for most steels used in prestressed concrete work will be very close to the elastic range.

The ultimate moment capacity of Eq. (15) is derived for rectangular sections. The same analysis can be extended to a section of any shape as shown in Fig. 4. The ultimate internal compressive force C_u and the moment of this force about the transverse axis O through the centroid of the pile section are given by

$$C_u = \int_0^c f_c \, dA \qquad (21)$$

$$M \text{ (due to } C_u) = \int_0^c f_c y \, dA \qquad (22)$$

Substituting the above values into Eqs. (19) and (15), the ultimate moment capacity of any pile section can be determined.

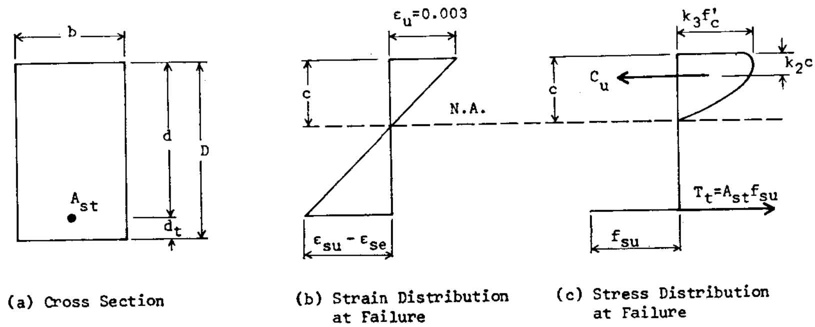


Fig. 5. Stress and strain conditions at ultimate load for nonuniformly prestressed sheet pile

Case B—Eccentric prestressing for bending. Whenever the maximum positive moments and negative moments over supports cannot be balanced, or wherever the sheet piling is subject to a significant difference in flexural tension on opposite faces, eccentric prestressing on the greater tension side under service loads is highly recommended. The necessary ultimate strength design approach follows.

The ultimate resisting moment M_u of a nonuniformly prestressed concrete member is obtained by assuming the stress and strain conditions at ultimate failure as shown in Fig. 5. From a study of the forces, the resultant compressive force at failure is

$$C_u = bck_1 k_3 f'_c \tag{23}$$

and the ultimate moment is

$$M_u = A_{st} f_{su} d \left[1 - \frac{k_2 A_{st} f_{su}}{k_1 k_3 bd f'_c} \right] \tag{24}$$

For the typical range of concrete strengths normally used for prestressed concrete piles, the parameters defining the shape of the stress block at failure and the ultimate strain capacity of concrete are adopted as follows: $k_1 = 0.8$; $k_2 = 0.4$; $k_3 = 0.85$; $\epsilon_u = 0.003$. Using the above values, the ultimate moment* is given by the following equation:

$$M_u = A_{st} f_{su} d \left[1 - \frac{A_{st} f_{su}}{1.7 f'_c bd} \right] \tag{25}$$

For making an accurate estimate of the ultimate moment, the proper value of f_{su} must be used as explained for the previous case of symmetrically prestressed piles (Case A).

Capacity reduction factor. ACI 318-71 requires that the ultimate moment capacity of members subjected to flexure be multiplied by a factor ϕ, called the capacity reduction factor, to allow for faulty workmanship, etc. The value of ϕ is 0.90 for flexure. Therefore, the ultimate moment capacities given by Eqs. (15) and (25) must be multiplied by 0.9 to obtain the design values.

OPTIMIZATION OF DETAILS

For optimum design, considera-

*Capacity reduction factor must be applied to obtain design value, as explained on p. 72.

tion[2] should be extended to a number of details. When the difference between positive and negative moment is too big for eccentric prestressing alone to resist, unstressed mild steel bars may be added on the tension side in regions of larger maximum design moment. To avoid breaking off the wings of the groove during driving, it is necessary to provide and extend mild steel bars into the wings.

To enable one pile to be driven to the same grade as the adjoining pile, a special helmet may be used or a short narrow extension may be cast on the head at the time of manufacture, to serve as a driving block for the hammer. After driving, this extension may be cut off and the tendons tied into a capping beam, or the space between extensions used for tie backs.

It is good practice to require that the tongue of tongue-and-groove cross-section piles lead in driving, to prevent plugging that would occur if the groove end was leading. When a double-groove joint is used, it is good practice to require a steel pipe splice to be driven with the pile and later withdrawn.

Except where there is no sand or water leakage, joints should be grouted, preferably using expansive cement, to make them tight. When accessible and necessary, hand packing may be required. In grouting through water, it is good practice to call for a canvas or polyethylene bag (tube) to be pushed down on a grout pipe, then the grout injected as the pipe is withdrawn.

SPECIFIC ADVANTAGES AND GENERAL COST COMPARISON

Prestressed high-strength concrete sheet piles can be produced at almost the same cost as unpainted steel sheet piles, and at least 5 percent cheaper than steel sheet piles painted for protection against corrosion[2].

The advantages of prestressed concrete sheet piles lie in their superior durability, excellent appearance, and good resistance to bending. Therefore, any economic comparison of prestressed concrete sheet piles with steel sheet piles must be made on the basis of capitalized cost or annual cost. In such a true economic comparison, prestressed concrete sheet piles can stand on their own merits.

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