AN ANALYTICAL INVESTIGATION OF PRESTRESSED CONCRETE GIRDERS WITH MINIMUM REINFORCEMENT

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ABSTRACT

It is widely accepted that the purpose of the minimum reinforcement requirement in prestressed concrete girders is to minimize their failure in brittle manner by providing adequate ductility and strength past the cracking limit state. This requirement, if quantified conservatively, can result in unnecessarily high amounts of longitudinal reinforcement, increasing the cost and possibly congestion. Moreover, increasing the longitudinal reinforcement without considering shear strength can also result in member failure due to shear or concrete compression in a brittle manner, defeating the purpose of meeting the minimum reinforcement requirement. AASHTO recently adopted a new minimum reinforcement requirement for prestressed girders based on reliability theory. However, this requirement is very different from other design codes and standards. This led to the investigation presented in this paper, which examines the minimum reinforcement requirement using a detailed analytical study. The main objective of this investigation is to analyze precast pretensioned concrete girders to determine whether the requirements specified in the AASHTO LRFD Bridge Design Specifications result in desirable performance and safety. The study found that cross sectional shape, concrete compressive strength, amount of prestressing used, and reinforcement type are important variables in specifying the minimum reinforcement requirement.

Keywords: Minimum Reinforcement, Prestressed Concrete Girder, Pretensioned Member, Parametric Study, AASHTO LRFD Bridge Design Specifications.

INTRODUCTION

Minimum flexural reinforcement requirements in prestressed or reinforced concrete girders are stipulated to provide sufficient strength and ductility of the member. That is, these requirements are meant to ensure that the girder does not fail in a brittle manner past its cracking limit state. To achieve this, most design codes, standards, and specifications require that a member is designed with sufficient flexural capacity above the cracking moment of the member. However, this cracking moment is dependent upon many factors, such as the concrete compressive strength, amount of prestressing in the girder, and the type and distribution of the reinforcement. Since current requirements merely focus on flexural strength, it is possible that the girder fails in other modes such as shear failure or a sudden failure of the concrete in compression, even though it has been designed to meet the minimum reinforcement requirements. Therefore, this study is aimed to gain a thorough understanding on the minimum reinforcement requirements for bridge girders.

In order to more accurately determine the influence of different variables and how they should be considered in minimum flexural reinforcement requirements, a literature review of past experiments and a parametric study are presented herein. The focus of the parametric study is to determine how a specific variable affects the sectional response of the specimen being analyzed. The variables analyzed include cross sectional shape, member span length, concrete compressive strength, the amount of prestress the tendons are subjected to, the type of reinforcement used, and the location of the reinforcement. These analyses were completed using Response 2000[1] software.

LITERATURE REVIEW

A review of previous experimental and analytical studies on beams with minimum reinforcement was conducted. Most of the literature found pertained to either reinforced concrete members or segmentally constructed members with an emphasis on validating or revising code provisions. One of the past experiments was conducted by Ozcebe et al.[2]. The specimens tested were six T-beams all reinforced with mild steel. From this experiment it was shown that requiring some amount of strength after the cracking of the beam is a possible criterion for minimum reinforcement requirements. While this test was completed on reinforced concrete beams, this idea could well be extrapolated to prestressed concrete beams.

Several other experiments were conducted to see what variables affected the behavior of reinforced concrete beams. An experiment conducted by Wafa and Ashour[3] used twenty rectangular, high strength concrete beams that were tested to determine the effects of concrete compressive strength on certain aspects of the beams’ behavior. This study concludes that as the concrete compressive strength increases, an increase in the reinforcement ratio is required to maintain a specified flexural reserve capacity. Another experiment by Rao et al.[4] tested twenty differently sized reinforced high strength concrete beams. It was observed that the depth of the beam affected the ductility capacity of the beam. Further, it was suggested that the minimum reinforcement ratio should consider a size scale effect.

In order to validate the analyses presented in this paper, the minimum reinforcement requirement as stated in AASHTO LRFD Bridge Design Specifications[5] has been used to represent minimally reinforced beams. According to Art. 5.7.3.3.2-1, that was established based on the recommendations from NCHRP 12-80[6], the minimum reinforcement required to ensure that the amount is adequate to develop a factored flexural resistance, M_r, at least equal to the lesser of:

1.33 times the factored moment required by the applicable strength load combination, or

$$M_{cr} = \gamma_3 \left[\left(\gamma_1 f_r + \gamma_2 f_{cpe} \right) S_c - M_{dnc} \left(\frac{S_c}{S_{nc}} - 1 \right) \right] \tag{1}$$

where:

- f_r = modulus of rupture of concrete specified in Article 5.4.2.6, i.e., $f_r = 0.24\sqrt{f_c'}$ for normal-weight concrete
- f_{cpe} = compressive stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at extreme fiber of section where tensile stress is caused by externally applied loads (ksi)
- M_{dnc} = total unfactored dead load moment acting on the monolithic or noncomposite section (kip.in)
- S_c = section modulus for the extreme fiber of the composite section where tensile stress is caused by externally applied loads (in^3)
- S_{nc} = section modulus for the extreme fiber of the monolithic or noncomposite section where tensile stress is caused by externally applied loads (in^3)

Appropriate values for M_{dnc} and S_{nc} shall be used for any intermediate composite sections. Where the beams are designed for the monolithic or noncomposite section to resist all loads, S_{nc} shall be substituted for S_c in the above equation for the calculation of M_{dnc}. The following factors shall be used to account for variability in the flexural cracking strength of concrete, variability of prestress, and the ratio of nominal yield stress of reinforcement to ultimate:

