

## SERVICEABILITY-BASED DESIGN METHOD FOR VERTICAL BEAM END REINFORCEMENT

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### ABSTRACT

*The application and diffusion of prestress force into a pretensioned bridge beam produces a considerable vertical tensile force near the beam end. From the owner's standpoint, the crack widths and lengths caused by this tension must be controlled in order for the beam design to satisfy the serviceability limit state. A crack's width is related to the tensile force in the reinforcement passing through the crack. The crack length propagating from the beam end is indicative of the length of the prestress force diffusion zone, and is strongly influenced by the vertical stiffness of the beam end. The vertical reinforcement area and distribution must be designed to give acceptable crack widths and lengths.*

*Researchers from the Virginia Transportation Research Council and the Virginia Military Institute have studied the crack widths and lengths in beams with varying vertical reinforcement, beam depth, and prestress force. The performance of the beam ends under study ranged from excellent, to poor and in need of repair. Designers frequently underestimated the length of the local zone in deeper beam sections. An analytical model was developed for the serviceability-based design of beam end reinforcement. This strut-and-tie model based design method was developed to produce acceptable crack lengths, and crack widths limited to 0.012" or less depending on the environment or engineer's preference.*

*A parametric study was conducted using the design method with the Virginia Bulb-T sections. Based on the study, the required total vertical reinforcement area was near the established minimum in the shorter beam sections, but was found to be substantially higher in the deepest beam sections. These results are especially important as deeper and more heavily prestressed beams are used more frequently.*

**Keywords:** Concrete, Bulb-T beams, cracking, reinforcement, strut-and-tie model

## INTRODUCTION

This paper presents a beam end reinforcement design method developed by the authors. It is based on a strut-and-tie model that is commonly used by engineers for the design of concrete reinforcement near post-tension tendon anchorage zones, and pretensioned strand transfer zones. The locations and distribution of the struts and ties have been calibrated to give both acceptable crack lengths and widths. These calibrations are based on the study of many pretensioned I-beams that varied in section, total prestress force, and vertical beam end reinforcement design. A detailed description of these case studies will be included in a more comprehensive paper presented at a later date.

A parametric study was also conducted using this method for the design of beam end reinforcement for the Virginia Bulb-T sections. The results of this parametric study provide engineers with the means for a quick check of a beam end reinforcement design. The results of this parametric study were not intended to replace design effort by the engineer.

## DESIGN METHOD

The beam end reinforcement design method developed by the authors is presented in this section with the aid of one of the case studies as an example. The beams shown in Figure 1 are 95.5 inch deep PCEF Bulb-T sections constructed with 8000 psi lightweight concrete. The concrete density is about 120 pounds per cubic foot. The tensile strength of this concrete is about two-thirds that of normal weight 8000 psi concrete based on the results of split cylinder tests. These beams have 56-0.5inch strands, with eight of the strands deviated. There is no debonding of the strands. The design called for an area of bars placed close to the beam end in accordance with the minimum specification from the 2004 AASHTO LRFD Bridge Design Specifications<sup>1</sup>. This minimum specification is based on the assumption that the total force in the vertical bar area is equal to at least four percent of the total prestress force, and that the vertical bars will be operating at 20ksi or less.

The vertical bar area proved to be insufficient for controlling crack lengths and widths, so additional bars were added to give the reinforcement shown in Figure 2. Although the cracking performance at the bulb-to-web interface was marginally acceptable with 0.008 inch to 0.012 inch widths, the beam end reinforcement was still not sufficient to control either the lengths (up to 69 inches) or the widths (up to 0.025 inches) of the diagonal cracks. The cracks were repaired by epoxy injection as seen in Figure 1.

