DEMYSTIFYING THE AASHTO LRFD CONCRETE SHEAR DESIGN PROVISIONS

Susan E. Hida, PE, California Department of Transportation, Sacramento, CA

ABSTRACT

The AASHTO LRFD Bridge Design Specifications[1] incorporate a simplified version of the modified compression field theory (MCFT) to design and analyze basic concrete members for shear. More effort can be required than when using the Standard Specifications for Design of Highway Bridges[2], but the author finds that the new provisions are feasible for routine use in bridge design offices, and appreciates being compelled to think about the internal flow of forces and resultant reinforcement required. AASHTO's "sectional method" results in shear-prevalent deep-beam members having more shear reinforcement than typical flexural members. This presentation attempts to guide bridge engineers through the new provisions in order to evaluate shear capacity and adjust reinforcement as required for shear demand in selected components. Examples are also provided.

Keywords: Shear, Modified-Compression-Field-Theory, Concrete Bridge Design, AASHTO LRFD

INTRODUCTION

The *AASHTO LRFD Bridge Design Specifications*[1] incorporate modified compression field theory (MCFT) where shear *capacity* is thought of in triangles: diagonal compressive stress in the concrete, stirrups or ties, and longitudinal reinforcement. Expressions required to estimate concrete shear capacity have been analytically derived based on force equilibrium, stress-strain relationships, and compatibility of deformations. For design of typical flexural members in bridges, though, the mechanics have been streamlined. The design procedure is referred to as the "sectional method" in *AASHTO LRFD Specifications*.

Shear *demand* causes strain and sometimes cracking. Application of the sectional method requires expression of the shear demand as stress, normalized for concrete strength (v_u/f'_c). The longitudinal strain (ε_x) is estimated, which in turn suggests a crack angle (θ) and a coefficient (β) in concrete capacity $V_c=\beta\sqrt{f'_c}b_v d_v$. A table correlating the demand to ε, β, and θ is provided. It is assumed that the angle of diagonal compressive stresses equals the resultant crack angle.

Longitudinal steel must be adequate to carry the horizontal component of the diagonal compression force i.e. shear capacity, as well as force due to flexure. While straight-forward for members exhibiting strictly beam behavior, steel requirements are uncustomary (large) for shear-prevalent members. The latter is illustrated in the case of a rigid frame bent cap. Strut-and-tie methods would have been more appropriate than the sectional method to model the given flow of forces, although steel requirements would still have exceeded those according to the *Standard Specifications*.

The changes to present practice can be overwhelming at the onset, but the time has come for bridge designers to consider the internal mechanism that affords shear capacity. Once one develops "a feel" for likely strains, crack angles, and resultant diagonal compression forces, "back-of-the-envelop" calculations are possible for flexural members. That is, one could approximate a value for β between 1.5 and 6.0, and easily evaluate V_c. The new provisions are feasible for routine use in today's design office where analytic tools such as spreadsheets are commonplace--and expected--and to the same extent that trigonometry tables and slide rules were a part of the practice decades ago.

This paper uses "10 Steps" to guide bridge engineers through the new provisions in order to evaluate shear capacity and adjust reinforcement as required to accommodate shear demand in undisturbed regions. Examples are provided for a conventionally reinforced bent cap and column, prestressed I-girder made continuous for live-loads, and an inverted-T bent cap. Recommendations for further investigation and implementation are made.

USING THE AASHTO LRFD SHEAR PROVISIONS

A "step-by-step" approach for using the *AASHTO LRFD Specifications 2nd Edition with '99, '00, '01, '02 Interim Revisions*[1] to evaluate shear at a given location is discussed below:

1. Determine the shear depth, d_v, measured perpendicular to the neutral axis between the resultants of the tensile and compressive forces due to flexure (capacity). The greatest of *0.9*d_e, *0.72h*, or *AASHTO* Eqn. C5.8.2.9-1, $d_v = \dfrac{M_n}{A_s f_y + A_{ps} f_{ps}}$, are suggested. The effective depth, d_e, is from flexure, *AASHTO* Eqn. 5.7.3.3.1-2, $d_e = \dfrac{A_{ps} f_{ps} d_p + A_s f_y d_s}{A_{ps} f_{ps} + A_s f_y}$.
   [Previous editions of the *Standard Specifications*[2] permitted d_v to be taken as 0.8*h* for prestressed members. The 0.72 factor in *LRFD* comes from 0.8 x 0.9.]

