Circular Roofs with Radial Framing
Student Activities-Physical Education Building
Agricultural and Technical Institute, Farmingdale, N.Y.

by Eric C. Molke*

A. GENERAL DESIGN CONSIDERATIONS

CIRCULAR SLAB

It is much simpler to design a cover over a circle than over a square. A slab uniformly loaded and supported along a square perimeter is not uniformly supported all around, as it will be distorted by uplift and torsion in the corners. A circular slab under uniform load will, however, dish uniformly and will therefore have a typical set of radial and tangential moments which can be calculated easily.

Fig. 1 illustrates moment distribution in a square slab. Moments shown arc along strips on the four axes of symmetry. In spite of unrestrained edge bearing, reaction and moment will be negative in the corners due to the corner effect.

The circular slab in Fig. 2 illustrates radial moments along strips in the radial direction and tangential moments along circular strips. Such principal moments are acting on any square unit area cut from the slab.

Since the edge reactions are uniform all around, it is possible to draw a free-body diagram for one-half of the circular slab as shown in Fig. 3.

With $W/2$ denoting the uniform load over the half circle, we know the load center of gravity to be $\frac{2}{3}\times\frac{D}{\pi}$ from the center. Likewise the circular line A to B, constituting the support of the free-body, has its center of gravity at $\frac{D}{\pi}$ from the center. For equilibrium, the total moment around axis A-B must therefore be

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$$M_o = \frac{W}{2} \times \left(\frac{D}{\pi} - \frac{2}{3}\frac{D}{\pi}\right) = \frac{WD}{6\pi}$$

M_o represents the sum of all tangential moments M_t along axis A-B. From other more theoretical sources we know that M_t has its maximum value, M_t^c, at the center and its minimum value $M_t^p = \frac{2}{3} M_t^c$ at the periphery. The distribution between these values is parabolic as shown in Figs. 2 and 3.

The radial moment at the center, M_r^c, for reasons of symmetry, must be $M_r^c = M_t^c$. The distribution of the radial moment is parabolic as in a uniformly loaded simple beam. This information is sufficient to calculate all moments in a uniformly loaded circular slab.

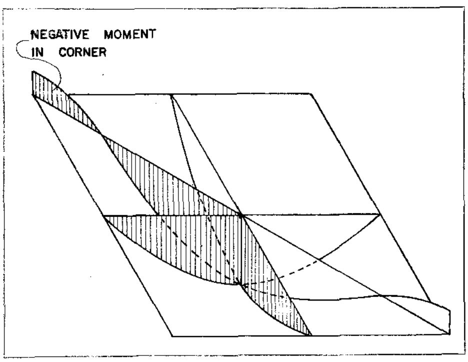

Fig. 1—Moments in Square Slab

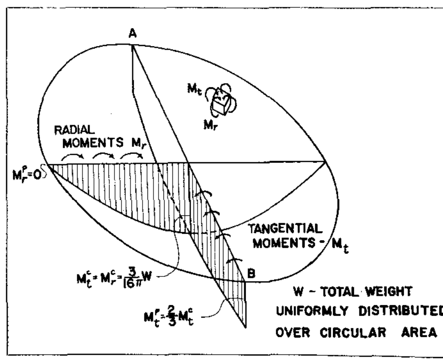

Fig. 2—Moments in Circular Slab

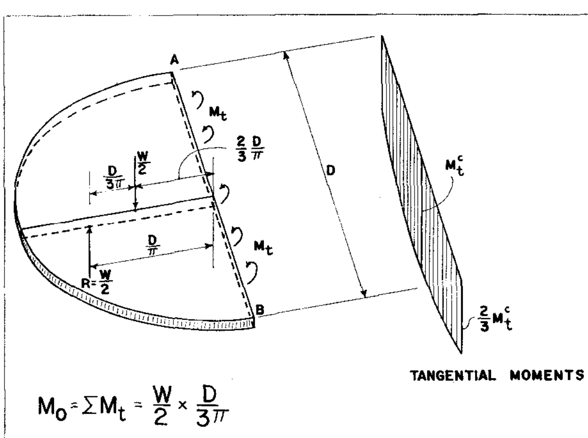

Fig. 3—Free-Body Diagram for Half Slab

RADIAL RIBS

Rectangular slabs become orthotropic when stiffening ribs are applied in one direction and become one way slabs when their stiffness and bending moments in the other direction can be neglected.

