Prestressed Continuous Structures

by

Robert L. Koons* and Gerald J. Schlegel**

I. Statically Indeterminate Structures

A statically determinate structure is defined as one in which the unknown quantities do not exceed the number of independent equations of static equilibrium that can be written for the structure. A statically indeterminate structure is one in which the number of unknown reactions exceed the number of equations of static equilibrium.

Structures, continuous or statically indeterminate, are found in certain types of trusses, arches, rigid frames, fixed-end beams and propped cantilevers.

The continuous structure, either in beams or slabs, is quite common in building construction. Many ingenious methods of construction have been evolved to take advantage of continuity—the mushroom slab, the continuous waffle slab and, most successful of all, the prestressed flat plate.

As will appear in future discussion, in continuous structures there are, as a rule, much higher moments at the points where the slab is continuous over the support. This has led to many forms of section design or tendon placement to satisfy these higher moments.

In cross-section design the members may have their depth increased by arching or with haunches; a member of uniform depth may be widened from the point of inflection to the support; hollow internal areas may reduce cross-section while maintaining exterior shape; heavier cantilever sections may support simple beams at inflection points; or precast members may be lapped past each other at supports.

In prestress placement: tendons may be lapped past each other over supports; sections of high moment may have added short tendons to increase both flexural and shear resistance; overloads above real live loads are rare, therefore reinforcements may be used in limited areas.

II. Concepts of Continuous Cable Structures

Consider in Figure II-a, two telephone poles, spanned with parabolic draped phone wire or cable. Line electrical engineers have known for many years that the tensile force in the cable is a function of the span and the load. It can be seen that the tensile force, P, times the deflection of the cable, a, is equal to $\frac{WL^2}{8}$, where L is the span and W is the weight of the cable (simple statics). All structural engineers are familiar with this formula. It is obvious that there is no bending in the cable as it hangs between the supports, and that there is no shear in

**Division Manager*
The Prescon Corporation
***Engineer*
The Prescon Corporation

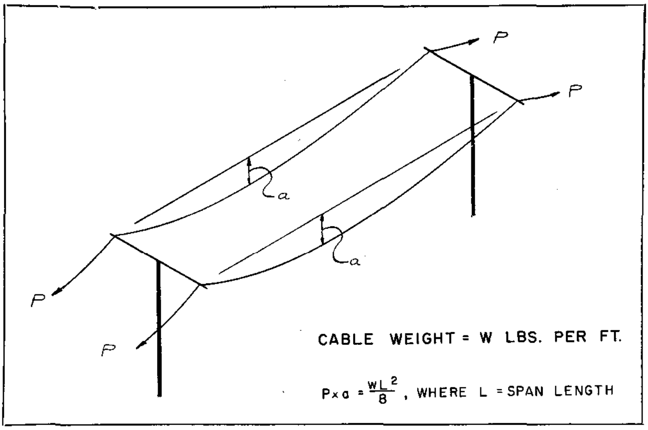


Figure II—a

the sense of diagonal tension as occurs in a beam, and that the loads are transferred directly into the supports.

Next consider a uniform load hung from a cable at some distance below it, Figure II-b. The uniform load could be a deep concrete slab. There would be no bending in the slab since it is uniformly supported. If the uniform load is made large enough, the cable will remain essentially in a parabolic curve, even though a moving live load may be superimposed.

The uplift force of the cable equals the supported uniform load and the gravity loads are transferred to the cable supports. This is the principle used in the design of suspension bridges.

To proceed further, assume a cable hung with small deflections from several supports; for example, columns of a building, Figure II-c. If the cable were enclosed in concrete it would be possible to put a tensile force in the cable, without changing radically the position of the cable, thus giving the cable an uplift force. If the cable tension is adjusted so that the uplift force of the tendon is equal to the downward load of the structure, the concrete would have no bending stresses, and therefore, no deflection. Accordingly, the stress in the concrete would be P/A if the cable is anchored to the con-

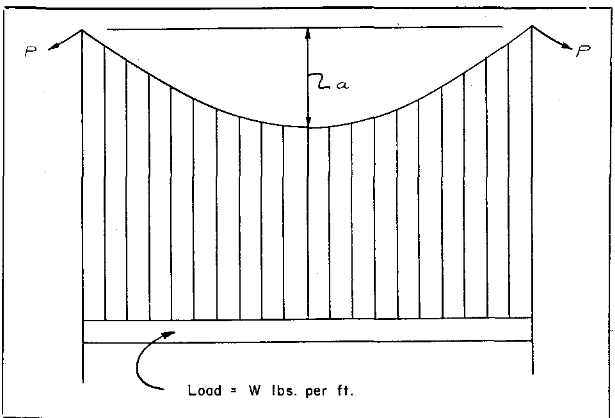


Figure II—b

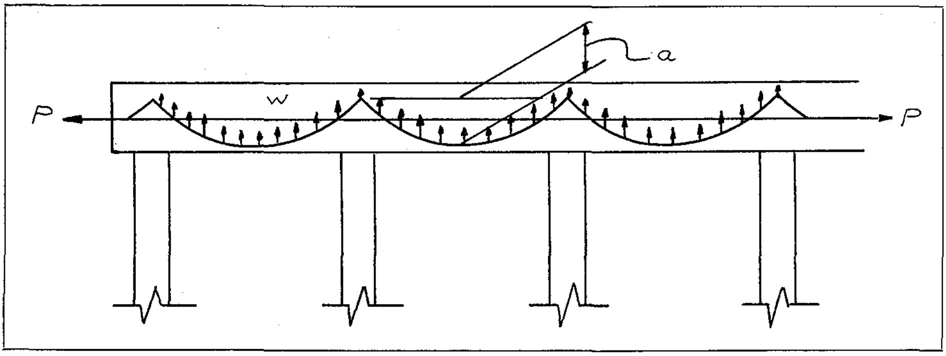


Figure II—c

crete by end plates. If the cable were anchored on each end to supports external to the building, there would be no stress in the concrete. The concrete would be level under this condition of loading and the reactions of the downward loads would be transferred directly into the reactions or the columns.

Any method of analysis that has been published in the past on the design of continuous prestressed structures using draped cables, is based on the fundamental concepts stated above. This method therefore, demonstrates a logical approach to the design of continuous self-anchored cable structures.

Properly designed cable structures (prestressed concrete) have many advantages over conventionally reinforced structures. Among the many advantages, the most important is the control of slab or beam deflections. Slab or beam deflection caused by gravity loads induce moments into monolithic columns or supporting members.

It is possible to eliminate or control very precisely the amount of deflection by varying the prestress force. This concept was discussed in Section I.

In addition, when the deflections are eliminated or reduced by the prestress force, the column moments caused by gravity loads are reduced or eliminated.

III. Idealized Cable Structures

An ideal cable supported concrete beam or slab is one in which the cables are placed as shown in Figure II-c. It is impossible to place continuous tendons in this manner. They must be placed in smooth continuous curves. Figure III-a shows a tendon as it is actually placed in the field. It can be seen that there are downward forces in the area over the support similar to the upward forces as shown in Figure II-c. In this example, the effect of the reversal has been neglected to simplify the explanation of the design.

