SIMPLIFIED SHEAR DESIGN

Nipon Jongpitakseel, Dept. of Civil Engineering, University of Nebraska, NE
Sherif A. Yehia, Ph. D.,P. E., Dept. of Civil Engineering, University of Nebraska, NE
Maher K. Tadros, Ph. D., P. E., Dept. of Civil Engineering, University of Nebraska, NE

ABSTRACT

A simplified shear design method is proposed for reinforced and prestressed concrete beams, based on analysis of the influencing factors and correlation with test results. The proposed method takes into account significant parameters influencing concrete contribution to shear capacity. Shear reinforcement contribution to shear capacity includes consideration of variable diagonal cracking angle. A non-iterative procedure and a simple formula for diagonal cracking angle, including effect of axial force, are proposed. The beneficial aspects of LRFD longitudinal reinforcement anchorage requirement and the maximum shear limit in LRFD are retained in this proposed method. In addition, the authors propose a design method to satisfy the LRFD longitudinal reinforcement anchorage requirement. A design example is given. The example shows how A_v/s *is calculated and demonstrates to difference in results of the proposed method with those of the AASHTO LRFD 1994, AASHTO LRFD 2002, and AASHTO Standard.*

Keywords: Shear Design, Reinforced Concrete, Prestressed Concrete, High Strength Concrete, Shear-span to Depth Ratio, Axial Load, Dowel Action, Size Effect, Diagonal Cracking Angle, Reinforcement Anchorage

INTRODUCTION

The introduction section of the ACI-ASCE Shear Committee 426 state-of-the-art report [1], published in 1973, states the following: “During the next decade it is hoped that the design regulations for shear strength can be integrated, simplified and given a physical significance so that designers can approach unusual design problems in a rational manner.” Before its publication, many shear test programs carried out had significantly improved the understanding of the complicated shear design problem. However, most of the test programs focused on beams with and without shear reinforcement and with depths of about 12 in. After the publication of the Committee’s report, more research should have been done on relatively large scale beams and with shear reinforcement, to integrate and simplify the shear design procedures of the ACI 318-71 code. Instead, most researchers have been trying to find a rational shear design theory aided with small scale beam testing and with diagonally loaded plate testing.

Shear behavior of beams is too complex to establish exclusively with theory. Available design methods attempt to offer semi-empirical procedures of varying theoretical rigor, accounting for as many of the significant influencing parameters as possible. A discussion is given below of such parameters.

INFLUENCING PARAMETERS

The significant parameters that contribute to shear resistance of concrete are the uncracked compressive zone of the member, the cracked concrete interface friction and aggregate interlock and the longitudinal reinforcement dowel action.

CONCRETE STRENGTH

Realizing that most code provisions were based on the results of many beam tests with relatively low compressive strength, varying mostly from 2000 psi to 6000 psi, and that there was an increasing use of concrete with strengths up to 12,000 psi, Mphonde and Frantz [2] initiated a program at the University of Connecticut. The main conclusion of the research was that the ratio of measured/predicted capacity using the then-current ACI equations for shear design decreased from 1.64 to 1.20 as the concrete strength increased from 3000 to 15,000 psi. This conclusion was further confirmed by researchers at Cornel University [3] and North Carolina State University [4] on mostly small beams without web reinforcement. Roller and Russell [5] carried out an experimental program on relatively large rectangular beams with web reinforcement and concrete strength of up to 18,000 psi. The main conclusions of the research were:

(1) For nonprestressed members subjected to shear and flexure only, the ACI 318-83 code provisions overestimated the nominal shear strength provided by the concrete contribution term when concrete compressive strength was greater than 16,970 psi; and

(2) The minimum quantity of shear reinforcement specified in ACI 318-83 code needed to be increased as the concrete compressive strength increased to compensate for the evident

lack of conservatism in the concrete contribution term at high concrete compressive strength levels.

It is proposed that apply the power 1/3 to f'c, i.e. $\sqrt[3]{f_c^{'}}$, rather than the conventional 1/2 power, i.e. $\sqrt{f_c^{'}}$, to predict shear capacity due to concrete contribution.

AXIAL FORCE AND MOMENT CORRECTION FACTOR

Tests [6] have shown that prestress has a positive impact on the shear strength of a structural concrete member. When prestress is introduced on the member, the member is subjected to a longitudinal compressive stress. This compressive stress can delay the occurrence of the diagonal cracking and flatten the diagonal cracking angle. In contrast, the sectional moment reduces shear capacity due to flexure-shear crack behavior. As a result in a concrete beam subjected to compressive force, a higher shear is required to cause principal tensile stresses at the extreme fiber equal to the concrete tensile strength in the zone of small moment, while in zone of large moment, a less shear is required. Obviously, these two types of applied loads affect the shear capacity in the related sense as shown in Figure 1. Therefore, the axial force and moment should be combined in one factor. The codes from some countries provide the formulas that affect the shear capacity due to axial force and/or applied moment as follows.

ACI Code [7]: $K_{AF} = 1 + \frac{N_u}{2000A_g}$ Compression

$K_{AF} = 1 - \frac{N_u}{500A_g} \geq 0$ Tension

Australian Code [8]: $K_{AF} = 1 + \frac{N_u}{14A_g}$ Compression

$K_{AF} = 1 - \frac{N_u}{3.5A_g} \geq 0$ Tension

Japanese Code [9]: $K_{AF} = 1 + \frac{M_o}{M_u} \leq 2$ Compression

$K_{AF} = 1 + \frac{2M_o}{M_u} \geq 0$ Tension

where M_o is decompression moment

The authors propose the term of

$$K_{AF} = 1 + \frac{A_{ps} f_{pe} h}{3M_u} \leq 1.75 \quad (1)$$

that is simplified from Japanese code [9] as the correction factor due to the effects of axial load, or prestressing force, and flexural moment [10].

where A_{ps} = total area of prestressing strand (in.^{2})
f_{pe} = effective prestressing force (kip)
h = height of the member subjected to prestressing force (in.)
M_u = factored moment at the section considered (in-kip)

Note that $A_{ps}f_{pe}$ may be replaced with N_u for reinforced concrete members subjected to axial compression. K_{AF} = 1 for member subjected to axial tension.