- γ_1 = flexural cracking variability factor
  = 1.2 for precast segmental structures
  = 1.6 for all other concrete structures
- γ_2 = prestress variability factor
  = 1.1 for bonded tendons
  = 1.0 for unbonded tendons
- γ_3 = ratio of specified minimum yield strength to ultimate tensile strength of the reinforcement
  = 0.67 for A615, Grade 60 reinforcement
  = 0.75 for A706, Grade 60 reinforcement
  = 1.00 for prestressed concrete structures

ANALYSIS METHOD AND VERIFICATION

Response 2000 uses the modified compression field theory to determine the sectional response of a given cross-section. To verify this software, the calculated cracking moment was compared to the result of an experiment by Bosco et al.[7]. The beam tested was a rectangular beam that was

200 mm (7.87 in) deep and 150 mm (5.91 in) wide. The reinforcement consisted of three 5 mm (0.20 in) diameter steel bars. Response 2000 estimated a cracking moment between 2.6 and 4.3 (mean = 3.45) kN-m while the experimental results yielded a cracking moment of 3.12 kN-m. Moreover, outputs of Response 2000 have also been verified using results from past experiments by Bentz[8].

PARAMETRIC STUDY

In order to determine what variables should be included in a minimum reinforcement requirement, it is first necessary to determine what has a significant impact on the response of the beam and how the response is affected. The variables analyzed included the shape and span length of the beam, the concrete compressive strength, the effect of the amount of prestressing used, and the placement of the reinforcement. Load deflection diagrams were calculated with a single point load located at the midspan.

Shape

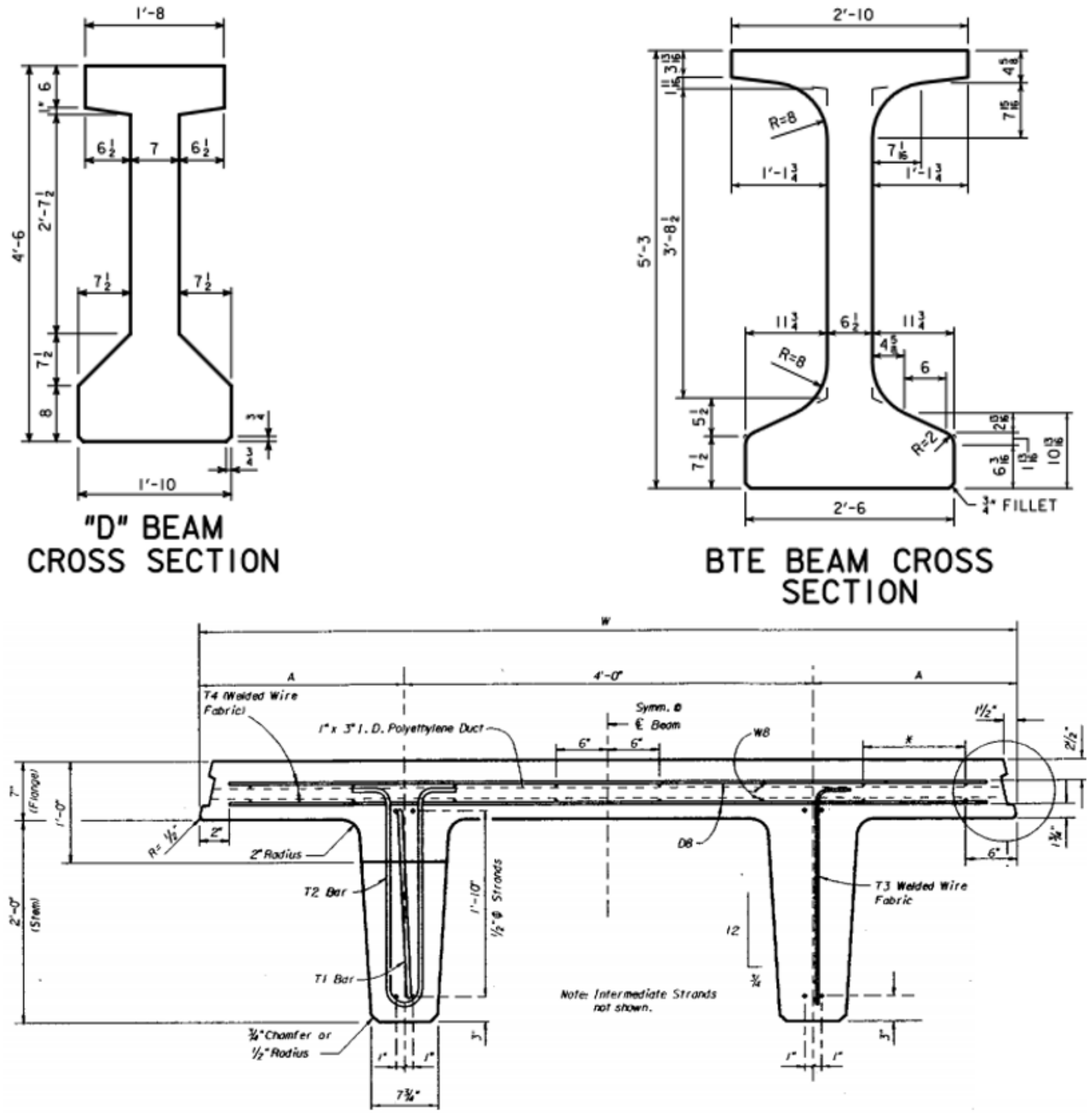
The first step in analyzing the effect of the cross-sectional shape on the response was to pick shapes that are comparable in strength. The shapes chosen in this parametric study are from the standard sections used by Iowa DOT[9] and Florida DOT[10], listed in Table 1 and shown in Figure 1. The double tee used (FDT 24) is no longer used by the Florida DOT, but was considered because it is not similar to the other two shapes.

