THE APPARENT MODULUS OF ELASTICITY OF PRESTRESSED CONCRETE BEAMS UNDER DIFFERENT STRESS LEVELS

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INTRODUCTION

The application of a sufficiently large prestressing force at a point below the neutral axis of a simply supported prestressed concrete beam will cause an upward deflection of the beam. This upward deflection is defined as camber. If the beam remains under these conditions and no additional loads are applied there will be some losses in the prestressing force due to shrinkage of the concrete and other factors. Normally under these conditions a decrease in camber would be expected, but in a prestressed concrete beam the opposite occurs. The camber of the beam will continue to increase as a function of time. This phenomenon is believed to be principally the effect of creep in the concrete.

Creep can be defined as the total time-dependent change in strain minus the shrinkage. A number of studies have been made to determine the rate and magnitude of creep in concrete. It has been shown that plain and reinforced concrete cylinders loaded in compression have continued to creep throughout a thirty year test period. (1)* It has also been demonstrated that the creep in cylinders at low stress levels is directly proportional to the stress in the concrete. (2) This linearity does not exist in cylinders loaded to a stress which is greater than 20-30% of their ultimate strength. Stresses which exceed this amount are commonly found in prestressed concrete.

Tests have been conducted to determine the effect of creep on the loss of prestressing force. (3) These losses were determined experimentally as creep coefficients or percentages of the initial elastic strain. The use of these factors as a method of calculating stress loss in the prestressing bars or strands has been generally accepted in design practice. (4)

Consider now the effect of creep on the Modulus of Elasticity of the concrete. The load-deflection relationship for a prestressed concrete beam would be linear up to the cracking load. This indicates that the Modulus of Elasticity of the beam is constant within this range. If the load-deflection relationship is studied over a period of time a different set of conditions will exist. Here the camber is increasing with respect to time, and the prestressing force is decreasing. From these two factors alone it can be seen that the Apparent Modulus of Elasticity of this beam is not a constant, but a function of time.

All of the factors that have been discussed up to this point enter into the problem of computing the deflections of prestressed concrete beams. There is no known way of determining the Modulus of Elasticity of the beam to any degree of accuracy. Cylinder tests will give an indication of the true value only. The actual beam Modulus is dependent on age, stress level, curing conditions, mix and many other factors. Also any deflection which has to be computed over a period of time becomes a function of the concrete creep. This creep is a function of stress level, moisture conditions, age and other factors. All this points out the extreme interdependence of a large number of variables in this one behavior alone.

Some recommendations have been made suggesting values of the Modulus of Elasticity to be used under various conditions. Where possible, these values will be used as a comparison with the results obtained in this study.

OBJECTIVE AND SCOPE OF THE STUDY

The primary purpose of this study is to present, from the results of a series of tests, the following factors:

1. The relationship between the beam Ap-

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*Numbers refer to references.

parent Modulus of Elasticity, the cylinder Modulus of Elasticity, and the beam Modulus of Elasticity;
2. The relationship between the total strain in the concrete, the creep, the shrinkage, and time;
3. The relationship between creep and the Apparent Modulus of Elasticity;
4. The effect of two different stress levels on all the above.

To accomplish this four beams were tested. During the primary testing period, the beams were loaded with only the prestressing force and their own dead load. The only condition varied was the prestressing force. Beams 1 and 2 were tested at approximately 0.40 f'_{ci}. Beams 3 and 4 were loaded to about 0.25 f'_{ci}. All other factors were kept as nearly constant as possible.

For the purpose of this series of tests the Apparent Modulus of Elasticity will be the Modulus as computed from the magnitude of camber and post-tensioning forces.

DESCRIPTION OF THE SPECIMENS

Four identical beams, 10 in. x 12 in. x 25 ft.-0 in. center to center of supports were cast for this test. The beams had a net area of 113.8 in.^{2} and a net moment of inertia of 1396 in.^{4}. Rollers were provided at the beam ends so that it was possible to cast the beams, instrument, and complete all primary testing without moving the beams. No stirrups were used. Pick-up hooks were provided at both ends of each beam.

The following design mix was used for the specimens:

Cement	620 lbs.
Coarse Aggregate	1798 lbs.
Fine Aggregate	1124 lbs.
Water	38.32 gals.

The fine aggregate was Interlachen sand with a fineness modulus of 2.14 and a specific gravity of 2.63. The coarse aggregate was Brooksville stone, one inch maximum size, with a specific gravity of 2.53. The concrete was provided by a local ready mix plant.

Internal vibration was used during the casting of the beams to facilitate the placing of the concrete. All of the specimens were cured for five days under wet burlap. At the end of this curing period the cylinder strength was in excess of 4800 psi. As this strength was satisfactory for the tests, no further curing was required. The forms were therefore removed and instrumentation begun at this time.

Prior to the pouring of the concrete it was necessary to place three Duoflex casings longitudinally in each of the beam forms. These casings were used to form the holes through which the post-tensioning bars were run. The holes were located so that the center of gravity of the post-tensioning steel was at the lower kern point of the beam.

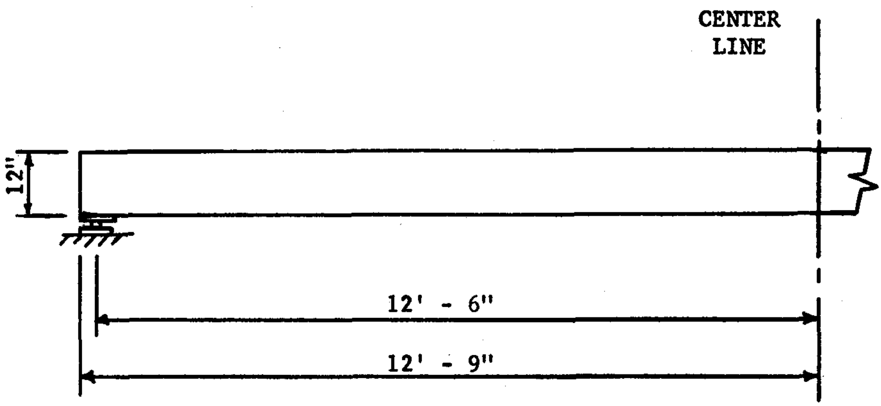
Three straight, unbonded, Stressteel bars, ¾ in. in diameter were used to provide the prestressing in each of the beams. These bars had a Modulus of Elasticity of 28.2 x 10^6 psi and a yield strength of 0.2% offset in excess of 130,000 psi.

Two shrinkage specimens, 10 in. x 12 in. x 5 ft.-0 in., were cast at the same time as the beams. They were cured and stored, at all times, under the same conditions as the beams. As these specimens were unstressed there would be no creep effect and any change in their strain would correspond to the change in strain of the beams due to shrinkage and temperature.

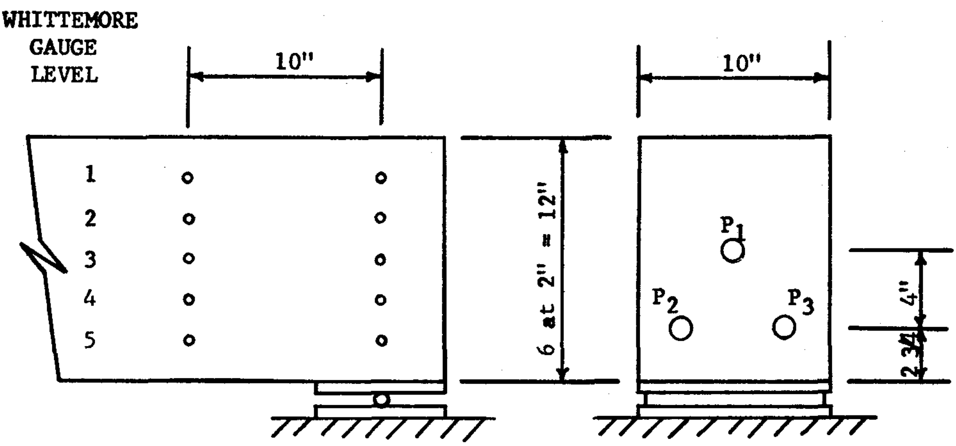
Thirty standard 6 in. by 12 in. cylinders were cast at the time of the concrete pour. All of these cylinders were cured for five days under wet burlap. At this time the forms were stripped and a few of the cylinders were placed in a curing room with a constant temperature of 70°F and a humidity of 100%. These cylinders were used to obtain the concrete properties at these ideal conditions. The remainder of the cylinders were stored under the same conditions as the beams and the shrinkage specimens, and were used to obtain the properties of the concrete under test conditions.

INSTRUMENTATION OF THE SPECIMENS

For convenience the beams were numbered 1 through 4. Beams 1 and 2 were tested as a pair at the higher stress level while beams 3 and 4 were tested as a second pair at the lower stress level. As the post-tensioning bars were jacked from one end only, that end was called the near end. The other end was called the far end. The post-tensioning bars and the Whittemore gauges were numbered as shown in Figure 1.



