Studies of Crack Widths and Deformation under Sustained and Fatigue Loading

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GENERAL CONSIDERATIONS

In connection with partial prestressing, cracks may develop and excessive deformation may take place. It is, therefore, essential to investigate safe limits of the maximum width of cracks to ensure freedom from corrosion of the steel and from excessive deflection.

In Fig. 1 the relationship between the deflection and concrete stress at the tensile face of a beam is illustrated. The load-deflection diagram is shown in a simplified manner comprising two straight lines: one based on the calculated deflection of a homogeneous section and the other parallel to the calculated deflection of a cracked section computed using the appropriate value of n corresponding to the modular ratio of the actual E-value of the steel and concrete. In fact, this deflection curve will be slightly rounded off near the change of incline in deflection beginning from the load at which micro-cracks develop. This corresponds to the tensile strength, i.e., a tensile stress of, say, 400 to 600 psi for high strength concrete; but the load at which the cracks become visible to the unaided eye is much higher and approximates the point at which the change in deflection occurs in the simplified deflection diagram. For well bonded and well distributed steel in the tensile zone this stress will be approximately 1,000 psi provided that the percentage of steel is not too small. At that stage cracks suddenly appear which have a width of 0.0003 to 0.0005 in. and extend into the concrete, reaching the steel and often penetrating further into the tensile zone.

For members of the same cross-section, the crack distribution depends on the efficiency of the bond, the distribution, size and cover of the steel. The bond resistance in turn is dependent on the concrete strength and on the shape of the reinforcement and on its surface conditions. The crack distribution also influences the stiffness of the section. That is to say the deflection of similar members in a cracked state may vary in spite of the same calculated EI-value from the computed deflection and become larger in the case of bad crack distribution.

In Fig. 1 it is indicated that with repeated loading the load at which the deflection curves of a homogeneous and cracked section change direction, may gradually be lowered to that corresponding to zero stress at the tensile face where the effective prestress becomes zero. This results in an appreciable increase in maximum deflection. A relatively small permanent set of the deflection after repeated load-

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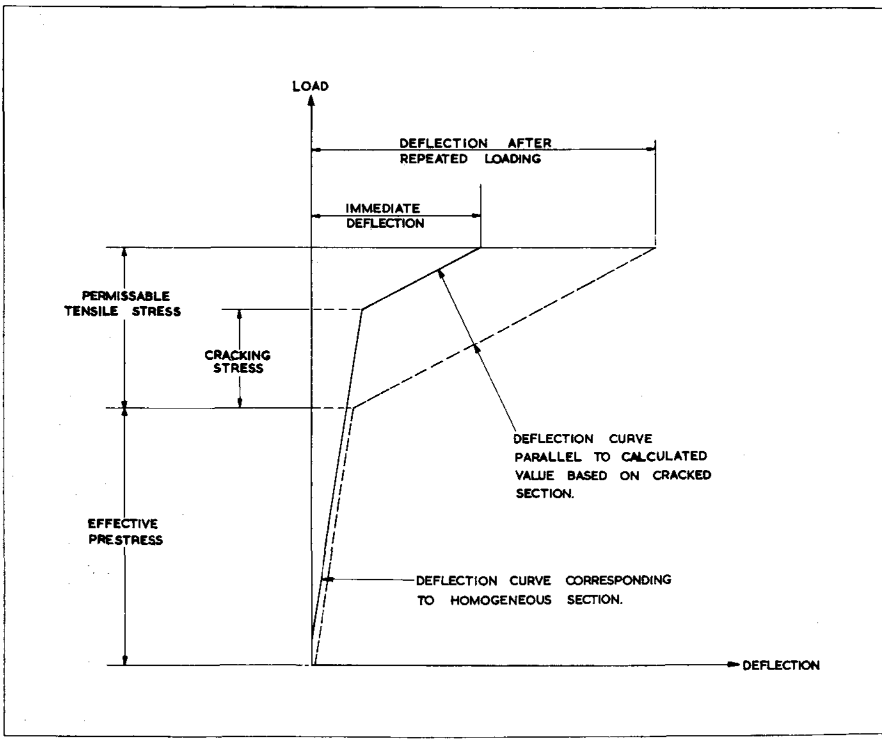

Fig. 1—Load-Deflection Curves

ing is also indicated in the figure.

