COMPUTER-GENERATED FORCE FLOW PATHS FOR CONCRETE DESIGN: AN ALTERNATIVE TO TRADITIONAL STRUT-AND-TIE MODELS

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ABSTRACT

Strut and tie models (STM) are widely used by designers of reinforced concrete and prestressed concrete structures. Selection of an efficient model, however, becomes a challenging task for complex design domains, such as 3d domains with cutouts. Topology optimization has therefore been promoted as means of automating the development of highly efficient (minimum strain energy) STM. Current drawbacks of such methods are that solutions may complex and fail to properly account for secondary tensile stresses; that is, the case where the major principal stresses are compressive and minor principal stresses are tensile. A hybrid truss-continuum topology optimization scheme was recently developed to overcome these challenges in 2D concrete design. That work is modified and extended herein to three-dimensional domains and mechanics models. The stiffness of the elements are formulated such that truss elements carry only tensile forces and thus represent straight steel rebar, while the continuum elements carry only compressive forces and thus represent the concrete load paths. The latter is achieved using a stress-dependent orthotropic material model. The design goal is then to optimize the STM by minimizing strain energy in the system. The algorithm is demonstrated on several benchmark design examples. Results are shown to produce more efficient STM than traditional designs.

Keywords: Structural optimization, Topology optimization, Strut-and-tie model, Reinforced concrete, Prestressed concrete, Bimodulus materials

INTRODUCTION

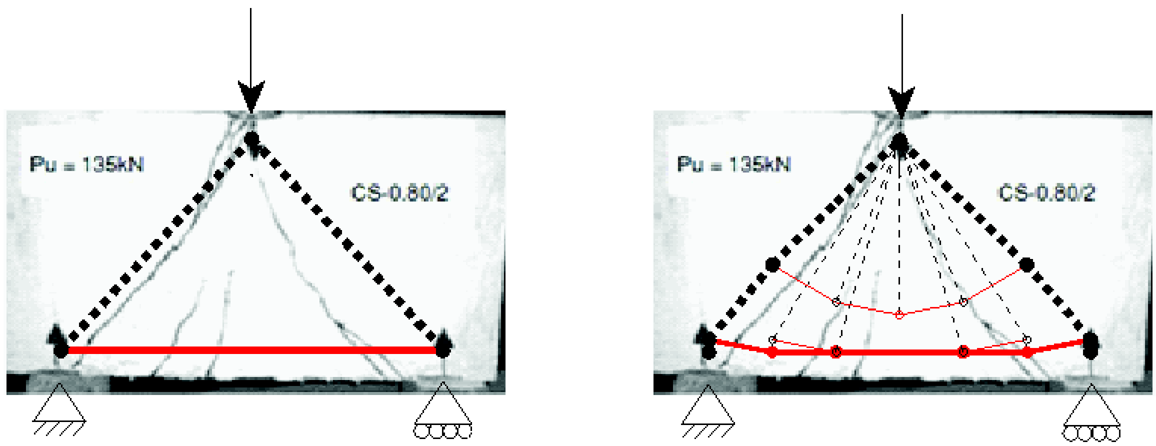
It is well-known that reinforced and prestressed concrete structures can in general be divided into two regions: regions where beam theory is valid, often referred as B-regions, and regions where the strain distribution is significantly nonlinear (e.g. near concentrated loads, corners, openings, etc.) known as discontinuous-regions, or D-regions. A common approach to designing B-regions is to assume the flow of forces can be represented as a truss. This truss analogy, first proposed by Ritter [1] and Mörsch [2], assumes that the cracked concrete structure acts as a truss with top and bottom longitudinal chords and an inclined web composed of concrete strut. D-regions, on the other hand, have been designed using rules of thumb or past experience for many years. The landmark paper by Jörg Schlaich and his colleagues at the University of Stuttgart (Schlaich et al. [3]) proposed generalizing the truss analogy to apply it in the form of strut-and-tie-models (STM) to both B-regions and D-regions. STM is a general truss model that consists concrete compression struts, steel tension ties, and joints. Based on the lower-bound theorem of plasticity which states that the capacity of STM is a lower bound on the strength of the actual structure, STM has been introduced into many design specifications and widely used in practical design for the past two decades.

There are two main challenges when using traditional STM. First, there are an infinite number of possibilities for the STM configuration, making it difficult to identify efficient solutions that mimic the internal stress trajectories. Conventional methods can then be used to solve the STM, such as the load path method (Marti [4]). Secondly, the geometry and topology (connectivity) of the STM is strictly related to a particular load configuration and cannot be used for other loads without modification (Schlaich [5]).

Topology optimization has been promoted as means of automating the development of highly efficient STM. The idea of this approach is that the design problem is posed and solved as an optimization problem with the governing mechanics embedded in the formulation. The optimizer then works to optimize the distribution of material (steel and load-carrying concrete) throughout the domain. Topology optimization is gaining momentum in the structural engineering community, with several firms using it to generate concepts for tall buildings (e.g., Baker et al. [6]; Sarkisian et al. [7]).