A flat plate structure is sketched

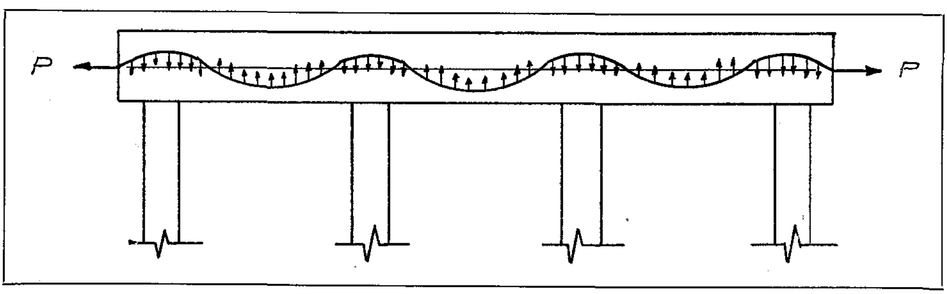


Figure III—a

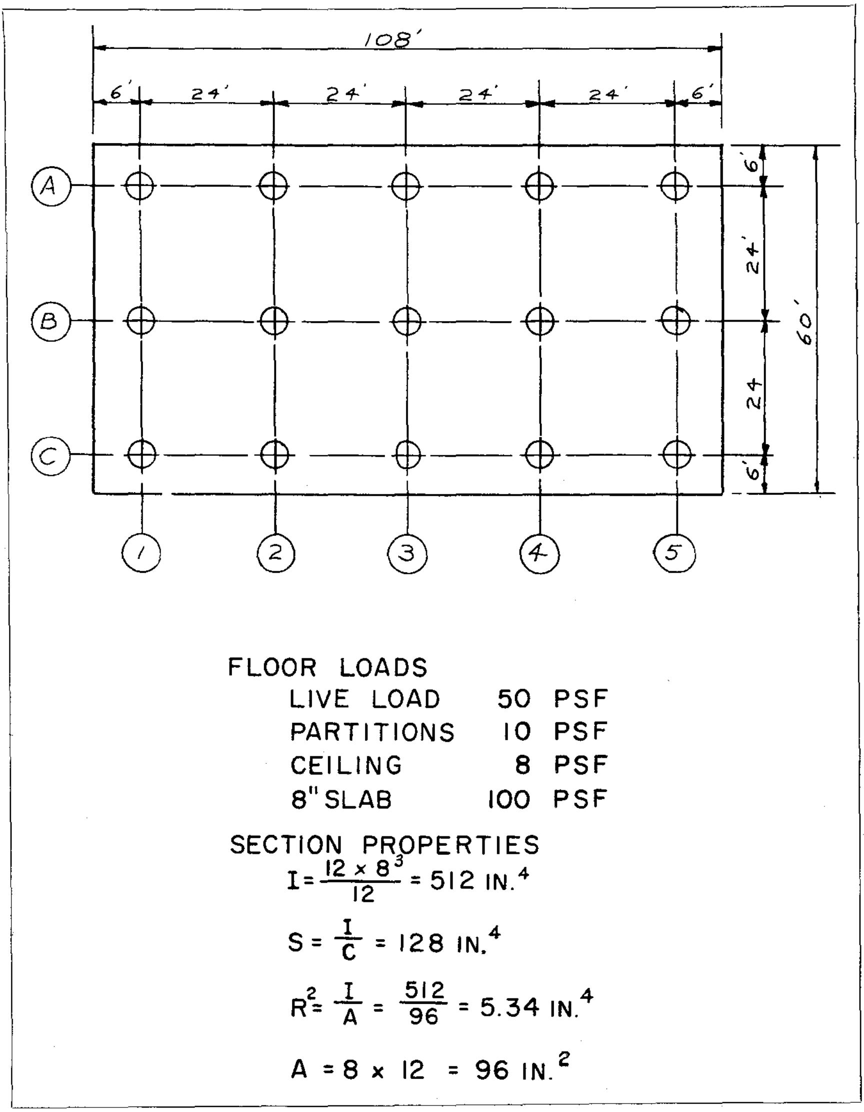


Figure III—b

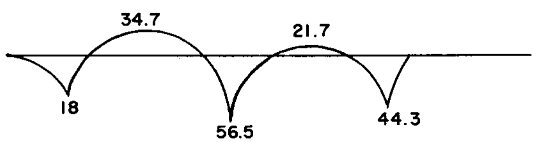
in Figure III-b. It has a column spacing of 24 feet, a cantilever of 6 feet, and a slab thickness of 8 inches. This structure could be an office building, an apartment house, or even a warehouse. In this particular case, assume that it is an office building. The slab could be cast-in-place or lifted. The design method presented applies to continuous beams as well as to slabs.

The first step is to determine the floor loads. In this case, the following loads were assumed: a partition load of 10 psf, a ceiling load of 8 psf, an 8 inch concrete slab at 100 psf, and a live load of 50 psf. The physical constants of a one-foot wide strip such as moment of inertia, section modulus, etc., are shown in Figure III-b. A unit load of a one wide strip was assumed. The fixed-end

MOMENT CURVES FOR UNIFORM LOADS
ASSUME UNIFORM UNIT LOAD ON A STRIP 1' WIDE

6'	24'		24'		24'		24'		6'
	100%	50%	50%	50%	50%	50%	50%	50%	100%
FEM-18	+48	-48	+48	-48	+48	-48	+48	-48	+18
	-30	-15					+15	+30	
	+3.75	+7.5	+7.5	+3.75	-3.75	-7.5	-7.5	-3.75	
	-3.75	-1.88					+1.88	+3.75	
		+.94	+.94			-.94	-.94		
	+18	-56.44	+56.44	-44.25	+44.25				

$K = \frac{I}{L} = 1$ FEM $= \frac{wL^2}{12} = \frac{(1)24^2}{12} = 48$ CAN. M. $\frac{wL^2}{2} = \frac{(1)6^2}{2} = 18$



MOM. UNIFORM DL = 56.5 x 0.118 = 6.68 $^{K'}$
MOM. UNIFORM DL + LL = 56.5 x 0.168 = 9.52 $^{K'}$

NEGLECT EFFECT OF COLUMNS

Figure III—c

moments were balanced as shown on Figure III-c. The effect of column restraint can be neglected. In the example shown, the slab was designed to be level under dead load.

On Figure III-d, a cable location was plotted in direct proportion to the dead load moment curve. The maximum eccentricity *(C)* was assumed at the point of maximum moment.

Eccentricity "*e*" is defined as the distance from the center of gravity of the cross-section to the actual physical position of the tendon. Calculations for other positions of the tendon are shown in the figure. The dimensions shown in Figure III-d, are given in feet. All calculations are carried in kips and feet, therefore, all dimensions and units will be similar. The tendon is a concordant cable as defined by Guyon and Lin. A concordant cable is one which will cause no secondary moments under the prestress force. A concordant cable can also be defined as a cable located so that it is coin-

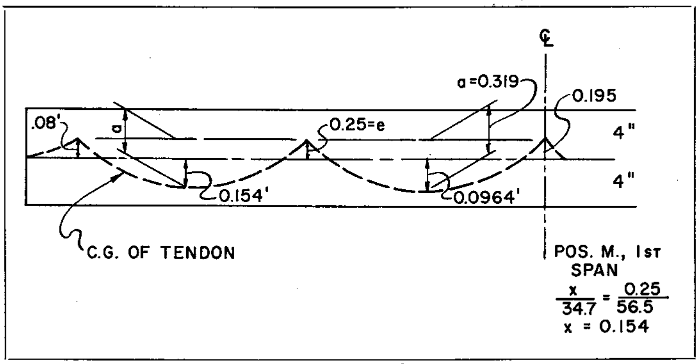


Figure III—d

cident with the line of compression of the cable. It is also possible to plot a nonconcordant cable and determine the effects of the secondary moments. Generally, it is not necessary to know the secondary moments, only the total moments produced by the cable.

To prove that no secondary moments exist in this instance, continue to study Figure III-d. A *Pe* moment curve is drawn in Figure III-e in terms of *P*, where *P* is the prestress force. The upward force *w* in K/Ft of span is equal to $\frac{8Pa}{L^2}$, where *a* is equal to .319 feet. Hence $w =$ $.00443P$. This is a uniform load. This uniform upward load in terms of *P* is in direct proportion to a uniform downward load. Place this uniform load on the structure, calculate the fixed-end moments, and balance them as shown in Figure III-f. In this case, the *Pe* moments equal the balanced moments. This is expected, since no secondary moments are induced in the slab by the concordant tendon force. If any position other than a concordant position were used, secondary effects would be evident in the balanced moments.

