DIRECT CALCULATION OF DEFLECTIONS AND STRESSES IN PRECAST PRESTRESSED CONCRETE BEAMS DURING LIFTING

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ABSTRACT

This paper introduces a calculation-based method for determining deflections and stresses in a precast concrete beam during lifting. The method is implemented with a freely available spreadsheet tool that outputs beam roll angle, internal forces and moments, weak-axis and strong-axis deflections, and the cross-sectional angle of twist. A field study of 128 precast concrete beams provides typical beam sweep and lift support eccentricity magnitudes that are inputs to the spreadsheet. An example demonstrates how to calculate tensile and compressive stresses in a PCI-BT-77 precast concrete beam and check these demand stresses against allowable limits.

Keywords: Beam lifting, lateral stability, sweep, lifting support eccentricity

INTRODUCTION

Precast prestressed concrete beam spans can exceed 200 ft because of improvements in material properties, the introduction of new girder shapes, larger prestressing strands, and design method advancements; however these long spans are accompanied by shipping, handling, and erection concerns.[1] States are reaching these long spans with optimized section shapes, for example, the California Wide-Flange Girder[2] and the Nebraska University (NU) I-girder.[3] The record length for a precast concrete plant-cast girder is currently held by a 213 ft (65 m) long, l9 ft 3 in. (2.8 m) deep spliced girder used for the Bow River Bridge near Calgary, Alberta, Canada.

A consequence of increased beam spans is an increase in depth, and therefore to reduce beam weight, web and flange width are narrowed. As a result, long-span beams tend to have lower minor-axis and torsional stiffness compared to typical precast beams. This makes them susceptible to minor-axis bending and twist when a beam is curved in plan (sweep) from eccentric prestressing or thermal gradients, which causes rolling during lifting and increases the likelihood of catastrophic instability[4-6].

Existing codes and publications on the subject of lifting stability of precast prestressed concrete beams do not offer explicit and easy-to-use formulas for calculating displacements, forces, and moments during lifting that could readily be utilized in practice. The goal of this paper is to provide the precast community with an accurate, accessible method for predicting behavior during lifting. Deflection, rotation, twist, and internal forces and moments are calculated with a freely available Microsoft Excel calculation sheet, *VT Lifting Analysis*[7]. An example problem is provided at the end of the paper where internal beam forces are used to obtain demand stresses during lifting that are checked against the same allowable stress limits in tension and compression using in flexural design.

CURRENT PRACTICE

The *PCI Design Handbook* (6th Edition)[8] emphasizes the importance of a lateral stability check in Section 5.4.1. The *PCI Bridge Design Manual* (2003) [9] addresses the lateral stability of slender members in Chapter 8.10, outlining a procedure for calculating a factor of safety against cracking for a hanging beam. The PCI calculation method is based a specific example and beam cross-section[10], however it is not generally applicable to other girder cross-sections and lifting conditions, especially the calculation of the maximum permissible tilt angle, θ_{max}. The *AASHTO LRFD Bridge Design Specifications* (2007)[11] and the *AASHTO LRFD Bridge Construction Specifications* (2004)[12] do not provide specific guidelines for investigating lateral stability of beams when hanging. Section 5.14.1.2.1 of the *AASHTO LRFD Bridge Design Specifications* (2007) assigns the responsibility for safe shipping and erection to the contractor. Additionally, Section 5.14.3.3 underscores the need for considering the possibility of buckling in tall, thin web sections. It is clear that an accurate and accessible calculation-based approach could improve erection safety for the precast concrete industry.

LIFTING ANALYSIS CALCULATION

A new method for investigating the behavior of beams during lifting was recently developed to calculate roll angle, twist, displacements, internal forces, internal moments, and stresses in a doubly symmetric curved beam during lifting by two cables. The formulas derived by Plaut and Moen (2012) [13] for a circularly curved beam can readily be employed in practice, offering engineers a means of determining the resulting stresses that will occur in beams during lifting, and using the results to prevent damage and failure.