Similarly, by using heavy radial ribs on a circular slab, we can neglect the tangential moments M_t and so carry the whole load by the radial ribs alone. Each radial rib will thus work independently from the other and will carry pie-shaped loads, linearly diminishing from the periphery to a zero value at the center.

If "n" denotes the number of radial ribs, each rib will carry two such pie shaped load segments $\frac{W}{n}$. The resulting rib moment is shown in Fig. 4 and its maximum value is

$$M^r_{max} = \frac{W}{n} \times \frac{D}{6}$$

All ribs are connected at the center with a combined moment at the center obtained by projecting the M^r_{max}-value of each radial beam onto the axis A-B, or

$$M_o = M^r_{max}\left[sin\,\frac{2\pi}{n} + sin\,\frac{4\pi}{n} + sin\,\frac{6\pi}{n} + \ldots + sin\,\frac{n\pi}{n} \right]$$

The average value of the terms of the series is $\frac{2}{\pi}$. Thus we obtain

$$M_o = M^r_{max}\frac{n}{2} \times \frac{2}{\pi} = M^r_{max}\frac{n}{\pi} = \frac{WD}{6\pi}$$

which, of course, is the same value of M_o on axis A-B, obtained with the free-body diagram in Fig. 3 for the solid slab.

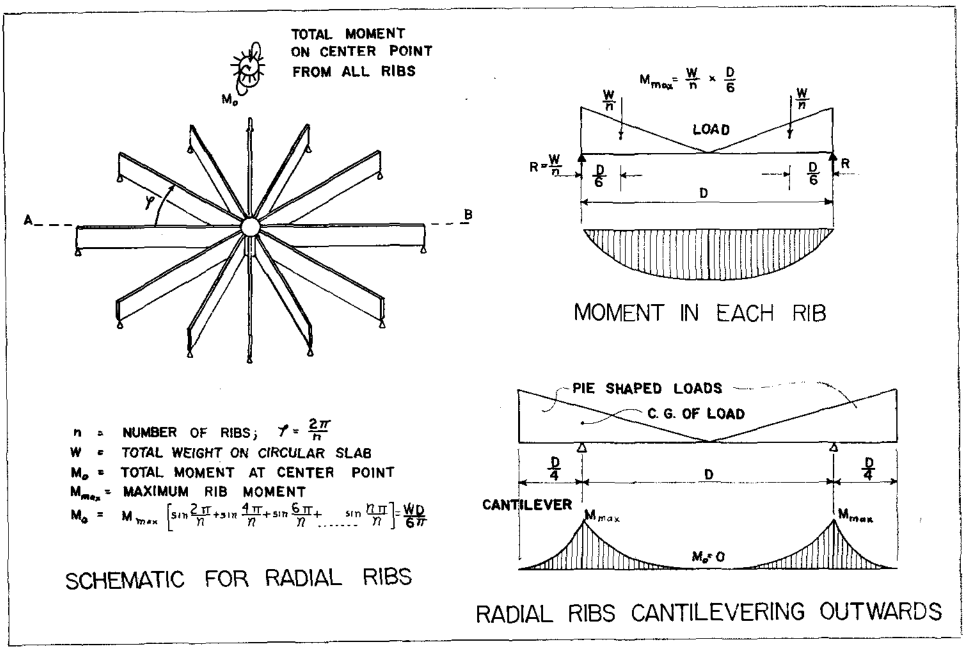

Fig. 4—Bending in Radial Ribs

With a radially ribbed slab we therefore have to provide the same moment stiffness concentrated at a central point as for the solid slab distributed all along line A-B.

TENSION AND COMPRESSION RINGS

It would be an easy matter to provide suitable prestressing tendons in each rib, were it not for the construction detail, caused by the central intersection of all the radial tendons at the point of maximum moment. Because tendons should be located as low as possible, this precludes their vertical stacking.

It becomes therefore logical to cut all tendons in the center and to anchor them to a tension ring, located as low as possible. The tension ring must be large enough to accommodate all anchorage hardware. This will govern the size of the tension ring. (See basic scheme in Fig. 7.)

In addition to the tension ring it is also necessary to provide a complementary compression ring, to balance the compressive concrete stresses occurring at the central end of each rib.

