Combined Bending and Axial Load in Prestressed Concrete Columns

by Paul Zia and E. C. Guillermo*

INTRODUCTION

Columns in a framed structure are often required to resist bending moments as well as axial forces. The bending moment may be induced by wind and earthquake forces, eccentric loads, or frame action. In precast concrete construction, bending in columns also occurs during handling. In such cases, prestressing may be of considerable value in providing greater strength and stiffness to the column.

A concrete column may be fully prestressed, partially prestressed or tri-axially prestressed. A fully prestressed column is one in which all the high strength reinforcement is fully tensioned. In a partially prestressed column, either all the high strength reinforcement is partially tensioned or the fully tensioned high strength reinforcement is supplemented by non-prestressed reinforcement. The tri-axial prestressing of a column is achieved by combining spiral reinforcement with longitudinal prestressing. The spiral reinforcement may be prestressed either by the Poisson's ratio effect resulting from longitudinal prestressing or by chemical prestressing with expansive cement. The following discussion is confined to the first two types of prestressed concrete columns.

Theoretical and experimental studies of fully and partially prestressed concrete columns have been reported by various authors (see references). These studies have dealt almost exclusively with rectangular sections containing symmetrically placed reinforcement. The effects of eccentricity, slenderness ratio and percentage of steel have been the major considerations.

This paper presents the results of a study examining the ultimate strength of the column under combined bending and axial load as affected by the variation of column cross section, amount and location of prestressing reinforcement, and concrete strength. Nine widely different column cross sections were selected for analysis (Fig. 1). It should be noted that these cross sections are, in reality, standard AASHO-PCI pile sections with only a minor modification. In order to have a symmetrical section for the convenience of analysis, all sections contain an even number of prestressing strands. A computer program to obtain the interaction curves was prepared in a very general manner so that it is possible to analyze the columns with a wide range of variation of the different parameters. A total of 54

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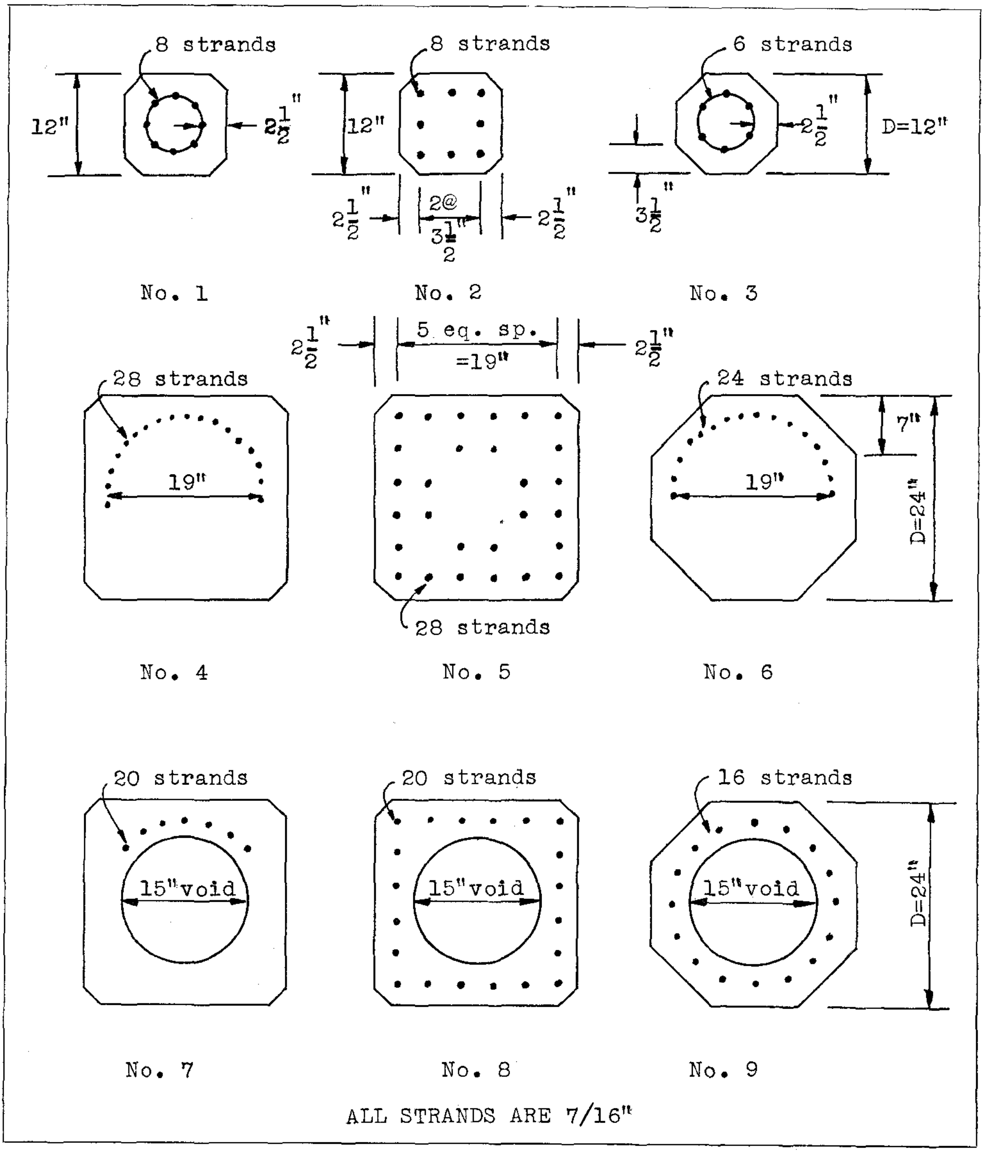

