ALLOWABLE TENSILE STRESSES FOR PRESTRESSED CONCRETE

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Shortly after the ACI Building Code (ACI 318-63) was revised in 1963, there was concern that some of the Code provisions for prestressed concrete products made it difficult, in many instances, for the products to meet serviceability requirements. This committee was formed to recommend higher allowable tensile stresses in prestressed concrete flexural members while maintaining serviceability requirements. The work of the committee was largely responsible for the changes in allowable tensile stresses for prestressed concrete in the draft of the revised ACI Building Code (see ACI Journal, February 1970). The committee report gives the rationale behind the use of higher tensile stresses in flexural members. Discussion of this report is invited.

—George G. Goble, *Chairman*

NOTATION

A = overload factor
f_c = apparent cracking stress*
f_t = allowable tensile stress
f_o = difference between cracking stress and allowable service load tensile stress
f_s = stress change in the bottom fiber due to superimposed load
I_c = cracked section moment of inertia**
I_g = gross section moment of inertia
$R = \dfrac{I_g}{I_c}$
Δ = allowable deflection at overload
δ = deflection due to superimposed service load computed using I_g
δ_g = deflection at which cracking occurs
δ_c = added deflection after crack-

**For normal percentages and good distribution of bonded steel,* f_c *can be taken as* $10\sqrt{f'_c}$.

***The cracked section moment of inertia shall be calculated neglecting the effect of the prestress force on the location of the neutral axis.* I_c *is approximately equal to* $nA_sd^2(1-\sqrt{p})$, *an empirical relationship that may be used in lieu of a more exact analysis.*

ing under overload

d = depth to centroid of tensile steel at midspan

p = $\frac{A_s}{bd}$

b = width of compression flange

A_s = area of tensile steel at midspan

n = modular ratio

W_s = superimposed service load

W_c = load at which cracking occurs

W_o = difference between cracking load and service load

INTRODUCTION

The design of a satisfactory prestressed concrete beam must be such that it provides sufficient safety factor with satisfactory behavior under service conditions. The 1963 ACI Building Code[1] sought to achieve this goal by placing limits on bending strength, shear strength, cracking strength and bending stresses. Live load deflections were also limited. Historically, in the design of prestressed concrete beams (as well as most other structural elements), allowable stresses have been specified to control serviceability. Although stress limitations have a creditable record in controlling service behavior, it appears more realistic to place direct limitations on deflection, crack width, or dynamic response.

The use of stress limits in prestressed concrete design will not always provide satisfactory behavior. For example, if the allowable tensile stress is small and the live load contains a large temporary load, such as snow, considerable camber growth can occur. In this case, use of a larger tensile stress may be required to achieve satisfactory behavior. Caution should be used in increasing allowable tensile stresses because of the effect on other serviceability considerations. Deflection is of particular interest because it is directly affected by changes in allowable stresses.

In addition to the problem of camber growth, unsatisfactory deflection behavior can also occur after cracking under some conditions. Certain types of prestressed beams can exhibit a very substantial loss of stiffness when loaded above the cracking level. In view of these two extreme cases, it seems apparent that any recommended allowable tensile stress is dependent upon the deflection characteristics of the beam both above and below the cracking load.

A simplified rational method of determining an allowable tensile stress, with a bi-linear assumption of load-deflection behavior, is presented here. It is based on the post-cracking behavior of bonded prestressed concrete beams.

LOAD-DEFLECTION BEHAVIOR

Typical prestressed beam load-deflection characteristics are illustrated in Fig. 1. Up to load level H linear behavior is typical. In this region the beam is elastic. Its stiffness is predictable from the concrete modulus of elasticity and the cross section properties. Cracking has not occurred. Above load level H some non-linearity is observed. At this point, cracking has occurred but still is not visible. With increasing load, visible cracks appear. Beams with identical stiffnesses prior to visible cracking can exhibit substantially different behavior at loads above the cracking load. One type, shown by the dashed line, exhibits non-linear behavior up to failure load. In the other case, after a substantial loss in stiffness, a nearly linear behavior occurs for a large range of deflection. The two curves shown do not necessarily represent limits of behavior nor is there a sharp line differentiating the two types of members. In the first case, failure generally occurs at relatively smaller deflections than the second case. The characteristic of the load-deflection curve above *H* is determined by the cross section, percentage and distribution of steel, cracking strength of concrete, crack distribution and probably many other variables.

After a prestressed beam has cracked and particularly if steel stresses are in the inelastic region, reloading curves differ from initial loadings. However, it is shown(2) that when the load reaches the level of the maximum previous load, the load-deflection curve will then continue on its previous path.

The proposed equation for allowable tensile stress is based on an assumed bi-linear curve for load-deflection behavior. To develop this curve, three parameters had to be established—the slope of the initial and final portions of the curve and the point of change in slope. The initial part of the curve is based on the moment of inertia of the gross section. The second stage stiffness is assumed to be proportional to the cracked moment of inertia (calculated as if the prestress force were zero). Although this assumption cannot be supported by rational analysis it agrees reasonably well with deflection test data. The stress at visible cracking represents a reasonable point of transition between the two curves.