The optimization objective is to design a STM with minimum internal strain energy or compliance (maximum stiffness) for the given load and domain. This idea is widely supported, including by statements by Schlaich et al. [3] that an effective model will represent a minimum energy distribution through the D-regions, numerical results obtained by Ali and White [8] who demonstrated with nonlinear finite element modeling to collapse ultimate strength increases as truss stiffness increases, and experimental results obtained by Kuchma et al. [9]. Different loading conditions can be considered simultaneously by incorporating multiple load cases in the optimization algorithm.

An example from Moen and Guest [10], given in Fig. 1, is used to illustrate how topology optimization can be used to visualize force paths and to develop strut-and-tie models. The STM, based on the traditional method, is developed for a reinforced concrete deep beam in Fig. 1a and superposed over experimental results from Nagarajan and Pillai [11]. The steel reinforcement is orthogonal to cracks at midspan, but loses efficiency near the supports where cracks are diagonal. Fig. 1b shows an alternative STM developed by minimum compliance topology optimization method. The optimizer here places steel orthogonal to the compression struts, creating a steel reinforcement layout that orthogonally bridges cracks (indicated in the background experimental images), thereby increasing flexural capacity.



(a) Traditional design (b) Minimum compliance design

Fig. 1. Compare (a) traditional STM and (b) minimum compliance STM derived with topology optimization. Black dashed lines represent compression carried by the concrete, red solid lines represent tension carried by the reinforcing steel. Experimental results provided in the background are taken from Nagarajan and Pillai [11].

Moen and Guest [10], Kumar [12], Ali [13], Biondini et al. [14, 15], Ali and White [8, 16] have used a truss ground structure technique to develop STM. The idea is that the design domain is densely meshed with truss elements and topology optimization is used to identify elements that should be removed. The advantage of using truss topology optimization is that ties are guaranteed to be straight and optimal layouts can be made practical by limiting complexity in the initial ground structure (e.g., Gaynor et al. [17]). A coarse ground structure mesh leads to a simple optimal topology, which has the merit of ease of construction, but it potentially excludes highly efficient solutions. On the other hand, a fine ground structure mesh, which likely increases the STM efficiency, often ends up with complex topology composed of a large number of inclined steel rebar. Thus, it dramatically raises the labor cost for placing them.

Continuum, or free-form, topology optimization has also been used to optimize STM (Kim and Baker [18, 19], Guan [20], Liang [21, 22], Leu et al. [23,] Kwak and Noh [24], Lee [25], Guan and

Doh [26], Nagarajan and Madhavan Pillai [27], Bruggi [28, 29]). Disadvantages of this method are tension regions are not defined as discrete bars, requiring post-processing of the continuum results to produce truss representations to size concrete reinforcement. Design complexity is also more difficult to control directly, though can be influenced indirectly by controlling member length scales (Guest and Moen [30], Gaynor et al. [17]).

An assumption in the previous truss and continuum topology optimization methods is that the elastic moduli are the same for steel reinforcement and concrete. Victoria et al. [31] proposed a heuristic approach for optimizing STM topologies considering different mechanical properties for tensile (steel) and compressive (concrete) regions. Bogomolny and Amir [32] used material-dependent elastoplastic models with the goal of enhancing performance at ultimate limit state. Gaynor et al. [17] used truss and continuum topology optimization methodologies to create a hybrid STM in which truss elements only carry tensile forces, thus representing steel reinforcement, and continuum elements only transfer compressive forces, representing concrete.

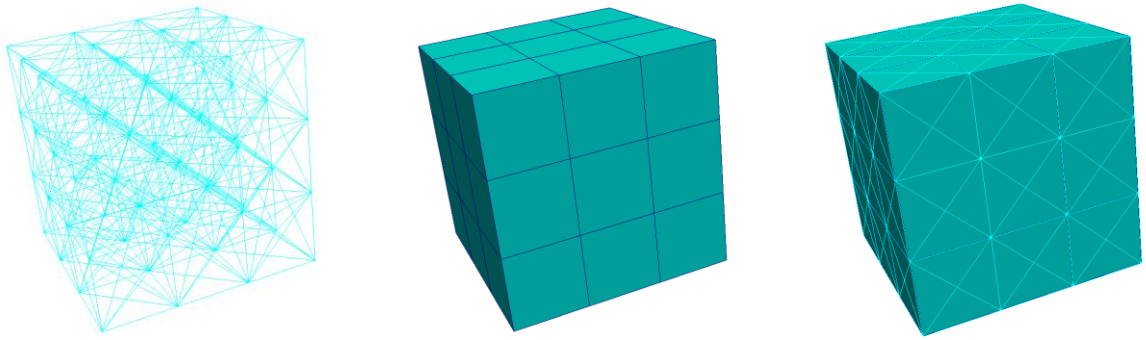
The vast majority of the above referenced literature considers two-dimensional structures. Only a few papers, including Leu et al. [23] and Bruggi [28], have investigated generating three-dimensional STM. It is the goal of this work to create a new three-dimensional automated tool for visualizing the flow of forces in reinforced concrete and prestressed concrete structural members and for designing optimized STM. The hybrid model initially developed by Gaynor et al. [17] is modified and generalized to account for more complex 3D stress state based on the bilinear stress dependent materials theory.