Table 1 Shapes Analyzed for Beam Span of 50 ft

Shape	**Standard**	**Standard Reinforcement Ratio**	**Prestrain (ms)**	**Concrete Strength (ksi)**	**Gross Area (in^2)**	**I_g (in^4)**
I Girder	D50 (Iowa DOT)	0.41%	6.75	7.5	639	214,974
Bulb Tee	BTE 60 (Iowa DOT)	0.38%	6.75	5	807	422,790
Double Tee	FDT 24 (Florida DOT)	0.42%	6.75	5	1014	78,269

The minimum reinforcement ratios for these shapes were calculated according to requirements in AASHTO LRFD Bridge Design Specifications (referred as AASHTO minimum reinforcement). An average of these calculated reinforcement ratios was taken and applied to each of the cross sections to examine the effects on minimally reinforced beams as currently considered by the specifications. The load vs. deflection diagrams for the three cross-sections can be seen in Fig. *2* for girders with standard reinforcement and Fig. *3* for girders with AASHTO minimum reinforcement.

These results indicate that shape has an effect on both strength and the ultimate deflection of the beam. The beams had different moment of inertias that caused a different stiffness in each of the beams. The difference in the yield and ultimate strengths is due to the different amounts of steel since the sections have the same reinforcement ratios in the second plot, but not the same areas of steel.



FDT24 BEAM CROSS SECTION

Fig. 1 Shapes Analyzed in the Parametric Study[9, 10]

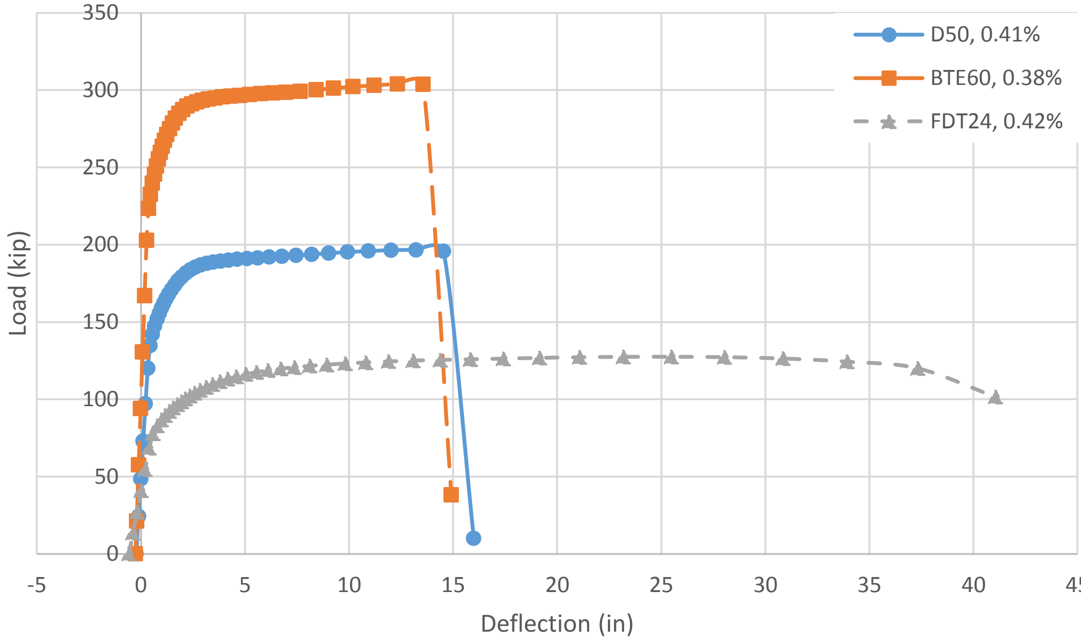


Fig. 2 Load vs. Deflection Diagrams for Shape Comparison of Beams with Standard Reinforcement