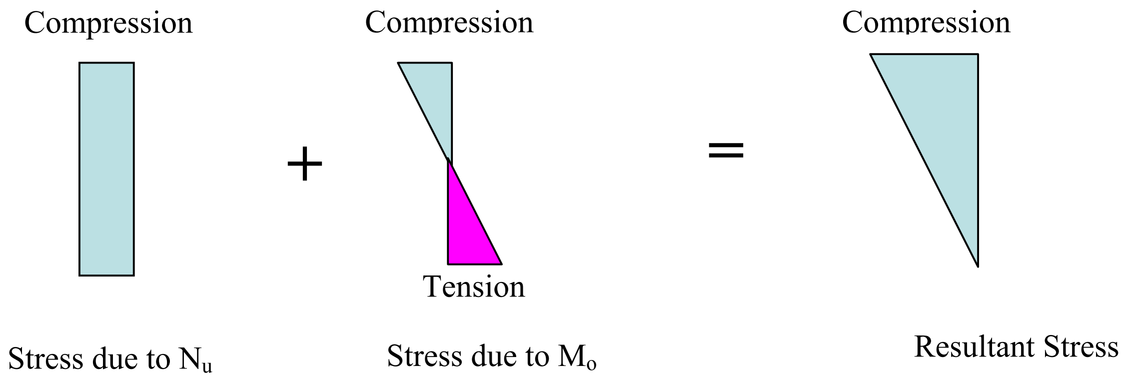

Figure 1 Effect of Axial Force and Moment on Tensile Stress at Extreme Fiber.

MEMBER SIZE CORRECTION FACTOR

Some researchers [11, 12] have demonstrated that shear strength of the concrete component of the section decreases as depth of member increases. The factors due to size effect are provided in some codes as shown in Figure 2.

To consider the member size effect, the parameter

$$K_{SIZE} = \frac{150}{100 + d_v} \quad (2)$$

is proposed to be applied to the concrete contribution to shear resistance. The formula proposed above have been shown through parametric analysis to produce results that correlate well with full scale tests and with rational theories of reinforced concrete shear [6, 10]. As shown in Figure 2, the proposed formula gives a curve that closes to Japanese and Australian codes for the most used member depth, and also gives a conservative view for all range of current available depth when compare to other codes. The effective shear depth d_v, in Equation (2) has a unit of inch.

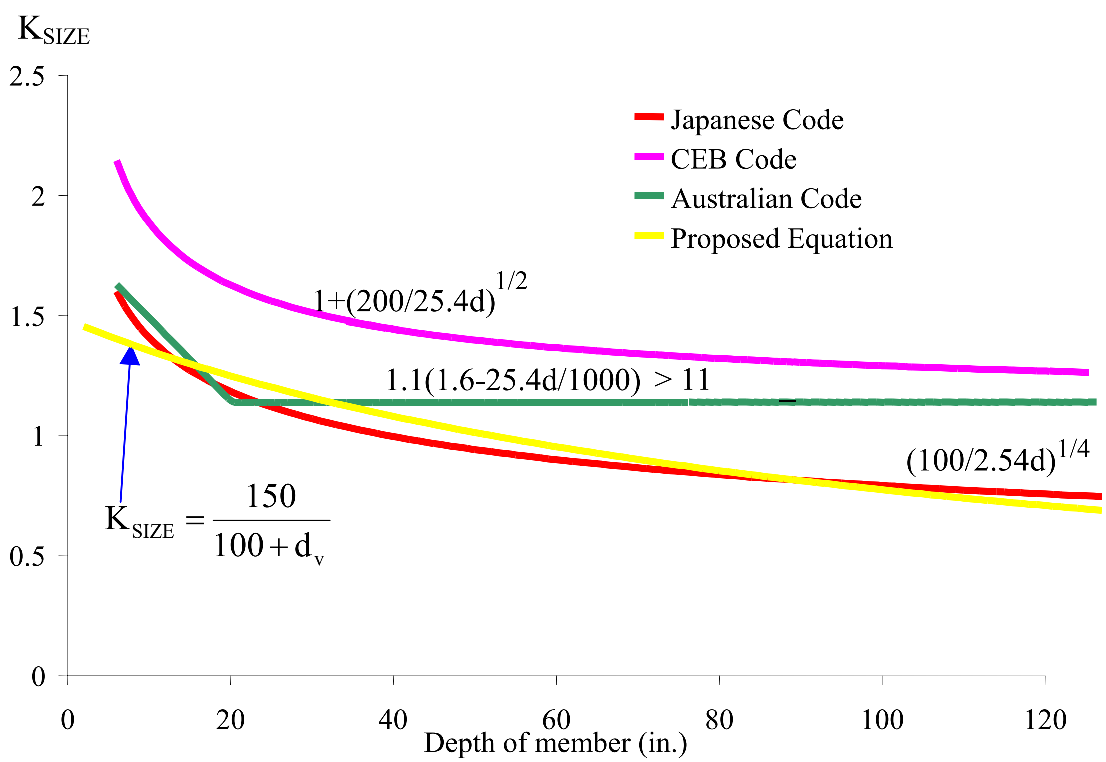

Figure 2 Relationships between Size Effect Factors and Depth of Member [8, 9, 13]

LONGITUDINAL REINFORCEMENT CORRECTION FACTOR

Greater amounts of longitudinal reinforcement produce higher dowel action forces in the member, and higher shear capacity. As the longitudinal reinforcement increases, flexural stress in the reinforcement decreases. When flexural stress decreases, flexural cracking width also decreases, and shear strength improves. It is proposed that the parameter

$$K_{LR} = 4.5\left(\sqrt[3]{\frac{A_{ps}+A_s}{b_v d_v}}\right) \geq 0.7 \qquad (3)$$

be used to represent the effect of longitudinal reinforcement. This formula is the same as in Australian [8], Japanese [9], and CEB-FIP Model Codes [13].

where A_{ps} = area of prestressing steel. (in.^{2})
A_s = area of mild tensile steel. (in.^{2})
b_v = web width (in.)
d_v = effective shear depth (in.)