HALF SIDE VIEW



TYPICAL END VIEWS

FIG. 1 - BEAM DETAILS

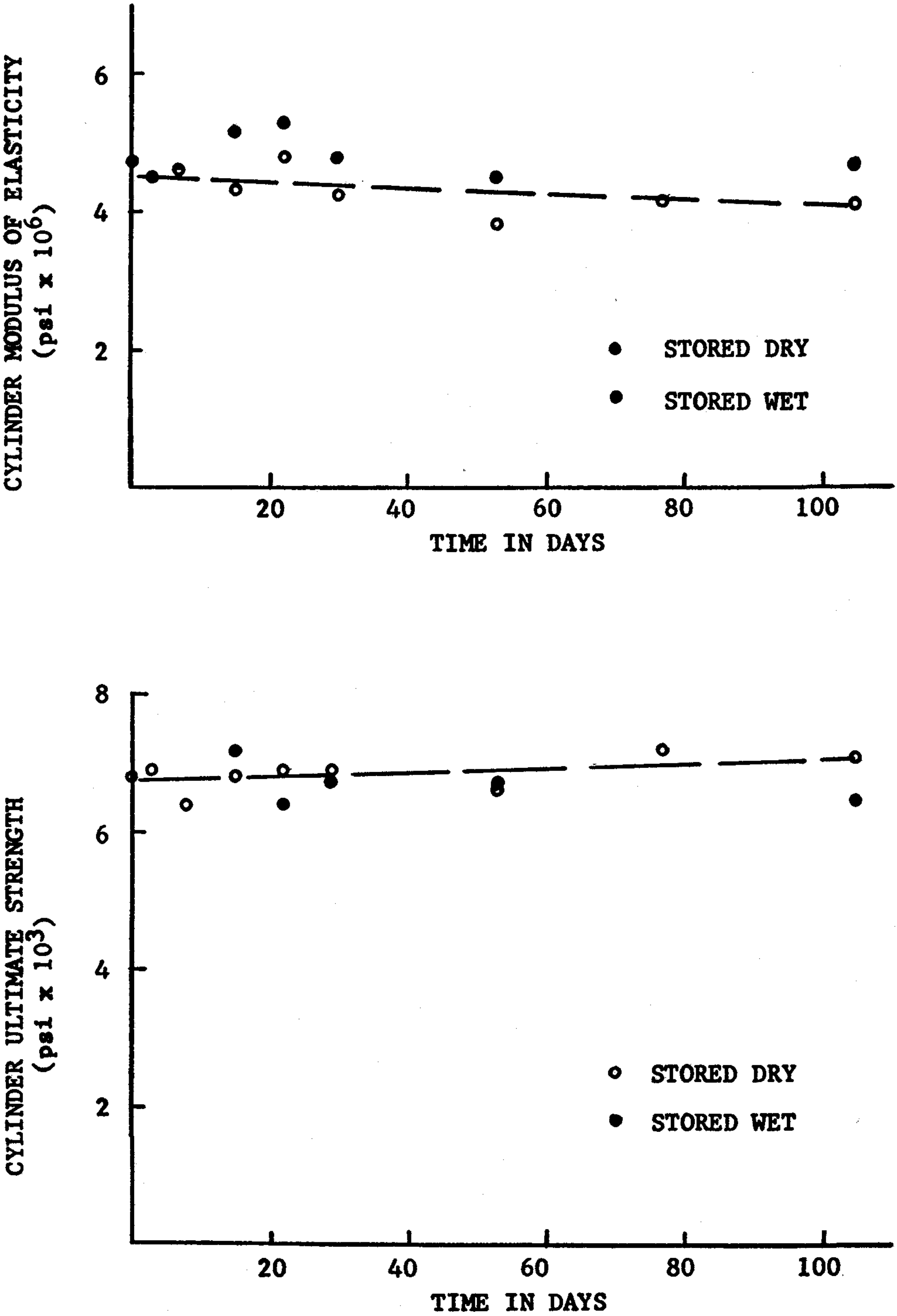
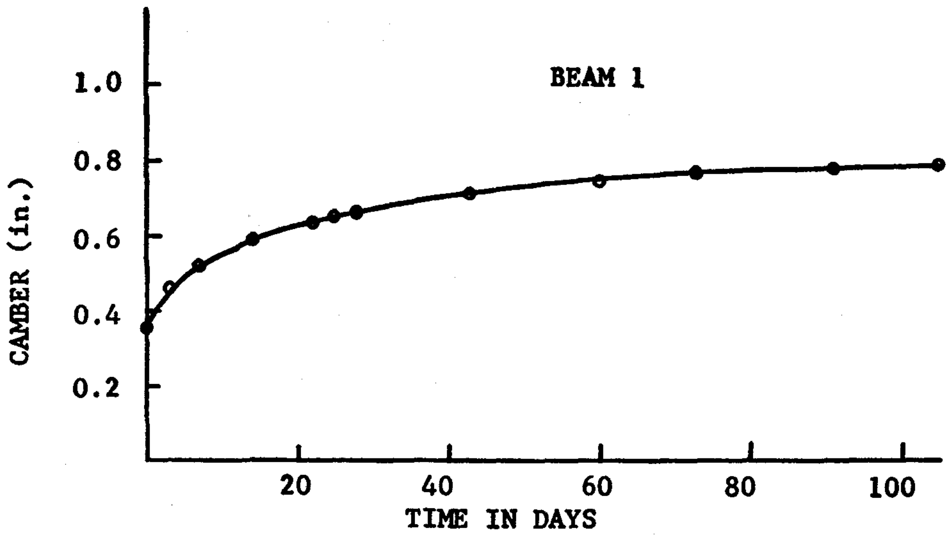


FIG. 2 - CONCRETE PROPERTIES



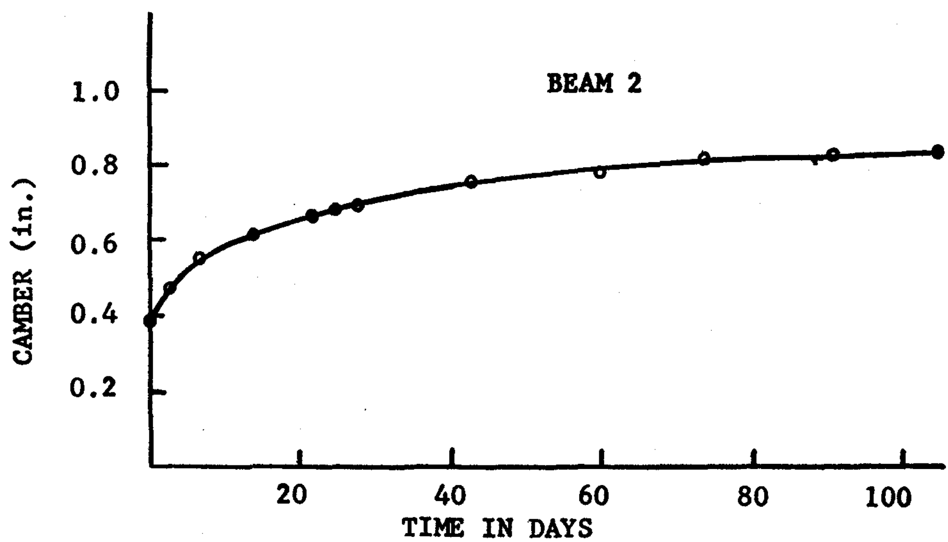
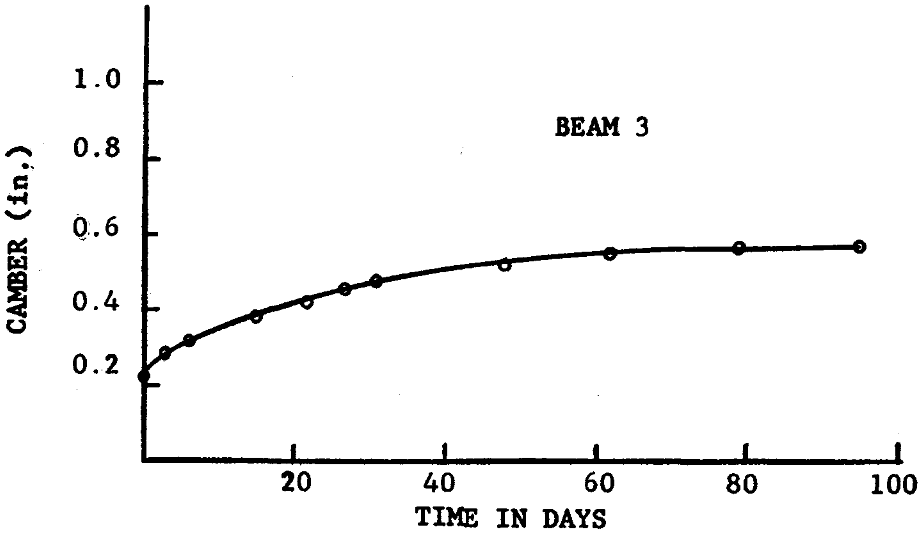


FIG. 3 - CAMBER CURVES, BEAMS 1 AND 2



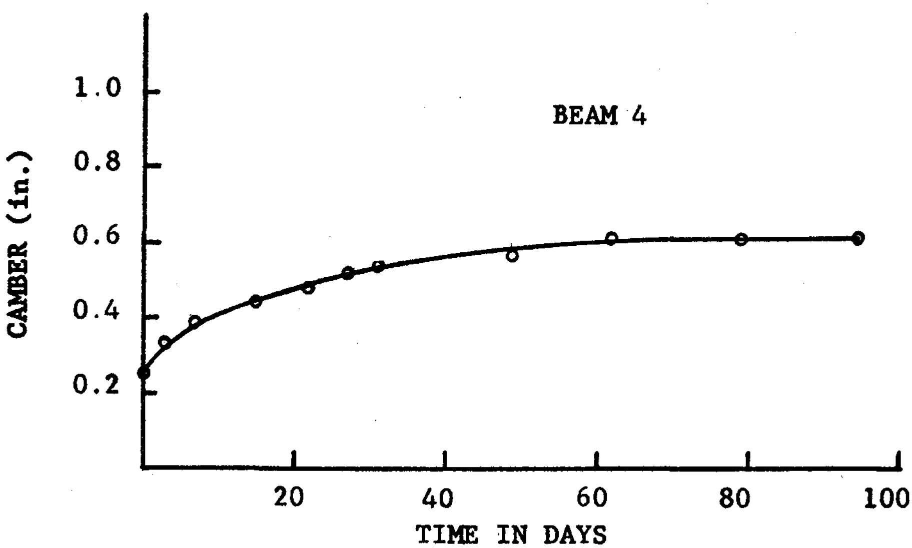


FIG. 4 - CAMBER CURVES, BEAMS 3 AND 4

Baldwin Lima Hamilton SR 4 Strain Gauges (type PA 3) were attached to each end of the post-tensioning bars. After the bars were in place these gauges were located about 12 in. inside the beams. The gauges were thoroughly waxed and covered with a wrapping of sheet rubber to protect them from abrasion and moisture. As the Stressteel system for post-tensioning requires that the ends of the bars be anchored in bearing plates, small grooves were cast in the ends of the beams leading from the sides to the Duoflex casings to provide access holes for the lead wires to the electrical strain gauges.

The strains in the bars were read directly from these gauges with a Baldwin Lima Hamilton SR 4 strain indicator. This instrument gave the strain in the bars directly in microinches. As the Modulus of Elasticity of the steel was known the load in each bar could be accurately determined. All electrical gauges were corrected for zero drift.

Ames dials, reading to 0.001 in., were used to measure the deflection at midspan of the beams. The dials were attached to brackets which were bolted to the beams. Steel plates were grouted to the floor for the dials to bear on. With this arrangement the vertical deflection was measured directly.

All concrete strain measurements were made with a Whittemore gauge. A 10 in. gauge length was used throughout the test. The gauge points were located at the ends and at midspan of both sides of each beam. In order to obtain a vertical strain distribution the gauges were placed at different levels as shown in Figure 1. The gauge points were placed at levels 1, 3 and 5 at all six locations on each beam. Levels 2 and 4 were instrumented at the near end and the centerline of the east side of each beam only. These gauges served as a check on the strain distribution and also provided additional concrete strain data. The shrinkage specimens were instrumented with Whittemore gauge points at all five levels at their midspan on both sides.

TEST PROCEDURE

The four beams were tested under the effect of the post-tensioning forces and their own dead load only. Periodic readings were taken to determine the change in the camber, post-tensioning forces, and the concrete strain. The specimens were tested in place and were not moved until the load-deflection tests were conducted at the end of the testing period.

The actual test period for a beam began at the time it was post-tensioned. Readings were taken on all gauges just prior to the pulling of the bars. The beam was then prestressed and the first set of data was taken. For recording purposes this was the zero day reading for that beam.

A definite creep effect was expected. As a typical creep curve shows a rapid rate of change during its early period, subsequent readings were taken at 3, 7, 14, 28, 60, and 90 days. In some cases readings were taken at more frequent intervals.

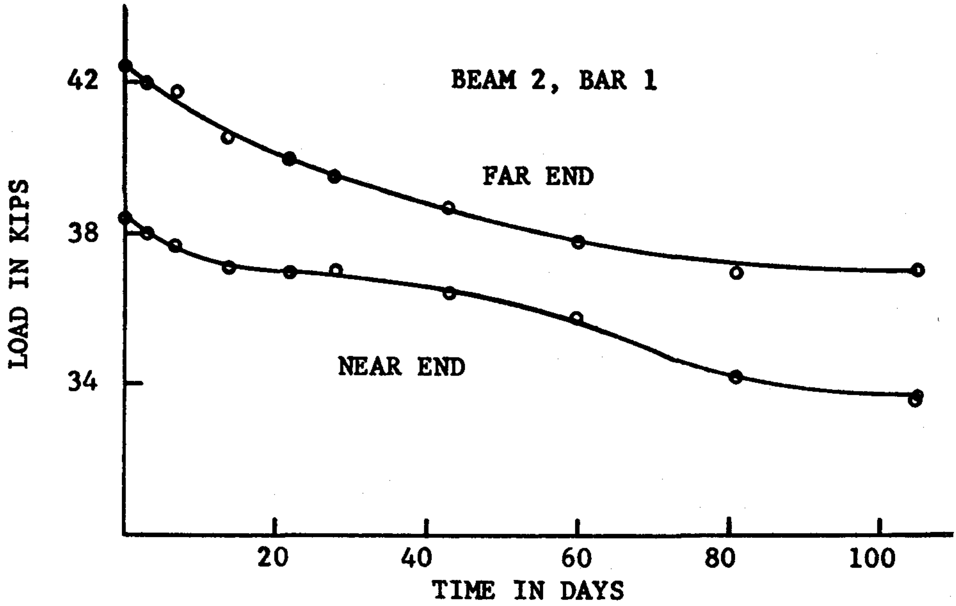
To eliminate, as much as possible, the effect of local temperature changes, all readings were taken during that time of the day when the ambient temperature in the laboratory was 72°F.