A series of tests was performed at the University of Illinois(3). While the primary purpose of these tests was the determination of the strength of prestressed members, deflection

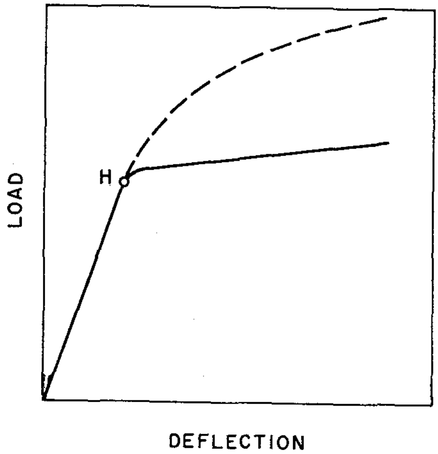

Fig. 1. Typical prestressed beam load-deflection behavior

and moment-curvature data were obtained. The Illinois tests were on beams with a rectangular cross section. Only part of the bonded beams are examined here. These tests included designs with both high and low percentages of steel. Tendons were straight. Some were fully tensioned, some only partially tensioned. Many of these beams did not show the bi-linear behavior char-

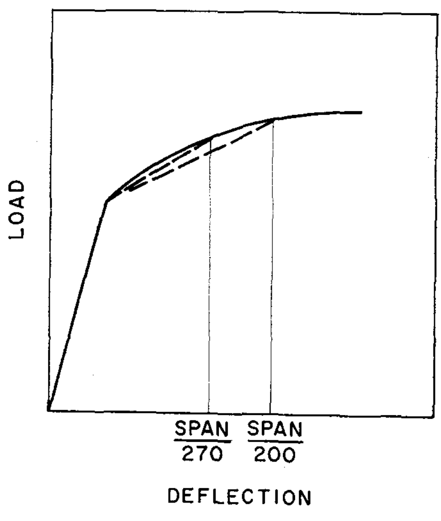

Fig. 2. Method for evaluating non-linear deflections

Table 1. Summary of results of Illinois tests

Beam	p %	Theoretical cracked stiffness(1) k/in.	Cracked stiffness(2)	
			@ L/270 k/in.	@ L/200 k/in.
OB.24.189	0.815	23.6	24.1	20.4
OB.24.168	0.579	19.5	20.8	18.8
OB.24.190	0.475	14.3	13.1	10.3
OB.24.061	0.177	6.5	3.9	3.5
OB.14.157	0.870	27.6	20.9	20.9
OB.14.175	0.656	20.0	20.2	19.1
OB.14.066	0.418	20.5	17.9	17.0
OB.14.030	0.107	5.5	2.8	3.4
OB.34.038	0.312	15.5	16.3	14.2
OB.34.043	0.284	14.1	14.4	11.7
OB.34.027	0.101	6.6	1.2	1.4
OB.34.115	0.943	28.5	47.5	37.3
OB.34.076	0.424	20.0	20.0	16.5
OB.34.120	0.413	20.0	19.8	16.2
OB.34.346	0.440	19.6	21.7	16.9
OB.34.122	0.746	23.7	31.0	28.4
OB.34.159	0.942	27.0	37.0	30.5
OB.44.140	0.873	27.1	40.2	32.0
OB.44.158	0.647	20.8	33.1	24.9
OB.44.032	0.108	5.7	0.6	0.6
RB.34.031	0.166	9.0	6.1	5.3
RB.34.093	0.369	18.5	22.0	18.1
RB.34.126	0.658	29.8	38.5	31.9

1. Theoretical cracked stiffness based on theory presented in Fig. 3.
2. Slope of line drawn between point of visible cracking and the load on the member where deflection equaled either L/200 or L/270 (see Fig. 2).

acteristics desirable for the proposed theory, but after cracking exhibited a gradual reduction in stiffness similar to the dashed curve in Fig. 1. However, those rectangular beams with low steel percentages (high I_g/I_c ratio) lost stiffness drastically at cracking.

The results of a number of the Illinois tests are shown in Table 1. The cracked slopes were obtained by constructing a straight line from the cracking load (the point where the curve first deviates from linear) to deflections of $\frac{1}{270}$ and $\frac{1}{200}$ of the span. The method is illustrated in Fig. 2.

A limited number of tests on full scale beams are summarized in Table 2. These results confirm the mathematical approximation of the behavior fairly well. Furthermore, the load-deflection curves followed the bi-linear behavior quite closely.

In general, the correlation between the proposed method and the tests is reasonably close. In cases with low I_g/I_c ratios, experimental results show that post-cracking stiffness is greater than predicted. The beams having high I_g/I_c ratios are more flexible than the method indicates and for very high ratios the difference is substantial.