CONCRETE WEIGHT CORRECTION FACTOR

Lightweight aggregate concretes having density from 90 – 120 pcf are generally used in precast concrete industrial. Their tensile strengths are significantly less than that of normal weight concrete at the same compressive strength. The tensile strength of lightweight concrete ranges 70 to 100% of that of normal weight concrete. The fracture surface of lightweight concrete is also smoother than that of normal weight concrete. As a result, the inclined cracking load of the lightweight concrete beam is usually less than that of the normal weight concrete beam for the same concrete compressive strength. Therefore, the shear capacity for the concrete beam will be multiplied by the following factors.

K_{LW} = 1.00 for normal weight concrete
= 0.85 for sand-lightweight concrete
= 0.75 for all-lightweight concrete

Note that these factors are the same as those of ACI 318-02 Section 11.2 [7].

SHEAR SPAN CORRECTION FACTOR

Formulas for K_{AF}, K_{SIZE}, and K_{LR} are primarily adapted from international codes as discussed above. With these parameters set, the best fit of previous test results [2, 14-27] as shown in Figure 3 is utilized to develop a formula for the shear span correction factor, β_{SS}. The shear capacity significantly decreases when the shear span increases, while all other parameters remain the same. From this phenomenon, the so-called shear failure is not caused by shear alone, but a combination of shear and moment. For simply supported beam, when the shear span increases, the moment near the applied load also increases. Shear is constant, however, if the beam's self-weight is ignored. With the increase of the moment, diagonal cracking increases and the uncracked compressive zone decreases. The increased crack width reduces the interface shear transfer capacity. The shear resisted by the compression zone also decreases because of the reduced compressive zone.

Thus, the parameter

$$\beta_{SS} = 0.80 \qquad \text{when } \frac{M_u}{V_u d_v} \geq 3 \text{ “shear/flexure failure”} \qquad (4a)$$

$$\beta_{SS} = 5\left(\frac{V_u d_v}{M_u}\right)^2 + 0.25 \leq 1.5 \qquad \text{when } \frac{M_u}{V_u d_v} \leq 3 \qquad (4b)$$

has been found to represent the effects of shear-span to depth ratio on concrete contribution to shear resistance as shown in Figure 3, where d_v is effective shear depth, and V_u and M_u are factored shear and factored moment at the section being considered, respectively.

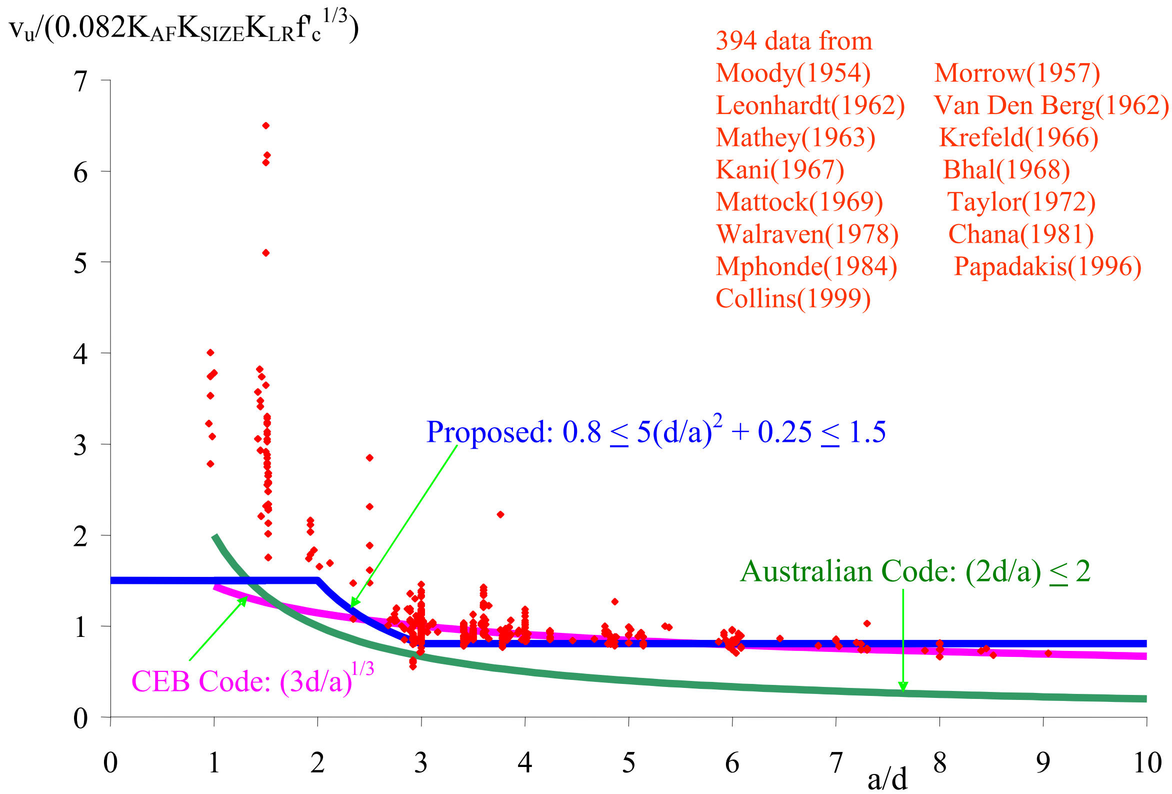

Figure 3 Proposed Shear Span Correction Factor Formula [2, 14-27]

PROPOSED SHEAR DESIGN EQUATIONS

The nominal shear resistance, V_n, is determined as:

$V_n = V_c + V_s + V_p$ (LRFD Eq. 5.8.3.3-1) [28] (5)

but not greater than

$V_n = 0.25f'_c b_v d_v + V_p$ (LRFD Eq. 5.8.3.3-2) [28] (6)