The beams were post-tensioned with a Stressteel Hydraulic Center Hole Tensioning Unit. The load on each bar was checked by three methods; measuring the total elongation of the bar, measuring the unit elongation of each bar with the electrical strain gauges, and reading directly from a calibrated indicator on the jack. All bars were overstressed a small amount then released to the proper load. All initial elongation readings were made with a load of 500 lbs. on the bars to insure that the anchor end was seated properly.

Beams 1 and 2 were post-tensioned 20 days after they were poured. The computed bottom fiber stress at the ends of the beams at this time was 2400 psi. Beams 3 and 4 were post-tensioned 12 days later. Their computed fiber stress was 1500 psi. For both pairs of beams, the method of test was identical. The only difference between the two pairs was the prestressing force applied.

TEST RESULTS

The properties of the concrete used in this test are given in Figure 2. During the entire testing period both the ultimate strength and the Modulus of Elasticity of the cylinders remained almost constant. Stress-strain curves were plotted for the cylinders. These curves were straight lines up to stress values well above those in the beams. The storage conditions apparently had little effect on the ultimate strength, but the cylinders stored with the beams had a slightly lower Modulus of Elasticity than



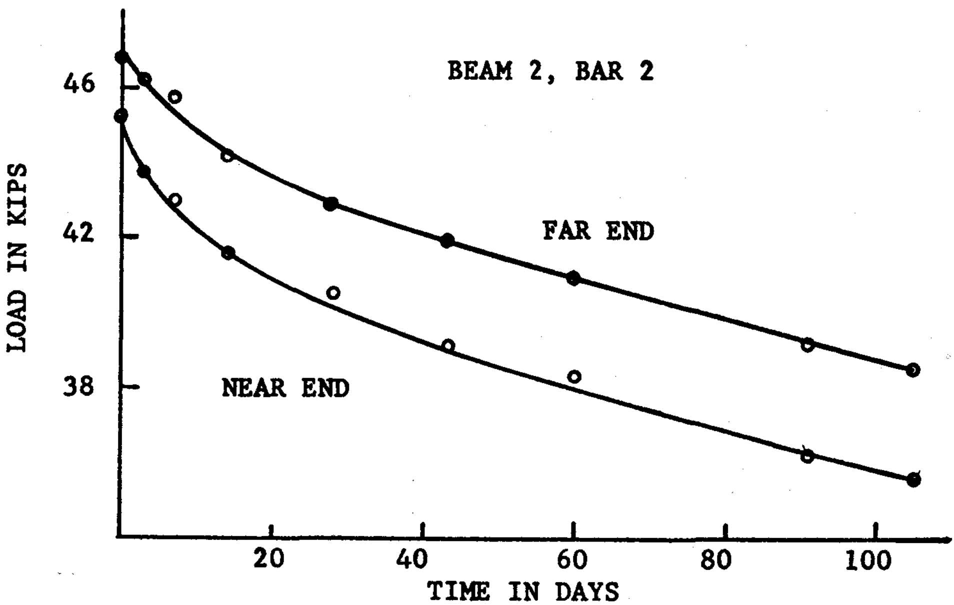


FIG. 5 - POST-TENSIONING FORCES, BEAM 2

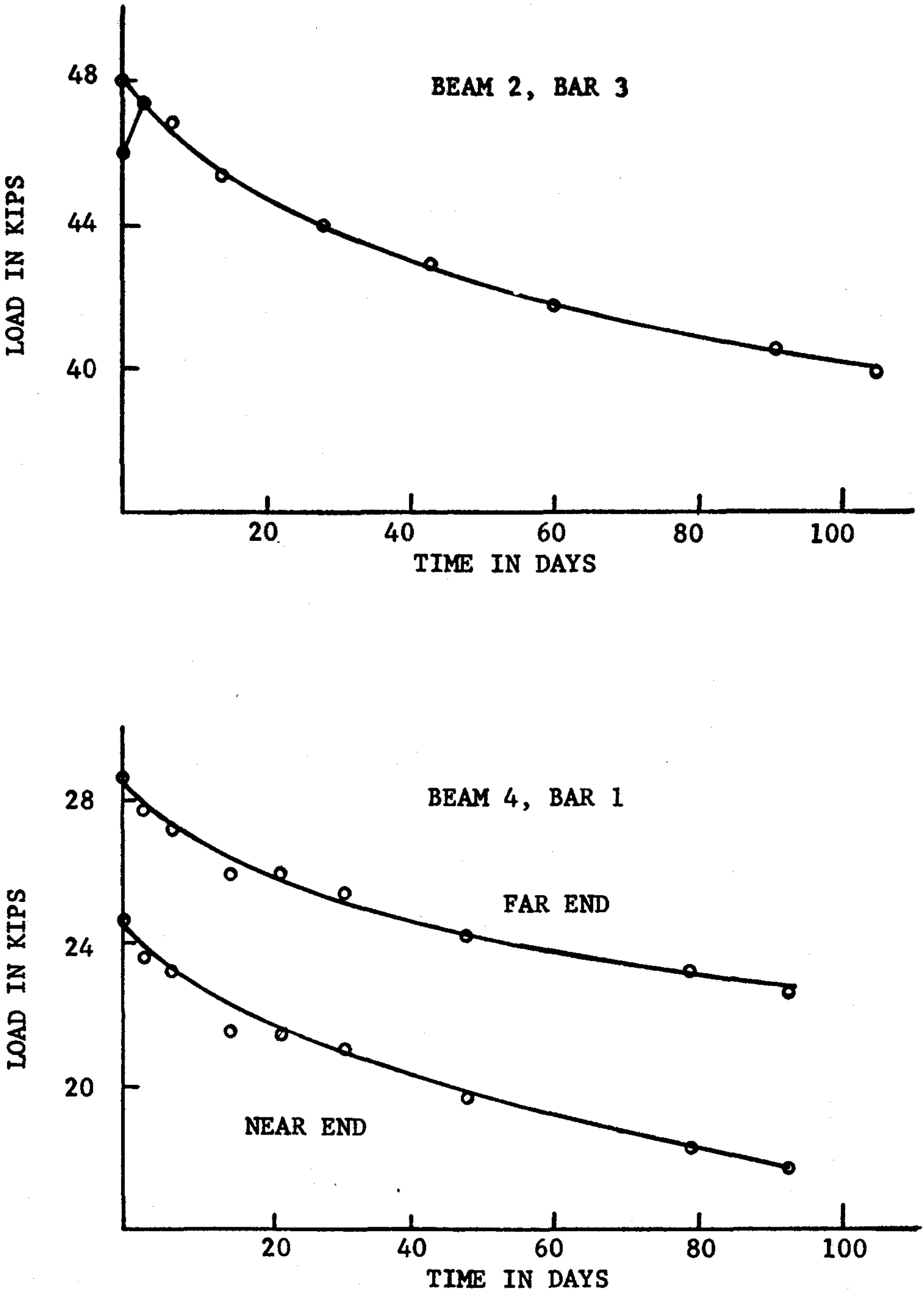


FIG. 6 - POST-TENSIONING FORCES, BEAMS 2 AND 4

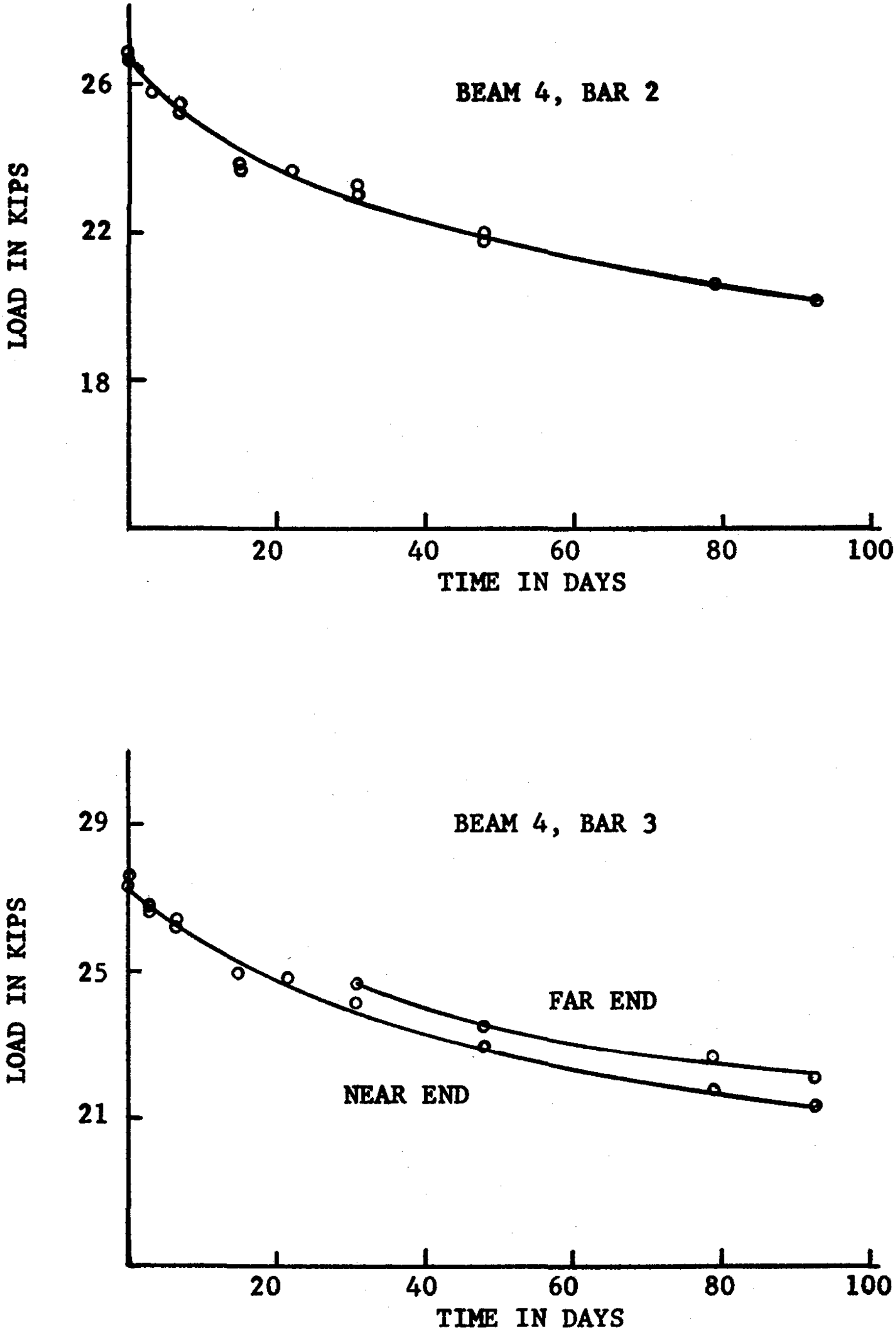
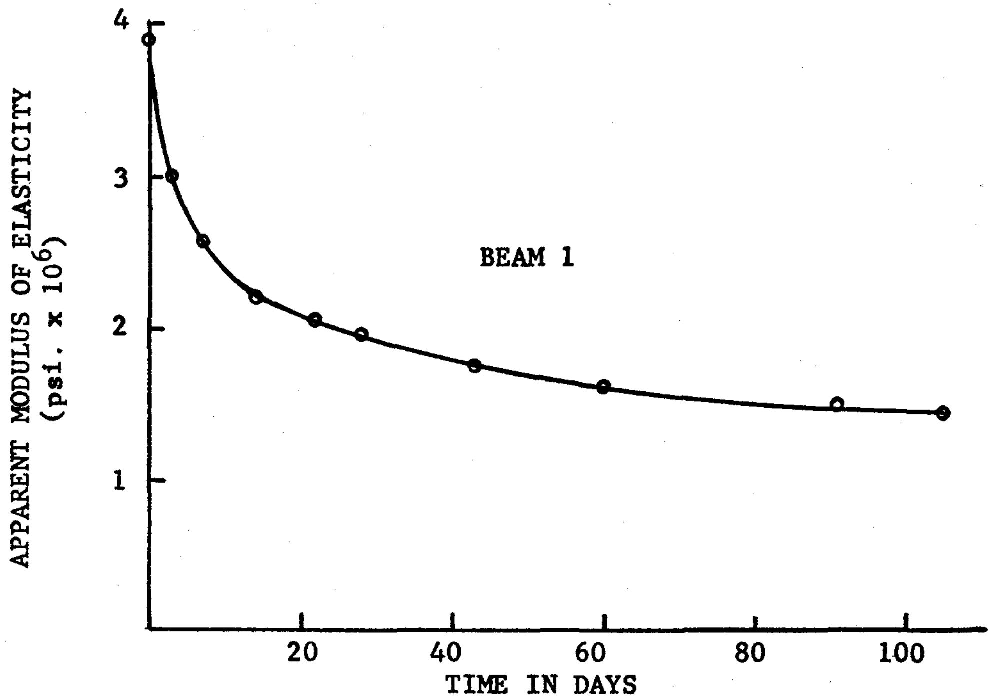