Note on Fig. 4 that the multiplier to obtain the central moment M_o from the maximum rib moment M^r_{max} was $\frac{n}{\pi}$. Consequently we can also use this multiplier to obtain the stresses in tension and compression rings, from the adjoining rib stresses.

If we have 24 radial ribs at an angle of 15°, the multiplier would be $\frac{24}{\pi} = 7.64$. The result, applies to the two faces of each ring in the cut along line *A-B*. The total tension for one face of the ring will be half the value or $\frac{n}{2\pi}$ times the tendon force in one rib.

This $\frac{n}{2\pi}$-factor also applies to the stresses in one face of the compression ring. Consequently we must design the compression ring to be $\frac{n}{2\pi}$ times as strong as the inner end of one radial concrete rib.

The large ring stresses in the center can be reduced by cantilevering the radial ribs outward past the supporting columns. If the length of the beam projection becomes $D/4$, the moments at the center would become zero as shown in Fig. 4, and no rings or central connection would theoretically be necessary for an exactly uniform load. Omitting the central rings would be very dangerous because of the possibility of nonuniform load distribution. The latter is not called for in most codes for wind or snow, but cannot be overlooked with circular roofs.

The roof described in Section B was also designed for a load of 20 psf, alternately applied in two quadrants, which required somewhat greater ring stiffness than would be necessary with the simplified approach given above.

CIRCULAR SPACE STRUCTURES

The analysis using the free-body diagram described in Fig. 3 for the circular slab can also be used to determine stresses in space structures, such as ribbed cupolas or catenary roofs, as illustrated in Fig. 5.

In a ribbed cupola the compression ring is high near the center, and the tension ring is low near the periphery.

In a catenary roof the tension ring is low near the center, and the compression ring is high near the periphery.

The magnitude of ring forces can be obtained by dividing the total moment, M_o, obtained from Figs. 3 and 4, by the vertical distance, H, between compression and tension rings.

The method of finding the total moment, M_o, with the free-body diagram does not apply only to a uniformly distributed load (such as from snow) as described in Fig. 3. It can also be applied to non-uniform load configurations, as long as the loads are distributed concentrically with respect to the polar axis and as long as the supporting columns are also arranged concentrically. All that must be adjusted is the distance $\frac{2}{3} \times \frac{D}{\pi}$ from axis A-B to the actual center of the load $\frac{W}{2}$.

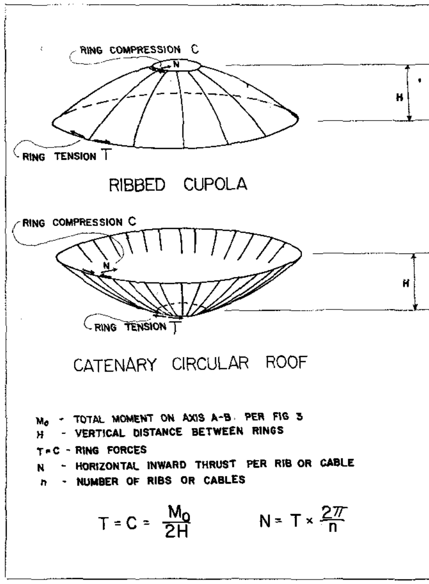

Fig. 5—Circular Space Structures

B. APPLICATION TO ROOF OVER THE STUDENT ACTIVITIES-PHYSICAL EDUCATION BUILDING

HORIZONTAL RIBBED ROOF

When we were first asked in the fall of 1961 by architects Urbahn and Brayton for suggestions on a structural roof framing over a circular gymnasium roof of 143-ft. diameter, centered over a lower roof area of 250-ft. diameter, we naturally thought of applying a dome structure for the high inner area, Fig. 6.

We had to change our thinking when informed that this area had to be separated into two halves by a folding partition, hung and operated from an overhead track. A horizontal track beam suspended from a dome was not only considered unsightly by the architects, but would also have destroyed a simple, straightforward dome action. The track beam would have to carry the load of the movable partition, as well as the weight of a crescent shaped permanent partition between a horizontal track and the curved underside of a dome surface.

The use of a horizontal ribbed roof, composed of 12 intersecting diametral girders, resting on 24 columns was therefore conceived, to allow for a simple mounting of the track beam, Fig. 7.

Precast, prestressed concrete construction appeared economical, considering the possible repetition in formwork due to the circular symmetry. It became particularly economical when basing the design concept for the ribs on the use of a single, central shoring tower for construction.