Equation (6) represents the upper limit of V_n to assure that the concrete in the web will not crush prior to yield of the transverse reinforcement. V_p is component of the effective prestressing force in the direction of the applied shear. The concrete contribution to the nominal shear resistance is computed using the following equation:

CONCRETE CONTRIBUTION TO SHEAR CAPACITY

The contribution of the concrete is given by the equation:

$V_c = (K_{AF}K_{SIZE}K_{LR}K_{LW})V_{CS}$ (7)

where $V_{CS} = 0.082\beta_{SS}\sqrt[3]{f'_c}\,b_v d_v$ (8)

β_{SS} = shear span correction factor as given by Equation (4a) and (4b)
K_{AF} = axial force correction factor; ranges from 1 to 1.75
K_{SIZE} = member size correction factor; ranges from 0.7 to 1.3
K_{LR} = longitudinal reinforcement correction factor; ranges from 0.7 to 1.3
K_{LW} = concrete weight correction factor; 0.75, 0.85 or 1.0

WEB REINFORCEMENT CONTRIBUTION TO SHEAR CAPACITY

The contribution of the web reinforcement is given by the general equation:

$$V_s = \frac{A_v\, f_y\, d_v}{s}(\cot\theta + \cot\alpha)\sin\alpha \qquad \text{(LRFD Eq. 5.8.3.3-4)}^{28} \qquad (9)$$

where θ = angle of inclination of the diagonal crack (degree)
α = angle of the web reinforcement relative to the horizontal beam axis (degree)
s = stirrup spacing in the direction of horizontal beam axis (in.)

ANGLE OF INCLINATION OF THE DIAGONAL CRACK

In 1996, the FIP [29] proposed a formula to calculate the inclined crack angle for members with axial compression or prestress as follow.

$$\cot\theta = 1.20 + 0.2\frac{N_u}{A_c f_t} \qquad (10)$$

Equation (10) is plotted in relationship between inclined crack angle, θ, and the term of $\frac{N_u}{A_c f_t}$ as show in Figure 4. The authors have modified Equation (10) as follow:

$$\theta = 45\left(1 - 0.21\sqrt{\frac{N_u}{A_c f_t}}\right) \qquad (11)$$

Equation (11) is proposed to make consistent between the inclined crack angle, 45 degree, generally used for reinforced concrete beams and the flatter angles used for concrete beams subjected to axial force. As shown in Figure 4, Equation (11) agrees well with Equation (10). The equation can be further modified in term of prestressing force and concrete compressive strength, f'_c, as:

$$\theta = 45\left(1 - 0.75\sqrt{\frac{A_{ps} f_{pe}}{A_c f'_c}}\right) \qquad (12)$$

where Ac = concrete area subjected to prestressing force or axial compression force

Note that for reinforced concrete members subjected to external axial compression, $A_{ps}f_{pe}$ will be replaced by N_u in Equation (10) to obtain θ. If the reinforced concrete members subjected to external axial tension, θ is equal to 45 degree.

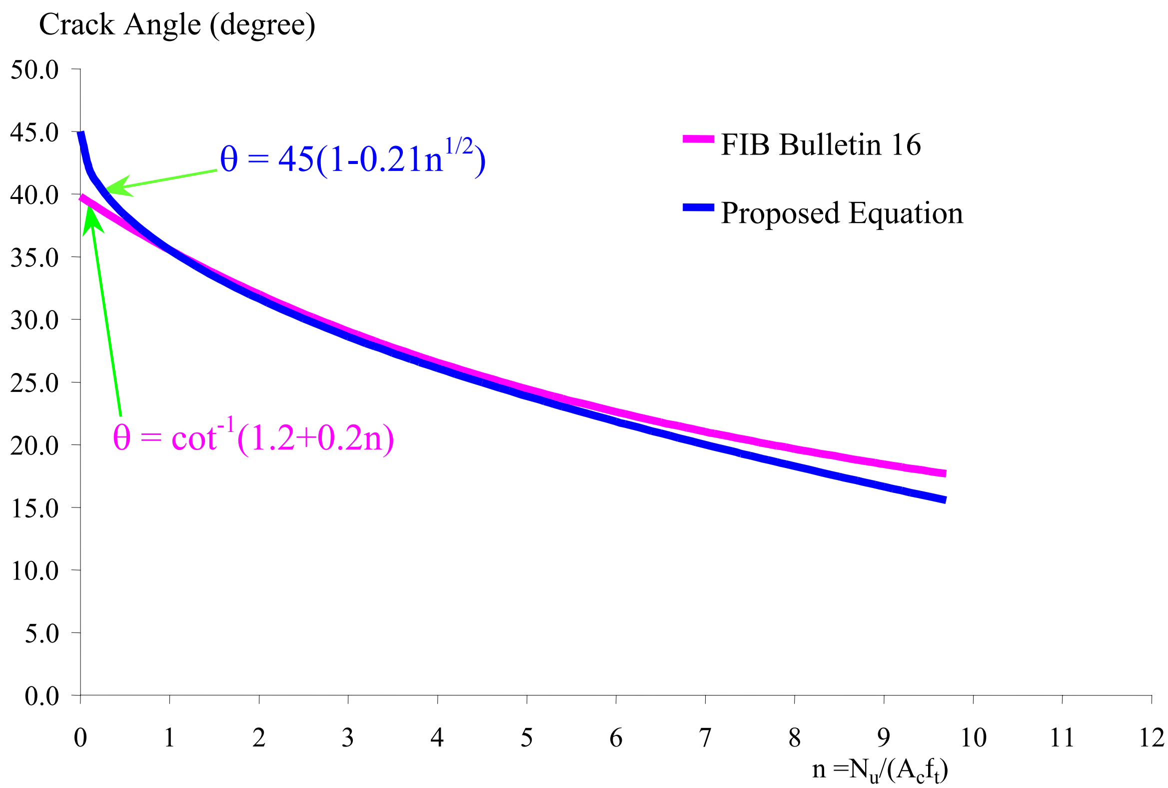

Figure 4 Relationship between Inclined Crack Angle and Ratio of Axial Stress to Concrete Tensile Stress

LONGITUDINAL REINFORCEMENT ANCHORAGE REQUIREMENTS

According to section 5.8.3.5 of AASHTO LRFD Bridge Design Specifications [28], tension force at each section shall not be greater than the capacity of the flexural tensile reinforcement. If the strands are cut at beam end, required tensile anchorage capacity may not be adequate. One can do the following options:

1. Accept reduced longitudinal capacity and increase θ and stirrups to compensate the provided longitudinal reinforcement.
2. Embed strands in a concrete diaphragm at abutment.