Obviously if there is a sustained loading this permanent set may become much greater due to creep. The purpose of this paper is to discuss the effect of sustained and repeated loading on the deflection and maximum crack width and to consider the conditions on which an appropriate design should be based to limit both deflection and crack width to a permissible limit.

In the following, first the question of permissible deflection is investigated and then that of permissible crack width.

PERMISSIBLE DEFLECTION

Before this question is discussed in detail one problem may be mentioned; i.e., the case at which the deflection becomes too large and results in permanent sagging of a roof which may result in the formation of a pond in rain, generally called "ponding". This in turn may cause a further increase in sagging and result in dangerous deformation requiring special relief. Often ponding is attributed to a design in partial prestressing. However, usually the main cause of ponding is the limited depth of the section with a depth to span ratio which is very small. For a roof, whether full or limited prestress is applied, the design should always be made in such a way that ponding does not occur when there is a possibility that the deflection would be too large. Thus such ponding can be avoided by casting the concrete with an immediate camber or by the provision of a suitable surfacing which ensures the necessary fall for the water.

With regard to deflection of a reinforced or prestressed concrete floor or roof, different views prevail. One opinion is that the entire deflection should be limited in order to ensure a certain rigidity of the construction. The other view is that it is mainly the appearance which matters, i.e., only the deflection occurring below a straight line connecting the supports (below the level where the floor or roof is horizontal) is of importance. Often, the limit of such proportion is given as 1/350 of the span and sometimes 1/250 is considered sufficient. This obviously ought to include the effect of creep, and the possible deflection should be within the limit specified. A simple way of overcoming any excessive deflection consists in the previous provision of a camber so that the entire deflection between the original stage and the final position may be much greater than the limited value as only that part matters which is a downward deformation below the horizontal level. This seems to be a much more practical interpretation and gives prestressed concrete a great advantage by the fact that by prestressing a camber is obtained and it is not the entire deflection which matters but only the part of the maximum possible deflection which causes a downward deflection below the horizontal level as mentioned before for reinforced concrete.

The A.C.I. Standard Building Code (ACI 318-63) does not take into account the deflection due to dead load but requires that "the allowable limit for the sum of the immediate deflection due to live load and the additional deflection due to shrinkage and creep under all sustained loads computed as above shall not exceed *l/360*". This applies to a "floor or roof construction intended to support or to be attached to partitions or other construction likely to be damaged by large deflections of the floor". For roofs which do not support plastered ceilings the allowable immediate deflection is 1/180 of the length and for roofs which support plaster ceilings or for floors which do not support partitions the allowable immediate deflection is 1/360 of the span. These latter values are very small particularly as the deflection may be doubled due to creep in the case of a sustained loading. Moreover, the deflection due to the dead load is of some importance and consequently the entire deflection may become rather excessive if the immediate deflection due to live load only is as much as 1/180 of the span. As already stated, prestressed concrete has the great advantage that from the very beginning by prestressing an artificial camber is obtained and, consequently, the deflection due to superimposed loads may be greater without resulting in a large downward deflection below the horizontal level. The view that only the live load deflection matters and that the camber due to prestress has to be disregarded in order to obtain a greater stiffness is, in the author's view, not quite justified because it is not essential to have a certain rigidity for prestressed concrete. On the contrary, a greater flexibility under higher loading is a great advantage because of the possibility of absorbing impact, almost entire recovery taking place on removal of the load.

PERMISSIBLE CRACK WIDTH

In concrete beams reinforced with mild steel with ordinary permissible stresses, cracks of 0.001 to 0.015 in. may occur due to the working load, shrinkage and creep, as has been established by Emperger some thirty years ago.[1] It depends entirely on the property of the concrete whether such cracks become dangerous from the point of view of corrosion. With porous concrete there is a danger of corrosion independent of whether there are cracks or not and independent of the cover of reinforcement. However, with dense concrete a relatively small cover is sufficient to prevent corrosion even if the cracks reach a width of 0.001 in. as the author was able to establish based on corrosion tests carried out in connection with spun concrete members.[2] On the other hand too small cover as, for example ¼ in., is insufficient if the concrete is exposed to weather conditions. Rain and sunshine may affect the outer skin, in combination with carbonation shrinkage, with the consequence that the outer skin of the concrete may become porous.