FIG. 7 - POST-TENSIONING FORCES, BEAM 4



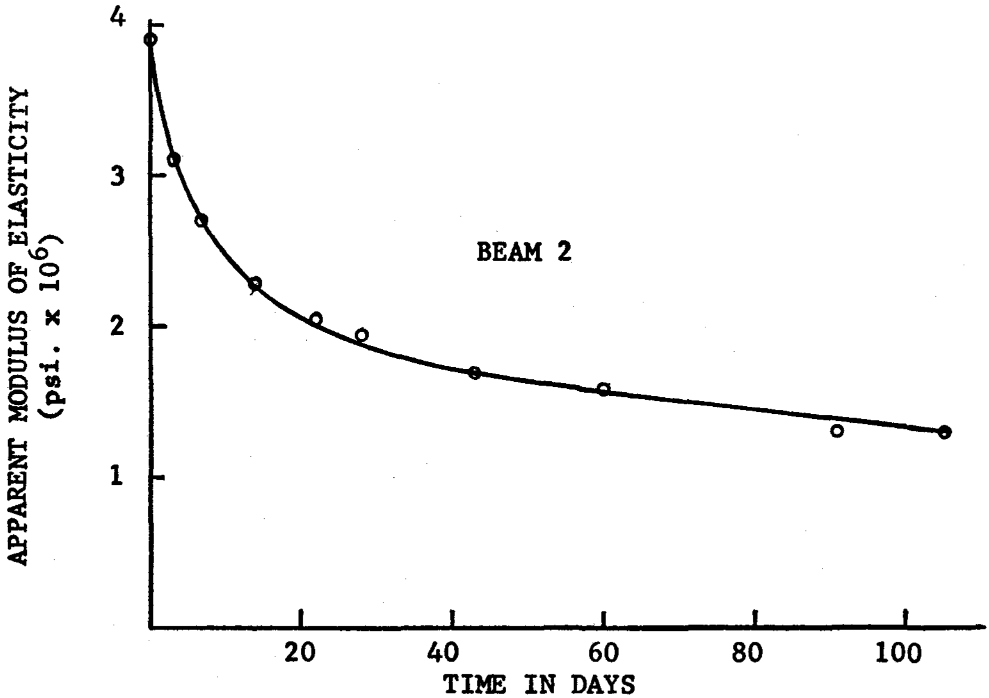
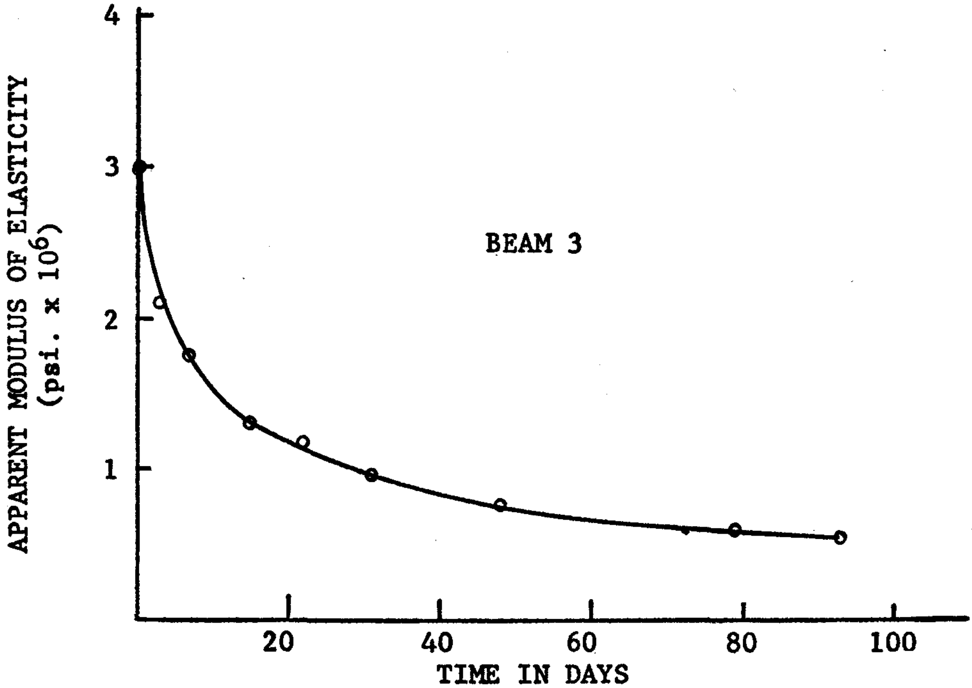


FIG. 8 - APPARENT MODULUS OF ELASTICITY, BEAMS 1 AND 2



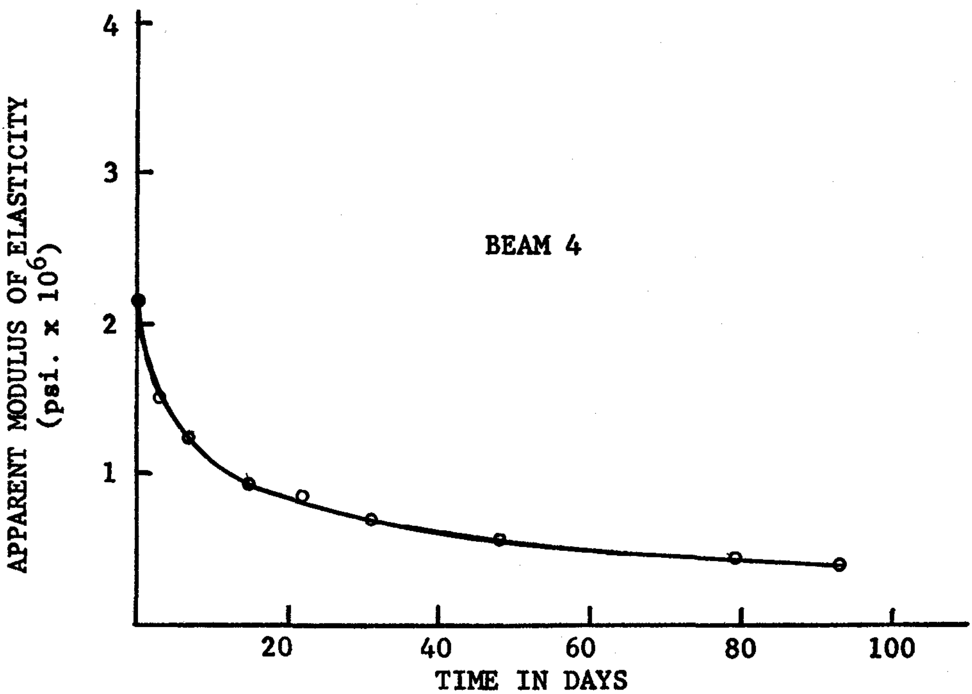


FIG. 9 - APPARENT MODULUS OF ELASTICITY, BEAMS 3 AND 4

those stored in the curing room. The cylinders that had been stored in the curing room were tested immediately after their removal from this storage area.

The camber curves for the four beams are given in Figures 3 and 4. The camber showed a large rate of increase during the early portion of the testing period, but by the end of the test it was increasing very slowly. The percentage increase in camber is of particular interest as it has a definite effect on the Apparent Modulus of Elasticity curves. Beams 1 and 2 had an initial mean camber of 0.375 in. and a camber at 90 days of 0.798 in. This was an increase of 108% of the original value. Beams 3 and 4 had a mean initial camber of 0.238 in and a final camber of 0.584 in. for an increase of 146%. Therefore, even though beams 1 and 2 gave the higher initial and final values, their percentage of increase was less than that of the beams stressed at the lower stress level.

Typical load versus time curves for a number of the post-tensioning bars are given in Figures 5 through 7. The bars for beam 2 all showed a definite friction loss at the time of prestressing. This loss was expected as the conduit allowed only ⅜ in. clearance around the bars. As the initial camber exceeded this value, it was expected that the bars would be in contact with the beam at the center. Bar 3 was apaprently able to overcome the frictional resistance sometime during the first three days by slipping, because for the remainder of the test the values of stress at both ends were identical. The initial camber for beams 3 and 4 did not exceed the ⅜ in. clearance around the bar, so no friction loss was expected. For some reason this was not true for bar 1 which showed a definite loss throughout the test. Bars 2 and 3 gave values from both ends which were very close, but after the camber had increased so that the bars were touching the conduit there was some variance. This was particularly noticeable in bar 3 after the 31-day reading.

As in the camber curves, there is a point of interest here that should be noted, that is, the percentage of stress loss in the beams. The mean prestressing loss for beams 1 and 2 at 90 days was 14.2%. Beams 3 and 4 had a loss of 23.9% over the same period. This shows again that the beams with the higher initial prestressing had a lesser percentage change than did the beams at the lower stress level.

The Apparent Modulus of Elasticity is computed by using the Moment-area method. The moment diagrams of the beams, the section properties, and the deflections are all known. The only unknown is the Modulus of Elasticity. As these values are to be used primarily for comparison, a number of simplifying assumptions were made. The eccentricities of the post-tensioning bars were considered constant and the effect of the bending upward of the bars was neglected. The equation derived using this method is:

$$E_a = 27.04\,(P_2 + P_3)/c - 5.21\,P_1/c - 0.788 \times 10^6/c$$

E_a = Apparent Modulus of Elasticity (psi)
P_n = Total stress in bar "n" (lbs.)
c = Camber (in.)

The curves determined by this equation are given in Figures 8 and 9.