This paper proposes to embed non-tensioned strands into the end diaphragm to meet anchorage requirement. The detail proposal of this method can be found in Reference 30. According to section 5.8.3.5 of the AASHTO LRFD Bridge Design Specification [28], the tensile capacity of the reinforcement of the flexural tension side has to be greater than or equal to the tensile force at the considered section calculated as:

$$T \geq \frac{M_u}{\varphi d_v} + 0.5\frac{N_u}{\varphi} + \left(\frac{V_u}{\varphi} - 0.5V_s - V_p\right)\cot\theta \qquad \text{(LRFD Eq. 5.8.3.5-1)}^{28} \qquad (13)$$

where T = tension force in longitudinal reinforcement (kip)

M_u = factored moment at section corresponding to factored shear force (in-kip)
N_u = applied factored axial force (kip)
d_v = effective shear depth (in.)
ϕ = resistant factor for shear
V_u = factored shear force at critical section (kip)
V_s = shear resistance provided by shear reinforcement at given section (kip)
V_p = component of the effective prestressing force in the direction of the applied shear (kip)
θ = angle of inclination of diagonal compressive stress (degree)

To calculate the number of bent strands, the tensile force in the reinforcement is set to equal the tensile force developed by strands as:

$$T = A_{ps} f_{ps} \tag{14}$$

where A_{ps} = total cross-sectional area of embedded bent strand (in.^{2})
f_{ps} = stress in embedded strands at first crack location (ksi)

Note that $f_{ps} = 0.8f_{pu}$ (216 ksi) may be used if a total length of 60 strand diameters is embedded into abutment diaphragm. Otherwise the proposed f_{ps} formula can be found in Reference 30.

CONCLUSIONS

AASHTO LRFD shear design has good features, including: (1) Anchorage of longitudinal reinforcement, (2) High maximum shear limit, and (3) Variable compression strut angle. The most difficult and least significant variable in design of I-beam, and similar thin-web members, is concrete contribution V_c. A proposed non-iterative calculation of V_c is offered. It takes into account more significant factors than AASHTO LRFD Method. A non-iterative procedure and a simple formula for inclined crack angle, including effect of axial force, are proposed. In addition, embedding strands into abutment diaphragms are is highly effective in satisfying longitudinal reinforcement anchorage requirement.

REFERENCES

1. ACI-ASCE Committee 426, "The Shear Strength of Reinforced Concrete Members," *Journal of Structural Division*, ASCE, 99 (ST6), 1973, 1091-1187.
2. Mphonde, A.G. and Frantz, G.C., "Shear Tests of High- and Low-strength Concrete Beams without Stirrups," *ACI Journal*, 81 (4), July-Aug 1984, 350-357.
3. Elzanaty, A. H. et al., "Shear capacity of prestressed concrete beams using high-strength concrete," *ACI Journal*, 83 (3), 1986, 359-368.
4. Ahmad, S. H. et al., "Shear Capacity of Reinforced High-strength Concrete Beams," *ACI Journal*, 83 (2), 1986, 297-305.
5. Roller, J.J. and Russell, H.G., "Shear Strength of High-Strength Concrete Beams with Web Reinforcement." *ACI Structural Journal*, 87 (2), 1990, 191-198.

6. Tadros, M., Yehia, S., "Shear Limit of NU I-Beams," *Nebraska Department of Roads, Project Number SPR-PL-1 (35) P517*, University of Nebraska, November 2001, 379 pp.
7. ACI Committee 318, *Building Code Requirements for Structural Concrete (318-02) and Commentary (318R-02)* American Concrete Institute, Farmington Hills, MI
8. Warner, R., Rangan, B., Hall, A., and Faulkes, K., *Concrete Structures*. Longman, South Melbourne, Australia, 1998.
9. Japanese Concrete Committee, *Standard Specification for Design and Construction of Concrete Structures-1986* Part 1 (Design) Japan.
10. Jongpitakseel, N., *Behavior of End Zone of Precast Pretensioned Concrete Bridge Girders.* Dissertation presented to The University of Nebraska at Lincoln, USA, in partial fulfillment of the requirements for the degree of Doctor of Philosophy, May 2003.
11. Bazant, Z.P. and Kim, J.K., "Size Effect in Shear Failure of Reinforced Concrete Beams," *ACI Journal*, 81 (5), 1984, 456 - 468.
12. Bazant, Z.P. and Kazemani, M.T., "Size Effect on Diagonal Shear Failure of Beams without Stirrups," *ACI Structural Journal*, 88 (3), May-June 1991, 268-276.
13. Comite Euro-International du Beton (CEB), *CEB-FIP Model Code 1990* Bulletin D'Information No. 204, Lausanne, Switzerland, 1991
14. Moody, K., Viest, I., Elstner, R., and Hognestad, E., "Shear Strength of Reinforced Concrete Beams, Part 1-Test of Simple Beams," *ACI Journal Proceedings*, 51 (3), 1954, 317-332.
15. Morrow, J., and Viest, I., "Shear Strength of Reinforced Concrete Frame Members without Web Reinforcement." *ACI Journal*, 53 (9), 833-870
16. Leonhardt, F and Walther, R., "Schubversuche an einfeldrigen stahlbetonbalken mit und ohne schubbewehrung zur Ermittlung der Schubtragfahigkeit und der oberen schubspannungsgrenze." DAfStb, Heft 151, W. Ernst u. Sohn, Berlin, 1962. (in German)
17. Van Den Berg, F., "Shear Strength of Reinforced Concrete Beams without Web Reinforcement Part 2-Factors affecting Load at Diagonal Cracking." *ACI Journal*, 59 (11), November 1962, 1587-1600
18. Mathey, R. and Watstein, D., "Shear Strength of Beams without web Reinforcement," *ACI Journal, Procedings* 60 (2), February 1963, 183-208.
19. Krefeld, W., and Thurston, C., "Studies of Shear and Diagonal Tension Strength of Simply Supported Reinforced Concrete Beams," *ACI Journal Proceedings*, 63 (4), April 1966, 451-476.
20. Kani, G., "Basic Facts Concerning Shear Failure," *ACI Journal Proceedings*, 63 (6), June 1966, 675-692.
21. Bhal, N., "Uber den Einfluss der Balkenhohe auf Schubtragfahigkeit von einfeldrigen Stahlbetonbalken mit und ohne Schubbewehrung." PhD Dissertation, Universitat Stuttgart, Germany (in German) 1968.
22. Mattock, A. "Diagonal Tension Cracking in Concrete Beams with Axial Forces." *Journal Structural Division*, ASCE, 95 (9), 1969, 1887-1900.
23. Taylor, H., "Shear Strength of Large Beams," *Journal of Structural Division,* ASCE, 98 (11) November 1972, 2473-2490.
24. Walraven, J., "The Influence of Depth on the Shear Strength of Lightweight Concrete Beams without Shear Reinforcement." *Stevin Lab. Rep. No. 5-78-4*, Delft University of Technology, Delft, The Netherlands, 1978.