2. Calculate V_p, the vertical component of prestressing that contributes to shear capacity, if any.

3. Check that the shear width, b_v, where 25% of grouted duct width or 50% of ungrouted duct width has been deducted from the actual beam width, satisfies *AASHTO* Eqns. 5.8.2.1-2 and 5.8.3.3-2: $V_u \le V_r\ [= \varphi V_n = \varphi(0.25 f'_c b_v d_v + V_p)]$. φ, the resistance factor for shear, is 0.90 (*AASHTO* 5.5.4.2.1). Girders must often be flared adjacent to supports. This ensures a ductile failure in the shear reinforcing prior to crushing of the web.

4. Evaluate shear stress, $v = \dfrac{V_u - \varphi V_p}{\varphi b_v d_v}$ per *AASHTO* Eqn. 5.8.2.9-1. Divide by concrete strength f'_c, to get v/f'_c ratio.

5. Use an estimated value for longitudinal strain, ε_x, and the previously calculated value for v/f'_c, to identify a value for crack angle θ, in Table 5.8.3.4.2-1 of the *AASHTO LRFD Specifications*. Or, assume that θ=26.5° (which makes $0.5\cot\theta$=1.0), as noted in the Commentary to Article 5.8.3.4.2 ('03 Interims), and proceed to the next step.

6. Calculate strain, ε_x--which is the ratio of the vertical-to-horizontal forces, and then V_c:

$$\varepsilon_x = \left[\frac{\dfrac{M_u}{d_v} + 0.5N_u + 0.5(V_u - V_p)\cot\theta - A_{ps} f_{po}}{2(E_s A_s + E_p A_{ps})} \right] \le 0.002 \quad (\textit{AASHTO}\ \text{Eqn. } 5.8.3.4.2\text{-}1)$$

$$\varepsilon_x = \left[\frac{\dfrac{M_u}{d_v} + 0.5N_u + 0.5(V_u - V_p)\cot\theta - A_{ps} f_{po}}{2(E_c A_c + E_s A_s + E_p A_{ps})} \right] \le 0.002 \quad (\textit{AASHTO}\ \text{Eqn. } 5.8.3.4.2\text{-}3)$$

   If the sum of the vertical forces (numerator) is negative, the section is in compression and the concrete contribution must be considered in the denominator; the second of the two equations apply. The factor of 1/2 is due to taking strain at mid-height. For f_{po}, the tendons' modulus of elasticity multiplied by strain difference with surrounding concrete, use $0.7f_{pu}$ for usual levels of prestressing. When rating or evaluating existing structures that contain less than the minimum amount of shear reinforcement, use *AASHTO* Eqn. 5.8.3.4.2-2 and Table 5.8.3.4.2-2.