HYBRID TRUSS-CONTINUUM STRUT-AND-TIE MODELS

MOTIVATION AND ASSUMPTIONS

Truss topology optimization typically begins with a densely meshed domain, referred to as ground structure, and cross-sectional areas are to be optimized. The optimal STM generated by this method is straightforward and relatively easy to be designed, although it often produces structures with large degrees of indeterminacy. Continuum topology optimization is quite capable of producing highly efficient strut-and-tie models when the objective function is minimum strain energy, or compliance (maximum stiffness), but continuum tension regions are not defined as discrete bars, requiring postprocessing to determine sizing of reinforcing steels.

The idea of the hybrid truss-continuum STM, shown in Fig. 2, is that the truss members represent (tensile) steel reinforcement and thus are straight with complexity controlled by the designer at the ground structure level, and that the continuum members represent (compressive) concrete load paths and thus may have any complexity. A key advantage of this method is the ability to capture transverse tensile stresses that develop in the concrete phase as a result of load spreading. This is commonly seen in bottle-shaped struts where the width of struts is not constant. Traditional linear elastic topology optimization approaches are not capable of capturing this effect.



(a) Truss model (b) Continuum model (c) Hybrid truss-continuum model

Fig. 2. Hybrid mesh scheme

BILINEAR MATERIAL BEHAVIOR

In order to direct tensile forces to the steel and compression forces to the concrete, the approach taken here is to use negligible tensile strength and stiffness for the concrete, and negligible compressive strength and stiffness for the steel. We note this will prevent the appearance of compression steel, but this is consistent with the STM approach. A key challenge in such an approach is that the elastic moduli are not only nonlinear (bilinear), but that the continuum concrete models are dependent on the relative orientation of the principal stresses, adding a rotation dependency. Young's modulus and Poisson's ration are E_t and υ_t, respectively, when the corresponding principal stress is in tension along certain direction; while Young's modulus and Poisson's ration are E_c and υ_c, respectively, when the corresponding principal stress is in compression along certain direction. Material properties are thus related to the material, geometry, and boundary conditions of the system. In 2D, Gaynor et al. [17] captured this effect using an orthotropic material model proposed by Darwin and Pecknold [33]. Ambartsumyan and Khachatryan [34] proposed a 3D the constitutive tensor defined in the principal plane with a key assumption that shear modulus is ignored. Liu and Meng [35] discussed the influence of shear modulus on the convergence of numerical calculations and showed that the solution is not stable without considering shear modulus. Mathematically, the improved constitutive equation can be shown as follows [35]:

$$\{\varepsilon_p\} = [a]\{\sigma_p\} = [d]^{-1}\{\sigma_p\}$$
(1)

where, $\{\varepsilon_p\} = \lfloor \varepsilon_{p1}, \varepsilon_{p2}, \varepsilon_{p3}, 0, 0, 0 \rfloor^T$ and $\{\sigma_p\} = \lfloor \sigma_{p1}, \sigma_{p2}, \sigma_{p3}, 0, 0, 0 \rfloor^T$ denote the stress and strain vectors in the principal stress coordinate system, respectively. The constitutive tensor can be computed corresponding to principal stresses as follows:

(a) if $\sigma_{p1} > 0, \sigma_{p2} > 0, \sigma_{p3} > 0$: $a_{ii} = \frac{1}{E_t} \quad (i = 1, 2, 3);\; a_{ik} = -\frac{\upsilon_t}{E_t} \quad (i, k = 1, 2, 3, i \neq k)$

(2.1)

(b) if $\sigma_{p1} < 0, \sigma_{p2} < 0, \sigma_{p3} < 0$: $a_{ii} = \frac{1}{E_c} \quad (i = 1, 2, 3);\; a_{ik} = -\frac{\upsilon_c}{E_c} \quad (i, k = 1, 2, 3, i \neq k)$

(2.2)

(c) if $\sigma_{p1} > 0, \sigma_{p2} > 0, \sigma_{p3} < 0$;

$$a_{11} = a_{22} = \frac{1}{E_t},\; a_{33} = \frac{1}{E_c},\; a_{12} = a_{21} = a_{31} = a_{32} = -\frac{\upsilon_t}{E_t},\; a_{13} = a_{23} = -\frac{\upsilon_c}{E_c}$$