25. Chana, P., “Some Aspects of Modeling the Behavior of Reinforced Concrete under Shear Loading.” *Tech. Rep. No. 543*, Cement and Concrete Association, Wexham Spring, 1981.
26. Zararis, P., and Papadakis, G., “Diagonal Shear Failure and Size Effect in RC Beams without Web Reinforcement,” *J. of Structural Engineering*, ASCE, July 2001, 733-742.
27. Collins, M. and Kuchma, D., “How Safe Are Our Large, Slightly Reinforced Concrete Beams, Slabs, and Footings?,” *ACI Structural Journal*, 96 (4), 1999, 482-490
28. AASHTO LRFD 1998 and 2000-2002 Interim Bridge Design Specifications, Second Edition, American Association of State Highway and Transportation Officials, Washington, DC.
29. CEP-FIP *Design Examples for the 1996 FIP recommendations* ‘Practical Design of Structural Concrete’ by Reineck, K., Technical Report Bulletin 16, January, 2002
30. Noppakunwijai, P., Jongpitakseel, N., Ma, Z., Yehia, S., and Tadros, M., “Pullout Capacity of Non-Prestressed Bent Strands for Prestressed Concrete Girders” *PCI Journal*, 47 (4), July-August 2002
31. Precast/Prestressed Concrete Institute, *Precast Prestressed Concrete Bridge Design Manual*, Chicago, IL, 1997.

DESIGN EXAMPLE

This example illustrates in detail the design of a typical interior beam at the critical section in shear based on the new proposed method. This example is adapted from Example 9.4 of PCI Bridge Design Manual [31]. In addition to calculation of shear design at the critical section, Figure 6 shows comparison on A_v/s between the AASHTO LRFD 1994, AASHTO LRFD 2002, AASHTO Standard, and the proposed method.

GIVEN INFORMATION

Cast in place slab:	Total thickness	=	8.0	in.
	Structural thickness	=	7.5	in.
	Concrete strength at 28-days	f'_c =	4,000	psi
Precast beams: AASHTO-PCI 72 inch bulb-tee				
	Concrete strength at release	f'_{ci} =	5,800	psi
	Concrete strength at 28-days	f'_c =	6,500	psi
	Concrete unit weight	=	150	pcf
	Overall beam length	=	121.0	ft
	Design span	=	120.0	ft

Cross section properties:

non-composite	$A = 767$ in.^{2}	composite	$A_c = 1419$ in.^{2}
	$I = 545894$ in.^{4}		$I_c = 1100320$ in.^{4}
	$y_b = 36.60$ in.		$y_{bc} = 54.77$ in.
	$S_b = 14915$ in^3.		$S_{bc} = 20090$ in^3.
	$h = 72$ in.		$h_c = 80$ in.
	$b_v = 6.0$ in.		$b_e = 108$ in.

Pretensioning strands: 0.5 in. diameter, seven wire, low relaxation, 270 ksi
Number of strands = 48 with 12 strands being draped.
Cross sectional area of one strand = 0.153 in.^{2}
The effective final prestress = 149.0 ksi.
Factored shear and moment at the critical section (d_v from face of support):
V_U = 321.8 kips M_U = 1803.4 ft-kips

SOLUTION

Calculations are shown here for only one section, which is the critical section at a distance d_v from face of support.

where d_v = effective shear depth (= 57.95 in.)
= max (d_e-0.5a, 0.9d_e, 0.72h_c)
d_e = the corresponding effective depth from the extreme compression fiber to the centroid of the tensile force in the tensile reinforcement (= 60.58 in.)
a = depth of compression block (= 5.27 in.)
h_c = total height of the section (= 80 in.)

The contribution of the concrete to the nominal shear resistance is:

$$V_c = (K_{AF}K_{SIZE}K_{LR}K_{LW})V_{CS}$$

where $V_{CS} = 0.082\beta_{SS}\sqrt[3]{f'_c}\, b_v d_v$

Since $\frac{M_u}{V_u d_v} = \frac{1803.4x12}{321.8x57.95} = 1.16 < 3$, $\beta_{SS} = 5\left(\frac{321.8x57.95}{1803.4x12}\right)^2 + 0.25 = 3.96 > 1.5$ use 1.5