For convenience, the equations derived in Reference 13 are organized in a user-friendly calculation spreadsheet, *VT Lifting Analysis*[7], which is freely available to the precast community in both U.S. and metric units. The calculation sheet can accommodate beams with vertical and inclined cables, with an initial curvature due to sweep, and with eccentric lifting supports as shown in Fig. 1.

The spreadsheet requires the following inputs:

- Material properties: modulus of elasticity E_c, specific gravity *SG* (typically 2.4 for reinforced concrete), and modulus of rigidity G.
- Beam properties and dimensions: beam length L, cross-sectional area A, strong-axis and weak-axis moments of inertia I_z and I_y, torsion constant J, and self-weight w. (A method for computing J for typical prestressed concrete girders is presented in Reference 14, or it can be calculated using cross-section analysis computer programs).
- Lifting device information: location a of lifting device from the ends of the beam, height H of yoke to cable attachment points above the shear center of the beam, global eccentricity e_s of lift supports, and the inclination angle ψ of the cables.
- Initial normalized sweep imperfection δ/L.

Based on the received input, the calculation sheet computes the following values at any location along the length of the beam:

- Roll angle
- Twist angle
- Internal forces (weak-axis shear, strong-axis shear, and longitudinal axial force)
- Internal moments (twisting moment, weak-axis bending moment, and strong-axis bending moment)
- Deflections (weak-axis and strong-axis)