With ordinary reinforced concrete the ACI Standard Building Code requires that ". . . the average crack width at service load at the concrete surface of the extreme tension edge, does not exceed 0.015 in. for interior members and 0.010 in. for exterior members". This seems to be a rather generous allowance, as it relates to the average crack width because the maximum crack width which matters may be double this value. On the other hand, it is not the width on the outer tensile face which is of essenal influence with regard to corrosion, but rather the maximum width at the level of the reinforcement. The European Concrete Committee has suggested that this should be 0.012 in. for the interior of protected structures, 0.008 in. for the exterior of structures generally and 0.004 in. for the exterior of structures which are particularly exposed to aggressive influences. This might be a more rigorous condition but again it depends on the side cover of the reinforcement. If the side cover is small the actual width of the crack at the position of the steel will not be much less than that measured at the outside face, but if it is very large the actual width at the steel may be considerably less than that measured at the outer side face, at the level of the steel.

With prestressed concrete the question of corrosion seems to be somewhat more serious because of the smaller size of the steel and the possibility that by slight corrosion a substantial part of the cross-sectional area may become ineffective. This appears to have been one of the main reasons for avoiding visible cracks altogether. There is also the question of possible stress corrosion which has been raised. There is, however, no doubt that stress corrosion does not occur if there is not a danger of corrosion itself. In

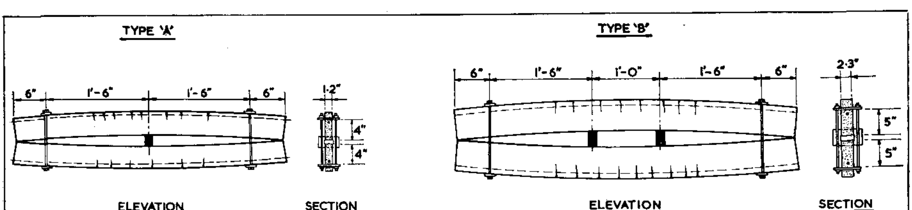

Fig. 2—Specimens for Corrosion Tests

all known instances of trouble there was definite cause for corrosion mostly due to chlorides which affected concrete and steel. During his association with British Railways, Eastern Region, the author carried out a number of tests in which prestressed concrete members with artificially open cracks were exposed to steam and to sea water. These tests have indicated that a crack width of 0.01 in. appears to be harmless with regard to corrosion if the concrete is very dense as is usual with prestressed concrete. The results of some of these tests, relating to accelerated tests in highly polluted air, have been published in this Journal.[3]

After his retirement from the British Railways, Eastern Region, the author has been able to introduce new corrosion tests with the Railways in which pairs of small elements, containing pre-tensioned 0.2-in. and 0.276-in. diameter wires and having covers of ½ in. and 1 in. respectively, were bent against fulcrums as indicated in Fig. 2. These test specimens have cracks of varying widths; exact measurements have been taken of the width of crack at the position of the tendon in the tensile zone and at both sides of the wire where the cover is ½ in. and 1 in. respectively. The stress in the steel in these specimens approaches its strength, since appreciable forces were needed to obtain the cracking required. Pairs of such specimens are being investigated at certain intervals after exposure to smoke in an engine shed (Figs. 3 and 4) and exposure to seawater between low and high tides (Fig. 5). The cracks in the specimen are of varying width up to 0.02 in. and thus it should be possible to establish the limit of width at which substantial corrosion takes place.

These tests should show interesting results although the effect of the steam is not quite uniform with re-