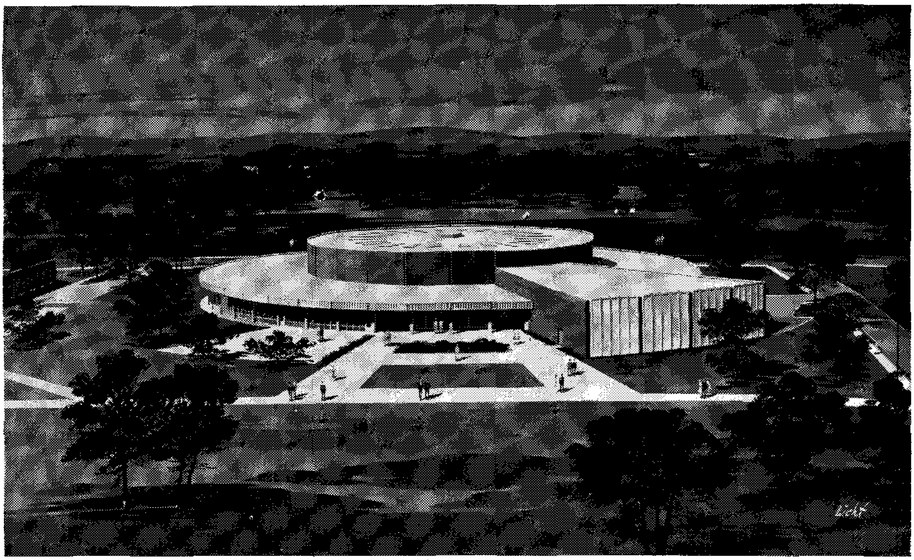

Fig. 6—Student Activities-Physical Education Building
New York State University, Agricultural & Technical Institute
Farmingdale, Long Island, New York

The 12 diametral girders actually consist of two radial ribs each, which must be tied together at the center compression ring, before the shoring tower can be removed.

We decided to conventionally reinforce the ribs, just strong enough so that they could carry themselves between the central shoring tower and the columns near the periphery. Post-tensioning would then be applied to raise the ribs off the center support and to make them strong enough to allow the casting of a closure slab.

Basic design and construction would, therefore, consist of three principal stages:

1) Dead load of ribs carried between columns and central shoring.
2) Casting of the compression ring on the central shoring, and application of prestressing to the ribs to make them strong enough to carry forms and fresh concrete for the closure slab without depending on the shoring tower.
3) Balancing applied dead loads and snow load carried on the composite beams, that consist of the prestressed ribs and the hardened concrete closure slab.

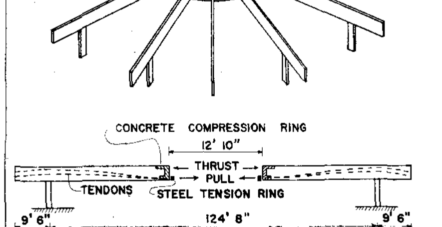

Fig. 7—Basic Scheme

The prestressed girders project outward beyond the supporting columns to make a roof over an annular mechanical space all around an upper gymnasium wall. This girder projection acts like a cantilever beam and thus reduces bending in the center of the roof.

The tendons in the prestressed concrete beams act exactly like the cables in a catenary roof, and carry the concrete beam into which they are embedded.

Unlike with a cabled catenary roof, an outer compression ring is not necessary. The inward pull of the tendons is resisted by the radial concrete ribs, which are all anchored to a smaller, central compression ring.

When radial steel ribs or cables are used for a circular roof, it is usually possible to arrange a steel spool in the center, from which the radial members are diverging outwardly. Making such a spool is not as simple with prestressed concrete construction, since the high strength tendons need considerable space for their central anchorage.

The architects required a ventilator in the center. The inner diameter of the ventilator opening was set at 13 ft., which seemed to be the minimum necessary to obtain sufficient space for the cable anchorage to the tension ring and for joining the concrete beams to the concrete compression ring.

PRECAST RIBS

Fig. 8 illustrates the shape of a precast rib unit; also shown are the cross-section through the cast-in-place compression ring and the construction of the tension ring.

Basically the 5-ft. high ribs are single tees with a maximum flange width of 8 ft., tapering to a width of 1ft. 1½ in. at the center.