The downward loads are set equal to the upward loads and the pre-

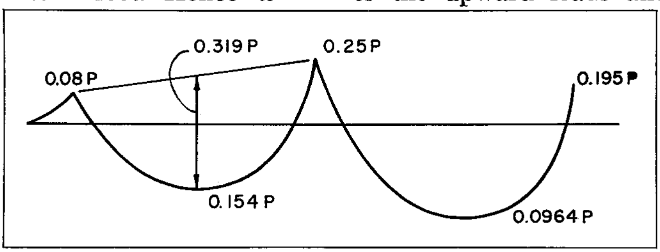


Figure III—e

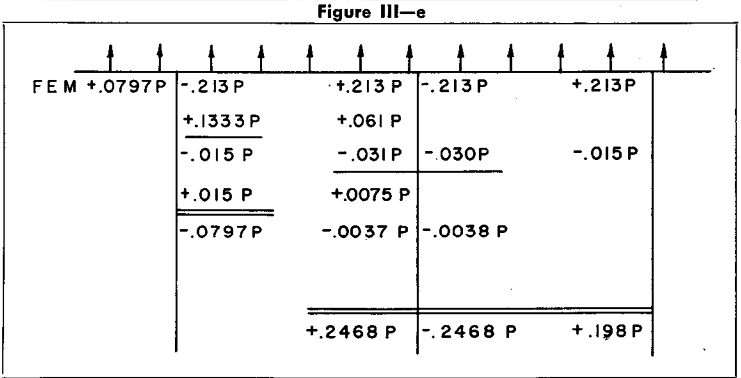


Figure III—f

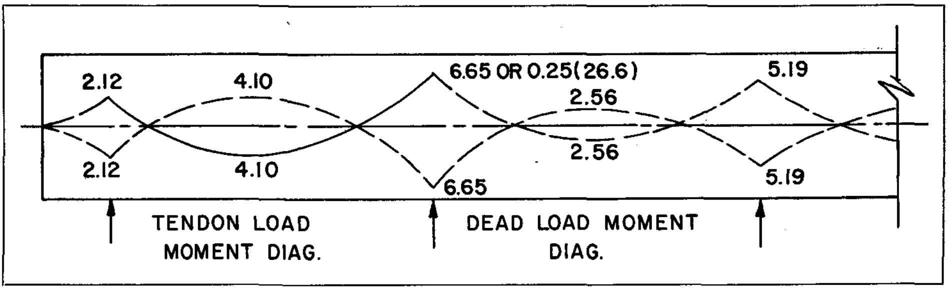


Figure III—g

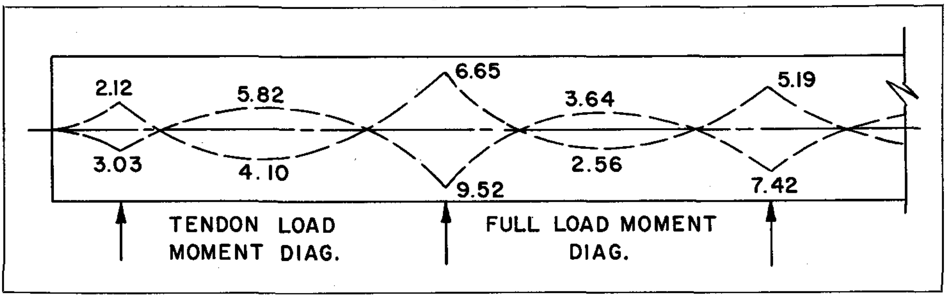


Figure III—h

stress force is calculated directly. Hence, $.25P = 56.5 \times .118$ where .118 is dead load in kip/ft. Therefore $P = 26.6K$. Since the downward loads equal the upward loads, the stress in the slab is P/A. In Figure III-g, the moment diagram of the dead loads has been superimposed on the moment diagram of the tendon load. It can be seen that the moments are equal and opposite in sign. Therefore, the bending stresses are zero and the only existing stress is the P/A stress.

In Figure III-h, the moment diagram of the dead plus the live load has been superimposed on the moment diagram of the tendon load. Moment of $DL + LL$ equals the moment of the unit load times .168 kips per foot. The algebraic sum of the bending moments along the slab gives the residual moment that is resisted by the concrete rather than the cable. The resulting bending stresses in the concrete due to the residual bending moments are generally overcome by the effect of the self-anchoring of the cable, P/A. To explain the physical action mathematically:

$$P/A => \frac{M_{Residual}}{S}$$

In this example refer to Figure III-h. The residual moment is $9.52 - 6.65 = 2.87$ K-Ft.

$$\text{Hence } fc = \frac{P}{A} \pm \frac{M_R}{S}$$

$$= \frac{26.6}{96} \pm 270 = \begin{matrix} 8\ psi\ Top \\ 548\ psi\ Bottom \end{matrix}$$

In this section the simplified action of a cable on a concrete member has been discussed.

IV. Cable Geometry

A few of the practical aspects in the placing of tendons in the slab or girder will be discussed in this section. It can be seen that it is impossible to place the tendons in the configuration in figure II-c. Figure III-a illustrates the shape of tendon curves in practice. It should be noted that tendons are placed in smooth curves. These curves are continuous parabolas exerting upward forces where concave upward, and downward forces where concave downward. Cables composed of ¼ in. wires drape naturally with the point of reversal of curvature approximately $.1L$ to $.125L$ from the support.

In Figure IV-a, is sketched a cable or tendon with a reversal of curvature in the span. Since the curve is continuous, the slopes at the point of intersection of the two curves must be equal. If the point of reversal is a distance of K_1L from the support, from the properties of a parabola the distances marked $a_1 = a_1'$ and $a_2 = a_2'$ By similar triangles

$$\frac{2a_1}{K_1L} = \frac{2a_2}{K_2L} \text{ or } a_1 = \frac{a_2K_1}{K_2} \text{ and } \frac{2a}{L/2} =$$

$$\frac{2a_1}{K_1L}\text{, so } a_1 = 2aK_1 \text{ and } a_2 = 2aK_2$$

There are now two parabolic curves with spans equal to $2K_1L$ and $2K_2L$. From the above information, the forces exerted by the cable on the concrete are:

$$W_1 = \frac{8Pa_1}{(2K_1L)^2}(down) \text{ and } W_2 =$$

$$\frac{8Pa_2}{(2K_2L)^2}\ (upward) \text{ from } Pa = \frac{WL^2}{8}$$

$$\text{which is } W_1 = \frac{2Pa_1}{K_1^2L^2} \text{ and } W_2 = \frac{2Pa_2}{K_2^2L^2}$$

$$\text{Then } W_1 = \frac{2P(2aK_1)}{K_1^2L^2}$$

$$\text{and} \quad W_2 = \frac{2P(2aK_2)}{K_2^2L^2}$$

$$\text{or} \quad W_1 = \frac{4Pa}{K_1L^2} \qquad \textbf{Formula–I}$$

$$\text{and} \quad W_2 = \frac{4Pa}{K_2L^2} \qquad \textbf{Formula–II}$$

(where "a" is the total drape in the cable). From these equations, write:

$$W_1K_1L^2 = 4Pa \text{ and } W_2K_2L^2 = 4Pa$$

$$W_1K_1L = W_2K_2L$$

In other words, the total uplifting force exerted by the cable is equal to the total downward force of the cable in the span. This produces no exterior reactions on the structure. However, if secondary moments exist, equal and opposite shears are induced in the span.

By the use of the above equations, the tendon force, P, can be converted into uniformly distributed loads on the structure. For example, in Formula II, if $K_2 = .5$, and $K_1 = 0$ (See Figure IV-a), then $W_2 = \frac{8Pa}{L^2}$, which is the same formula as shown for a simple hung cable. W_1 in this case becomes infinite. This is because there is no uniform downward load but rather a point load at the end