(2.3)

(d) if $\sigma_{p1} > 0, \sigma_{p2} < 0, \sigma_{p3} < 0$;

$$a_{11} = \frac{1}{E_c},\; a_{22} = a_{33} = \frac{1}{E_t},\; a_{12} = a_{13} = a_{23} = a_{32} = -\frac{\upsilon_c}{E_c},\; a_{21} = a_{31} = -\frac{\upsilon_t}{E_t}$$

(2.4)

The constitutive tensor $[d]$ can be obtained by inverting the flexibility matrix $[a]$. The remaining undetermined terms d_{44} , d_{55} and d_{66} can be obtained by assuming the following

$$d_{44} = d_{55} = d_{66} = \frac{\eta E_t + (1-\eta) E_c}{2\eta(1+\upsilon_t) + 2(1-\eta)(1+\upsilon_c)}$$

(3)

where η is equal to the ratio of the sum of positive principal stresses and the sum of absolute value of all principal stresses, thus $0 \le \eta \le 1$. Then the constitutive tensor in global coordinate system, denoted as $[D]$, can be obtained as follows;

$$[D] = [L]^T [d][L]$$

(4)

$$[L]=\begin{bmatrix} l_1^{\,2} & m_1^{\,2} & n_1^{\,2} & l_1m_1 & m_1n_1 & n_1l_1 \\ l_2^{\,2} & m_2^{\,2} & n_2^{\,2} & l_2m_2 & m_2n_2 & n_2l_2 \\ l_3^{\,2} & m_3^{\,2} & n_3^{\,2} & l_3m_3 & m_3n_3 & n_3l_3 \\ 2l_1l_2 & 2m_1m_2 & 2n_1n_2 & l_1m_2+l_2m_1 & m_1n_2+m_2n_1 & n_1l_2+n_2l_1 \\ 2l_2l_3 & 2m_2m_3 & 2n_2n_3 & l_2m_3+l_3m_2 & m_2n_3+m_3n_2 & n_2l_3+n_3l_2 \\ 2l_3l_1 & 2m_3m_1 & 2n_3n_1 & l_3m_1+l_1m_3 & m_3n_1+m_1n_3 & n_3l_1+n_1l_3 \end{bmatrix}$$

(5)

where, l_i, m_i and n_i are the direction cosines of the i-th principal stress to global x-, y- and z-directions, respectively. The stiffness matrix is then formulated in the standard manner.

The elastic properties for concrete and steels used herein are illustrated in Fig. 3, Young's moduli for the concrete are assumed 24.9 GPa (3600 ksi) in compression and 2.0 GPa (290 ksi) in tension, while moduli for the steel are assumed 200 GPa (29000ksi) in tension and zero in compression. We emphasize that these stiffnesses are chosen to focus tensile forces in the steel and compressive forces in the concrete, consistent with STM methodology. It is also noted that the tensile stiffness of the concrete is negligible but nonzero to prevent singularities in the global stiffness matrix.