The web thickness of the girders is only 6 in. This is the minimum thickness required for covering the large tendons, and was adopted to keep the dead loads down. A tapered bottom flange had to be provided near the center of the roof to provide room for the flaring of the tendons into a single horizontal layer, where they are spliced to the tension ring.

The principal difficulty with radial prestressed concrete beams is that their tendons must be anchored or spliced near span center, the point of maximum bending moment, where the eccentricity of the prestressing force must be as large as possible. It is therefore not desirable at this point to stack the tendons vertically, as with anchorages at outer beam ends. This led to the use of large, compact prestressing tendons and to the placing of the tension ring lower than the beam bottom.

PRESTRESSING

Two post-tensioned tendons per girder were required with a final specified prestressing force of 266,000 lb. each.

Thus with a final prestressing force of 532 kips per rib, and applying the multiplier $\frac{n}{2\pi} = 3.822$, we arrive at a ring force of approximately $532 \times 3.822 = 2000$ kips tension.

The design called for three concentric, welded steel rings, each of 20 x 1½ in., high strength, low alloy steel plate, ASTM A242-55. The maximum thickness of 1½ in. was chosen not only for ease in bending, but also because heavier plates have a lower guaranteed yield strength. To avoid the necessity of a machine fit, the rings were set 3 in. apart and the gap was filled with 5000-psi nonshrink concrete. Further, to avoid a

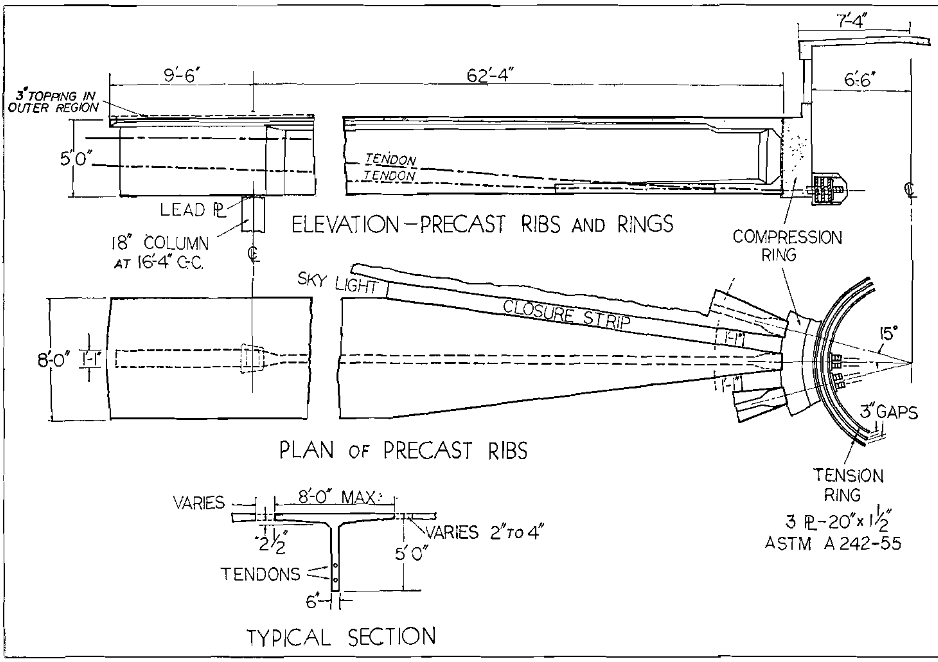

Fig. 8—Construction Details

machine fit between the anchorage base plates and the inner steel ring, the contractor was permitted to fill a similar gap with Sika's epoxy "Colma-Dur" mortar.

The contract drawings showed only tentative details for the anchorage and left the choice of the post-tensioning system up to the contractor, with the proviso, that the tendon supplier must be made responsible for the stressing operation.

Freyssinet tendons consisting of 12 strands, ½-in. diameter, Type 270 K steel wire, were selected, with suitable shop details for their anchorage to the tension ring.

Stressing was done from both ends of each rib and opposite ribs were stressed simultaneously. Only about 50 percent of the required tension was applied initially, as the jacks were moved around inside the ring, to avoid high bending stresses in the ring.

At the time of stressing, the ring was freely suspended inside the compression ring to permit its elastic elongation without touching the surrounding concrete. The tension ring should be considered to be only a part of the anchorage system, as it is merely acting as a splice for the tendons.