It is apparent that this equation does not represent the true elastic properties of the beams. It does show the relationship between the camber as it changes with time, and the prestressing losses. It is in this relationship that the percentage change of the variables is so pronounced. The mean Apparent Modulus of Elasticity for beams 1 and 2 at the start of the test was found to be 3.90×10^6 psi. At 90 days the Apparent Modulus of Elasticity was 1.42×10^6 psi or 36.4% of the initial value. For beams 3 and 4 the same relative values of Modulus were 2.56×10^6 psi at zero days, 0.46×10^6 psi at 90 days, or 18% of the initial value. This shows the large difference in the change in Modulus under the two conditions. It was evident that the relative percentage prestress losses and increase in camber were cumulative in their effect on the Apparent Modulus of Elasticity.

A check on the validity of the simplified equation for the computation of the Apparent Modulus of Elasticity was made. The exact values of the Apparent Modulus, including the effects of the changing eccentricity and the bending upward of the post-tensioning bars, were calculated for all beams at zero and 90 days. Computations were also made on two of the beams at several intermediate points. The results of the exact solution and the simplified equa-

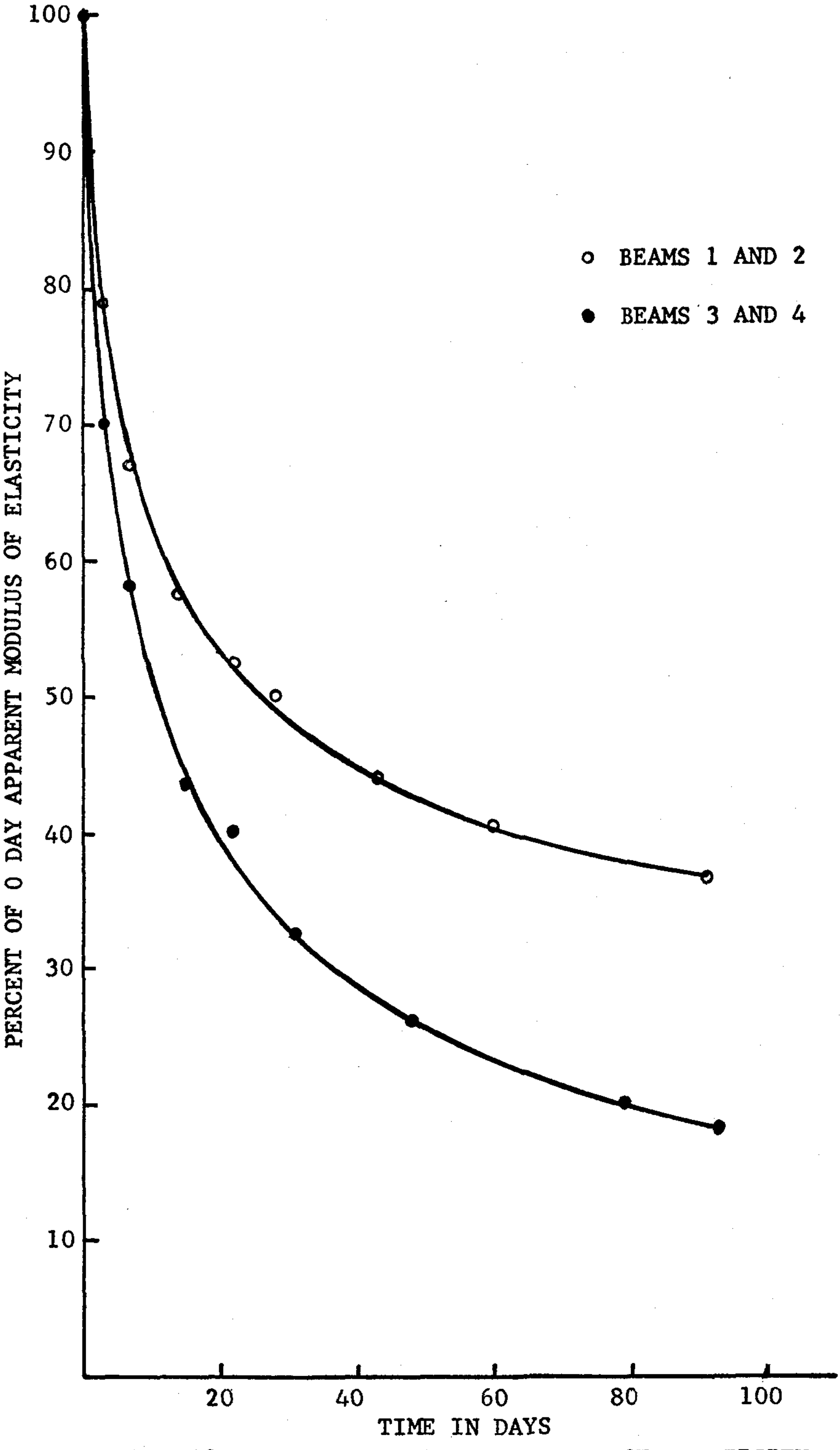


FIG. 10 - PERCENT APPARENT MODULUS OF ELASTICITY

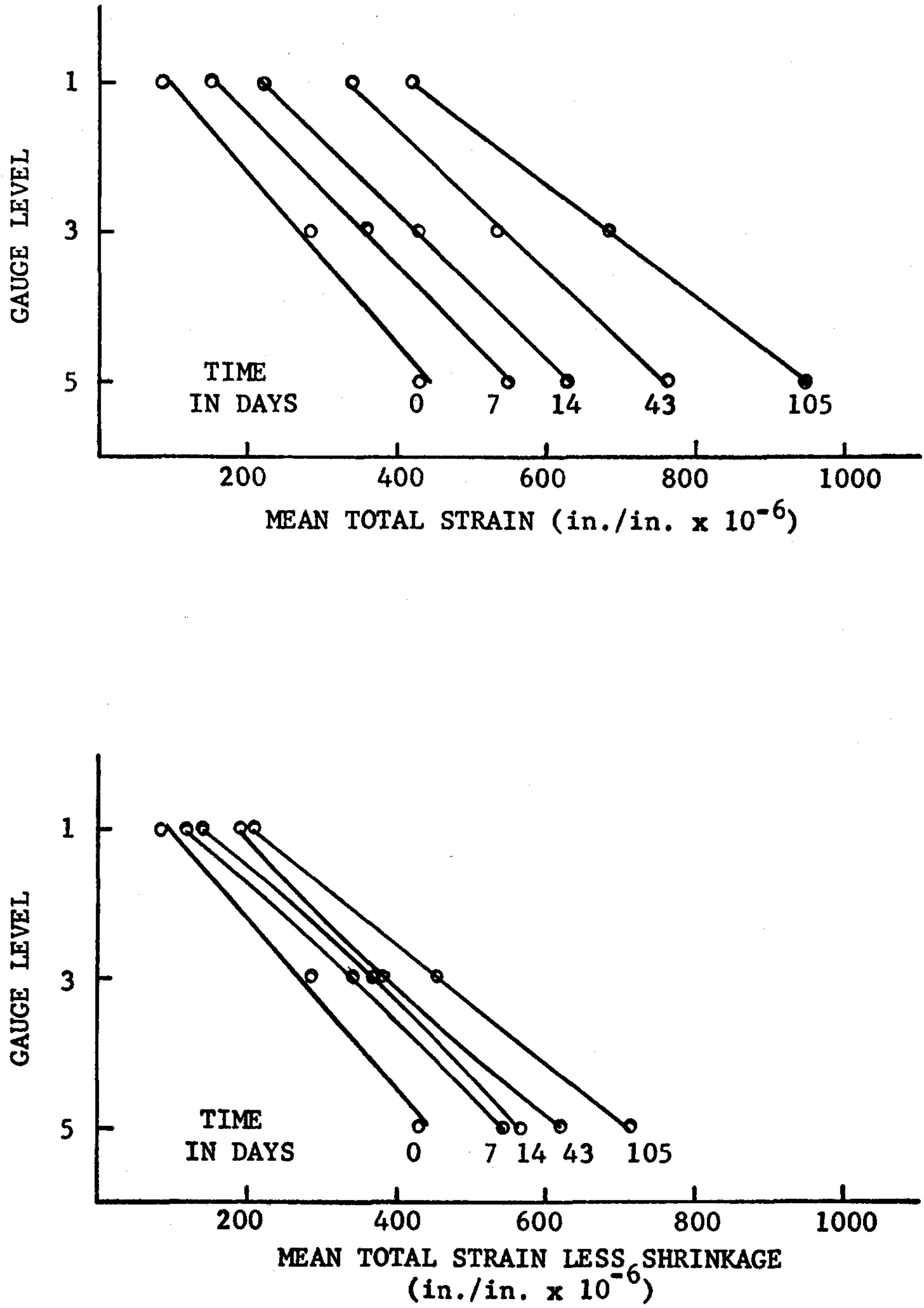


FIG. 11 - CONCRETE STRAIN DISTRIBUTION, BEAM 1, NEAR END

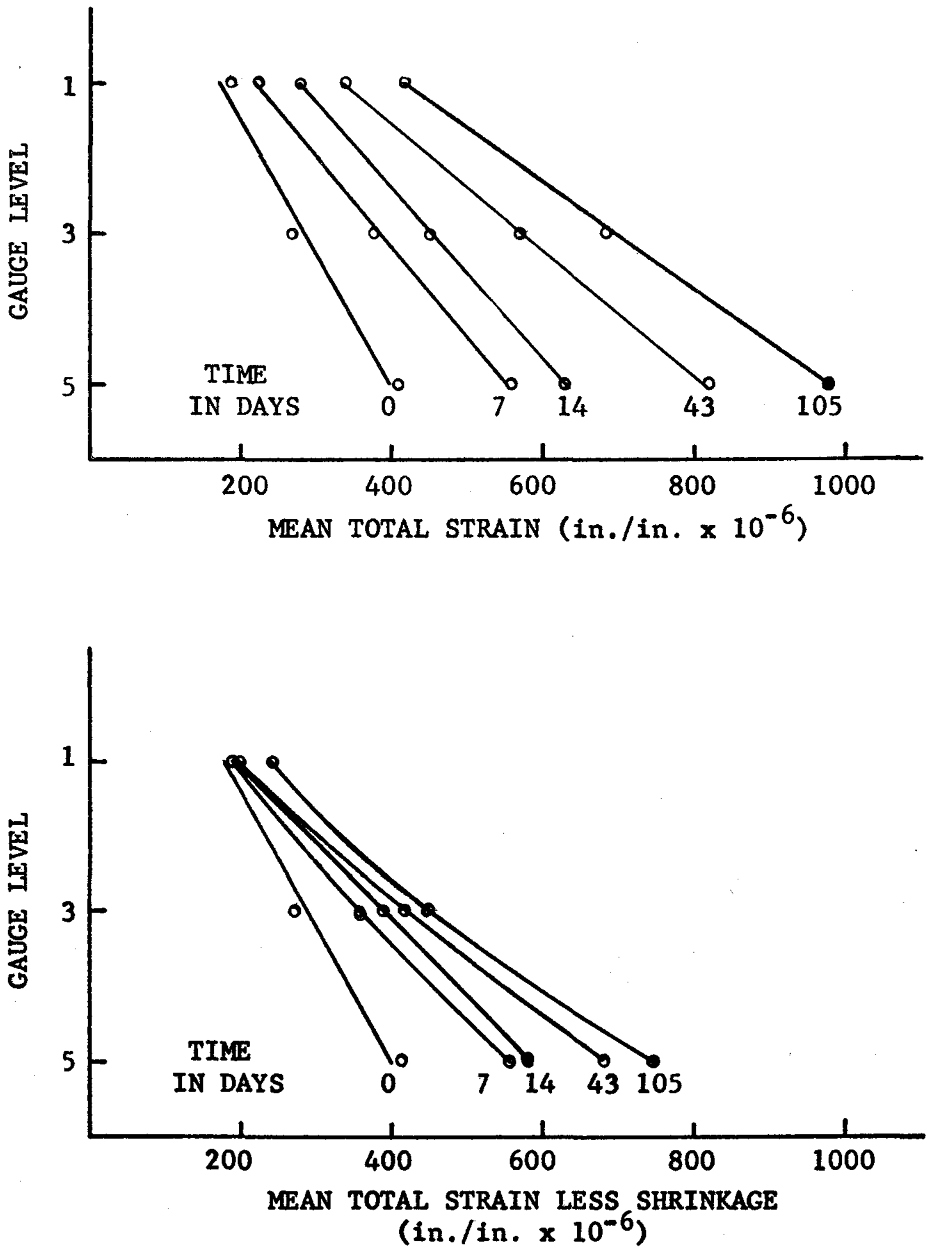


FIG 12 - CONCRETE STRAIN DISTRIBUTION, BEAM 1, CENTER LINE tion for zero and 90 days are given in Table 1.