Fig. 1—Column Cross Sections Included in Analytical Study

different cases was studied. For each of the nine columns, analysis was performed for $f'_c = 5000$ psi and $f'_c = 7000$ psi as follows:

1. The strands are all prestressed to an effective prestress of 70 ksi.
2. One half of the strands are prestressed to an effective prestress of 140 ksi, and the remaining half of the strands are left untensioned.
3. An effective prestress of 140 ksi is applied to all the strands.

THEORETICAL FORMULATION

The theoretical formulation was based upon the following assumptions:

1. Compressive stress distribution in concrete is replaced by the equivalent rectangular stress block as permitted by the ACI Building Code (318-63).
2. Actual stress-strain curve of the strand as furnished by the manufacturers is used.
3. Tension in concrete is neglected.
4. The compressive stress in the concrete is computed without regard to the small amount of concrete displaced by the prestressing steel.
5. The ultimate strain in concrete is taken as

$$\epsilon_u = 0.003$$

and the modulus of elasticity as

$$E_c = w^{1.5}\, 33\sqrt{f'_c},\; w = 145 \text{ pcf}$$

The basic conditions of static equilibrium and strain compatibility for a prestressed column section under ultimate load are given in Fig. 2. These conditions lead to the governing equations

$$\Delta\epsilon_i = \epsilon_u \left(\frac{d_i - k_u d}{k_u d}\right) \qquad (1)$$

$$T_i = A_i E_s (\epsilon_{se} + \epsilon_{ce} + \Delta\epsilon_i) \qquad (2)$$

$$P_u = C - \Sigma\, T_i \qquad (3)$$

$$M_u = P_u e' = M_c + \Sigma\, M_{Ti} \qquad (4)$$

where ϵ_{se} = steel strain under effective prestress
ϵ_{ce} = concrete strain under effective prestress
A_i = area of i^{th} steel
M_c = moment of C about column center line
M_{Ti} = moment of T_i about column center line

The solution of this type of problem is most expeditiously executed by first selecting a value of $k_u d$ and then calculating P_u and M_u from Equations (1) through (4).

INTERACTION CURVE

The results of the study can be most conveniently expressed in the form of dimensionless interaction curves as in the case of reinforced