Similarly, the compression ring should be considered to be merely a splice for the concrete ribs. It will be automatically prestressed when the ribs are stressed, since the compressive rib forces will introduce corresponding compressive stresses in the ring. The relation between prestress in the rib and prestress in the ring is again governed by the multiplier $\frac{n}{2\pi}$.

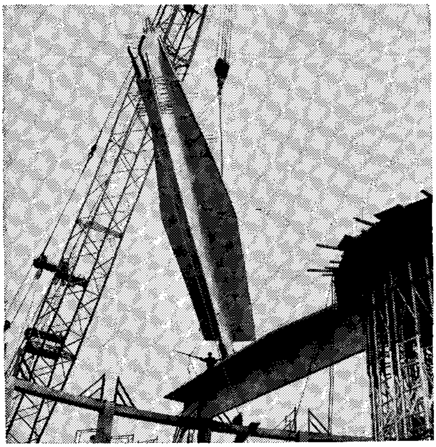

Fig. 9

The prestress raised the compression ring and the ribs off the shoring tower. With the tension ring fully elongated, the space between compression and tension ring was then filled with concrete.

ERECTION

All prestressed members were manufactured by Braenstress, Inc., Wyckoff, New Jersey, using steel forms and high-early strength cement. The large tees, weighing 25 tons, were trucked 50 miles from the plant to the site and were erected by the manufacturer, Fig. 9. The contract drawings had specified pick-up points and type of lifting inserts. Note the slender 6-in. width of the 5-ft. deep stems with the 125-ft. beam spans, and the special dowels at their inner end, which are to make connection to the compression ring which will be cast between them.