The curves produced by the exact solution would assume the same shape as those given by the simplified equation. The ordinates at zero days would be equal to those given in Table 1 and the exact curve would rapidly approach, almost asymptotically, the one given by the simplified solution. For the purpose of this test it appears that the simplified solution is adequate for comparison.

TABLE 1
CORRECTED APPARENT MODULUS OF ELASTICITY

Beam Number	Time of Reading days	Apparent Modulus of Elasticity psi $\times 10^6$	Corrected Apparent Modulus of Elasticity psi $\times 10^6$
1.	0	3.92	4.83
1.	90	1.47	1.52
2.	0	3.89	4.72
2.	90	1.39	1.41
3.	0	2.97	3.57
3.	90	0.54	0.73
4.	0	2.15	2.69
4.	90	0.37	0.49

The exact Apparent Modulus at zero days is of value since it represents the true Elastic Modulus at that time as no creep or shrinkage had taken place to affect the readings.

For comparison, the values of the Apparent Modulus of Elasticity of each beam were divided by the Apparent Modulus of that beam at zero days. The reason for using this representation is that the initial Apparent Modulus very nearly represents the true elastic characteristics of the beam. These values are plotted, as percentages, in Figure 10. It was found that the values determined for beams 1 and 2 were almost identical, the 90 days results giving the Apparent Modulus at 36% of the original value. Beams 3 and 4 were also nearly identical but were only 18% of the original value at 90 days. These curves point out two distinct behaviors. Even though the beams did not act alike in all respects, the percentage values of the Apparent Moduli for the beams prestressed at approximately the same stress level were almost exactly the same. Also the beams at the higher stress level had a higher percentage of Apparent Modulus than did the beams stressed at the lower stress level.

At the conclusion of the testing period (the specimens were in excess of 100 days old) the beams were loaded statically and load-deflection relationship was obtained. From this data it was possible to compute the values of the beam Modulus of Elasticity which were 2.08×10^6, 1.55×10^6, 1.73×10^6, 1.20×10^6 psi for beams 1 through 4, respectively. These values, in all cases, were higher than the Apparent Moduli and there was no consistency between two pairs of beams or the initial values computed for the Apparent Moduli.

Typical curves from the data obtained from the concrete strain readings showing the Mean Total Strain and the Mean Total Strain Less Shrinkage are given in Figures 11 and 12. These curves show the strain as the abscissa and location of the gauge as the ordinate for Beam 1. The mean value refers to the average of both sides of the beam.

As expected the extension of the Mean Total Strain curves at the zero day readings show zero strain in the top of the beam at the ends. This indicates that the steel was located, as computed, at the lower kern point. This is not true in the center of the beams where there were dead load stresses due to the weight of the beam after prestressing. It can be seen that in nearly all cases the Mean Total Strain curves were actually straight lines. This is in accord with the known test data which shows that plain surfaces of beams remain plain after bending.

The curves for the Mean Total Strain Less Shrinkage do not give straight lines after the zero day reading. This is due primarily to the uneven rate of shrinkage at the various levels in the beam. The Mean Total Strain Less Shrinkage actually represents the elastic strain due to post-tensioning plus the creep. It is apparent from the relationship between these two curves that where the shrinkage was great, the creep was less than normal. This would necessarily be true if the curve of the Mean Total Strain at a section is to be a straight line. In other words, it appears that the

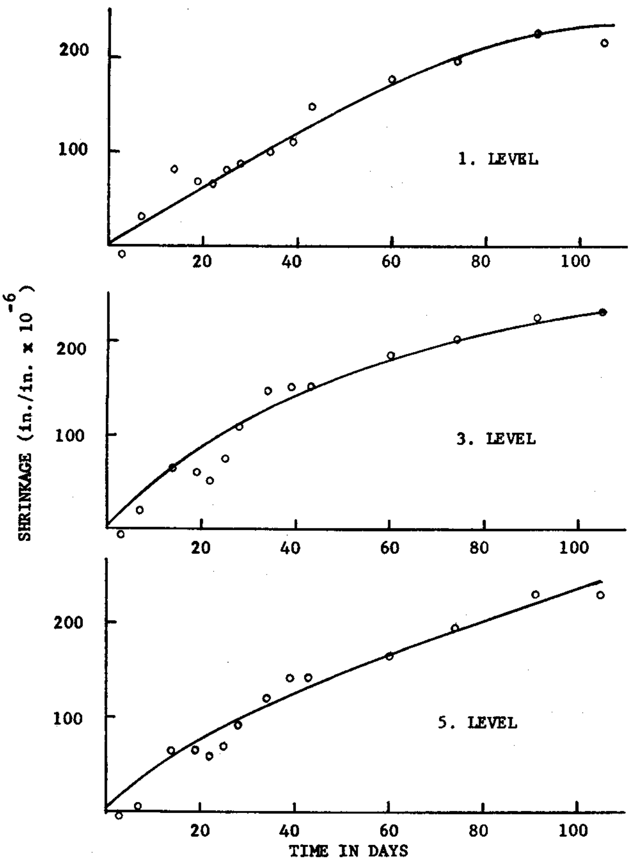


FIG. 13 - CONCRETE SHRINKAGE CURVES

shrinkage plus the creep at any given time can be expressed as the sum of two terms. These are the shrinkage at the neutral axis of the beam, plus some function of time multiplied by the distance from the neutral axis to any point on the section. The variable would have to represent the creep characteristics and the different rate of shrinkage of the section. The variable would be a constant for any given time, thus giving the equation of a straight line.

The shrinkage curves used in these calculations are given in Figure 13. It can be seen that at the end of the three months covered by this testing period, all of the curves had reached about the same level. The only major difference was the rate of shrinkage. In this case level 3 showed the fastest rate of shrinkage in the early portion of the test but leveled off sufficiently for the concrete at levels 1 and 5 to catch up. This difference in rate of shrinkage is responsible for the curving of the Mean Total Strain Less Shrinkage curves.

Creep curves were plotted for all locations. The curves are given in Figures 14 through 19. All of the curves appeared to follow a normal pattern with the creep at the 1 level being the least and the creep at the 5 level the greatest. Probably the most significant behavior indicated by this set of curves is the fact that there is no marked difference in the creep values determined at respective points for the two differently stressed pairs of beams. There was no marked difference in creep or the rate of creep at all corresponding locations. This suggests that additional information and testing is necessary to substantiate these results which

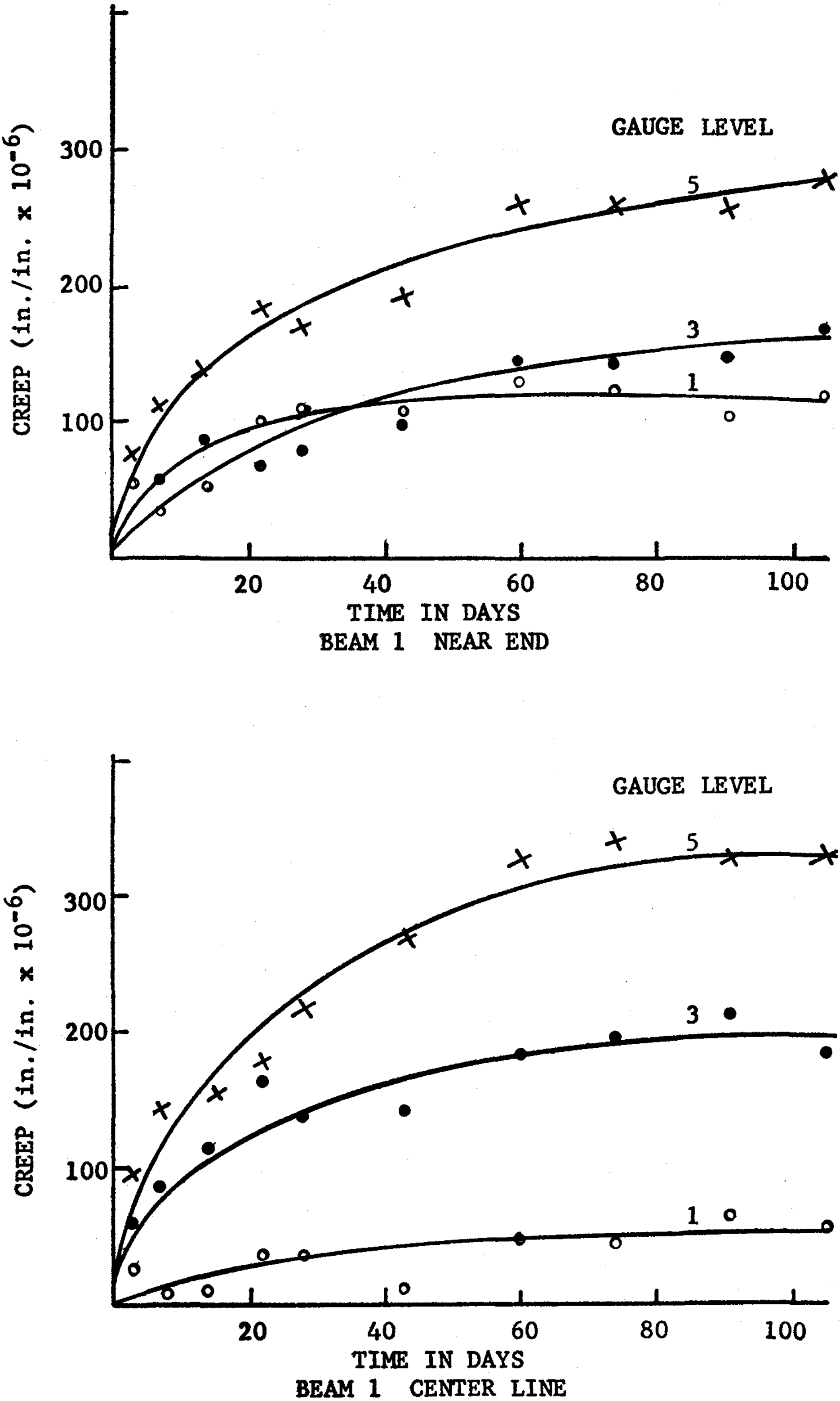
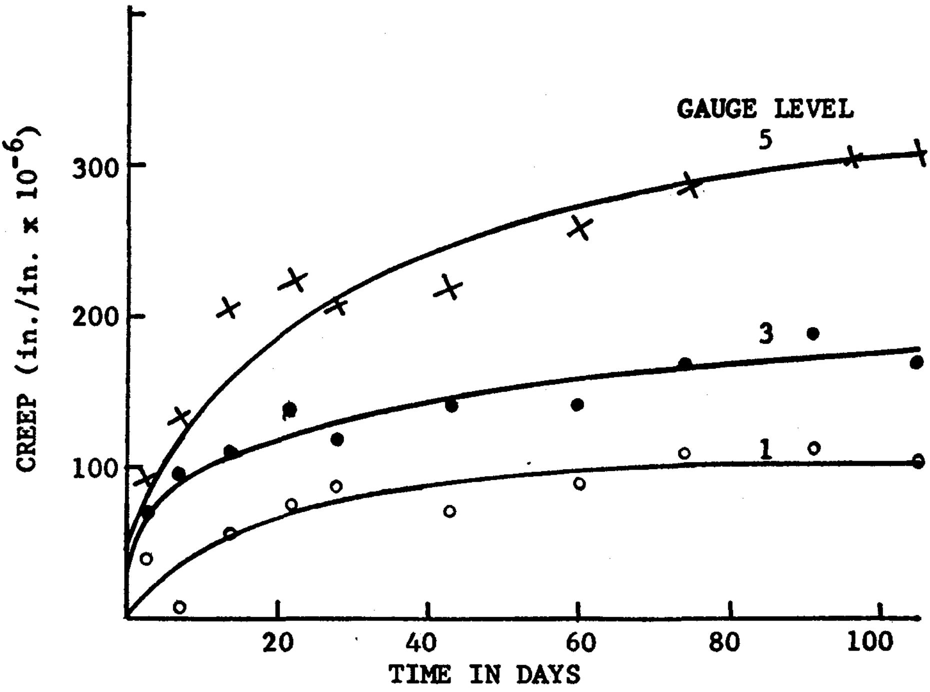
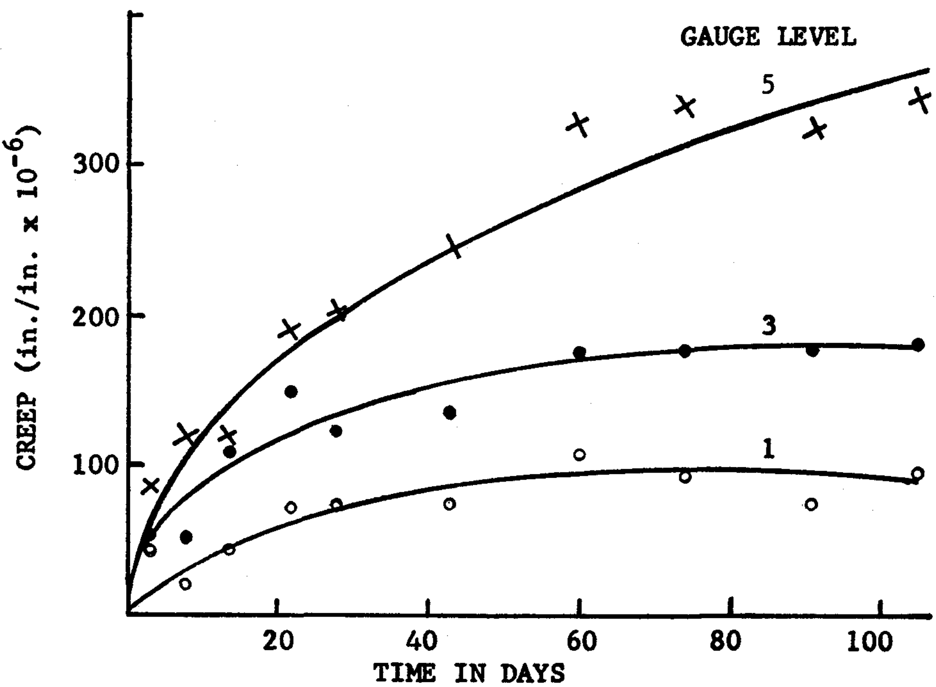