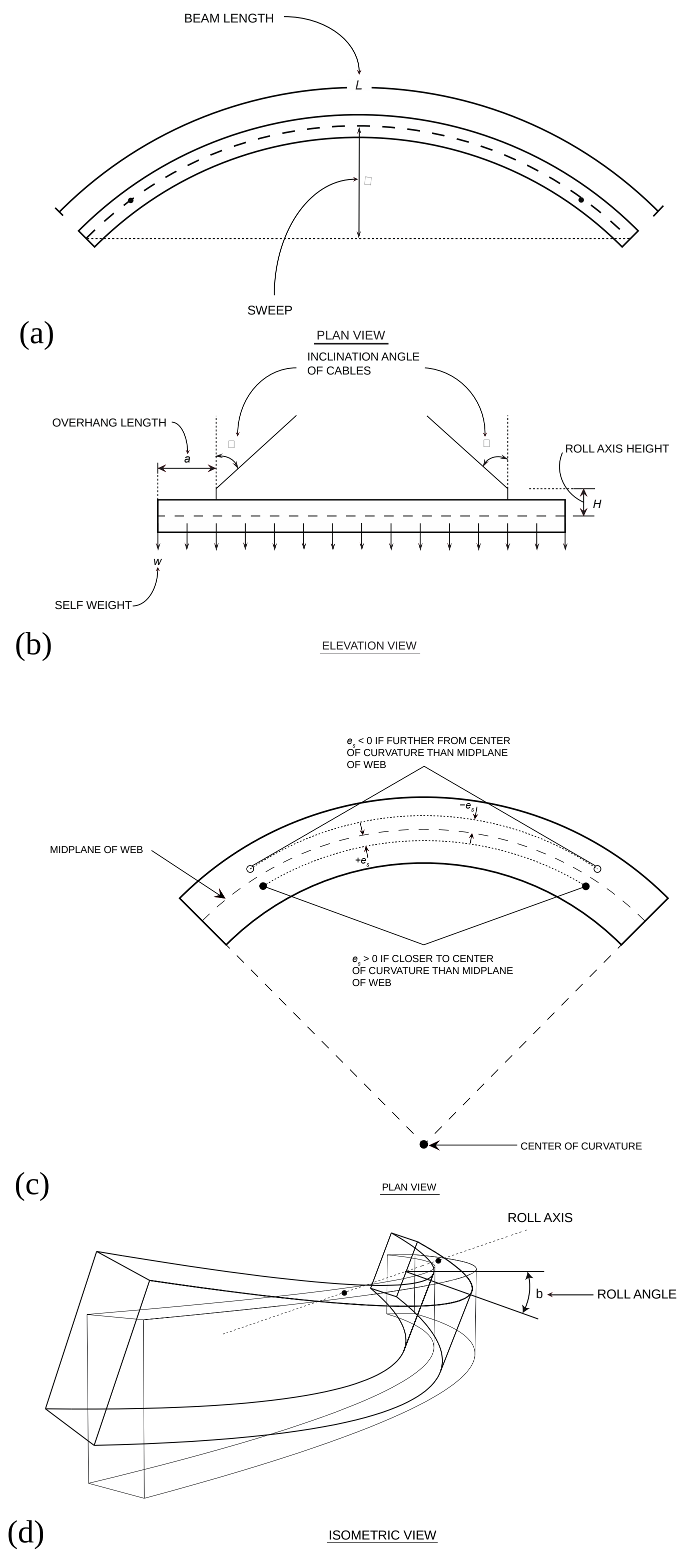

Fig. 1 Beam geometry definitions - (a) sweep imperfection; (b) lifting location and angle; (c) lifting eccentricity; (d) beam roll angle

LIFTING STRESS CHECK PROCEDURE

During lifting, a beam curved in plan experiences major-axis bending, minor-axis bending, and compression from prestressing and inclined lifting cables, all of which when considered together, can cause high tensile and compressive stresses. These stress magnitudes can be calculated with the following procedure.

Step 1: Input material properties (modulus of elasticity E_c, specific gravity SG, and modulus of rigidity G), beam properties (beam length L, cross-sectional area A, strong-axis and weak-axis moments of inertia I_z and I_y, torsion constant J, and self-weight w), lifting device information (location a of lifting device from the ends of the beam, height H of yoke to cable attachment points above the shear center, global eccentricity e_s of lift supports, and the inclination angle ψ of the cables), and initial sweep imperfection (δ/L) in the *VT Lifting Analysis* calculation sheet. Collect axial force and weak-axis and strong-axis bending moments acting on the cross-section at critical locations along the length of the beam (midspan, harp points, and lift points).

Step 2: The maximum tensile and compressive stresses during lifting can be approximated by calculating self weight stresses (using internal forces from *VT Lifting Analysis*) and prestressing effects separately with mechanics of materials, i.e., $\sigma = Mc/I$, $\sigma = P/A$, and then summing the stresses together. Alternatively, in a cross-section analysis program (e.g., XTRACT[15]), apply the axial force and weak-axis and strong-axis bending moments on the beam's cross-section. Add the effect of the prestressing. Record the resulting maximum tensile and compressive stresses acting on the cross-section.

Step 3: Check the resulting lifting stresses at the two critical locations of the rotated cross-section: the corner of the downward top flange in tension and the corner of the upward bottom flange in compression (Fig. 2). Compare these values with maximum allowable stresses per code specifications to ensure that cracking in tension or overstressing in compression do not occur. Perform this check at critical locations along the length of the beam: midspan, harp points, and lifting points.

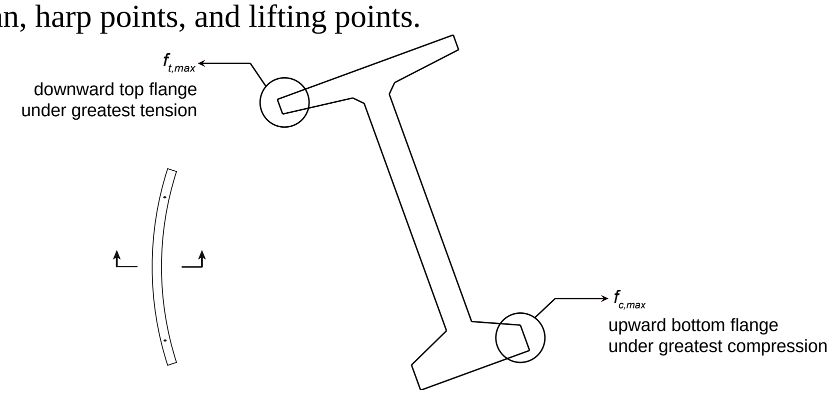

Fig. 2 Maximum tensile and compressive stress locations during beam lifting

This procedure for evaluating precast prestressed concrete beams during lifting is demonstrated in the following section.