There were several important conclusions based on this and the other case studies that were instrumental in the development of a strut-and-tie model for the beam end reinforcement design. The first is that the beam end cracking occurs within a length from the beam end that is equal to the height of the web or less. The second is that the dense reinforcement closest to the beam end will not control the diagonal cracks that will form in the web. The third is that the vertical stiffness provided by the reinforcement will control not only the crack widths, but the length of the local zone and the resultant crack

lengths. The fourth is that the force from the beam self-weight should not be included in the analysis. All of the beam end cracks were noted to have formed or to have extended when the beams were lifted off of the soffit forms. Note the location of the lift devices in Figure 2 and Figure 3.



Figure 1. 95.5inch PCEF Bulb-T beams with crack repairs.



Figure 2. Reinforcement distribution near beam end.



Figure 3. Lifting 145 foot long by 95.5 inch deep PCEF Bulb-T.

The basic strut-and-tie model developed for Bulb-T beams is shown in Figure 4. Note that there are actually two strut-and-tie models that will be solved separately. The upper strut and smeared tie model is solved to design a total bar area and distribution to control the diagonal cracks in the web. The bars are assumed to be evenly spaced along a length equal to about 75 percent of the beam height. This ignores the fact that the bar density will actually be increased near the beam end. These vertical bars are all assumed to act at a uniform stress level chosen by the engineer.

The lower strut and smeared tie model is solved to design a bar area and distribution to control the cracks at the bulb-to-web interface, and along the end of the beam in the web. These bars are assumed to be evenly spaced along a length equal to about 33 percent of the web height in deeper sections, or about 25 to 33 percent of the beam height in shorter sections.

The most important calibration to the model is the length of the local zone. Based on inspections of beam performance, as well as analytical experimentation, it was decided to make the length of the strut-and-tie model the same as the beam is deep. The model length is 95.5 inches in the example shown in Figure 4. This length is highly influential to the total bar area produced by the solution, and to the vertical stiffness of the beam end. The length selected by the authors will force cracks to terminate within the heavily reinforced beam end. An overly short model length produces a result with too much bar area near the beam end, and with the potential for cracks to propagate beyond the beam end reinforcement. Longer models produce a result that is not stiff enough for proper crack width or length control.