FIG. 14 - CONCRETE CREEP CURVES, BEAM 1

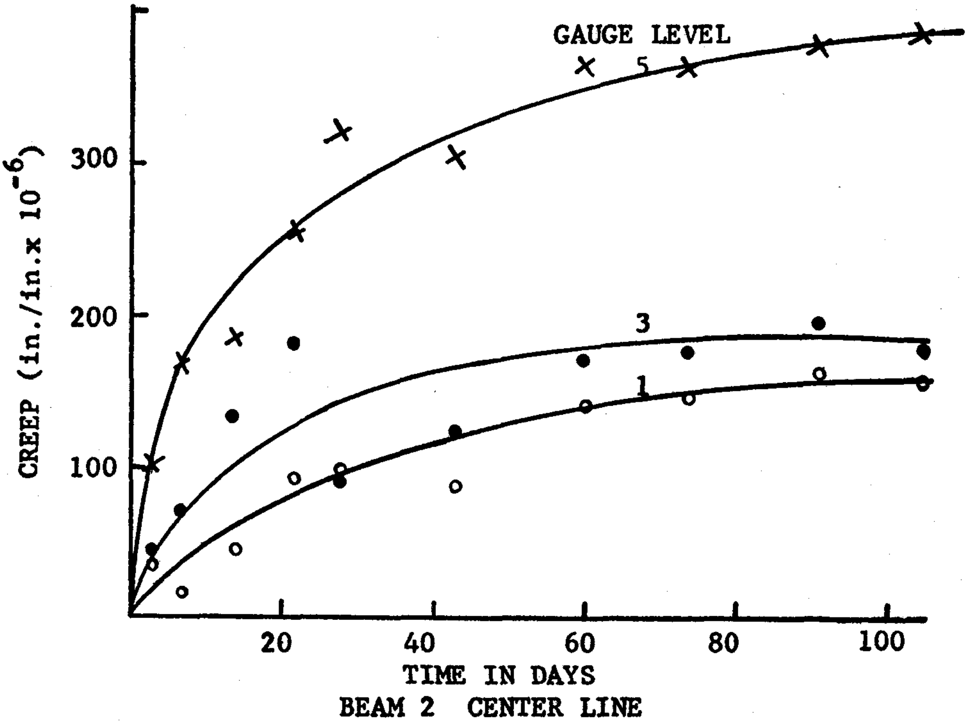


BEAM 1 FAR END



BEAM 2 NEAR END

FIG 15 - CONCRETE CREEP CURVES, BEAMS 1 AND 2



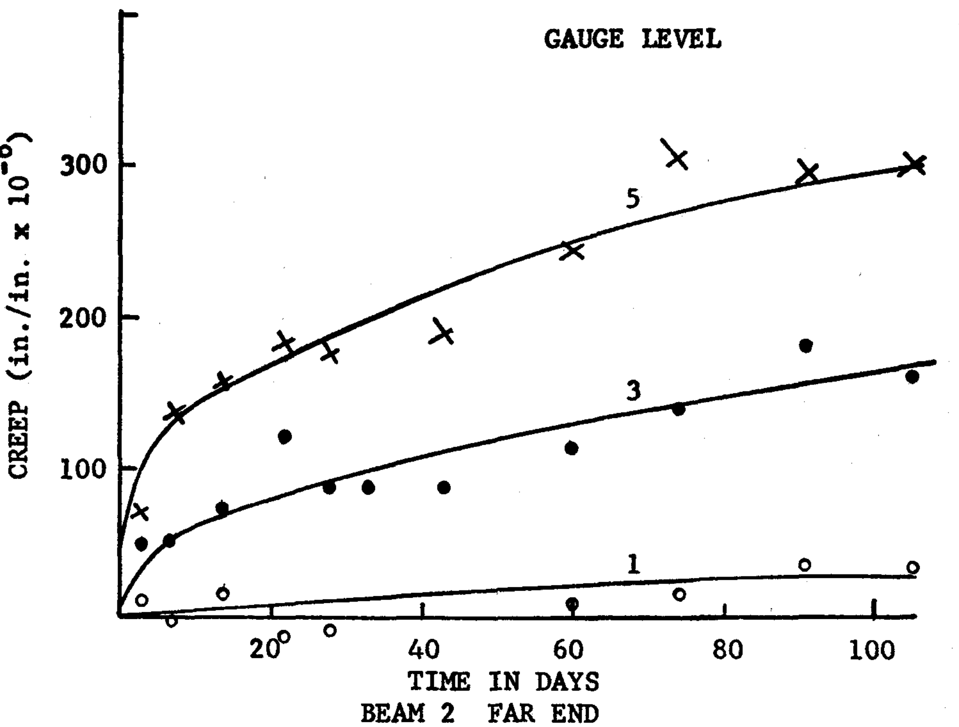
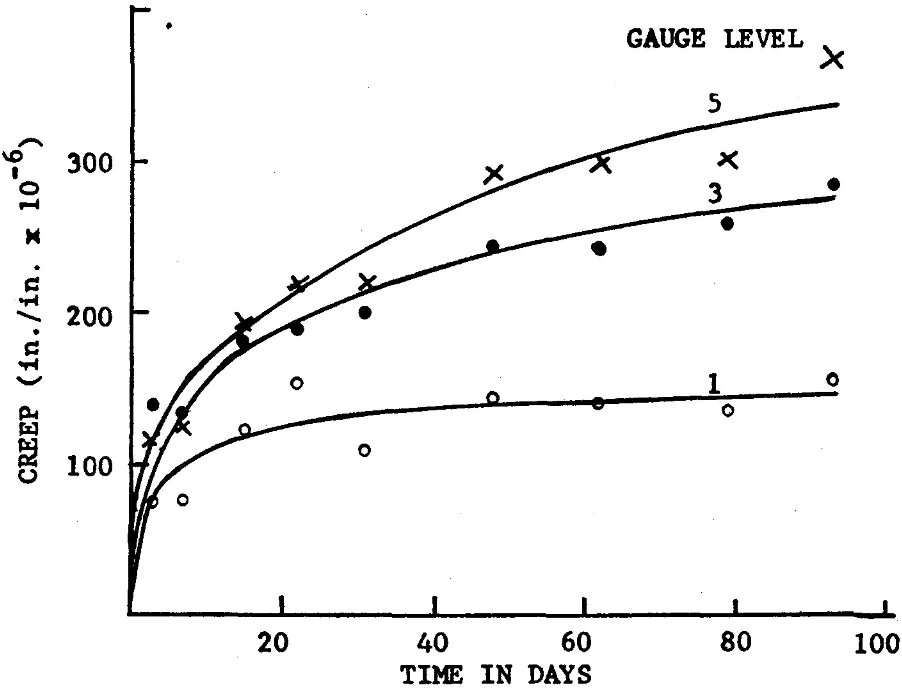
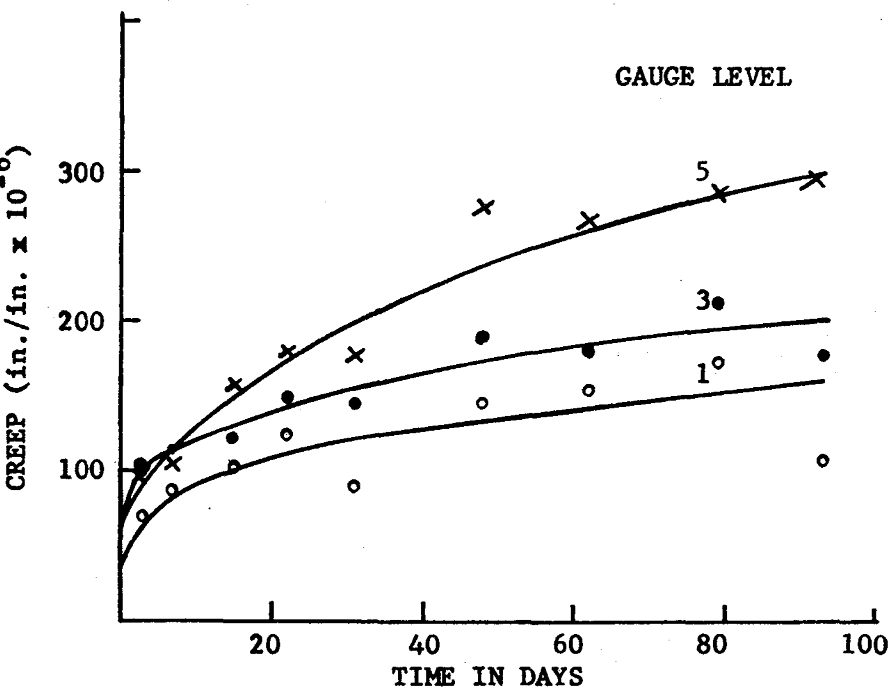