EXAMPLE PROBLEM – 77 in. PCI BULB TEE

The first example is a PCI-BT-77 beam that was cast in 2011 for the North Carolina Department of Transportation. The beam has L = 139 ft, A = 970.7 in.^{2}, strong-axis moment of inertia I_z = 789,500 in.^{4}, weak-axis moment of inertia I_y = 63,600 in.^{4}, torsion constant J = 34,560 in.^{4}, and self-weight w = 0.084 kip/in. The lift point location is a = 90 in. at each end. The beam is assumed to be lifted by inclined cables (ψ = 45°) which will cause axial compressive force during lifting, and the roll axis height is H = 39 in. above the shear center. The lifting loops are located on the vertical centerline of the beam, i.e., the lifting supports have zero eccentricity with respect to the midplane of the web: e_s = 0. The specified 28-day strength of the concrete is f'_c = 8,000 psi, and the release strength is f'_{ci} = 6,500 psi. The unit weight of the concrete is 150 pcf. The beam is prestressed using 56 - Grade 270 low-relaxation prestressing strands with a 0.60 in. diameter. The strands are harped at 5 ft from midspan. The initial jacking force is 43.90 kips per strand. Strands are released one day after casting. Assume 7 percent losses at the time of strand release (equivalent stress in the strands after release is $0.7f_{pu}$). The beam has six draped strands, and the harp points are located 5 ft from midspan in both directions. Detailed drawings for the beam dimensions and the locations of the prestressing strands at the three critical locations: midspan, harp points, and lift points are provided in Fig. 3.

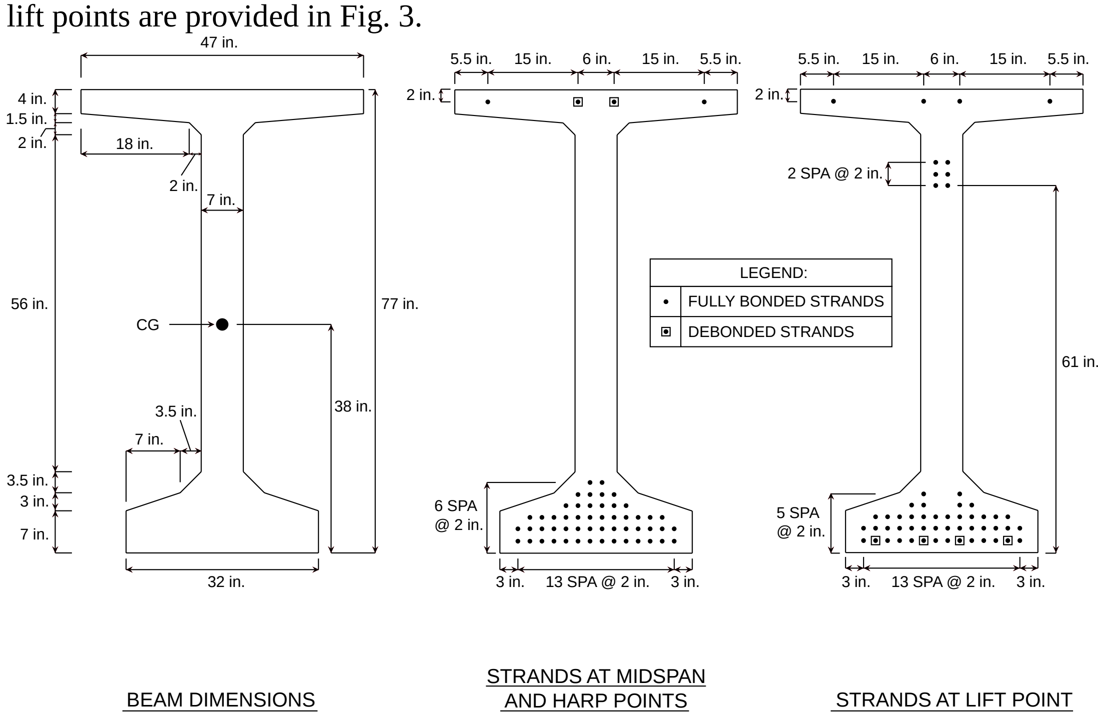

Fig. 3 PCI Bulb Tee details, note: 1 in. = 25.4 mm

Required:

Find the maximum stresses acting on the beam during lifting and compare with the maximum allowable stresses per code specifications.