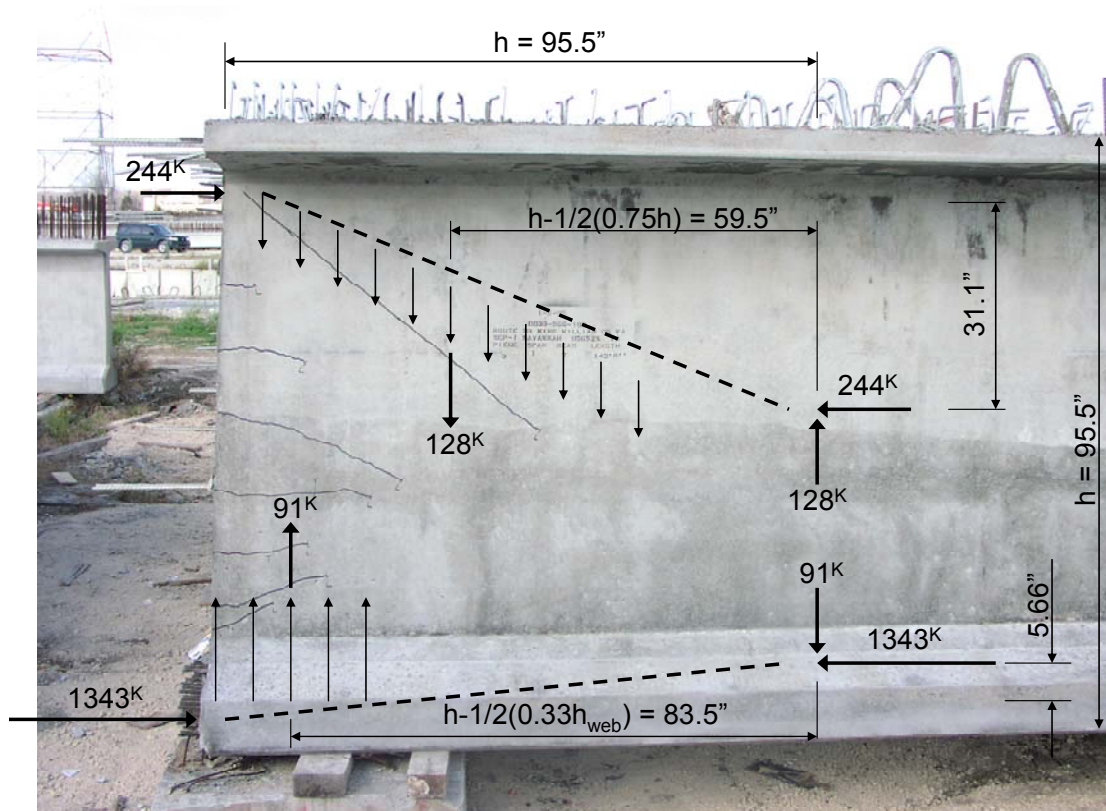


Figure 4. Strut-and-Tie Model for the 95.5 inch deep PCEF Bulb-T.

The location and magnitudes of the forces shown in Figure 4 are calculated by performing a transformed sectional analysis at the section 95.5 inches from the beam end. This sectional analysis assumes that the section is not warped, but in fact it will be warped to some extent due to the section's proximity to the crack tips among other factors. This warping is to be ignored in the sectional analysis. Also, the only forces that need to be accounted for in the calculation are those from the prestressing.

The total force in lower strand group is found by integrating stress over the strand area at the section 95.5 inches from the beam end. This force, 1343 kips in the example in Figure 4, is applied to the strut-and-tie model as if it were an externally applied force. The elevation of this resultant load is found by integrating the concrete stress over the concrete area at the section 95.5 inches from the beam end. Start the integration at the soffit and integrate upward through the bulb and web until 1343 kips has been reached. The 1343 kip force is applied to the strut-and-tie model at the center of gravity of the force resultant in the concrete. Figure 4 shows that the resultant 1343 kip force has migrated upward 5.66 inches from the center of gravity of the strand group at the beam end to the section 95.5 inches from the beam end. The moment couple to be balanced by the vertical bar force is 1343 kips times 5.66 inches. If the analysis is done correctly, the moment to be balanced in the upper strut-and-tie model will be the same as for the lower model.

In order to find the bar area, the engineer must choose an appropriate average operating stress level for the reinforcing bars. Based on the case studies and engineering judgment, the authors recommend using a stress of 18 ksi for Bulb-T beams made from normal weight concrete in non-aggressive environments, a stress of 12ksi for beams made from lightweight concrete or for normal weight beams in aggressive environments, and 8ksi for the most extreme cases. For the lightweight concrete example in Figure 4, the 12 ksi operating stress recommendation requires #5 stirrups at 4 inch spacing out to 74 inches from the beam end, and bundled #5 stirrups at 4 inch spacing out to 26 inches from the beam end. This rather extreme example case requires a total bar area that will carry the equivalent of 10 percent of the total prestress jacking force at 12ksi. These recommended operating stress levels for the vertical bars were selected in order to produce crack widths no larger than 0.012 inches at transfer.

### **PARAMETRIC STUDY**

The results of the parametric study are shown in Figure 5. This study was performed by applying the design method described above to typical Virginia Bulb-T beam designs. All cases used draped strands with no debonding. The total bar area of all vertical beam end reinforcement was assumed to be operating at the design stress level. Then this force was divided by the total force in all the prestressing strands upon jacking, and plotted on the Figure. The lower line represents the force in the bars placed near the beam end. The traditional minimum of 4 percent of the effective total prestress force corresponds very closely with the result in Figure 4, but only for the shorter sections and for the reinforcement near the beam end. The value from the lower line can be subtracted from that of upper line in Figure 4 to allow computation of the bar area that needs to be placed farther away from the beam end.