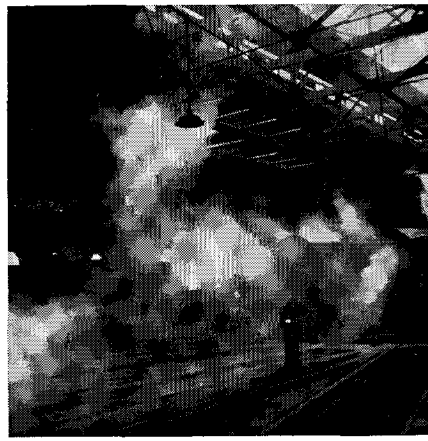

Fig. 3—Specimens Suspended from Roof of Engine Shed

Fig. 4—Specimens Suspended from Roof of Engine Shed

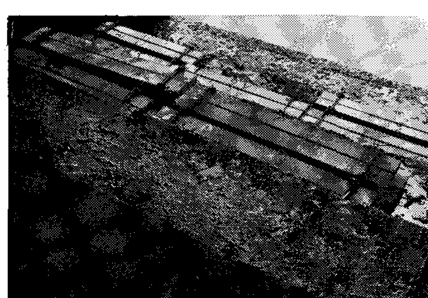

Fig. 5—Specimens in Seawater between Low and High Tide

gard to all members and some sealing of the cracks occurs in the specimens exposed to seawater owing to biological conditions. It is intended to carry out additional accelerated tests in which these influences are to be avoided. Based on his previous experience in this connection, the author is quite confident that these tests will confirm that a crack width at the level of the steel of 0.01 in. is not dangerous from the point of view of corrosion even if very heavy chemical influences have to be considered and the cover is only ½ to 1 in. This obviously relates to the maximum possible width of crack. Still to be investigated is the question of what maximum width under static loading will finally result in a maximum width of 0.01 in. under sustained or repeated loading.

SUSTAINED AND REPEATED LOADINGS

The greater part of creep occurs at an early age and influences the deformation of prestressed concrete. Thus, if a sustained loading is applied at an age when the concrete has gained most of its strength, it has little effect on the stiffness, as is shown in Fig. 6.

This example relates to a prestressed concrete beam which remained unloaded for approximately two and one half years during which time the initial camber increased to more than double its value. When the beam was loaded to such an extent that tensile stresses of approximately 700 psi occurred, the immediate deflection was not substantial and the further increase in deflection during 420 days was relatively small as can be seen from the figure. At this loading which corresponded to 45% of the static failure load, micro-cracks, which became visible after about a year, occurred. This beam was later loaded to 63% of the static failure load corresponding to a nominal tensile stress of 1980 psi, and in 720 days the deflection due to creep increased, but not excessively because the load was applied at a later stage. If it had been loaded at an early age the effect of creep would have been much more pronounced. From the figure it is seen that at the increased loading the widths of maximum crack became 0.002 in. and increased during the 720 days to 0.006 in. A similar loading was applied to a second beam; a maximum load of 80% of the static failure load was applied and the original maximum width of crack of 0.005 in. gradually increased to 0.015 in. within a year. More particulars are described in Reference 4. It may be pointed out that the maximum downwards deflection in Fig. 6 was less than 1/500 of the span. These beams are loaded in the open air at the railway station at Derby where the air is highly polluted. Thus, as they have cracks, they are also undergoing test with regard to the actual effect of heavy chemical influences with regard to corrosion.

Fig. 7 shows that a maximum width of a crack of approximately 0.004 in. width which occurred at a static loading, and increased to 0.01 in. after 1 million repetition of the load. This related to a composite T-section containing two post-tensioned strands and two non-tensioned ½-in. diameter strands, as indicated in the figure.

From these tests it seems appropriate to limit the permissible maximum crack width at static loading to 0.004 in., which might result in a width of 0.01 in. under sus-