Step 1:
Using the *VT Lifting Analysis* calculation sheet and the beam information given above, the axial force and the weak-axis and strong-axis moments acting on the cross-section due to lifting are determined. Three different sweep magnitudes are investigated: the PCI limit of $L/960$, the 99th percentile sweep imperfection according to the probability density function in Reference 16, and the sweep magnitude at which cracking first occurs for this particular beam ($L/320$).

The compressive axial force in the beam due to the prestress is 2,205 kip at midspan and harp points, and 2,123 kip at the lift points. The moment due to the prestress is -66,027 kip-in. at midspan and harp points, and -44,168 kip-in. at the lift points. The resultant compressive axial force due to the inclined cables is 70 kip, which is the same at midspan, harp points, and lift points. The calculated weak-axis and strong-axis moments due to lifting for all three sweep magnitudes are recorded in Table 1. Results are presented at midspan, harp points, and lift points. Additionally, the roll angles of the beam for the three imperfection magnitudes are calculated.

Step 2:
The axial compressive stresses and weak and strong axis bending stresses are calculated and summed together. The resulting maximum tensile and compressive stresses acting on the cross-section are recorded in Table 1. To show the change in the stress state of the beam when lifted, Fig. 4 depicts the stress distribution on the cross-section at the harp points for the beam when resting on the ground (supported at its ends) and when hanging. The lifting stresses illustrated in Fig. 4 (right side) are for the case when the beam reaches its cracking limit, which occurs at a sweep magnitude of $L/320$.

Step 3:
Compare the resulting lifting stresses with the maximum allowable stresses per code specifications to ensure that cracking does not occur.

For the purpose of this example problem, the allowable stresses are computed in accordance to the *PCI Bridge Design Manual* (2003). It is assumed that the beam is lifted from the casting bed within one day of casting. Therefore the compressive strength f'_{cm} at the time of lifting is taken as the release strength f'_{ci} = 6,500 psi. The allowable tensile stress is $f_t=7.5(f'_{cm})^{0.5}$=605 psi and the allowable compressive stress is $f_c=0.60\,f'_{cm}$ =3900 psi.

As seen in Table 1, for a sweep imperfection magnitude of $L/320$, the maximum tensile stress in the beam at the harp points reaches the concrete's tensile modulus of rupture. For all three sweep magnitudes, the maximum allowable compression stress is exceeded at midspan, harp points, and lift points. Cases where the allowable compression stress is exceeded can occur, especially when low release compressive strengths are specified. A fresh look at allowable stresses in compression may be warranted for this lifting limit state. Cracking in tension or overstressing in compression can occur at sweep imperfection values greater than $L/320$ (5.2 in.) which is unlikely based on the sweep imperfection survey in Reference 16.