For shorter beam depths, the necessary vertical reinforcement will change little from that used in standard practice. For medium depth beams such as the 45 inch Virginia Bulb-T, the stirrup spacing needs to be such that bars will intersect the diagonal crack that will usually form in the web away from the beam end. As beam depth is increased, and the bulb is filled with prestressing strands, the required density of the vertical bars away from the beam end is considerable.

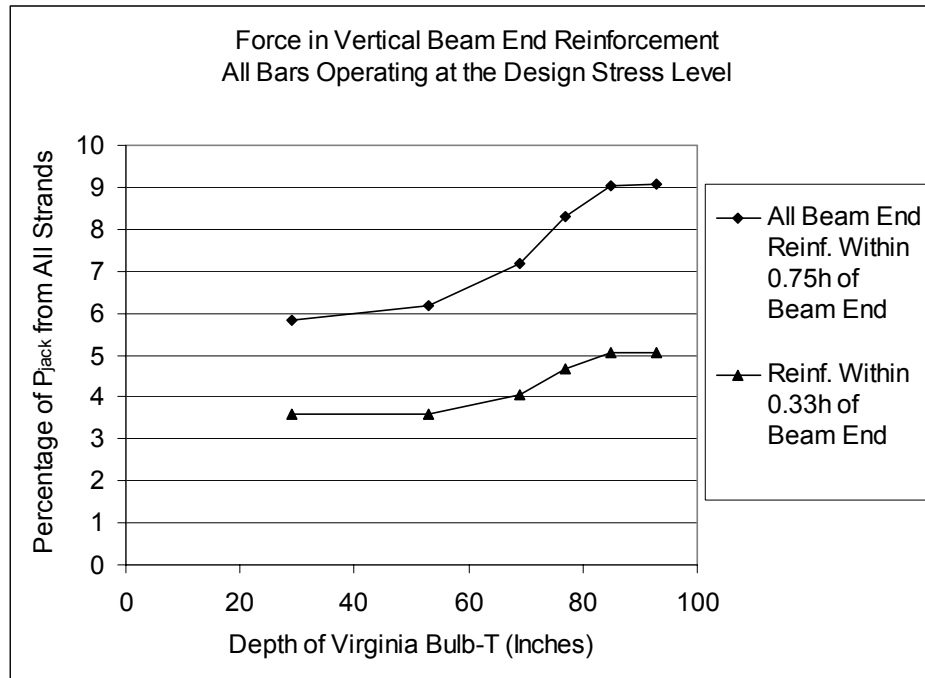


Figure 5. Results of parametric study for Virginia Bulb-T's.

## CONCLUSIONS

The increased use of Bulb-T beams in Virginia, combined with greatly varying beam end reinforcement designs seen in practice, encouraged the authors to take a closer look at the serviceability of the beam ends. Great differences have been seen between the beam end reinforcement designs performed for different states, and even between designs for the same state and for the same beam section. The serviceability of the beam ends examined by the authors ranged from poor to excellent, with the performance in the shorter sections generally being good. Given the increased use of deeper and heavily prestressed Bulb-T beams, the design method above provides a systematic approach for finding vertical beam end reinforcement area that will give acceptable serviceability.

The results of the parametric study indicate that the traditional amount and distribution of vertical beam end reinforcement is effective in controlling crack widths and lengths in the shorter beam sections. But as the beam section deepens, both the traditional minimum vertical reinforcement area and distribution will be insufficient to control cracks. Diagonal cracks in lightly reinforced webs of the deeper beam sections are seen to extend to approximately 75 percent of the beam depth from the beam end. Proper reinforcement limits the crack damage to within about 50 percent of the beam depth from the beam end.

## REFERENCES

1. AASHTO LRFD Bridge Design Specifications, Article 5.10.10.1, American Association of State Highway and Transportation Officials, 2004.