Thus, $V_c = (1.75x0.95x1.24x1.0)0.082x1.5x\sqrt[3]{6.5}\,x6.0x57.95 = 164.5$ kips

where $K_{AF} = 1 + \frac{A_{ps}f_{pe}h}{3M_u} \le 1.75 \quad = 1 + \frac{48x0.153x149x72}{3x1803.4x12} \quad = 2.21$ Use 1.75

$$K_{SIZE} = \frac{150}{100 + d_v} \quad = \frac{150}{100 + 57.95} \quad = 0.95$$

$$K_{LR} = 4.5\left(\sqrt[3]{\frac{A_{ps} + A_s}{b_v d_v}}\right) \ge 0.7 \quad = 4.5\left(\sqrt[3]{\frac{48x0.153}{6.0x57.95}}\right) \quad = 1.24$$

K_{LW} = 1.0 for normal weight concrete

Check if $V_u > 0.5\,\phi\,(V_c + V_p)$ (LRFD Eq. 5.8.2.4-1)
where V_p = component of the effective prestressing force in the direction of the applied shear (= 23.4 kips)
ϕ = resistance factor (= 0.9)

$0.5\phi(V_c + V_p) = 0.5(0.9)(154.3 + 23.4) = 79.9$ kips < 321.8 kips. Therefore, transverse shear reinforcement must be provided. The contribution of the shear reinforcement to the nominal shear resistance is:

$V_s = \frac{A_v\, f_y\, d_v}{s}(\cot\theta + \cot\alpha)\sin\alpha$ (LRFD Eq. 5.8.3.3-4)

where A_v = area of shear reinforcement within a distance s (in.^{2})
s = spacing of stirrups, in.
f_y = yield strength of shear reinforcement, ksi
α = 90^o for vertical stirrups
θ = angle of inclination of diagonal crack

$$= 45\left(1 - 0.75\sqrt{\frac{A_{ps} f_{pe}}{A_c f_c^{'}}}\right)$$

$$= 45\left(1 - 0.75\sqrt{\frac{48x0.153x149.0}{767x6.5}}\right) = 29.2 \text{ degree}$$

Since $V_n = \frac{V_u}{\varphi} = (V_c + V_s + V_p)$ (LRFD Eq. 5.8.3.3-1)

$$V_s = \frac{321.8}{0.9} - 164.5 - 23.4 = 169.7 \text{ kips}$$

Therefore, area of shear reinforcement within a spacing (s) is

$$A_v = \frac{sV_s}{f_y\, d_v \cot\theta} = \frac{s(169.7)}{60x57.95x\cot 29.2} = 0.0273(s)$$

If s = 12 in., required A_v = 0.33 in.^{2}/ft
Check maximum spacing of transverse reinforcement
Check if $V_u \geq 0.1(f_c^{'}\, b_v\, d_v)$
$0.1(f_c' b_v d_v) = 0.1\ (6.5)\ (6)\ (57.95) = 226.0$ kips < 321.8 kips
Thus $s \leq$ 12 in. or $0.4d_v = 0.4 \times 57.95 = 23.2$ in.
The area of transverse reinforcement should not be less than

$$= 0.0316\sqrt{f_c^{'}}\frac{b_v s}{f_y} = 0.0316\sqrt{6.5}\frac{(6)(12)}{60} = 0.10 \text{ in}^2\text{/ft}$$ (LRFD Eq. 5.8.2.5-1)

Use # 4 bar double leg @ 12 in., A_v = 0.4 in^2/ft

Thus, $V_s = \frac{0.4\,(60)\,(57.95)\cot 29.2}{12} = 207.4$ kips

In order to assure that the concrete in the web of the beam will not crush prior to yield of the transverse reinforcement, AASHTO LRFD Specifications give an upper limit of V_n as follows:

$$V_n = 0.25\, f_c^{'}\, b_v\, d_v + V_p$$ (LRFD Eq. 5.8.3.3-2)

$$V_c + V_s \leq 0.25\, f_c^{'}\, b_v\, d_v$$

164.5 + 207.4 = 371.9 kips ≤ 0.25 (6.5)(6)(57.95) = 565.0 kips **OK**

THE REINFORCEMENT CAPACITY AT THE SECTIONS NEAR THE SUPPORTS

According to section 5.8.3.5 of the AASHTO LRFD Bridge Design Specification [28], the tensile capacity of the reinforcement of the flexural tension side has to be greater than or equal to the tensile force at the considered section calculated as:

$$T \geq \frac{M_u}{\varphi d_v} + 0.5\frac{N_u}{\varphi} + \left(\frac{V_u}{\varphi} - 0.5V_s - V_p\right)\cot\theta \qquad \text{(LRFD Eq. 5.8.3.5-1)}$$

As shown in Figure 5, the assumed crack plane crosses the centroid of the 36-straight strands at a distance of (6 + 4.22cot 29.2 = 13.55 in.) from the end of the beam. From section 9.4 of the PCI Bridge Design Manual [(31)], the following notation and values are used:

M_u = factored moment at section corresponding to factored shear force (= 331.9 ft-kips [by interpolation])

N_u = applied factored axial force, kips (= 0.0 kip)

d_v = effective shear depth (= 57.95 in.)

V_u= factored shear force at given section (= 345.2 kips [by interpolation])

According to AASHTO LRFD Section 5.5.4.2, ϕ = 1.0 for moment and axial tension.

Substituting all parameters:

$$T = \frac{M_u}{\varphi d_v} + 0.5\frac{N_u}{\varphi} + \left(\frac{V_u}{\varphi} - 0.5V_s - V_p\right)\cot\theta$$

$$= \frac{331.9}{57.95(1.0)} + 0 + \left(\frac{345.2}{0.9} - 0.5\times 207.4 - 23.4\right)\cot 29.2^{o}$$

$$= 464.6 \text{ kips}$$

Since the transfer length is 60 times the strand diameter, 30 in, the prestress in the strands along the transfer length is a fraction of the effective prestress, f_{pe}. If the strands were cut at the beam end, the prestress in the strands at the crack is only $(13.55/30)f_{pe}$= (13.55/30)(149)= 67.3 ksi.