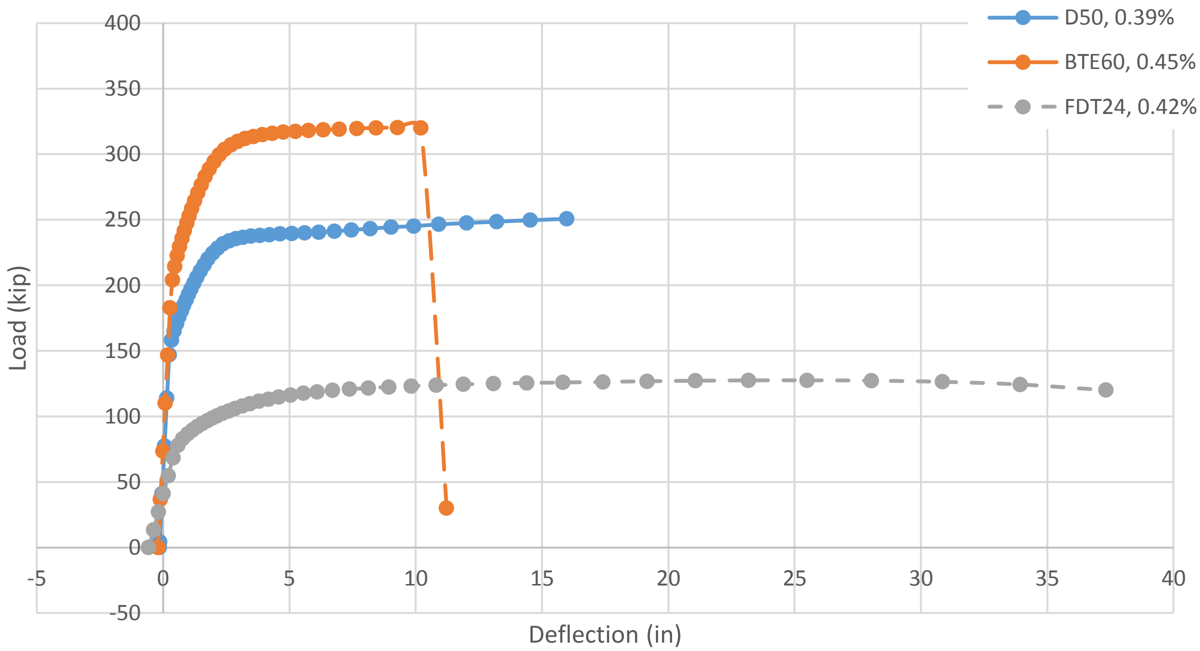


Fig. 3 Load vs. Deflection Diagrams for Shape Comparison of Beams with AASHTO Minimum Reinforcement

Span Length

After reviewing the results from the analyses of the different cross-sectional shapes, the BTE shape was selected to be used for further parametric study. The first was the effect the span size on a beam's response. The AASHTO minimum reinforcement requirement yielded a reinforcement ratio of 0.45%, as previously mentioned. The span lengths considered were 50, 100, and 150 ft. **Error! Reference source not found.** and **Error! Reference source not found.** show the moment vs. curvature and load vs. deflection diagrams for the three spans at a reinforcement ratio of 0.45%. The load vs. deflection diagram was created by utilizing the moment area method and assuming a simply supported condition where the load is applied at the midspan of the beam. The moment vs. curvature diagram yielded a similar response in all three of the beams tested, as expected. The load deflection diagrams produced the anticipated trend. The increase in strength of the smaller spans occurs only because the load deflection diagram calculates the load applied based on the span of the beam and the moments from the moment vs. curvature diagram. This means that since a smaller span was tested, more load is required to reach a given moment. This is also why the longer spans experience larger deflections before failure.

One parameter of interest that can be calculated from these moment vs. curvature diagrams is curvature ductility, which can be defined in one of two ways:

$$\mu_{\varphi cr}=\frac{\varphi_u}{\varphi_{cr}} \tag{2}$$

or

$$\mu_{\varphi y}=\frac{\varphi_u}{\varphi_y} \tag{3}$$

where

φ_u = the ultimate curvature or the curvature at failure
φ_{cr} = the curvature when the concrete first cracks on the extreme tensile face
φ_y = the yield curvature taken when the bottom layer of reinforcement first yields; this is taken as 10 milistrain for 270 ksi low relaxation strands per Collins and Mitchell[11]

This may also be applied to the load deflection diagram to calculate deflection ductility:

$$\mu_{\Delta cr}=\frac{\Delta_u}{\Delta_{cr}} \tag{4}$$

or

$$\mu_{\Delta y}=\frac{\Delta_u}{\Delta_y} \tag{5}$$

where

Δ_u = the ultimate deflection or the deflection at failure
Δ_{cr} = the deflection when the concrete first cracks on the extreme tensile face
Δ_y = the yield deflection taken when the bottom layer of reinforcement first yields

These factors have all been calculated and presented in **Error! Reference source not found.**. The ductility calculations based on yielding were not able to be calculated for the BTE155 due to the tendons not reaching a yielding strain. These factors demonstrate that the displacement ductility factor is approximately the same as the curvature ductility factor for any of the beams analyzed. Furthermore, while some of these factors are substantially large, they do not give any indication regarding how much strength is gained by the beam after cracking or yielding. This means that while the beam may undergo large deflections before failure, the beam may not take any significant increase in load before the beam fails. That is, the large deflections and eventually failure may occur without much addition to the load applied to the beam.