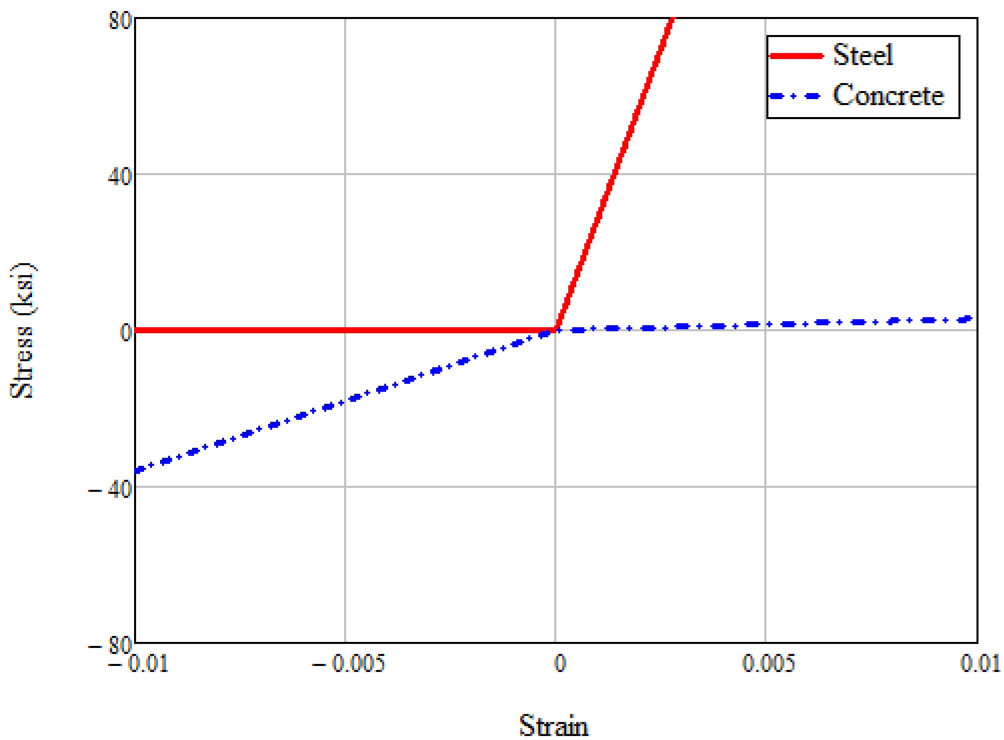


Fig. 3. Stress-strain relationship for continuum concrete and truss steel models

FINITE ELEMENT SOLUTION SCHEME

Since the constitutive tensor is a function of the stress state of a point, which is not known in advance, it is necessary to use an iterative solution strategy. As the moduli are bilinear, load stepping is not required. More simply a direct iterative method is used where the constitutive tensor for each element is adjusted and element stiffness re-computed based on the computed principal stresses in the previous FE iteration. We have found this approach to be simple and accurate in this and past work [17].

TOPOLOGY OPTIMIZATION FORMULATION AND SOLUTION ALGORITHM

The topology optimization formulation for minimum compliance (maximum stiffness) using the hybrid truss-continuum model can be expressed as follows;

$$\begin{aligned}
&\min_{\rho_c,\rho_t} f^T d \\
&s.t.\ Ku = f \\
&\quad \sum_{e\in\Omega_c} \rho_c^e v_c^e + \sum_{e\in\Omega_t} \rho_t^e v_t^e \le V^\circ \\
&\quad 0 \le \rho_c^e \le 1,\ \forall e \in \Omega_c \\
&\quad 0 \le \rho_t^e,\ \forall e \in \Omega_t
\end{aligned}$$
(6)

where the design variables ρ_c^e represent the volume fractions for continuum concrete and ρ_t^e the cross-sectional areas for the the truss elements, and v_c^e denotes the element volume for continuum and v_t^e the element length for the truss elements. V° is the total allowable volume of load-carrying material. Note that the truss and continuum members pull from the same total volume, allowing steel and concrete to be used as necessary to create the most efficient system. The global stiffness matrix is assembled in the standard manner

$$K = \underset{e\in\Omega_c}{A} K_c^e(\rho_c^e) + \underset{e\in\Omega_t}{A} K_t^e(\rho_t^e)$$
(7)

where $K_c^e(\rho_c^e) = ((\rho_c^e)^{p_c} + \rho_{\min}^e)K_{0c}^e$, $K_t^e(\rho_t^e) = (\rho_t^e)K_{0t}^e$, and K_{0c}^e and K_{0t}^e are the element stiffness matrices for unit design variable magnitude for the concrete and reinforcing steel, respectively. The variable $\rho_{\min}^e$ is a small positive number to maintain positive definiteness of the global stiffness matrix. The exponent p_c = 3.0 is a standard approach in topology optimization known as the Solid Isotropic Material with Penalization (SIMP) method to drive continuum volume fractions to the bounds (Bendsøe [36]). In this case, $\rho_c^e = 0$ indicates non-load carrying concrete and $\rho_c^e = 1$ indicates load carrying concrete.

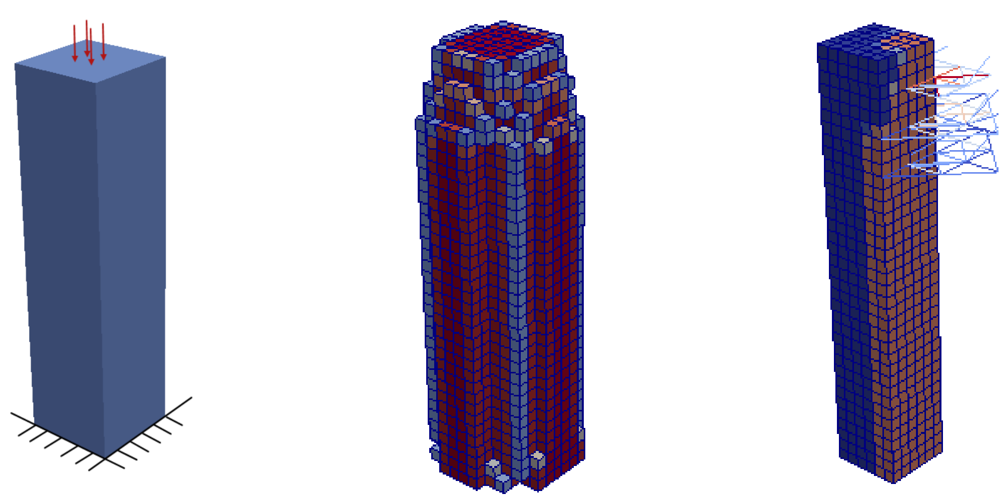
The Heaviside Projection Method (HPM) (Guest et al. [37], Guest [38]) is used to avoid well-known numerical instabilities of checkerboards and mesh dependency associated with continuum elements. Sensitivities are calculated using the adjoint method (see Gaynor et al. [17] for equations) and the gradient-based optimizer is chosen as the Method of Moving Asymptotes (MMA) (Svanberg [39]), which is very efficient for structural optimization. Full algorithmic details are available in Guest et al. [40].