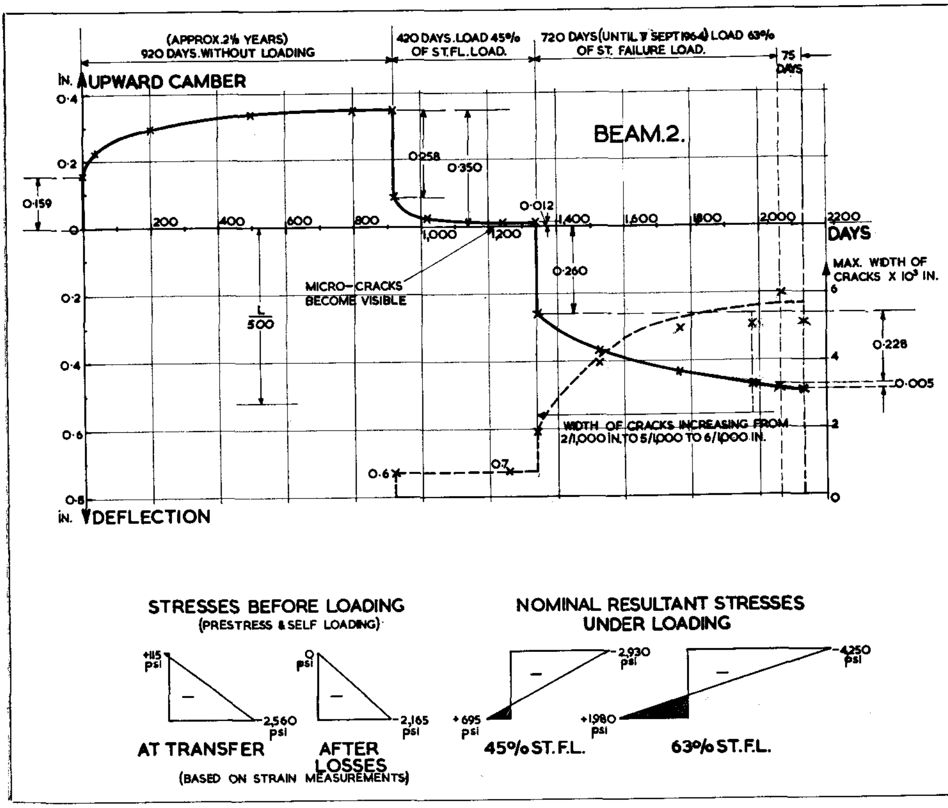

Fig. 6—Sustained Loading Tests—Derby Station, British Railways

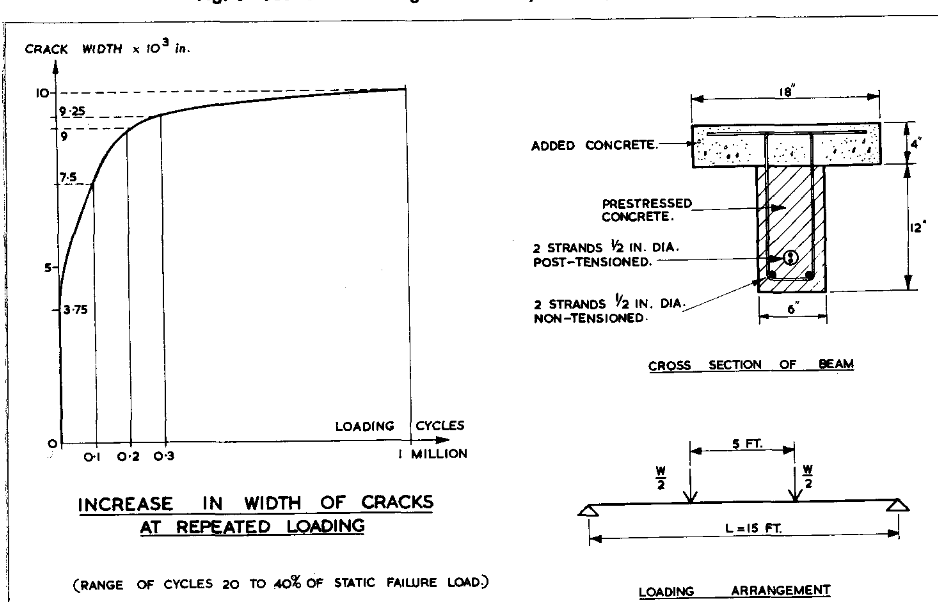

Fig. 7—British Comparative Tests—1963

tained or repeated loading. Obviously, if there is no danger of corrosion, a greater maximum crack width under static loading, say 0.01 in., may be considered, particularly if the load occurs only occasionally.

DESIGN BASIS—LIMITED CRACK WIDTH

Whereas the design for a maximum deflection can be carried out on the basis of existing design formulas, it is impossible to establish a simple design method based on the limitation of cracks to a definite width. This is because, as already stated, the maximum crack width depends on so many conditions among which are the size, distribution, surface condition, and percentage of the steel; the strength of the concrete; and the cross-sectional shape of the structural member. With ordinary reinforced concrete an endeavor has been made to relate the crack width to the steel stress in the cracked section. The following formula has been recommended within certain ranges.[5]