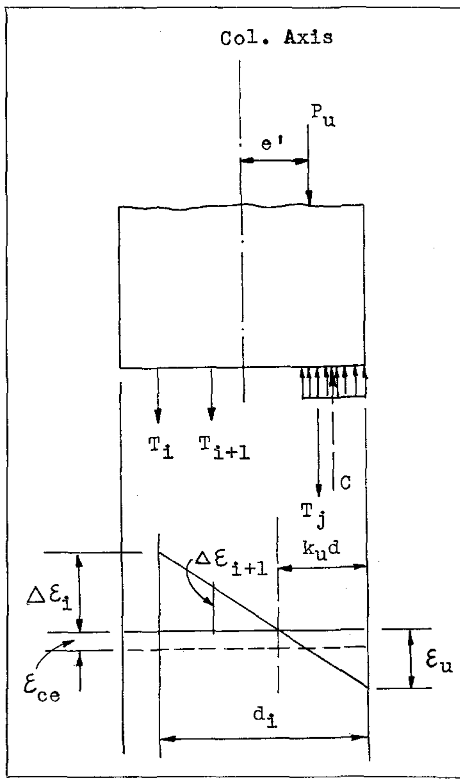

Fig. 2—Stresses and Strains on a Typical Column Cross Section

concrete columns. A typical set of the curves is shown in Fig. 3*. Several interesting observations can be made:

1. For the partially prestressed columns, whether the prestressing strands were only partially tensioned or a portion of the strands fully tensioned and the remaining portion untensioned, the ultimate strengths for the two cases are identical.

2. Full prestressing reduces the ultimate strength of the column as compared to 50% partial prestressing, and the reduction in load capacity is nearly constant regardless of the applied bending moment.

3. For columns subjected to large axial loads $(P_u > 0.55 f'_c A_g)$, a reduction of 50% of prestressing produces a significant increase in the bending capacity of the column, whereas for columns subjected to small axial loads $(P_u < 0.35 f'_c A_g)$, a reduction of 50% of prestressing would cause a slight reduction in the bending capacity of the columns.

4. In spite of the large variation in column cross section, amount and location of prestressing steel, and concrete strength, the dimensionless interaction curves for the various cases are nearly identical. For 50% partial prestressing, the point m corresponding to the maximum moment occurs at:

$$\left(\frac{P_u}{A_g f'_c}\right)_m = 0.375$$

$$\text{and} \left(\frac{M_u}{A_g D f'_c}\right)_m = 0.1125$$

For full prestressing, the similar point n occurs at:

$$\left(\frac{P_u}{A_g f'_c}\right)_n = 0.30$$

$$\text{and} \left(\frac{M_u}{A_g D f'_c}\right)_n = 0.1125$$

5. Although the interaction curve is similar in form to that of the ordinary reinforced concrete column, the mode of failure for prestressed columns is quite different. For reinforced concrete columns, an interaction curve of this type represents two modes of failure: compression failure for the range above the point m and tension failure for the range below the point m; the point m is referred to as "balanced failure point". For the cases of prestressed columns studied herein, the entire range of the interaction curve represents the mode of compression failure.

MODE OF FAILURE

The difference in the mode of failure for prestressed concrete columns as compared with that for reinforced concrete columns is an unusual feature that requires further explanation.

In Fig. 4(a), the moment resistance contributed by the steel, M_T, and the moment due to the compressive force in the concrete, M_C, are plotted against the depth of compression zone. Also plotted are, the total tension in the steel, T, and the compressive force in concrete, C. These curves are obtained from the computations of a typical case—the 24-in. octagonal column (solid section). It can be seen that as the depth of compression zone increases, the force C increases with a much greater rate than the rate of decrease of the force T. This is so because the force caused by the change in steel strain is insignificant compared to the initial prestressing force. The rapid increase of C causes the M_C curve to rise sharply until the neutral axis

* It should be emphasized that in these figures, the dimensionless quantities $P_u/A_g f'_c$ and $M_u/A_g D f'_c$ have been used, in which A_g is the gross cross-sectional area of the column.