HYBRID TOPOLOGY OPTIMIZATION RESULTS

Two simple examples are presented to show the effectiveness of the proposed hybrid topology optimization algorithm.

PRESTRESSED ANCHORAGE ZONE

The first example is the concrete block shown in Fig. 4, essentially a 3D version of the problem studied in Gaynor et al. [17]. The domain is subjected to a compressive force acting on the center 16% of the top surface. This structure is meant to represent the anchorage zone of a prestressed beam or a column, or concrete bearing component. As shown in previous work [17], the optimal topology using a linear elastic truss-only topology optimization model would indicate only compression forces and thus suggest steel is not required. The solution using the proposed bilinear hybrid truss-continuum model is shown in Fig. 4b and 4c. These figures clearly illustrate that the hybrid approach designs horizontal steel members to carry the principal tension stresses that develop due to force-spreading. The vertical distance that reinforcing steel is required is approximately equal to the width of the compression block.



(a) Design domain, loads, (b) Optimized model using (c) Cutaway view of and boundary conditions | hybrid truss-continuum topology optimization | solution (b)

Fig. 4. Topology optimized solutions for the prestressed anchorage zone design example (a). Traditional solutions would indicate only compressive load paths, while the hybrid model correctly indicates the presence of tensile stresses due to load spreading, as indicated by the placement of steel (c)

PILE CAP

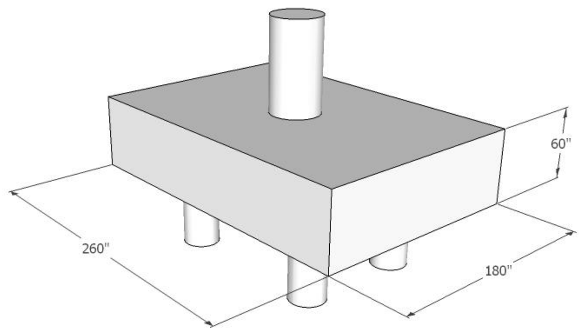
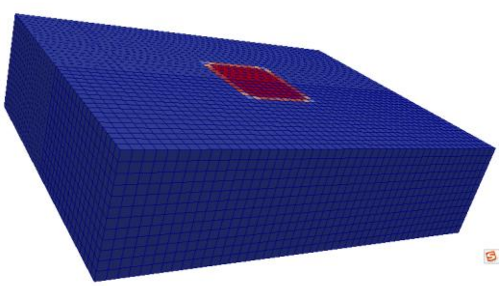
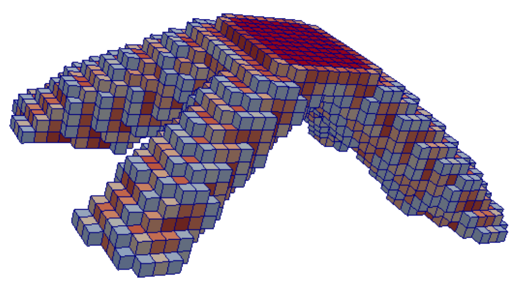


Fig. 5. Pile cap

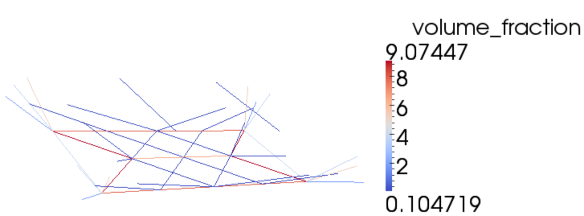
A pile cap, whose function is to transfer load from a column to piles, is shown in Fig. 5. The topology optimized STM is shown in Fig. 6. Compression is focused into four primary load paths (Fig. 6b) connecting each pile to the load, and concrete outside of these regions is not carrying load. This compression system is optimally balanced with a tension system that includes inclined members that connect to a central tension system plane near the bottom of the domain (Fig. 6d).