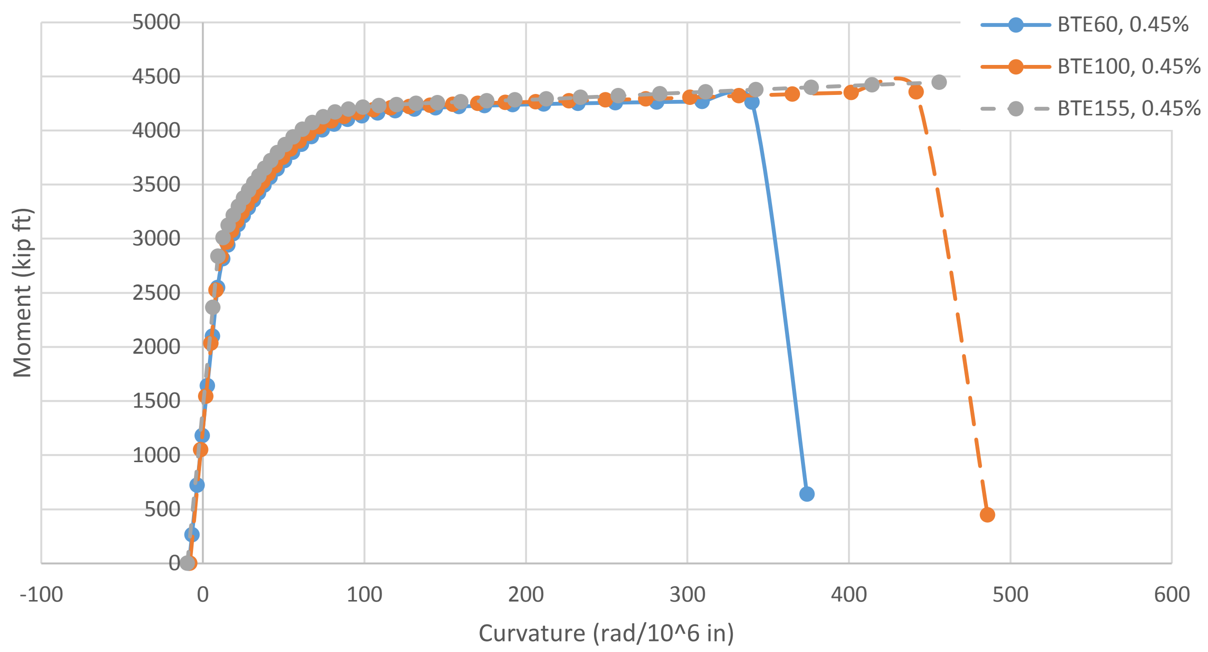


Fig. 4 Moment vs. Curvature Diagrams for Beams with Different Span Lengths

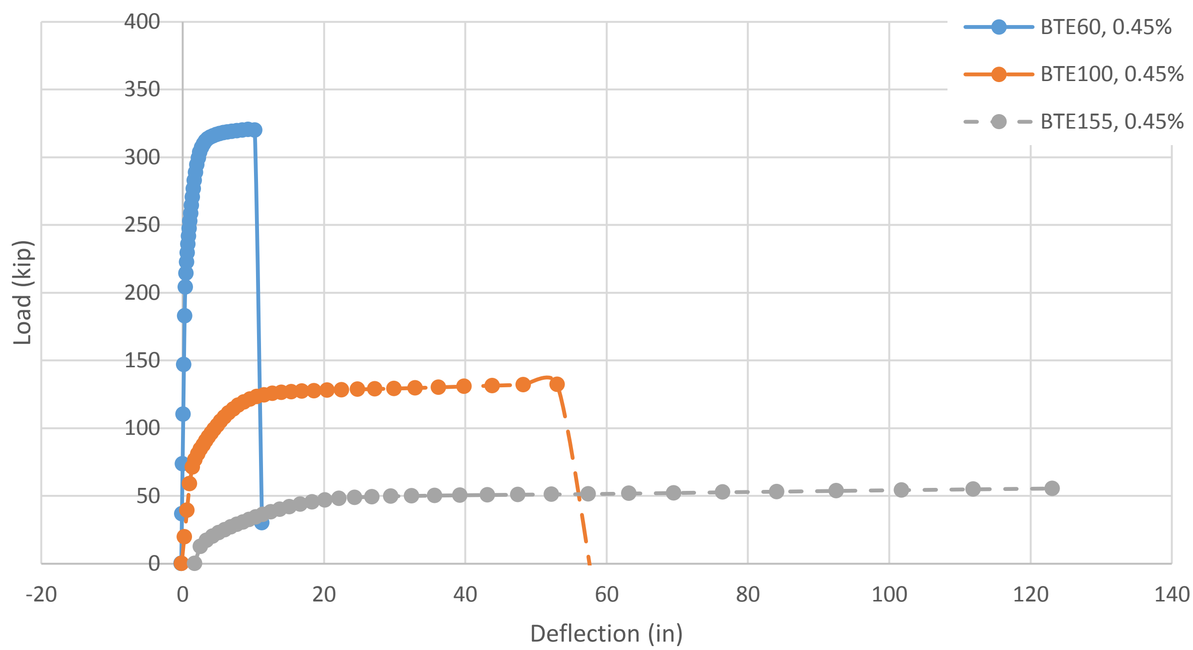


Fig. 5 Load vs. Deflection Diagrams for Beams with Different Span Lengths

Table 2 Ductility Factors for Varying Span Sizes

	BTE 60	**BTE100**	**BTE 155**
$\boldsymbol{\mu_{\varphi cr}}$	25.13	38.51	53.47
$\boldsymbol{\mu_{\varphi y}}$	3.45	5.05	N/A
$\boldsymbol{\mu_{\Delta cr}}$	25.13	38.50	53.48
$\boldsymbol{\mu_{\Delta y}}$	3.45	5.05	N/A