$$c_w = \left(4.5 + \frac{0.4}{p_e} \right) \frac{\phi f_s}{K'}$$

In this formula c_w is the width of crack; $p_e = A_s/A_{ct}$, the ratio of the area of steel to the area of the tensile zone of the concrete ($A_{ct} = 2b(t\text{-}d)$, where t is the entire depth* and d is the effective depth of the reinforcement); f_s is the tensile stress in the steel; and ϕ is the diameter of the steel bar. There is a further parameter K', which depends on the bond capacity of the steel, and is equal to 42,700 ksi for deformed bars and 29,500 ksi for round bars. This formula is hardly applicable to prestressed concrete when non-tensioned steel is used. It seems, therefore to be more appropriate to consider nominal permissible concrete tensile stresses as the basis for the design to be carried out for a nominal homogeneous section. However, it would be completely wrong to base the design on a definite value for the tensile stress. For example, the author has designed numerous bridges with pretensioned tendons for a permissible concrete tensile stress of 650 psi and roof members for a permissible stress of 750 psi. These constructions have been in use for 12 to 16 years and thorough examinations proved that visible cracks have not occurred. The same applies to precast prestressed members, 80 to 103 ft. in length, for three roofs which were built in 1952-53. These members contain post-tensioned tendons and were designed for a concrete tensile stress of 650 psi under working load. In all these cases non-tensioned prestressing wires were provided and the distribution of the steel was close to the tensile face of the members. Even if the concrete tensile stress had been increased in this type of construction to 1500 psi the crack width would have been within the suggested limit of 0.004 in. as tests have indicated. However, in a prestressed concrete member in which the percentage of steel is very small and/or the steel is not well distributed in the tensile zone, cracks under loading, may become wide immediately after they become visible. This may occur at a much lower stress than in the previous cases, e.g. at a tensile stress as low as 700 psi. Thus the tensile stress permissible for a definite crack width depends mainly on the distribution of the steel around the tensile zone and on the percentage, p, of the steel. The val-

**This is shown in the author's figures as D.*

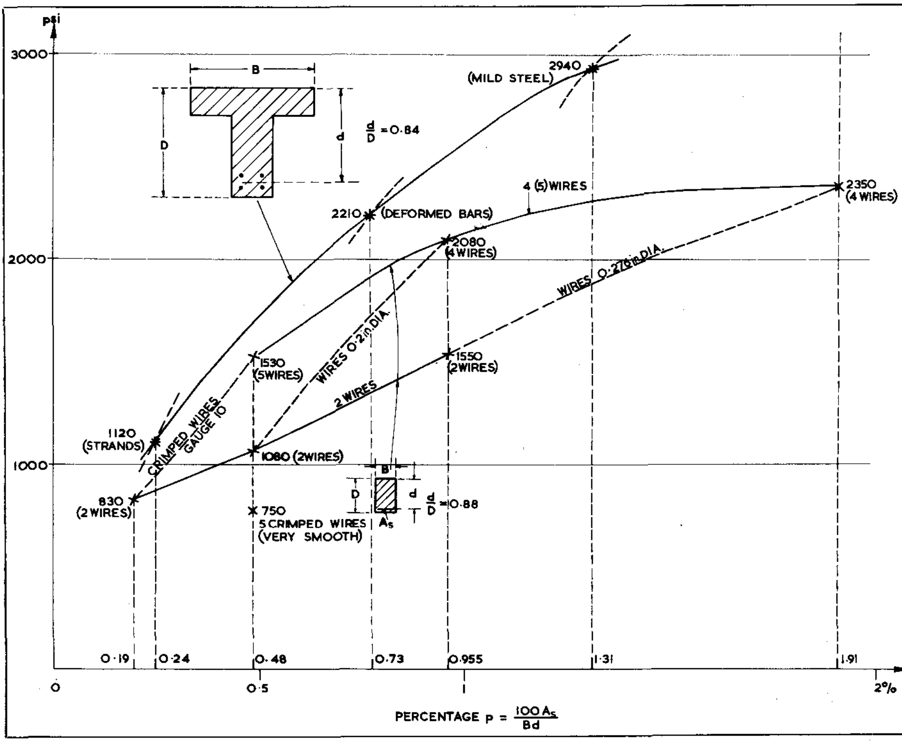

Fig. 8—Example—Allowable Nominal Tensile Stresses Corresponding to Maximum Width of Cracks of 0.004 in.

ue, p_e, which to a certain extent indicates the distribution of the steel, may also be of influence but the entire percentage seems to be of greater influence.