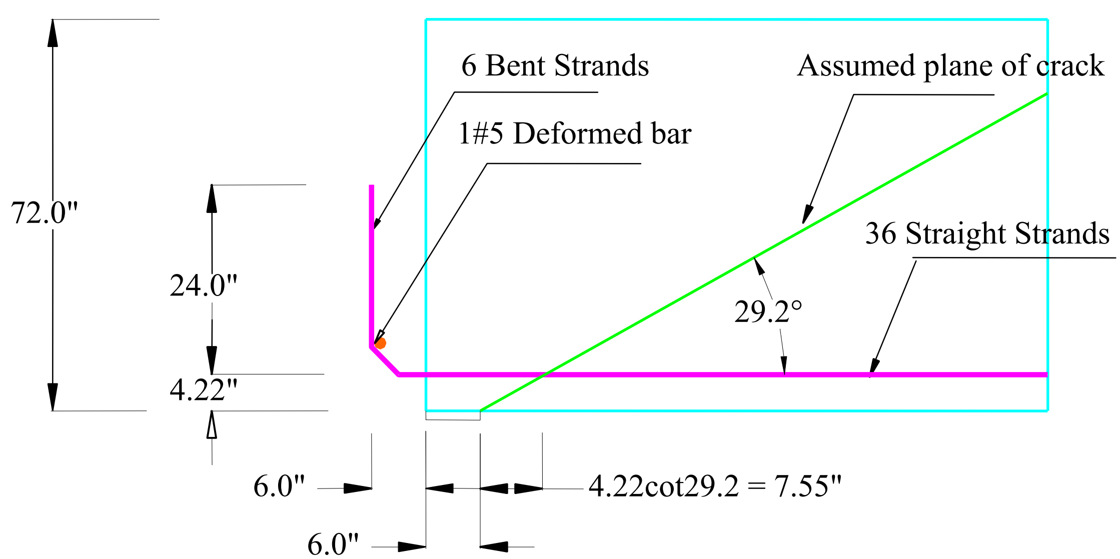

Figure 5 Bent Strand Details

Based on the bent strand pullout test study [30], if a total embedment length, L_e, of 30 in. with a horizontal embedment length, L_h, of 6 in. is bent into the diaphragms, the strand can develop a stress given by equation f_{ps} = (0.017x270x24/0.5) = 220 ksi, but not greater than $0.8f_{pu}$, 216 ksi.

Let n = number of bent strands

Since T ≤ The total tension capacity of embedded bent strands and straight strands

Thus, 464.6 kips ≤ n(0.153)(216) + (36-n)(0.153)(67.3)

n ≥ 4.1 strands

Therefore, bend 6 strands. The longitudinal force resisted by all strands is (6x0.153x216) + (36-6)(0.153x67.3) = 507.2 kips > 464.6 kips, which satisfies the requirement of Section 5.8.3.5 in the AASHTO LRFD Bridge Design Specifications.

Figure 6 gives a comparison between the results of the proposed method and those of other methods. Note that using the proposed methods requires no entry to tables, or charts and requires no iteration.

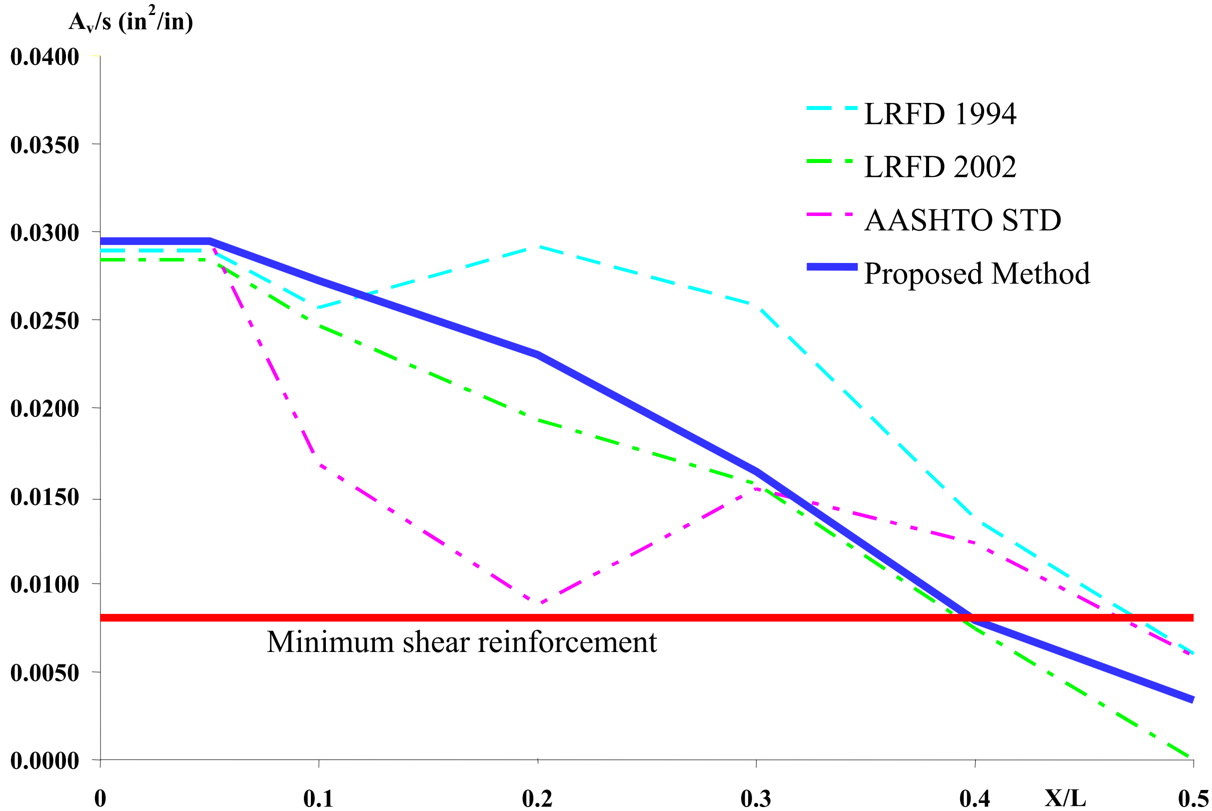

Figure 6 Comparison on A_v/s between AASHTO LRFD 1994, LRFD 2002, AASHTO Standard, and the proposed method