Concrete Compressive Strength

It is well accepted that altering the concrete compressive strength of the beam will alter the ultimate strength of a beam. This can, however, also change the failure mode of the beam. In order to analyze how this occurs, the BTE100 beam was analyzed using a 100 ft span and three different concrete compressive strengths. The AASHTO minimum reinforcement ratios as well as the standard reinforcement ratio per the Iowa DOT were used. The beams had compressive strengths of 4, 6, and 8 ksi with calculated AASHTO minimum reinforcement ratios of 0.69%, 0.67%, and 0.68%, respectively. The standard reinforcement ratio for the beam was 0.65%. Fig. *6* shows the moment vs. curvature diagrams for the beams using AASHTO minimum reinforcement. These beams act as expected; the curves have the same shape with an offset implying greater loads may be carried. A different behavior, however, is experienced for the 4 ksi beam. Fig. *7* presents the moment vs. curvature diagrams for the beams using the standard amount of reinforcement used for a BTE100 beam by the Iowa DOT. In both graphs, the first notable difference is the beam using 4 ksi concrete yielded a differently shaped response. The earlier failure in this beam can be attributed to the prestressing tendons not reaching yield before the concrete crushed. The other two beams reached the steel yield point before the concrete crushed. The difference in these responses is simply due to the amount of compressive stress the respective concrete strengths were able to withstand.

From these diagrams, it can be concluded that it is important to include concrete compressive strength as a factor when calculating minimum reinforcement. This is evidenced by the differences in ductility found in the standard beams. Finally, it is important to ensure that the steel reinforcement yields before failure to ensure sufficient ductility.

Amount of Prestressing

Another parameter examined is the amount of prestress in the tendons. This was done by taking the standard BTE100 beam cross section and creating multiple moment vs. curvature diagrams with varying percentages of the prescribed amount of prestress. The percentages of prescribed prestress force according to Iowa DOT standards used were 0%, 25%, 50%, 75%, and 100% of the prescribed prestress. Fig. *8* displays the described moment vs. curvature diagrams. In each of the diagrams, there are three points of interest: the cracking point, the steel yielding point, and the ultimate point. The diagram for the 100% prestress experiences cracking near steel yield, therefore making it difficult to see two distinct changes in stiffness. For the three diagrams between 0% and 100% of the prescribed prestress, three different stiffnesses can be noted. The first of these is before cracking, the second is after cracking but before steel yielding, and the third is after steel yielding until failure. The diagram with no prestress cracks under the dead load, thus the initial stiffness in the diagram is unrealistic.

From these diagrams, a couple of conclusions can be stated. First, as the amount of prestress decreases in the beams, curvature corresponding to the cracking moment reduces. Also, the amount of prestress has no effect on the stiffness of the beam, as anticipated. Finally, increasing the amount of prestress decreases the curvature ductility when based on cracking.

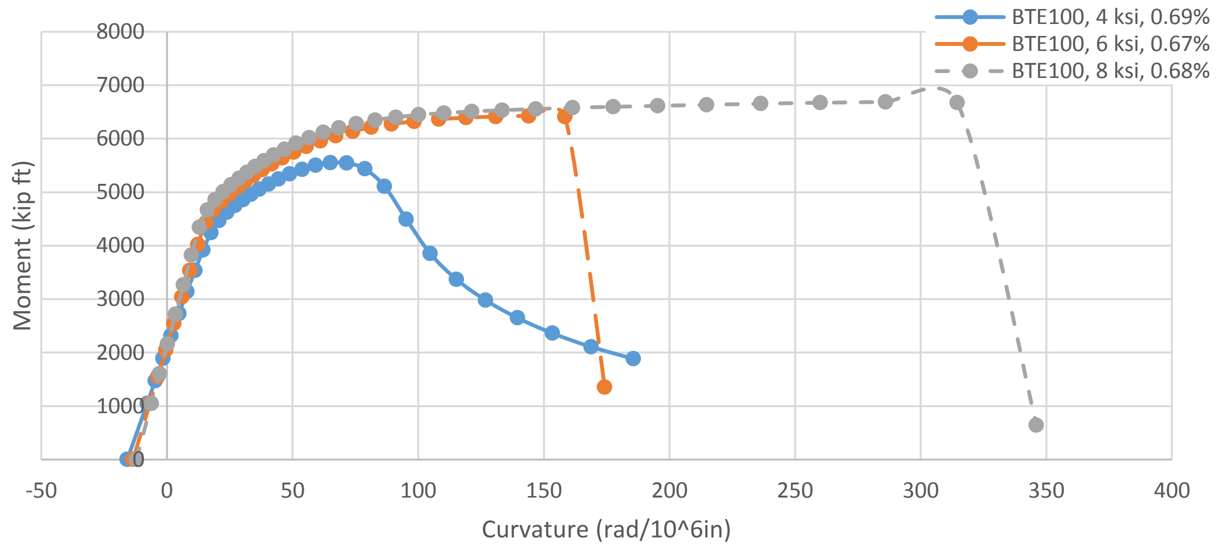


Fig. 6 Moment vs. Curvature Diagrams for Varying f'c on Beams with AASHTO Minimum Reinforcement