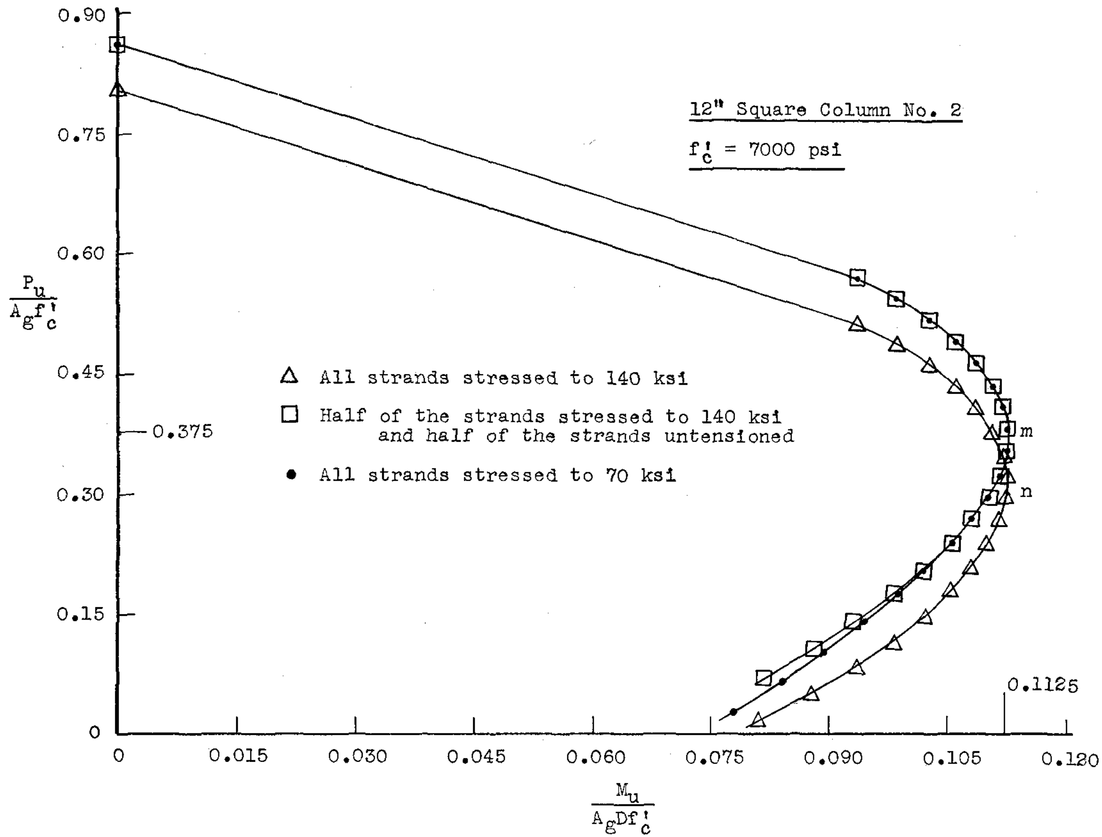

Fig. 3—Interaction Curve for 12-in. Square Column

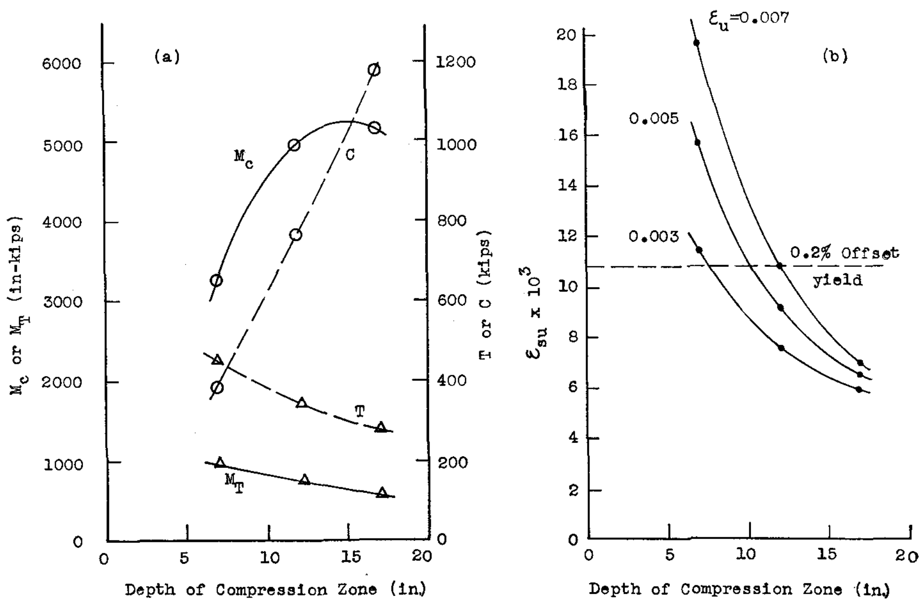

Fig. 4—Relation of Forces, Moments and Strains in Typical Column Cross Section to Depth of Compression Zone

reached the center line of the column. Thereafter the force C continues to increase at nearly the same rate, but its moment arm is reduced significantly. As a result, the M_C curve flattens out, reaching a peak value when the neutral axis is located in a position such that the depth of the equivalent stress block for the concrete is equal to one half of the depth of the column section.