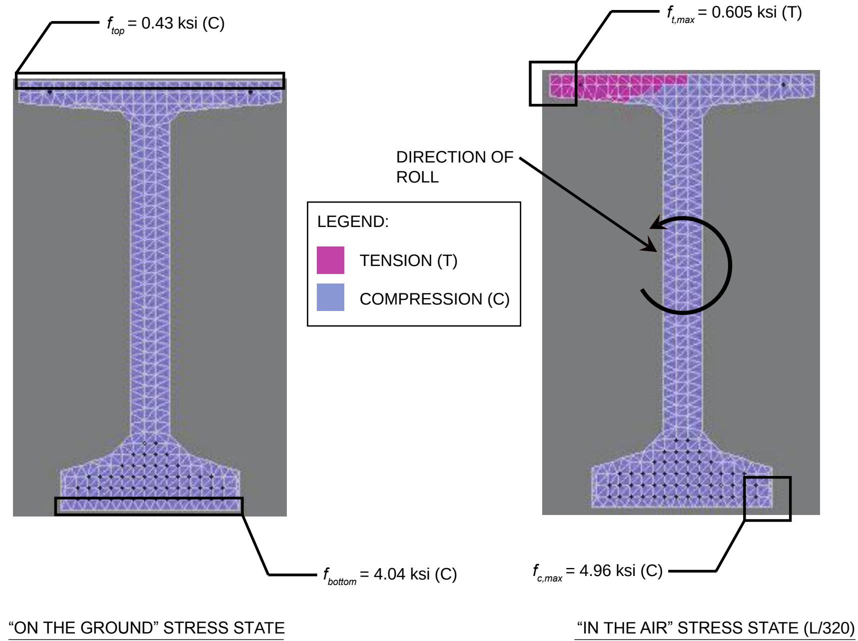

Fig. 4 Stress state for 77 in. PCI Bulb Tee. Note: The figure on the left depicts the stress state of the beam at harp points when resting on the ground; the figure on the right depicts the state of stress of the beam at harp points during lifting; f_{top} = stress in top fiber of the beam; f_{bottom} = stress in bottom fiber of the beam; $f_{t,max}$ = maximum tensile stress acting on the cross-section during lifting; $f_{c,max}$ = maximum compressive stress acting on the cross-section during lifting; 1 ksi = 6.895 MPa.

Table 1 Lifting moments and stresses including self weight (SW) and prestressing (PT)

Normalized sweep δ/L (Actual sweep δ, in.)	Location	Lifting moments (SW)		Maximum stresses (SW+PT)		Roll angle β, deg
		Strong-axis M_z, kip-in.	Weak-axis M_y, kip-in.	Tension: $f_{t,max}$, ksi	Compression: $f_{c,max}$, ksi	
L/960 (1.7)	Midspan	24,897	754	0.048	4.67‡	1.6
	Harp point	24,746	749	0.054	4.67‡	
	Lift point	-340	-9.7	n.a.	4.38‡	
L/472 (3.6)	Midspan	24,866	1533	0.330	4.82‡	3.3
	Harp point	24,715	1523	0.334	4.82‡	
	Lift point	-340	-19.8	n.a.	4.39‡	
L/320 (5.2)	Midspan	24,817	2,260	0.595	4.95‡	4.9
	Harp point	24,660	2,250	0.605†	4.96‡	
	Lift point	-339	-30	n.a.	4.39‡	

Note: † denotes a tensile stress value greater than7.5 $\sqrt{f'_{cm}}$; ‡ denotes a compressive stress value greater than $0.60 f'_{cm}$; f'_{cm} = compressive strength at time of lifting or transporting verified by test but shall not exceed design compressive strength (f'_c) at 28 days in psi + 1,000 psi; positive strong-axis bending moment produces compression in the top fibers and tension in the bottom fibers of the beam; positive weak-axis bending moment produces tension in the face farther from the center of curvature and compression in the face closer to the center of curvature; 1in. = 25.4 mm; 1 kip-in = 0.113 kN-m; 1 ksi = 6.895 MPa.

CONCLUSION

A new calculation-based method for the analysis of precast prestressed concrete beams during lifting has been presented. Using the procedure outlined in this paper, engineers and contractors can accurately calculate roll angle, twist, moments, forces, deflections, and most importantly the maximum tensile and compressive stresses acting on a beam during lifting, which can be compared to allowable limits to ensure that the beam lift can be performed safely.

In most cases lifting at the beam quarter points will produce the minimum stresses during lifting. Providing supplementary weak axis reinforcement to limit cracking as the beam rolls during lifting could improve lateral stability. Beam weak axis moment of inertia is important to consider for preventing large rolling deformations and stresses; these deformations and stresses can be calculated using *VT Lifting Analysis*. The allowable compression stress was exceeded in the PCI bulb tee example, even for a small sweep imperfection. A fresh look at allowable stresses in compression may be warranted for the lifting limit state.

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