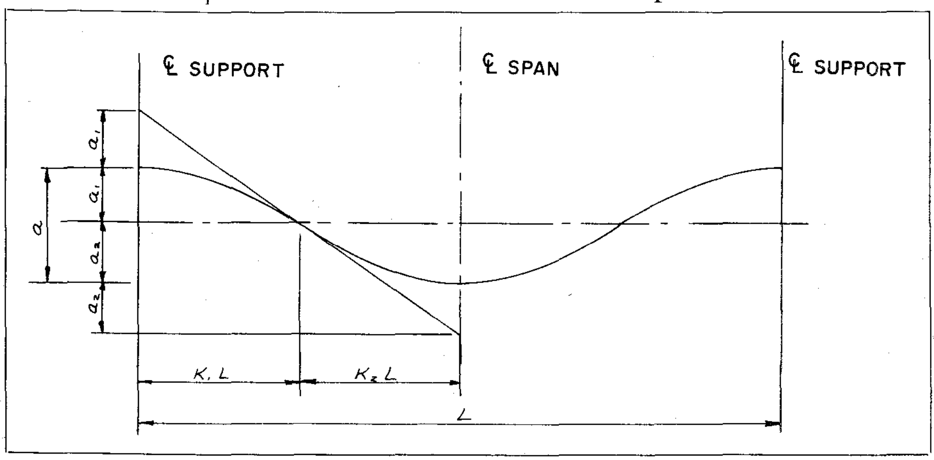


Figure IV—a

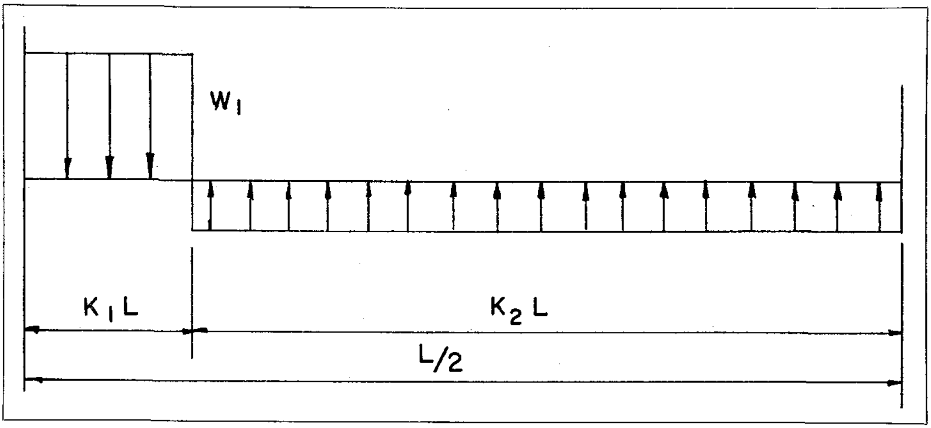


Figure V—a

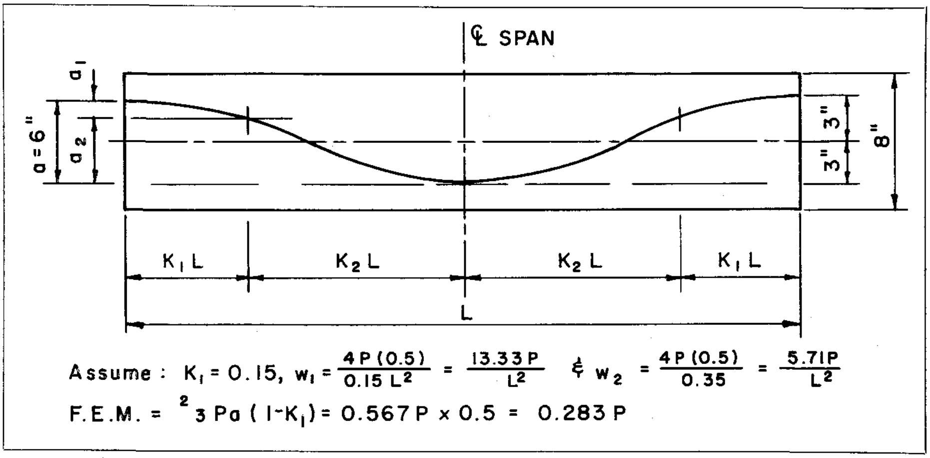


Figure V—b

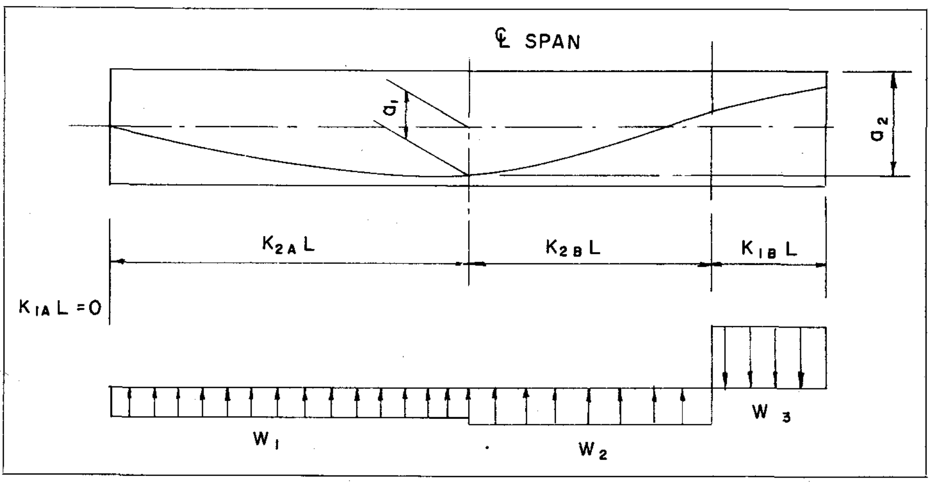


Figure V—c

of the cable.

V. Development of Design Charts

With the cable force converted into uniform loads on the structure, it is possible to compute from this the resultant fixed-end moments.

It is interesting to note from the above equations that the total drape *(a)* determines the forces exerted on the slab. In Figure IV-a is illustrated a tendon which is symmetrical about the center line of span and has a total drape "*a*". Figure V-a illustrates the uniform loads induced on the slab by the tendon shown in Figure IV-a.

In this case, the fixed-end moments can be determined by loading the span with the upward and the downward loads as determined for W_1 and W_2.

By the use of tables giving the FEM's for partially loaded spans, these end moments can be determined. For the case illustrated in Figures IV-a and Figure V-a above, in which the tendon location is symmetrical about the centerline, the fixed-end moments are given by the following formula: Fixed-end moment $= 0.667\ Pa[1\text{-}K_1]$ Equation III.

The fixed-end moment from Equation III varies directly with K_1. Chart I shows values of the fixed-end moment in terms *CPa* for known values of K_1. This chart is for the symmetrical case. Figure V-b illustrates a typical tendon profile for an interior span. Assume that $K = 0.15$, therefore the FEM is computed as follows:

$$M = CPa = .567P \times .5 = 0.283P$$

Since all tendons are not placed symmetrical, another condition will be investigated. Refer to Figure V-c. In this case the tendon consists of three portions of parabolic curves. One from the support to the centerline of the span, $K_{2A}L$; another from the centerline span, a distance $K_{2B}L$; and a third, the distance $K_{1B}L$.

For this span, the prestressing produces three uniform loads of different magnitudes as shown in the figure. Again the fixed-end moments can be worked out. For this condition the moment at each end can be computed by using the following formulas and adding the results for each half span:

$$M_A = \frac{Pa}{3(0.5 - K_1)}(0.6875) - \frac{Pa}{3(0.5 - K_1)}(K_1^2)(6 - 8K_1 + 3K_1^2) - 1/3\,(Pa)K_1(6 - 8K_1 + 3K_1^2)$$

$$M_B = \frac{Pa}{3(0.5 - K_1)}(0.3125) - \frac{Pa}{3(0.5K_1)}(K_1 3)(4 - 3K_1) - \frac{Pa}{3}K_1^2(4 - 3K_1)$$

The fixed-end moments can be computed by using Chart II, which gives the values for fixed-end moment coefficients based on the total drape in the half of the span considered. The general equation for the fixed-end moment on end *A* is $M_A = C_A\,Pa_1 + C_B\,Pa_2$, and on end *B*, $M_B = C_B\,Pa_1 + C_A\,Pa_2$. These equations summarize the effects of all the loads created by an unsymmetrical tendon. The charts are developed for curves having their low point at the centerline of their spans. For the load W_1, where $K_{1A}L = 0$ and $a_1 = 0.25$ ft. and $a_2 = 0.5$ ft., the effects on the fixed-end moments at *A* and *B* are:

$$M_A = 0.458(P)(0.25) = 0.115P$$

$$M_B = 0.208(P)(0.25) = 0.052P$$

Similarly, the total combined effect of w_2 and w_3 on the fixed-end mo-

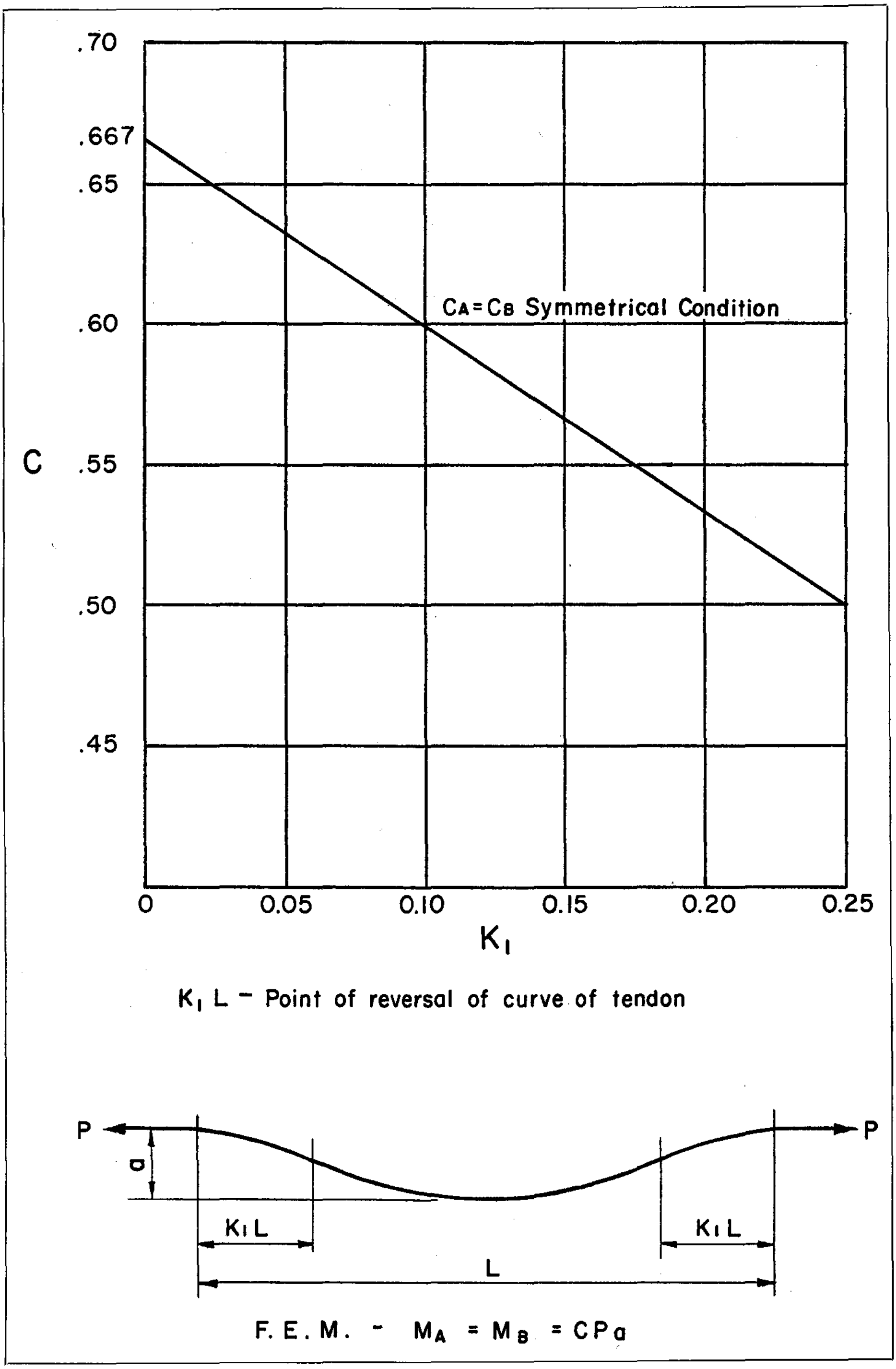

.70
.667
.65
.60
C
.55
.50
.45
0
0.05
0.10
0.15
0.20
0.25
K₁
CA=CB Symmetrical Condition
K₁ L − Point of reversal of curve of tendon
P
P
a
K₁L
K₁L
L
F. E. M. - MA = MB = CPa

Chart I

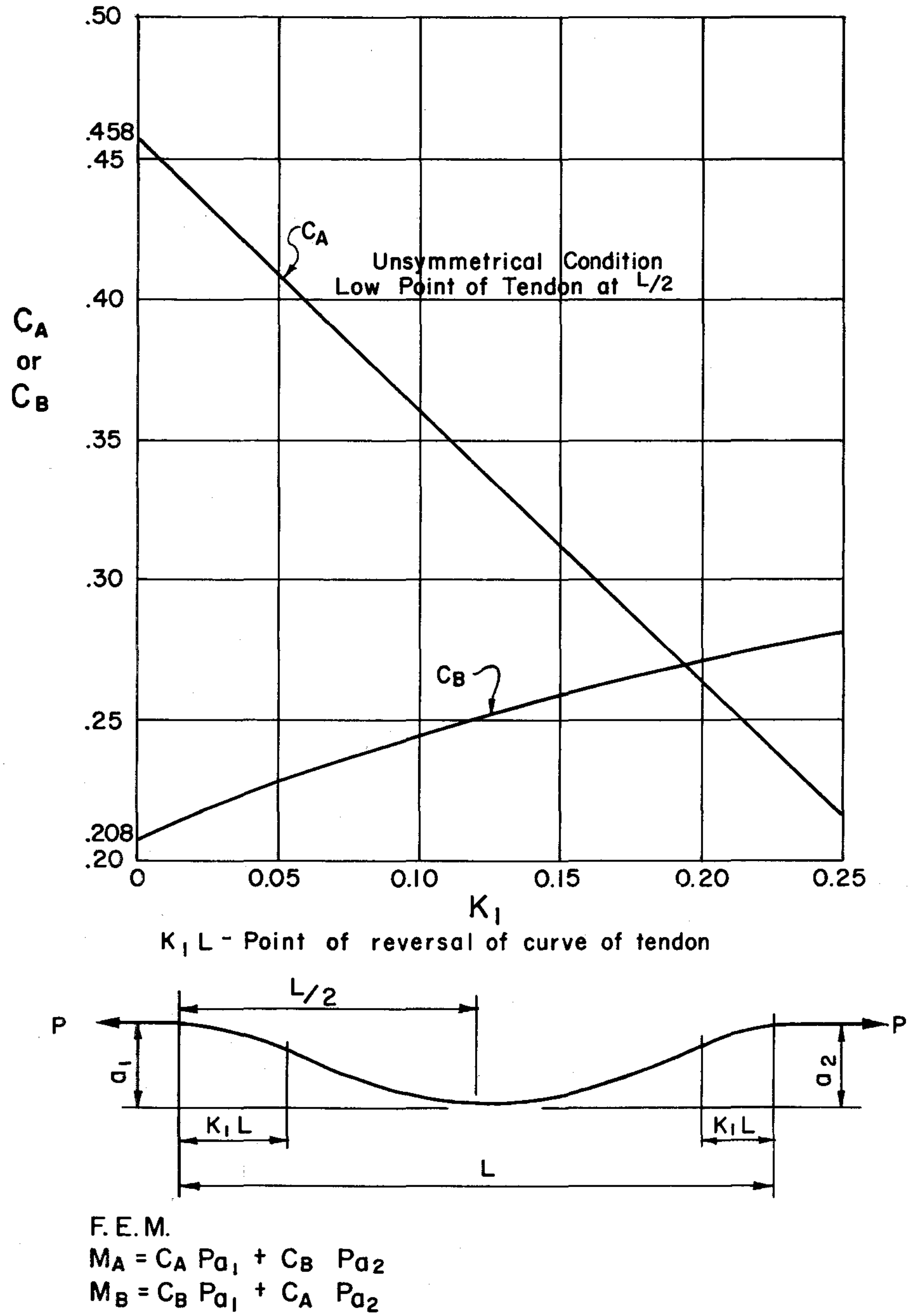
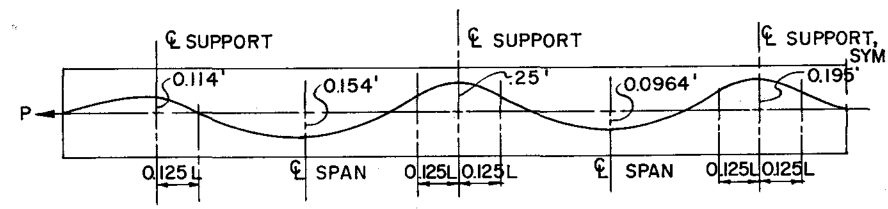

.50
.458
.45
.40
.35
.30
.25
.208
.20
C_A or C_B
C_A
Unsymmetrical Condition
Low Point of Tendon at L/2
C_B
0
0.05
0.10
0.15
0.20
0.25
K_1
K_1L - Point of reversal of curve of tendon
L/2
P
P
a_1
a_2
K_1L
K_1L
L
F. E. M.
$M_A = C_A Pa_1 + C_B Pa_2$
$M_B = C_B Pa_1 + C_A Pa_2$

Chart II



F.E.M. END SPAN M_L = P(0.114 + 0.134)(0.332) + P(0.154 + 0.250)(0.253) = 0.191P

M_R = P(0.268)(0.253) + P(0.404)(0.332) = 0.202P

CENTER SPAN − M_L = P(0.3464)(0.332) + P(0.2914)(0.253) = 0.189P

M_R = P(0.3464)(0.253) + P(0.2914)(0.332) = 0.184P

CANTILEVER MOMENT − 0.114P

CANT.	1.0	0.43	0.57	0.5	0.5
-0.114P	+0.191P	-0.202P	+0.189P	-0.184P	+0.184P
	-0.077P	0.006P	+0.007P	-	-
		-0.039P	0	+0.004P	-0.004P
		+0.017P	+0.022P	-	-
				+0.011P	-0.011P
				-	-

+0.114P -0.170P +0.218P -0.125P +0.169P

Figure VI—a

ments at A and B where $K_{1B} = 0.15$ is $M_A = 0.269P(0.5) = 0.13P$, and $M_B = 0.307P(0.5) = 0.154P$

Thence the total fixed-end moments;

$M_A = (0.115 + 0.13)P = 0.245P$ and;

$M_B = (0.052 + 0.154)P = 0.206P$

This illustrates the use of the charts. If the tendon does not follow the drape criteria as shown on the chart, the end moments can be computed using the various loads on the span.

VI. Design Procedure

Section III, IDEALIZED CABLE STRUCTURES, discussed tendons placed proportional to the moment diagrams and neglected the effect of the reverse curvature of the tendons. A tendon layout similar to that shown on Figure III-d is shown with reverse curves in Figure VI-a. The point of reverse curvature was chosen at 0.125L. The total drape shown is the same as the sketches of Figure III-d. Fixed-end moment coefficients are taken from the charts. Then the fixed-end moments are balanced.