Since the ultimate moment capacity is given by $M_u = M_C + M_T$, and the contribution of M_T is relatively small, the moment capacity, M_u of the column is mainly determined by M_C. As the depth of compression zone increases, the moment capacity of the column increases along with its ultimate load capacity, $P_u = C - T$ until it reaches the maximum value as indicated by the point m or n in Fig. 3. For the case in question, the corresponding depth of compression zone is 15 in. as can be seen from Fig. 4(a).

This behavior is similar to that of most conventional reinforced concrete columns. However, the corresponding mode of failure of the column depends entirely on the maximum strain, ϵ_{su}, developed in the steel which in turn depends on the ultimate strain, ϵ_u, in concrete.

In Fig. 4(b), the maximum strain in the steel, ϵ_{su}, is plotted against the depth of compression zone for various values of the ultimate concrete strain, ϵ_u. It is clear that for $\epsilon_u = 0.003$, ϵ_{su} is well below the strain corresponding to the 0.2% offset yield strength and therefore the column could only fail in compression. Had a greater value of ϵ_u been assumed, such as 0.005 or 0.007, it would have been possible for the column to fail in tension at least for a small portion of the range corresponding to the lower branch of the interaction curve.

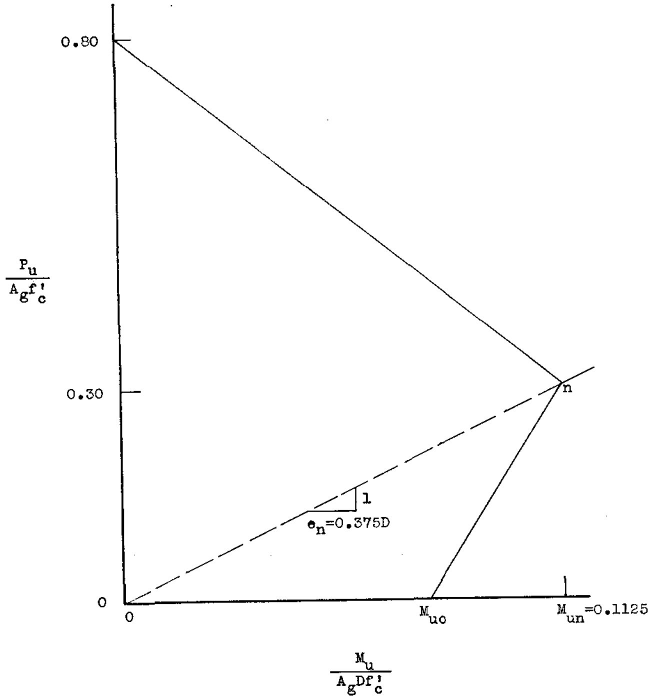

Fig. 5—Idealized Interaction Curve for Prestressed Concrete Column

CONCLUSIONS

Based on the results of this study, the following conclusions may be drawn:

1. The non-dimensional interaction curve of prestressed concrete columns under combined bending and axial compression resembles the similar curves for ordinary reinforced concrete columns, except that the mode of failure for the former is totally compressive if the maximum concrete strain is assumed to be 0.003.

2. Wide variations of the column cross section, of the concrete strength, and the arrangement of prestressing steel do not have any significant effect on the dimensionless interaction curve.

3. Partial prestressing increases slightly the load capacity of the column under a given bending moment. For an axial load $P_u > 0.55\ f'_c\ A_g$, partial prestressing causes a significant increase in the moment capacity of the column. On the other hand, for an axial load $P_u < 0.35 f'_c A_g$, full prestressing would result in a slightly higher moment capacity of the column.

4. A simple design method appears possible if one takes the idealized interaction curve for fully prestressed columns as the lower bound and follows the similar approach as the current design method for reinforced concrete columns under combined bending and compression. This method is illustrated in Fig. 5.

For $e < e_n$, $P_u \leqq 0.80f'_c A_g - \frac{4.45M_u}{D}$

For $e > e_n$, $P_u \geqq 0.3f'_c A_g - \left(\frac{M_u - M_{uo}}{M_{un} - M_{uo}}\right)$

where $M_{un} = 0.1125 f'_c D A_g$

M_{uo} = ultimate moment under pure bending

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