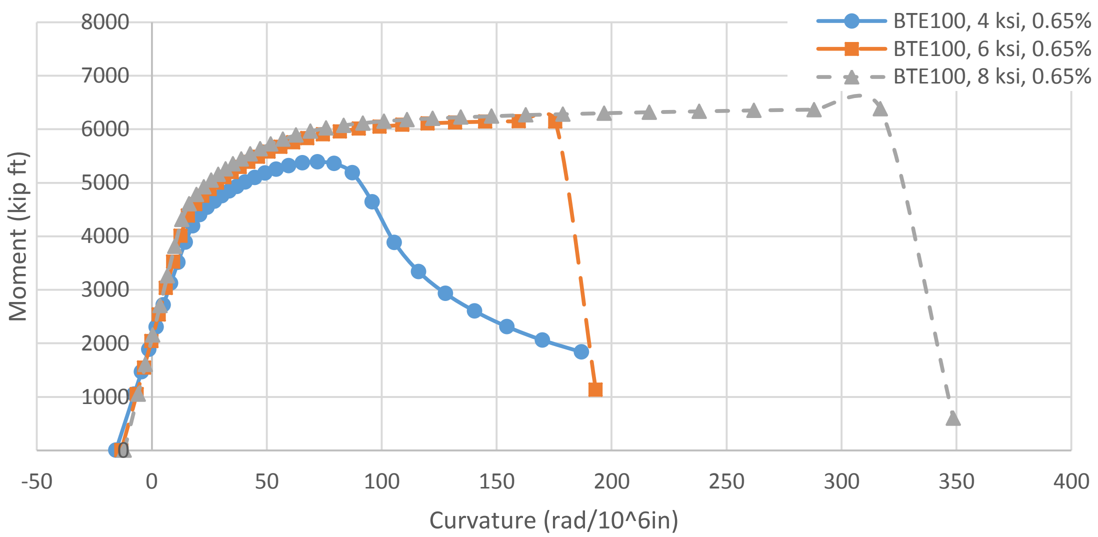


Fig. 7 Moment vs. Curvature Diagrams for Varying f'c on Beams with Standard Reinforcement

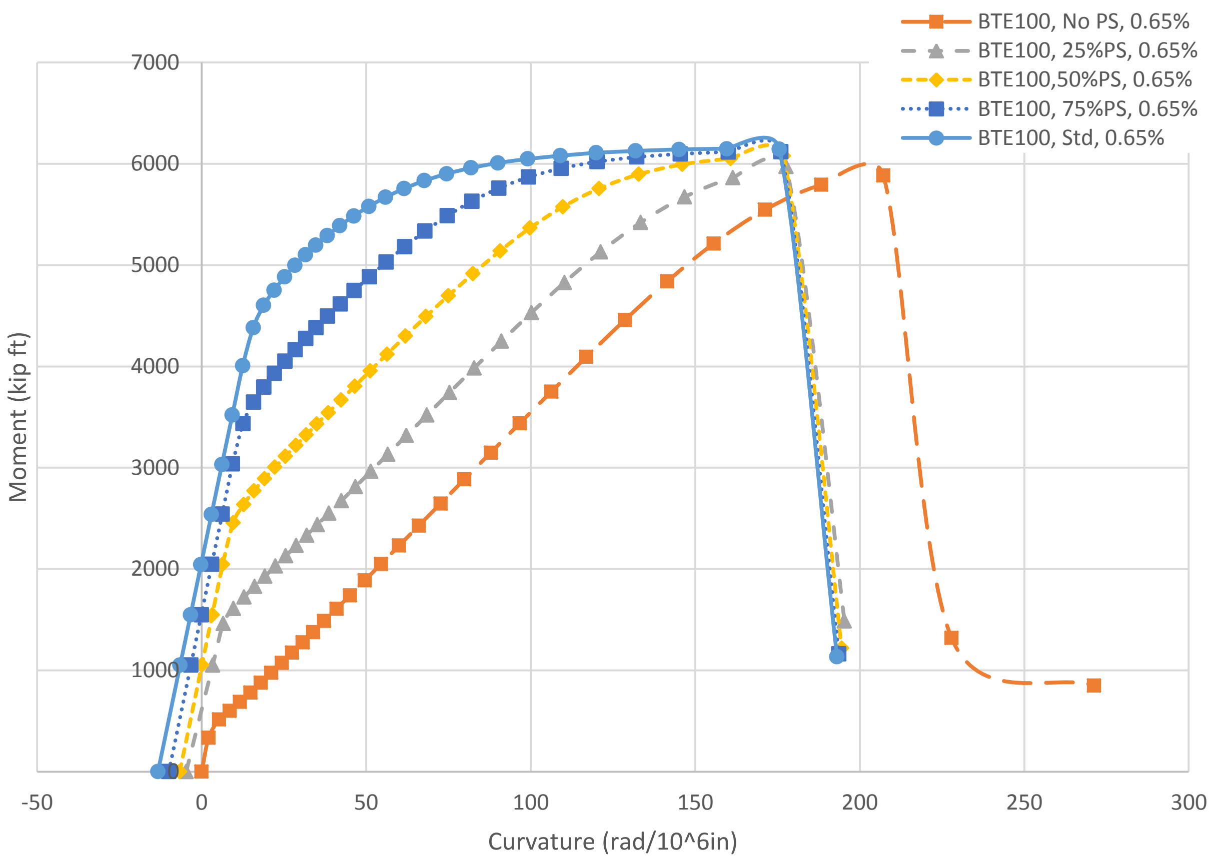


Fig. 8 Moment vs. Curvature Diagrams for Beams with Varying Amounts of Prestress

<u>Reinforcement Type and Distribution</u>

The type of steel used, whether it be mild steel or prestressing tendons, as well as the distribution of the steel were analyzed on the BTE100 cross-section. Three variations of this beam were analyzed alongside the standard cross section. The three variations included removing all prestress from the tendons, replacing the prestressing tendons with mild steel, and moving all tendons to the vertical center of gravity of the steel rather than distributing it in layers. The amount of mild steel used was calculated as follows:

$$A_{ms} = A_{pt} * \frac{f_{y,pt}}{f_{y,ms}} \quad (6)$$

where

A_{ms} = the equivalent area of mild steel
A_{pt} = the area of prestressing tendons
$f_{y,pt}$ = the yield stress of the prestressing tendons (220 ksi)
$f_{y,ms}$ = the yield stress of the mild steel (60 ksi)