In simple or statically determinate structures, the moment induced by the prestressing is always *Pe* where *e* is in feet or inches. The physical position of the tendon is always coincident with the center of force of the tendon. In a continuous structure, this is not necessarily true. The center of force can be moved up or down depending on the effect of continuity. In this example, the balanced moment at the first support is 0.218*P*. This moment is again in terms of *P* (Kips) and some distance, e', in feet. Since we are applying a constant force, *P*, the full length of the structure, the distance e' is $\frac{M}{P} = \frac{0.218P}{P} = 0.218$ feet. This distance, e', is the location of the center of force whereas the actual physical location of the tendon is 0.25 feet. The center of force (e') at any position in the span is found by dividing the balanced moment by *P*.

To determine the moment at other points, find the amount the center of force of the prestressing moves up or down at the supports. The line of force in the rest of the slab moves a proportionate amount.

In the end span, the position of the cable and the center of force are the same at the first support because the cantilever is statically determinate. At the other end of the span, the cable is at +0.25 ft. while the center of force is at 0.218 ft. This shows the center of force has been moved down .032 ft. at this support. At any point in the end span, the center of force is moved down by an amount $X/L(0.032)$. See Figure VI-b.

In the center span, the center of force of the tendon has been moved down −.032 feet at the left end of the span, and a distance $0.195 - 0.169 = -.026$ feet downward on the right end. The secondary moments are defined as the difference between the balanced moment *(Pe′)* and the *Pe* moments. At the left end of the second span, the secondary moment is −.032*P* and at the right end, it is −.026*P*.

In this span the location of the center of force of the cable can be determined at any point by the use of these distances. See Figure VI-c.

Refer to Figure III-f and find the balanced moments due to prestressing. These shall be recorded (Table I) and compared with the results of Figure VI-a.

By this comparison, it can be seen the effect of reversal of curvature. The example from Figure III-f assumes no point of reversal. The two curves intersect over the center line of the support. This is impossible to do in a continuous cable, but the result can be approximated by tying the cables down adjacent to the support and forcing it into, or nearly into, the desired shape. In any case, the results show an error in assuming the tendon in this idealized location.

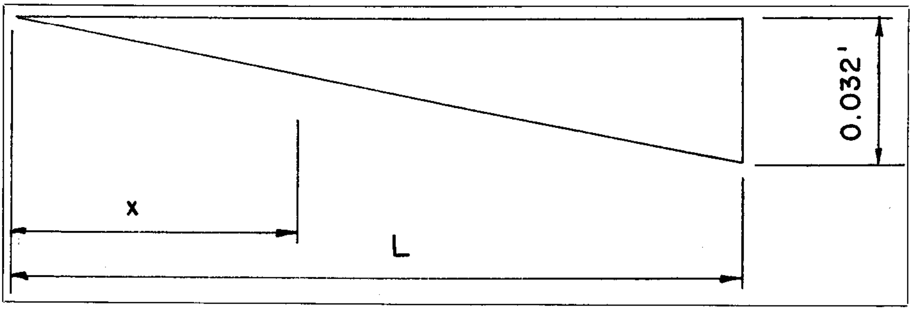


Figure VI—b

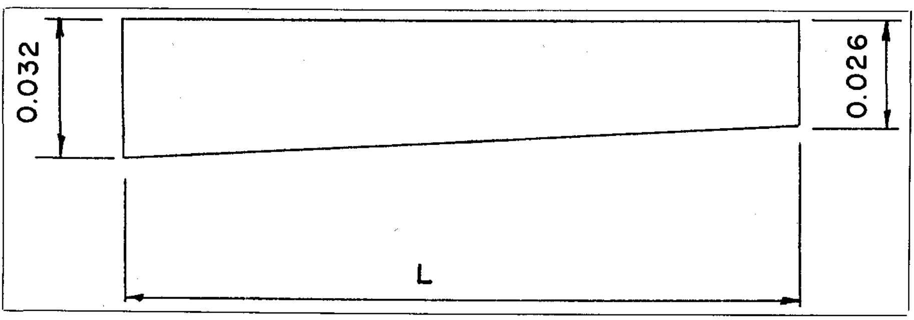


Figure VI—c

	1ST SUPPORT	CENTER LINE SPAN	2ND SUPPORT	CENTER LINE SPAN	3RD SUPPORT
From Fig. VI-a	+ 0.114*P*	− 0.170*P*	+ 0.218*P*	− 0.125*P*	+ 0.169*P*
From Fig. III-f	+ 0.114*P*	− 0.154*P*	+ 0.25*P*	− 0.0964*P*	+ 0.195*P*
% Error	0	10%	12.4%	13%	11.6%

Table—I

Now determine the prestressing force required. From Figure III-c, we have the moments (Table II) due to *D.L.* + *L.L.* The concrete stress due to loads are computed from

$$f_c = \frac{M_{DL+LL}}{S}$$

$$f_c = \frac{P}{A} \pm \frac{Pe'}{S} \pm \frac{M_{Loads}}{S} \quad \text{Where } f_c = 0$$

Reduces to

$$\frac{M}{S} = \frac{P}{A}(1 + \frac{e'y)}{R^2}$$

R = Radius of gyration
f_c = Stress in concrete
e' = True location of center of force

Solve for P

$$P = \frac{\frac{M}{S} \times A}{1 + \frac{e'y}{R^2}} = \frac{.890 \times 96}{1 + \frac{(.218 \times 12)4}{5.34}}$$

P equals 28.7^K with *"0" tension* stresses at point of maximum moment. The stresses due to prestressing and loads are summarized (Table III).

The prestress force P equal *to* 28.7 K Ft. was computed to give zero tension stress at the point of maximum bending stress in the concrete.

In the example discussed in Section III, IDEALIZED CABLE STRUCTURES, the prestress force calculated to give essentially the same results was 26.6 K Ft. This establishes the fallacy of neglecting the effect of the reversal of tendons over supports.

The two previous examples used a tendon location that was proportional to the moment diagram. The most economical tendon location uses a drape that is maximum for the loading conditions. Generally, for structures subject to uniform loads, such as office buildings, apartments, etc., the maximum drape can be used for all spans. The ACI code permits structures to be designed without analyzing for effects of alternate span loading when the superimposed loads do not exceed 75% of the dead load. Prestressed structures can be designed with the same limiting conditions. If the superimposed loads exceed 75% of the dead load, it will be necessary to investigate for the effects of alternate loads and adjust the cable location as required.

Figure VI-d continues with the same problem using the maximum drape possible consistent with the

	1ST SUPPORT	CENTER LINE SPAN	2ND SUPPORT	CENTER LINE SPAN	3RD SUPPORT
Moments	− 3.0	+ 5.8	− 9.5	+ 3.6	− 7.4
f_c Stresses in PSI	± 280	± 542	± 890	± 338	± 694

Table—II

	1ST SUPPORT	CENTER LINE SPAN	2ND SUPPORT	CENTER LINE SPAN	3RD SUPPORT
P/A Top / Bottom	+300	+300	+300	+300	+300
$\frac{Pe'}{S}$ Top	+309	−460	+590	−339	+457
Bottom	−309	+460	−590	+339	−457
Total Prestress					
Top	+609	−160	+890	−39	+757
Bottom	−9	+760	−290	+639	−157
D.L. + L.L.					
Top	−280	+542	−890	+338	−694
Bottom	+280	−542	+890	−338	+694
Combined Prestress Plus Loads					
Top	+329	+382	0	+299	+63
Bottom	+271	+218	+600	+301	+537

Table—III

placement of tendons in two directions in a slab.

The fixed-end moments were determined using the charts and balanced. This illustrates another method of determining the prestress force shown as follows. The same criteria of zero tension fiber stress at total load is used. Consider the prestress force inducing the following slab stresses, $f_c = \frac{P}{A}$ and $f_c = \pm \frac{Pe'}{S}$. These stresses in the slab may be equated to moments by the relationship, $f_c = \frac{M}{S}$. The moment caused by the prestressing is $M_p = f_c S = S\left[\frac{P}{12A} \pm \frac{Pe'}{S}\right]$ so $M_P =$