Fig. 8 shows the relationship between nominal tensile stress in a homogeneous section and percentage of reinforcement, p, based on some test data which gives some indication of the different influences. This figure is based on a permissible maximum crack width of 0.004 in. The test results on rectangular beams shown in the figure relate to high strength concrete beams reinforced with non-tensioned prestressing steel, made by the author at the University of Southampton in 1964. These tests on non-prestressed concrete beams reinforced with non-tensioned prestressing steel simulate the behavior of prestressed concrete beams at the stage of loading at which the prestressing force has become ineffective and at zero loading the stress at the tensile face of a beam has become zero.

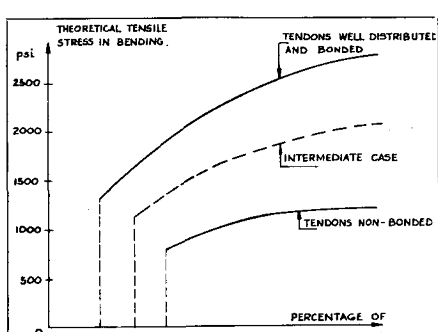

Fig. 9—Theoretical Tensile Stress Governed by Permissable Crack Width

The three T-shaped beams referred to in Fig. 8 contain two post-tensioned strands ½ in. in diameter, which have been grouted-in; and different kinds of non-tensioned steel, i.e., ½ in. diameter strands as seen in Fig. 7, ¾-in. high strength deformed bars and 1-in. diameter mild steel bars. These three beams carried approximately the same ultimate load, but the deflection and crack width increased very much as the entire percentage, p, decreased. Any non-tensioned steel causes a reduction of the pre stressing force because of the compressive force which must develop in the non-tensioned steel owing to stress redistribution due to shrinkage and creep of the concrete. The magnitude of this compressive force depends on the cross-sectional area of the non-tensioned steel; the larger the cross-sectional area of the non-tensioned steel, the greater becomes this compressive force, which in turn results in reduction of the effective prestressing force. Thus microcracks must develop earlier with a larger area of non-tensioned steel such as mild steel than would occur with the more economical much smaller reinforcement in high strength ½-in. diameter prestressing strands. However, as can be seen from Fig. 8, the much greater percentage of mild steel apparently favourably affects the gradual increase in width of visible cracks. In the meantime, the author has carried out further tests on 60 high strength concrete beams of two different sizes and reinforced with various types of non-tensioned prestressing steel. The great bond capacities of strands were noticed, and the author hopes to be in a position to show more detailed values as soon as the report on these tests is completed. In his view, it should be possible to obtain for a definite crack width and a definite shape of section, e.g. a rectangle, design curves which relate to three types of crack efficiency and are dependent on the distribution and bond capacity of the steel as indicated in Fig. 9. If there is no bond and the tendons are not well distributed near the tensile face, obviously a concrete tensile stress should not be permitted under design load. The same applies if the tendons are far from the tensile face and there is no steel close to the tensile zone acting as crack protection.

REFERENCES

1. Emperger, F. von, "Die Rissfrage bei hohen Stahlspannungen und die zulässige Blosslegung des Stahles", [The problem of (concrete) cracking under high stresses in the steel and the permissible exposure of the steel], *Mitteilungen über Versuche ausgeführt vom Österreichischen Eisenbeton-Ausschuss,* No. 16, p. 126 (1935).
2. Abeles, P. W., "Die Rostgefahr von Eisenbetonkonstruktionen bei Rissbildung", [The danger of corrosion in reinforced concrete after cracking.], *Zement,* Vol. 26, No. 7, February 18, pp. 107-111; No. 8, February 25, pp. 124-129; No. 9, March 4, pp. 146-148 (1937).
3. Abeles, P. W. and Filipek, S. J., "Corrosion of Steel in Finely Cracked Reinforced and Prestressed Concrete", *PCI Journal,* Vol. 10, No. 2, April 1965.
4. Abeles, P. W., *An Introduction to Prestressed Concrete,* Vol. II, (to appear this year).
5. Hognestad, Eivind, "High Strength Concrete Reinforcement, Part 2. Control of Flexural Cracking", *Journal of the PCA Research and Development Laboratories,* Vol. 4, No. 1, January 1962.

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