If a prestressed beam exhibits the sort of behavior illustrated by the dashed line in Fig. 1, there is no reason why the beam should not be allowed to approach or exceed the cracking load under design load for the usual applications of prestressed concrete. Because the beam loses very little stiffness on cracking, it could be loaded above the cracking load in the working condition (from the standpoint of deflections). However, for beams that undergo a significant loss of stiffness, as illustrated by the solid line in Fig. 1, it is undesirable to have the cracking load exceeded under working conditions.

PROPOSED FORMULA FOR *ALLOWABLE TENSILE STRESS*

The basis for the proposed formula is a load-deflection curve for a prestressed beam represented by a bi-linear curve. The slope of the first portion of the curve is determined by the moment of inertia of the gross concrete section. The slope of the second portion is determined by the moment of inertia of the cracked section analyzed as in reinforced concrete. The point of change in slope is determined by an assigned cracking stress and the prestress conditions.

When other structural materials, such as wood, steel and reinforced concrete, are used the designer can expect that no sudden change in load-deflection behavior will occur at loads near the design load. Either a gradual reduction in stiffness develops or a more rapid change occurs near the ultimate load. In the case of prestressed beams, the point of change in behavior depends on the stress state. Therefore, if beams having a low post-cracking stiffness are designed with service loads near the cracking load an undesirable behavior could result due to small overloads or a small accidental re-

Table 2. Full scale tests

Identification	p %	Theoretical cracked stiffness k/in.	Cracked stiffness @ L/270 k/in.	Cracked stiffness @ L/200 k/in.
PCA A66 Tee fully prestressed	0.0685	6.80	7.16	6.85
PCA B64 Tee some mild steel	0.133	12.50	9.80	9.50
PCA C64 Tee strand 2/3 stressed	0.0685	6.80	7.06	7.06
CMC 16TT 48 lightweight coarse agg.	0.226	0.024*	0.028	0.028
CMC 16TT 48 lightweight coarse agg.	0.172	6.33	5.50	5.50

* Based on a simulated uniformly distributed load

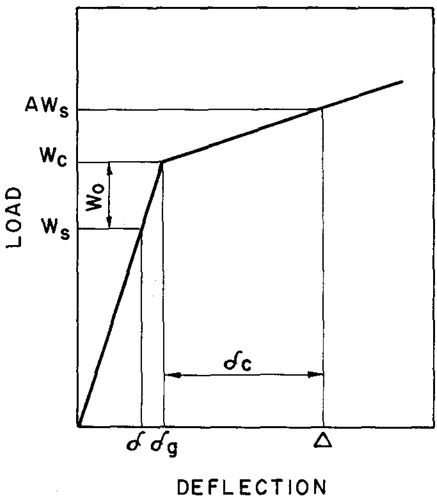

Fig. 3. Proposed theory for load-deflection relationship

duction of the cracking load. Thus, in addition to a check on deflection for service load, the deflection must be checked for a small overload compatible with the intended use of the structure. This latter requirement can be used to set the limiting cracking load and hence the allowable tensile stress. These overload deflection requirements can be satisfied in a particular design by increasing the uncracked stiffness, by increasing the cracked stiffness, or by increasing the cracking load. They cannot be satisfied by checking deflections under service load if this load is below the cracking load.

The proposed limitation on allowable tensile stress will be derived with the aid of Fig. 3. The limiting overload deflection can be stated

$$\Delta = \delta_g + \delta_c \tag{1}$$

By substituting values for δ_g and δ_c, it is found that

$$\Delta = \delta\left(1 + \frac{W_o}{W_s}\right) + R\,\delta\left(\frac{AW_s - W_s - W_o}{W_s}\right) \tag{2}$$

Equation (2) can be reduced to

$$\frac{\Delta}{\delta} = 1 + \frac{W_o}{W_s} + R\left(A - 1 - \frac{W_o}{W_s}\right) \tag{3}$$

It can be stated that

$$\frac{W_o}{W_s} = \frac{f_o}{f_s} \tag{4}$$

By substitution

$$\frac{\Delta}{\delta} = 1 + AR - R + \frac{f_o}{f_s}(1 - R) \tag{5}$$

Now

$$f_t = f_c - f_o \tag{6}$$

Then substituting in Equation (5) for f_o and rearranging terms

$$f_t = f_c - f_s\left[\frac{\frac{\Delta}{\delta} - AR}{1 - R} - 1\right] \tag{7}$$

DESIGN PROCEDURE FOR FLEXURAL MEMBERS

1. Select a beam cross section.
2. Determine the area of prestressing steel to meet ultimate requirements.
3. Select prestress force.
4. Check the following:
   a) Working stresses
   b) Camber and deflections including long term effects
   c) Overload deflections.

REFERENCES

1. "Building Code Requirements for Reinforced Concrete," ACI 318-63, June 1963, American Concrete Institute, Detroit, Mich.
2. Campbell, F. S., "Load-Deflection Behavior of Prestressed Concrete Beams" Master's Thesis, Case Institute of Technology, 1966.
3. Warwaruk, J., Sozen, M., and Siess, C., "Strength and Behavior in Flexure of Prestressed Concrete Beams", *Engineering Experiment Station Bulletin No. 464,* University of Illinois, 1962.