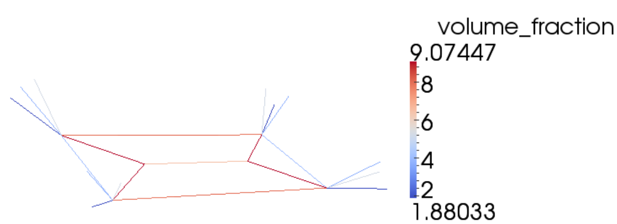
(a) Optimized domain with red indicating compressive load-carrying concrete and blue indicating non-load carrying concrete



(b) Optimal compression load paths (topology of load-carrying concrete)



(c) Optimal tension load paths (topology of



(d) The primary paths extracted from (c) reinforcing steel)

Fig. 6. Solutions for the hybrid topology optimization model for the pile cap example

CONCLUSIONS

Topology optimization has recently been shown as an effective design tool for visualizing the flow of forces in concrete and producing efficient STM. This paper uses a hybrid truss-continuum model, following the idea of Gaynor et al. [17], to focus tensile forces in the steel (truss) and compressive forces in the concrete (continuum). The work is extended herein to the generalized concrete material constitutive equations to account for more complex three-dimensional stress states and domains. Key advantages of the approach are that (i) the steel rebar is modeled by truss elements which directly determine the locations and the amount of reinforcing steel required, (ii) design complexity can be controlled through selection of the truss ground structure, and (iii) the bilinear hybrid model successfully captures tensile stresses that develop due to force spreading, an effect missed by linear elastic approaches. Future work will focus on investigating more structures with cutouts and minimizing as built costs.

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REFERENCES

1. Ritter W., "Die Bauweise Hennebique," Schweizerische Bauzeitung, V. 33, No. 7, 1899, pp. 59–61.
2. Mörsch E., “Concrete-steel construction,” E. P. Goodrich, translator, McGraw-Hill, New York; 1909.
3. Schlaich J., Schaefer K., Jennewein M., "Toward a consistent design of structural concrete," *PCI Journal*, V. 32, No. 3, May-June 1987, pp. 74-150.
4. Marti P., "Basic tools of reinforced concrete beam design," *ACI Structural Journal*, V. 82, No. 1, Jan. 1985, pp. 46-56.
5. Schlaich J., Schafer K., "Design and detailing of structural concrete using strut-and-tie models," *Structural Engineer*, V. 69, No. 6, Mar. 1991, pp. 113–125.
6. Baker W., Beghini A., Carrion J., Mazeika A., Mazurek A., “Numerical tools in structural optimization”, Proc. of the 6th International Conference on Computation of Shell and Spatial Structures, Ithaca, NY; May 2008.

7. Sarkisian M., Long E., Shook D., Doo C.S., “Optimization Tools for the Design of Structures.” Proceedings of the 2012 Structures Congress, Chicago, Illinois, ASCE, Reston, Virginia; Mar. 2012.
8. Ali M. A., White R. N., "Automatic generation of truss model for optimal design of reinforced concrete structures," *ACI Structural Journal*, V. 98, No. 4, July 2001, pp. 431-442.
9. Kuchma D., Yindeesuk S., Nagle T., Hart J., Lee H. H., "Experimental validation of strut-and-tie method for complex regions," *ACI Structural Journal*, V. 105, No. 5, Sept. 2008, pp. 578-589.
10. Moen C.D., Guest J.K., "Reinforced concrete analysis and design with truss topology optimization," Third International fib Congress and Exhibition, May 2010.
11. Nagarajan P., Pillai T. M. M., "Analysis and design of simply supported deep beams using strut and tie method," *Advances in Structural Engineering*, V. 11, No. 5, Oct. 2008, pp. 491-499.
12. Kumar P., "Optimal force transmission in reinforced concrete deep beams," *Computers and Structures,* V. 8, No. 2, Apr. 1978, pp. 223–229.
13. Ali M.A., "Automatic generation of truss models for the optimal design of reinforced concrete structures," PhD thesis, Cornell University, 1997.
14. Biondini F., Bontempi F., Malerba P.G., "Optimal strut-and-tie models in reinforced concrete structures," *Computer Assisted Mechanics and Engineering Sciences,* V. 6, 1999, pp. 279–293
15. Biondini F., Bontempi F., Malerba P.G., "Stress path adapting strut-and-tie models in cracked and uncracked R.C. elements," *Structural Engineering and Mechanics,* V. 12, No. 6, Jan. 2001, pp. 685–698.
16. Ali M.A., White R.N., "Formulation of optimal strut-and-tie models in design of reinforced concrete structures," *ACI Special Publications,* V. 193, Aug. 2000, pp. 979–998.
17. Gaynor A.T., Guest J.K., Moen C.D., "Reinforced concrete force visualization and design using bilinear truss-continuum topology optimization," *Journal of Structural Engineering,* V. 139, No. 4, Apr. 2013, pp. 607-618.
18. Kim H.A., Baker G., "Topology optimisation of reinforced concrete structures," In: Computational Mechanics - New Frontiers for the New Millennium: proceedings for the 1st Asian-Pacific congress on computational mechanics, V. 2, 2001, pp. 1251–1256.
19. Kim H.A., Baker G., "Topology optimization for reinforced concrete design," In: Mang HA, Rammerstorfer FG, Eberhardsteiner J (eds) WCCM V fifth world congress on computational mechanics. Vienna, Austria; July 2002.
20. Guan H., "Effect of sizes and positions of web openings on strut-and-tie models of deep beams," *Advances in Structural Engineering*, V. 8, No. 1, Jan. 2005, pp. 69–84.
21. Liang Q.Q., "Performance-based optimization of structures: theory and applications," Routledge, London, 2005, pp. 134–207.
22. Liang Q.Q., "Performance-based optimization of strut-and-tie models in reinforced concrete beam-column connections," In: 10th East Asia-Pacific conference on structural engineering and construction: materials, experimentation, maintenance and rehabilitation (EASEC-10). Bangkok, Thailand; Aug. 2006.