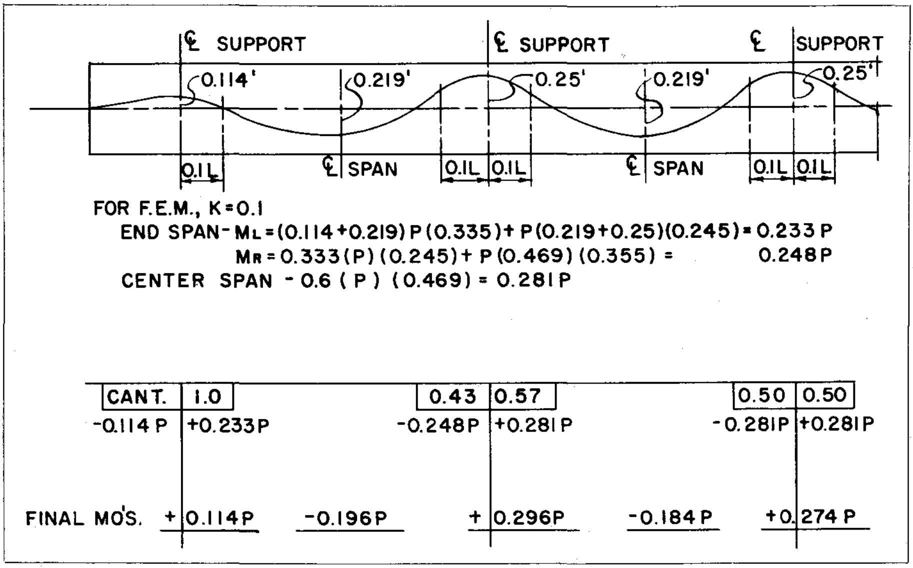


Figure VI—d

	1ST SUPPORT	CENTER LINE SPAN	2ND SUPPORT	CENTER LINE SPAN	3RD SUPPORT
$M_p = (0.111 \pm e')P$					
Top	$+0.225P$	$-0.035P$	$+0.407P$	$-0.073P$	$+0.385P$
Bottom	$-0.003P$	$+0.307P$	$-0.185P$	$+0.295P$	$-0.163P$
Top	$+5.26$	-2.0	$+9.5$	-1.71	$+9.0$
Bottom	-0.07	$+7.2$	-4.3	$+6.92$	-3.8
$M_{PL}{}^{+LL}$					
Top & Bottom	± 3.0	± 5.8	± 9.5	± 3.6	± 7.4

Table—IV

$P[\frac{S}{A(12)} \pm e']$. The factor $\frac{S}{A(12)}$ is constant for the slab. The "M_P" moment is the combination of the resistance offered by the slab through the axial prestress force and the uplift effects of the cable.

Since M_P is the total moment induced by the prestress force, it may be equated directly to the moment of the dead load plus live load.

In this instance $M_{DL+LL} = 9.5^{K\text{-}Ft.}$ at the first interior support (Figure III–h). The term $\frac{S}{12A}$ is always positive; i.e., produces compression on both top and bottom fibers. The term e' is indicative of the position of the tendon relative to the center of gravity of the section. Compression is produced in the extreme fibers nearest the tendon.

The load moments also cause tension or compression in the outside fibers. Consider + (plus) being compression and – (minus) being tension.

Hence $9.5 = P[\frac{128}{12 \times 96}] + P \times 0.296$

$\therefore P = 23.34^K$.

The P required to balance the maximum moment having been calculated is 23.34^K. Other moments due to this prestressing force will be tabulated (Table IV).

Stress at any point may be computed by adding algebraically the moments and dividing by the section modulus.

The economy that can be accomplished by using the maximum drape is evident. In the idealized tendon the required "P" force was 26.6^K, in the next example using the same drapes as the first example, the required force calculated to be 27.7^K, finally in the maximum drape example, the force calculated to be 23.34^K.

Further economy may be realized by using discontinuous tendons. Again consider the longitudinal direction. Since the design moments are highest in the first or exterior span, increase the prestress force in

	1ST SUPPORT	CENTER LINE SPAN	2ND SUPPORT	CENTER LINE SPAN	3RD SUPPORT
M_P					
Top	$+5.3$	-2.1	$+9.5$	-1.5	$+7.6$
Bottom	0	$+7.4$	-4.2	$+5.9$	-3.2
M_{DL+LL}					
Top Bottom	± 3.0 K′	± 5.3 K′	± 9.5 K′	± 3.6 K′	± 7.4 K′

Table—V

this span by the use of discontinuous tendons. The discontinuous tendons will be placed on the same profile as the continuous tendons. However, after they pass the first interior support, they will be carried to the centerline of the slab and stopped. This is illustrated in Figure VI-e. The continuous tendon is designated as P and the discontinuous tendon as $0.2P$. The tendon profile is shown below.

For the discontinuous tendon, $a_1 = 2(0.469)(0.1) = 0.0938$ ft.

The slope at 2.4 ft. out is $\frac{2(0.0938)}{24} = 0.0781$ Ft./Ft. Carry the tendon on at this slope to the intersection of the centerline. This distance is $\frac{0.25 - 0.0938}{0.0781} = 2$ ft. 0 in.

At this distance, $2.4 + 2.0 = 4.4$ ft. an upward component of the prestressing force exists. In this instance, the tangent of the angle was used rather than the sine. Calling this value R, $R = 0.2P(0.0781) = 0.0155P$.

The load on the interior span (due to the discontinuous tendon) is illustrated in Figure VI-f.

Referring to Figure VI-f, $W_1 = \frac{4(0.2P)(0.469)}{0.1(24)^2} = 0.0065P$. Therefore, the $FEM_L = \frac{0.0065P(24)^2}{12}(0.01)(6 - 0.8 + 0.03) = 0.0164P$, on the left end of the beam, and $FEM_R = \frac{0.0065P(24)^2}{12}(0.001)(4 - 0.3) = 0.001P$, on the right. For the concentrated load upward, the $FEM_L = \frac{R(4.4)(19.6)^2}{(24)^2} = 0.046P$, and the $FEM_R = \frac{R(19.6)(4.4)^2}{(24)^2} = 0.65R = 0.010P$. Fixed-end moments from the continuous cable arc:

End Span $M_L = 0.355(0.333)(1.2P) + 0.245(0.469)1.2P = (0.142 + 0.138)P = 0.280P$

$M_R = 0.245(0.333)(1.2P) + 0.355(0.469)1.2P = (0.098 + 0.200)P = 0.298P$

and

Center Span $0.6(0.469) = 0.281P$.

The combined moments in the center span will be:

$M_L = (0.281 + 0.046 - 0.016)P = 0.311P$

$M_R = (0.281 + 0.010 - 0.001)P = 0.290P$

Cantilever $M_O = 0.114(1.2P) = 0.137P$

The moment distribution:

To determine e', the balanced moments (Figure VI-g) are divided

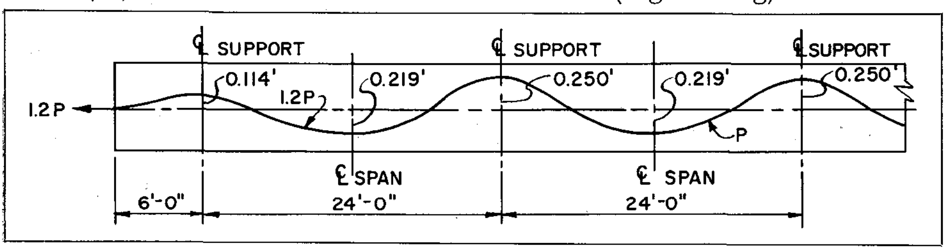


Figure VI—e

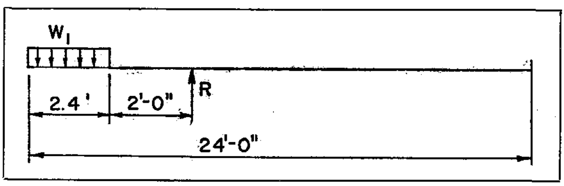


Figure VI—f

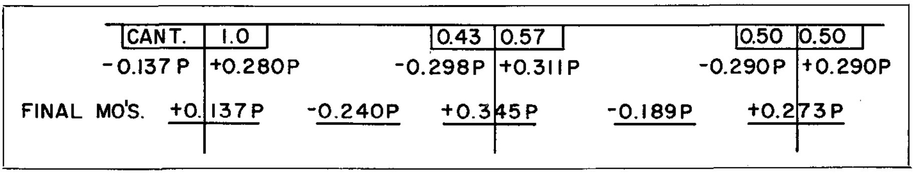


Figure VI—g

by the prestress force used at the particular section. For example: e' at the first support equals $e' = \frac{0.137P}{1.2P} = 0.114$ ft. At the first interior support, $e' = \frac{0.345P}{1.2P} = 0.288$ ft. As in the previous example, determine the $M_P = P[\frac{S}{12A} \pm e']$, at the first interior support. $M = 1.2P\ [\frac{128}{12 \times 96}] + 1.2P \times .288' = .480P$. To balance the load, set the moment of the load equal to the M_P, or $9.5 = .480P$. Therefore, the required P is 19.9^K. The moments are tabulated (Table V).

By combining the moments, it can be seen that there is no tension in the slab.

It is obvious that a stress concentration exists in the slab in front of the dead end plate. A small cage of mild steel bars should be used to help distribute the force from the tendon.

This example has demonstrated an efficient use of prestressing in a continuous slab of equal spans. In certain instances, it might be more desirable to consider the continuous and discontinuous tendons as two separate conditions and combine the results.