Following the above procedure yielded a mild steel reinforcement ratio of 2.35%. Fig. 9 provides the moment vs. curvature diagrams for these beams. The diagram for moving the steel to the center of gravity of steel results in the exact same response as the standard cross section, yielding that it has no effect on the response of the beam. The initial stiffness of the beam without any prestress represents the stiffness of the standard beam after cracking but before yielding. This is because the beam without prestressing cracked under the dead load. The change in stiffness for the beam with mild steel is clearly due to the yielding of the steel. The initial stiffnesses of the standard beam and the beam with mild steel are the same. The second stiffness of the beam with mild steel is slightly different than the standard beam. This is because the beam with mild steel cracks under the dead load but has a larger transformed moment of inertia due to the large difference in the area of steel.

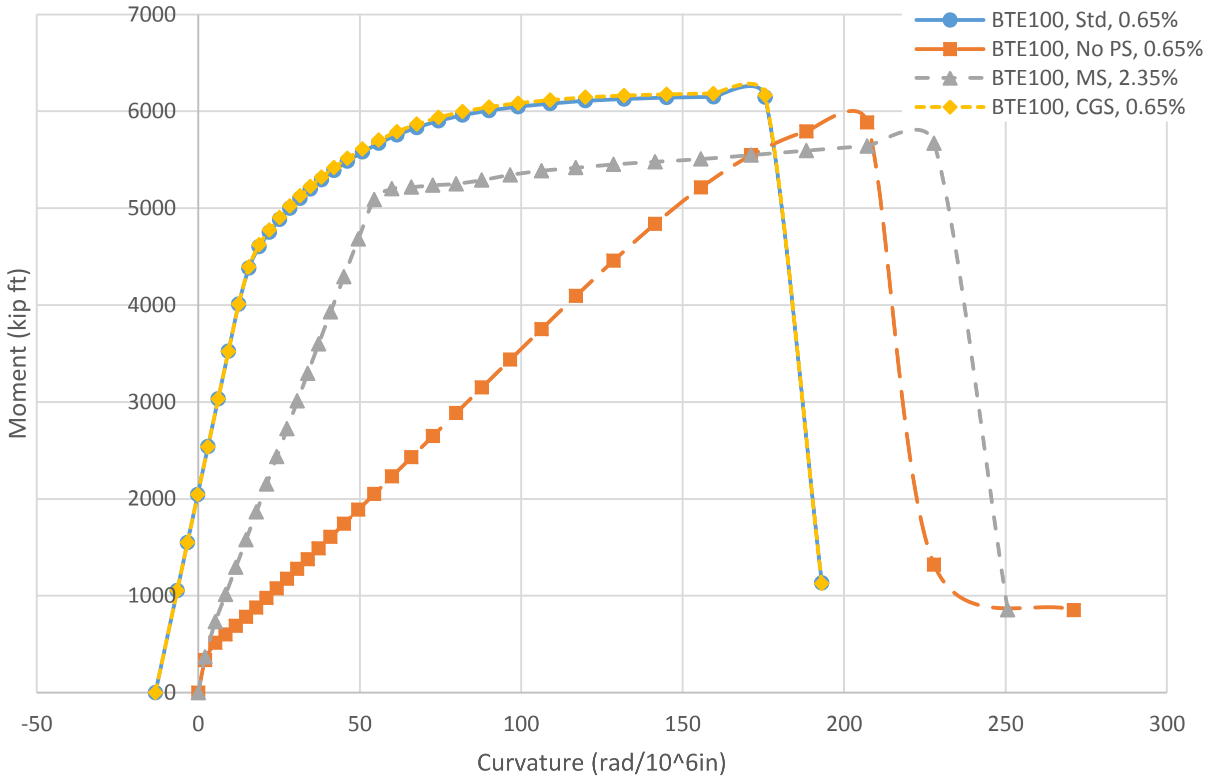


Fig. 9 Moment vs. Curvature Diagram for Beams with Different Reinforcement Types and Distributions

By comparing the results from beams reinforced with mild steel and with prestressing tendons, an observation can be made. For beams with mild steel reinforcement (i.e., reinforced concrete beams), it is meaningful to calculate the ductility based on the yield strength, since the beams exhibit nonlinear behavior past the yield point. However, the yield point is meaningless for prestressed concrete members since the yield capacity of the tendon is not exceeded in most cases. Therefore, calculation of ductility based on the cracking strength is deemed more meaningful.

CONCLUDING REMARKS

By examining the influence of different variables as presented above, some conclusions can be drawn. First, a beam may have a large displacement ductility but this does not necessarily imply that the beam will be able to sustain much of an increase in load before failure. Second, when calculating ductility of a member, a different equation should be used depending on the type of beam. For prestressed concrete beams, the curvature and displacement ductilities should be calculated based on the cracking strength instead of the yield strength, which is more suitable for reinforced concrete beams. Further analysis and experimental verification will follow in this study for recommendations regarding the minimum flexural reinforcement requirement. The study also found that cross sectional shape, concrete compressive strength, amount of prestressing used, and reinforcement type are important variables in identifying ductility.

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