23. Leu L.J., Huang C.W., Chen C.S., Liao Y.P., "Strut-and-tie design methodology for three-dimensional reinforced concrete structures," *Journal of Structural Engineering* V. 132, No. 6, June 2006, pp. 929–938.
24. Kwak H.G., Noh S.H., "Determination of strut-and-tie models using evolutionary structural optimization," *Engineering Structures,* V. 28, Aug. 2006, pp. 1440–1449.
25. Lee H.J., "Development of the stress path search model using triangulated irregular network and refined evolutionary structural optimization," *Journal of The Korean Society of Agricultural Engineers*, V. 49, No. 6, Nov. 2007, pp. 37–46.
26. Guan H., Doh J.H., "Development of strut-and-tie models in deep beams with web openings," *Advances in Structural Engineering,* V. 10, No. 6, Dec. 2007, pp. 697–711.
27. Nagarajan P., Madhavan Pillai T.M., "Development of strut and tie models for simply supported deep beams using topology optimization," *Songklanakarin Journal of Science & Technology,* V. 30, No. 5, Sept.-Oct. 2008, pp. 641–647.
28. Bruggi M., "Generating strut-and-tie patterns for reinforced concrete structures using topology optimization," *Computers and structures,* V. 87, Dec. 2009, pp. 1483–1495.
29. Bruggi M., "On the automatic generation of strut and tie patterns under multiple load cases with application to the aseismic design of concrete structures," *Advances in Structural Engineering* V. 13, No. 6, Dec. 2010, pp. 1167–1181.
30. Guest J.K., Moen C.D., "Reinforced concrete design with topology optimization," Proceedings of 2010 Analysis & Computation Specialty Conference, ASCE Structures Congress, Orlando, FL, USA, May 2010, pp. 1-10.
31. Victoria M., Querin O. M., and Martí P., "Generation of strutand-tie models by topology design using different material properties in tension and compression," *Structural and Multidisciplinary Optimization*, V. 44, No. 2, Aug. 2011, pp. 247–258.
32. Bogomolny M., Amir O., "Conceptual design of reinforced concrete structures using topology optimization with elastoplastic material modeling," *Numerical Methods in Engineering,* V. 90, No. 13, June 2012, pp. 1578-1597.
33. Darwin D., and Pecknold D. A., "Nonlinear biaxial stress-strain law for concrete," *Journal of the Engineering Mechanics Division*, V. 103, No. 2, Mar.-Apr. 1977, pp. 229–241.
34. Ambartsumyan S. A., and Khachatryan A. A., "The different-modulus theory of elasticity," *Mekhanika Tverdogo Tela*, V. 1, No. 6, 1966, pp. 64-67.
35. Liu X.B., Meng Q.C., "On the convergence of finite element method with different extension-compression elastic modulus," *Journal of Beijing University of Aeronautics and Astronautics*, V. 28, No. 2, Oct. 2002, pp. 231-234.
36. Bendsøe M. P., "Optimal shape design as a material distribution problem," *Structural Optimization*, V. 1, No. 4, Dec. 1989, 193–202.
37. Guest J.K., Prevost J.H., and Belytschko T., "Achieving minimum length scale in topology optimization using nodal design variables and projection functions," *International Journal for Numerical Methods in Engineering* V. 61, No. 2, July 2004, pp. 238 - 254.
38. Guest J.K., "Topology optimization with multiple phase projection," *Computer Methods in Applied Mechanics and Engineering,* V. 199, No. (1-4), Dec. 2009, pp. 123-135.
39. Svanberg K., "The method of moving asymptotoes—a new method for structural optimization," *International Journal for Numerical Methods in Engineering,* V. 24, Feb. 1987, pp. 359–373.

40. Guest J. K., Asadpoure A., and Ha S., "Eliminating betacontinuation from Heaviside projection and density filter algorithms," *Structural and Multidisciplinary Optimization*, V. 44, No. 4, Oct. 2011, pp. 443–453.