In all of the examples that have been discussed, the effects of column restraint has been neglected. The use of prestressing to reduce or eliminate column moments was discussed briefly in Section II. This is accomplished by slab or beam deflection control. This approach is more or less intuitive. The following example illustrates how to investigate those cases when the designer feels it is necessary to do so. See Figure VI-h. The example used is the flat plate illustrated in Figure III-a. The entire bay width will be used to compute the relative stiffness. Assume the columns to be 16 in. square and fixed at their lower ends. The combined dead load and live load moments are summarized in Figure III-a.

The tendons will be placed in the same manner as in the previous examples, using the maximum drape as illustrated in Figure VI-d. These moments have been balanced and are summarized in Figure VI-h.

The maximum combined dead plus live load moment is 213 K/Ft. The prestressing force can be determined by using the relationship $M_{DL+LL} = M_P = P[\frac{S}{12A} \pm e']$. The term $\frac{S}{12A}$ can be rearranged to $\frac{d}{72}$ which is true for all rectangular sections. Thence $P = \frac{213}{.111 + .278} = 548$ Kips, which is 22.9 per foot of slab. This force compares favorably to 23.34 K/Ft. that was determined by neglecting the effects of the columns.

The signs used in Figure VI-h, "Dead Load Plus Live Load Moments" indicate the direction of the moment on the end of the span, that is, Plus (+) is clockwise and

Minus (−) is counter clockwise. The moment in the center of the span is called positive, indicating tension in the bottom fibers. The two moments given at each point in Figure VI-i, "The Prestressing Cable Moments", indicate the effect of the prestressing. If the smaller moment is greater than the *dead* load moment, tension exists at that particular point.

The dead load plus live load moment in the exterior column is $+55^{K\text{-}Ft.}$ The prestressing moment is $-33^{K\text{-}Ft.}$ The combination reduces this column moment to $+22^{K\text{-}Ft.}$, demonstrating a reduction of slab deflection under total load compared to a conventional mild steel design.

Another effect of the prestressing is the slab shortening. In this example, the P/A stress is 233 psi. The slab will shorten $\frac{233(48)12}{E} = \frac{134 \times 10^3}{E}$ at the exterior column. The shortening effecting the next column is $\frac{67 \times 103}{E}$. The shortening is zero at the center of the building. This shortening will introduce moments into the columns. The fixed-end moments may be calculated by the formula

$$M_{FEM} = \frac{6EI\Delta}{L^2}$$

where Δ is the displacement in inches. The exterior columns will have fixed-end moments equal to $25.0^{K\text{-}Ft.}$ and $12.5^{K\text{-}Ft.}$ These moments are symmetrical with respect to the centerline of the building. Hence no side sway is introduced into the structure. The results are tabulated on the line drawing of the structure. Figure VI-j.

By examining the tabulated values for dead plus live load, it can be

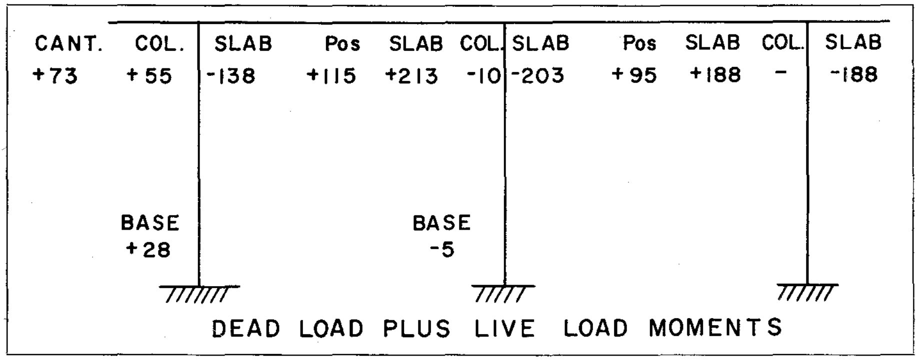


Figure VI—h

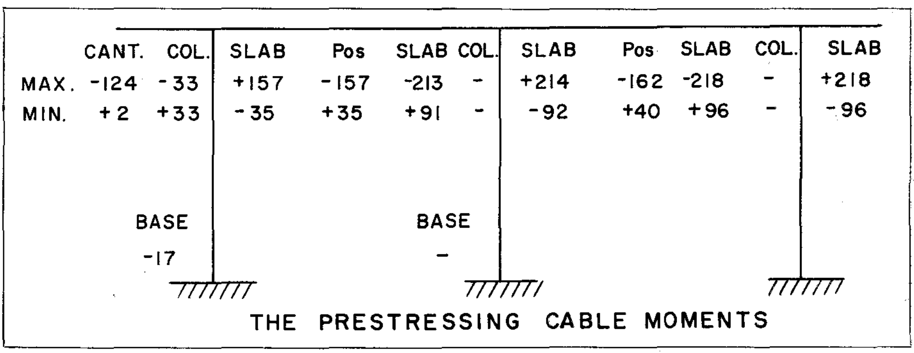


Figure VI—i

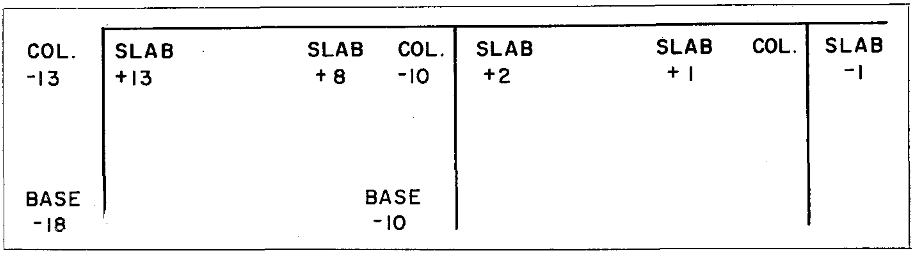


Figure VI—j

noted that the shortening decreases their values except on the exterior side of the first interior column. The magnitude of the moment is small in this example and may be neglected. Also, the shear in the columns is small, about 3 Kips in the exterior column. The shear effect may be neglected also.

The over-all effect of the columns in this instance has been insignificant. The original computation disregarding the columns yielded acceptable results. In structures with very stiff columns, the column restraint becomes a very important consideration and must be investigated. In many cases, it becomes necessary to free the columns from the beam and tie them together after the prestressing has been accomplished.

VII. Linear Transformation of Cables and Concordancy

Guyon discovered the phenomena of linear transformation. T. Y. Lin has studied the effect and written a set of Propositions and Corollaries governing the use of linear transformation and concordant cables. An idealized cable structure is sketched in Figure VII-a.

The cable exerts a uniform uplift force on the beam equal to $\frac{8Pa}{L^2}$. This uniform up-lift force is dependent upon the drape *"a"*, and independent upon the location of the cable in the beam so long as the ends of the cable are anchored at center of gravity. If the cable is lowered or raised at the interior support and the intrinsic shape of the curve or curves is not altered, the net upward load on the structure is not changed. The raising or lowering of the cable is called *Linear Transformation.*

Linear Transformation has many practical applications. The most common and useful is in slabs. The tendon may be moved either up or down to permit tendons from the other direction to by-pass. This can be accomplished in the field or office with no recalculation of stress being necessary.

Since the drape of the cable and the prestressing force is all that resists the gravity loads, it can be seen that the line of compression of the cable does not necessarily follow the cable. This fact has been demonstrated in all of the examples herein. The line of compression is determined by dividing the balanced moment of the tendon loads by the prestressing force. If the line of compression of the tendon falls coincident with the physical location of the tendon, the tendon is considered to be concordant. Conversely, if the line of compression falls other than on the physical position of the tendon, the tendon is considered to be non-concordant. It is interesting to note that for any one system of drapes, there is only one line of compression. It is also evident that the line of compression is independent of the numerical value

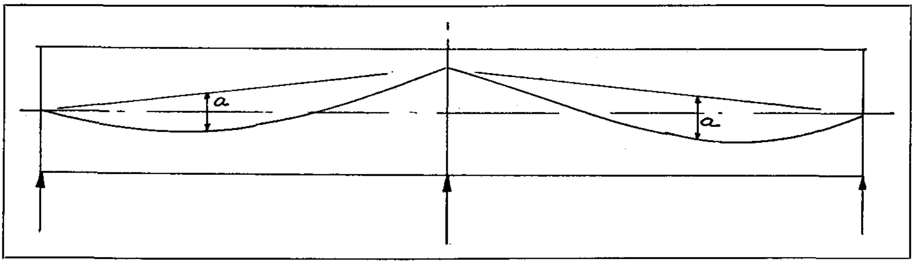


Figure VII—a

of the prestress force.

A cable may be linearly transformed into a concordant position if desired. Providing, of course, that the line of compression falls within the physical limits of the beam and the tendon has the proper cover. Also a concordant tendon causes no secondary moments.

There is no reason to choose a concordant position any more than a reason to choose a non-concordant position. The drape of the cable or tendon should be chosen for engineering reasons rather than concordancy reasons.