Fig 10 shows the placing of the outer rib end on a steel-framed lead

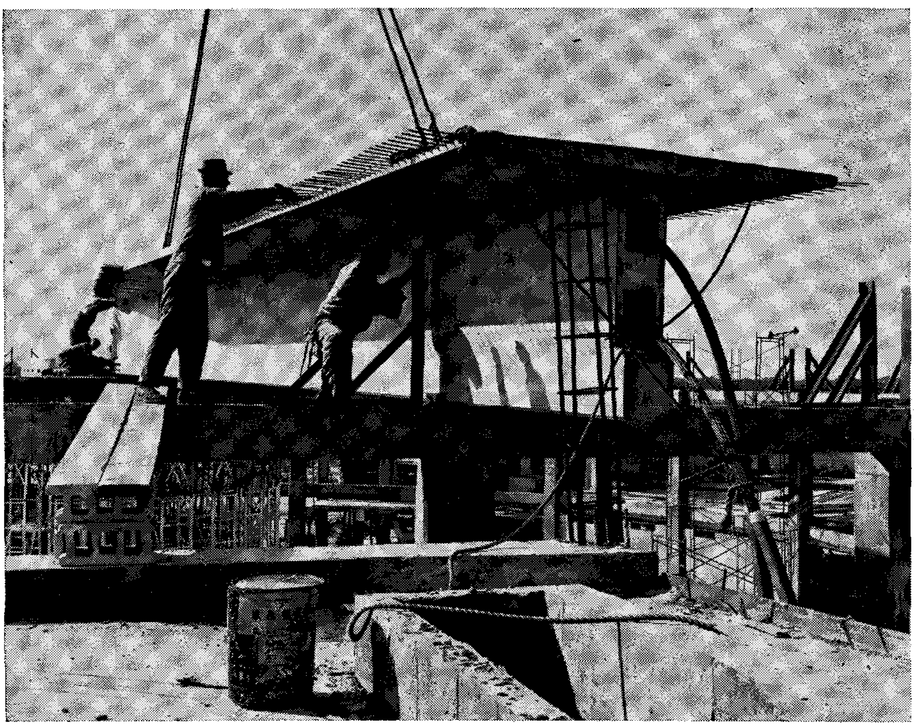

Fig. 10

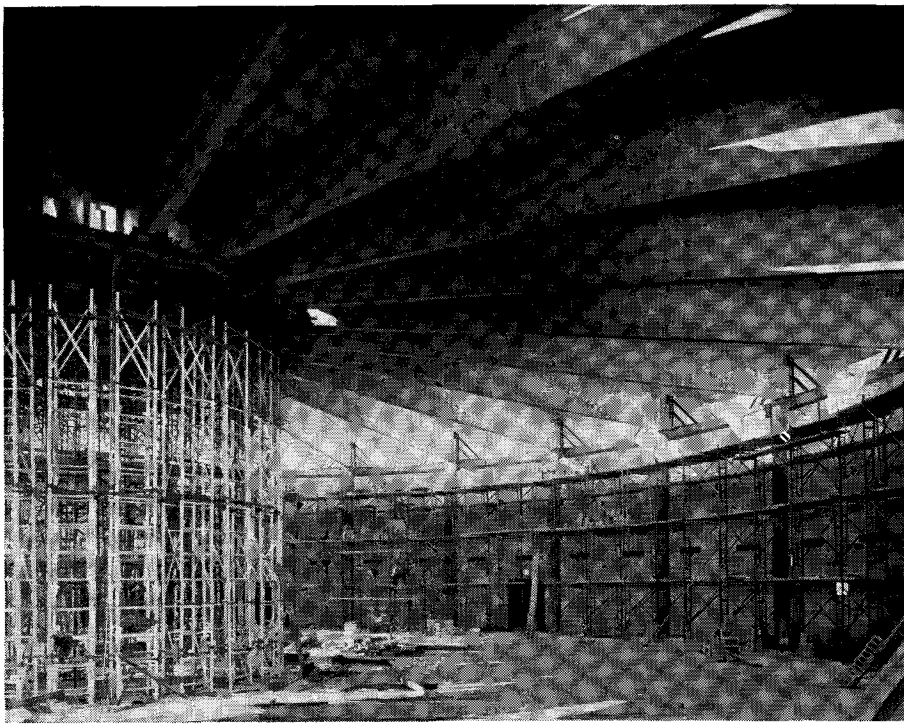

Fig. 11

bearing plate located on top of the cast-in-place column. Tees were temporarily held upright by steel templates, until they were braced against each other by the closure slabs. Note the dowels for the closure slabs projecting from the flanges and the omission of the dowels, where skylights are to be located between flanges.

Fig. 11 shows the placing of the inner ends of the ribs on the central shoring tower.

Fig. 12 is taken from the top of the crane boom and shows all concrete tees in place, spaced by temporary steel angles.

Fig. 13 shows the steel tension ring in place, with the pre-stressing tendons threaded through the holes in the plates. Anchor plates are not yet in place.

Fig. 14 is an interior view of the completed structure. It shows the horizontal curtain track which led to this unusual roof design.

Fig. 12

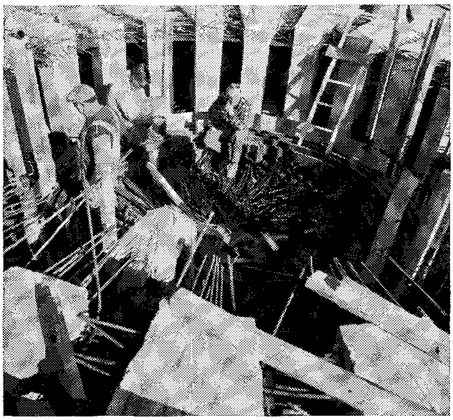

Fig. 13

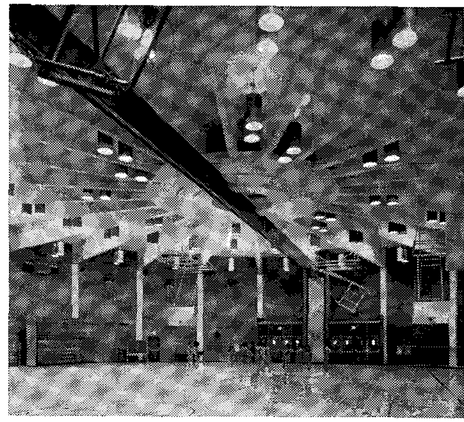

Fig. 14

The lower annular roofs are also of prestressed concrete construction (see Fig. 6) and consist of pretensioned single tees arranged in radial directions with a cast-in-place combination of topping and in-between filler slabs. These tees vary in length from 33 ft. to 67 ft. with a final specified prestressing force of over 500,000 lb. for the stem of the longer beams.

Architect for the project was the office of Max O. Urbahn. The writer, as partner of Summers and Molke, was the designing structural engineer. Anderson Construction Co. was the general contractor.

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Discussion of this paper is invited. Please forward your Discussion to PCI Headquarters before March 1 to permit publication in the June 1967 issue of the PCI JOURNAL