FIG. 16 - CONCRETE CREEP CURVES, BEAM 2



BEAM 3 NEAR END



BEAM 3 CENTER LINE

FIG. 17 - CONCRETE CREEP CURVES, BEAM 3

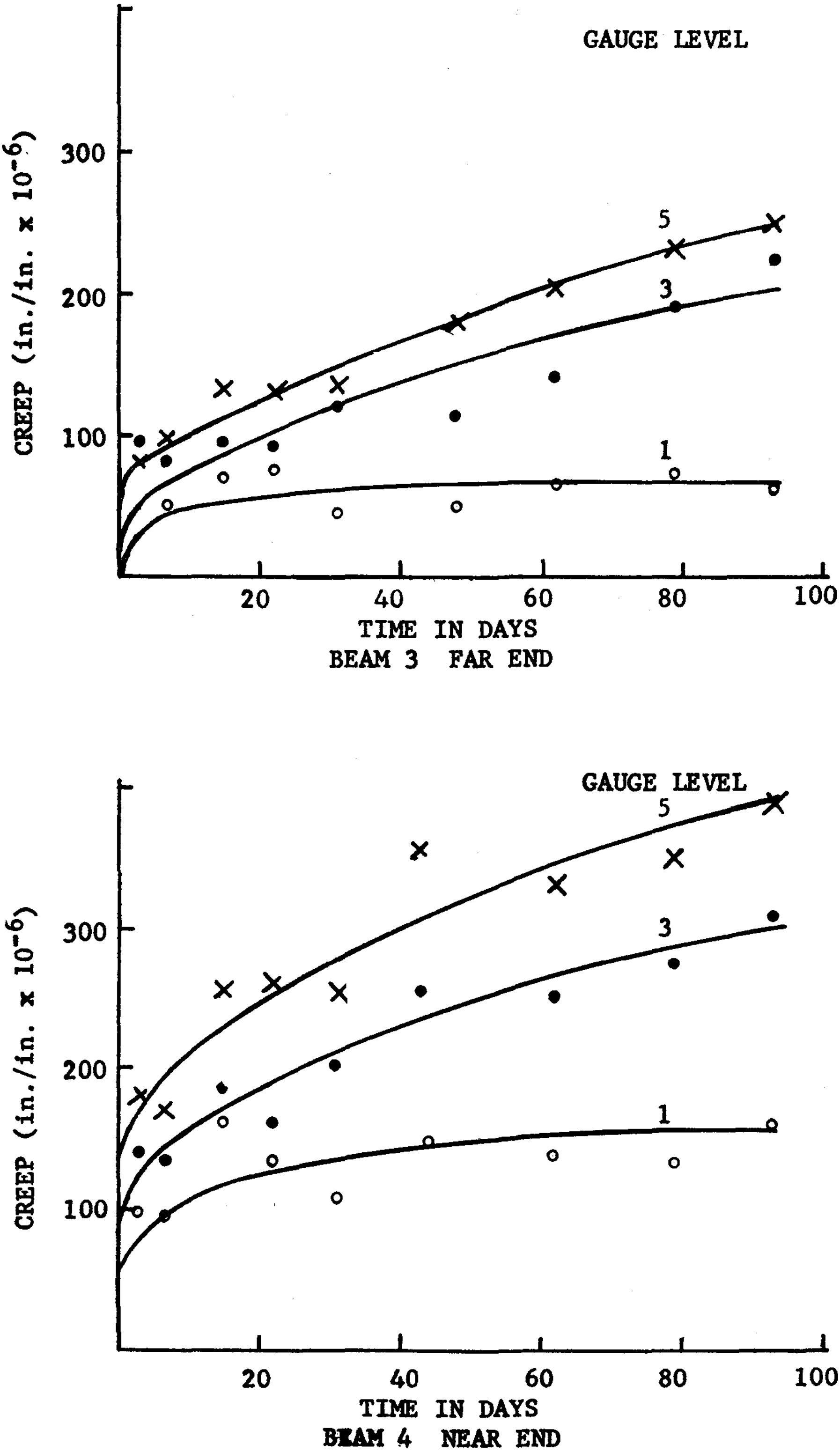


FIG. 18 - CONCRETE CREEP CURVES, BEAMS 3 AND 4

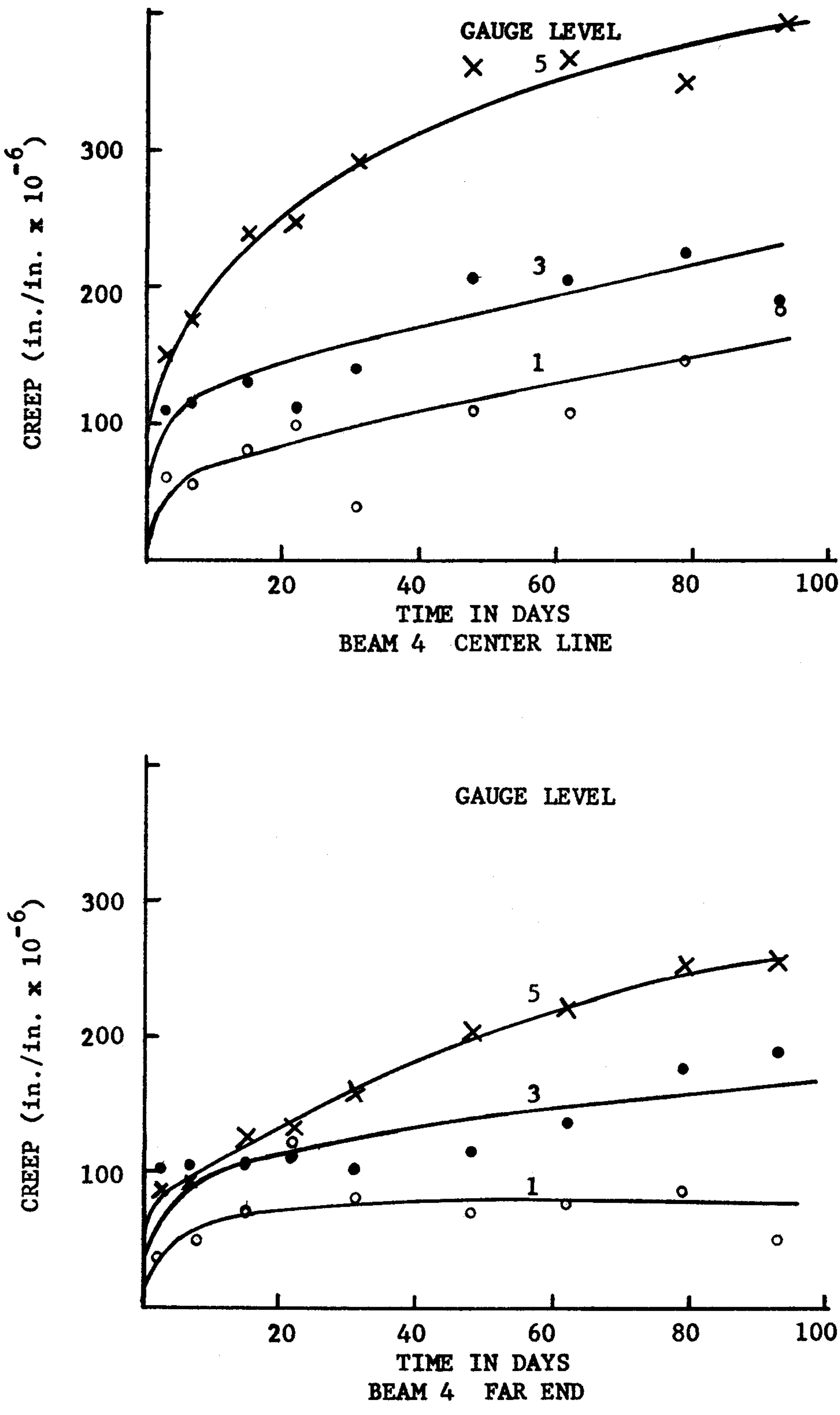


FIG. 19 - CONCRETE CREEP CURVES, BEAM 4

appear to be contrary to the accepted behavior.

The creep at level 5 varied from 250 to 400 microinches per inch for the beam ends. The elastic shortening of the beams at the same locations varied from 300 to 480 microinches per inch. Considering that the testing period covers only three months, the figures agree quite closely with the assumptions made for design. The ACI-ASCE Joint Committee 323 (5) assumes that the creep strain will vary from 100 to 300% of the elastic strain depending on the surrounding conditions.

The magnitude of shrinkage determined agrees closely with the design assumptions generally used. The value determined by test at 90 days was 225 microinches per inch. The values generally used in design vary from 200 to 300 microinches per inch depending on the surrounding conditions.

The suggested Modulus of Elasticity did not agree with the test data. The ACI-ASCE Joint Committee 323 recommends using;

$E = 1{,}800{,}000 + 500f'_c$.

For the concrete used in the test, this value becomes 5.2×10^6 psi. The Modulus of Elasticity in flexure, determined experimentally, varied from a high of 4.8×10^6 psi at zero days to a low of 1.2×10^6 psi at the end of the testing period. The Modulus suggested by Committee 323 is quite high compared to the experimental data.

CONCLUSIONS

From the test results of the particular specimens used in this program the following conclusions were reached:

1. The Apparent Modulus of Elasticity decreased with time. This change was definitely a function of the creep. In all cases, except for beams 1 and 2 at zero days, the Apparent Modulus of Elasticity was less than the cylinder Modulus. It was also less than the beam Modulus of Elasticity in flexure at the end of the testing period. The average corrected value for the Apparent Modulus of Elasticity for beams 1 and 2 at zero days was 4.78×10^6 psi. This agrees very closely with the cylinder Modulus of 4.70×10^6 psi determined at zero days.
2. The Apparent Modulus of Elasticity of the beams prestressed to 0.40 f'_{ci} was at all times higher than the value determined for the beams at 0.25 f'_{ci}. Expressed as a percentage, beams 3 and 4 had an Apparent Modulus of Elasticity at zero days equal to 66% of the value determined for beams 1 and 2. At 90 days this ratio had dropped to 30%.
3. There does not appear to be a definite relationship between the Apparent Modulus of Elasticity and the cylinder properties.
4. The creep strain plus the shrinkage was linear across the cross section.
5. The creep in the concrete at different locations of each beam appeared to be a function of both the stress magnitude and the relative shrinkage.
6. Paradoxically there was no marked difference in the creep or rate of creep readings taken at corresponding locations in beams prestressed to different stress levels (maximums of 0.40 f'_{ci} and 0.25 f'_{ci}). This was evidenced by the camber measurements of all beams which showed approximately the same amount of increase of 0.40 in. in camber during the 90-day test period.

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