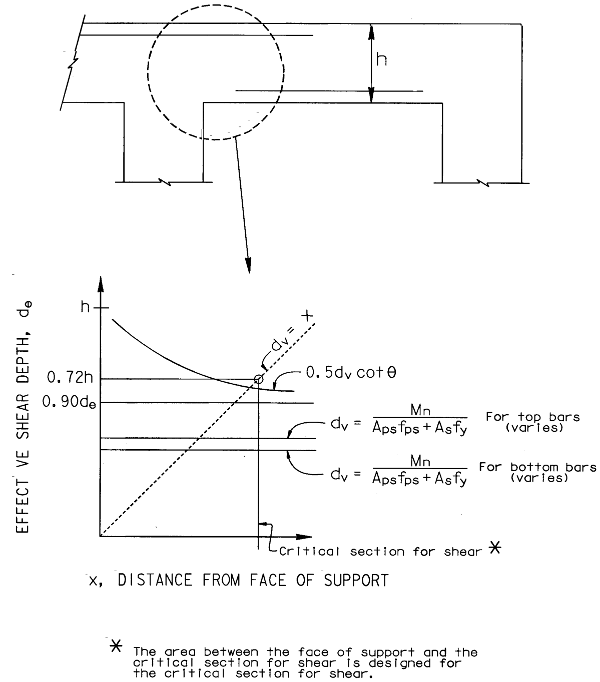
If the calculated value for ε_x is not in close approximation to the estimated value, recalculate ε_x using the new value for θ indicated on the table by the new value for strain. Convergence should take place in one iteration. Skip this check and recalculation if the "0.5cotθ=1.0" assumption had been made. In any case, note the corresponding value for β, in Table 5.8.3.4.2-1, and calculate

$$V_c = 0.0316\beta\sqrt{f'_c}\,b_v d_v \qquad (\textit{AASHTO}\text{ Eqn 5.8.3.3-3})$$

7. Determine shear strength needed from stirrups, $V_s = V_u - \varphi(V_p + V_c)$ (*AASHTO* Eqn. 5.8.3.3-2). Solve for $\frac{A_v}{s} \geq \frac{V_s}{\varphi f_y d_v(\cot\theta + \cot\alpha)\sin\alpha}$, *AASHTO* Eqn. 5.8.3.3-4 where α is the stirrup angle from horizontal. Select stirrup size and spacing. Check maximum spacing as directed in AASHTO 5.8.2.7:
$$s < s_{max} \begin{cases} = 0.8d_v < 24.0 \text{ in. for } v_u < 0.125f'_c \\ = 0.4d_v < 12.0 \text{ in. for } v_u \geq 0.125f'_c \end{cases}$$
   Check minimum reinforcement, $A_v = 0.0316\sqrt{f'_c}\,\frac{b_v s}{f_y}$ (*AASHTO* Eqn. 5.8.2.5-1).
8. Check that the longitudinal steel can develop the necessary tensile capacity for bending and shear, that is $A_{ps}f_{ps} + A_s f_y \geq \frac{M_u}{d_v\phi} + 0.5\frac{N_u}{\phi} + \left(\frac{V_u}{\phi} - 0.5V_s - V_p\right)\cot\theta$ (*AASHTO* Eqn. 5.8.3.5-1).. It is not necessary to provide bottom steel greater than that required for flexure at maximum moment locations if the loads causing the moment are "direct" i.e. applied on top of the girder. Also, it is not necessary to provide top steel greater than that required for flexure at direct supports such as bearings, or columns in a framed structure. In other words, this check does not apply where prospective cracking would be vertical.

   When approaching the point-of-inflection in continuous members, the designer is cautioned that the section is analyzed based on flexural tension either on the top or the bottom face, even though values for bending moment are small. Rigorous evaluation would require that the values for shear be checked for associated or maximum positive and negative moments. It may be necessary to either add stirrups or longitudinal steel.

9. The region between the face-of-support and the point of controlling shear need only be designed for the controlling point. To determine where this reduction applies, compare distance to the face-of-support, with the larger of d_v and $0.5d_v cot\theta$. If the distance is less, the shear capacity may be based on that from the controlling point. See Fig. 1. Note that this step is not addressed in the design examples shown here.



CRITICAL SECTION FOR SHEAR
(AASHTO LRFD 5.8.3.2)

10. Where force effects due to torsion are present and $T_u \geq 0.25\varphi T_{cr}$ (*AASHTO* Eqn 5.8.2.1-3, 10. check that shear reinforcement satisfies requirements for combined shear and torsion (*AASHTO* 5.8.3.6).

11. If applicable, one should check for horizontal shear capacity. In the case of girders, 11. demand will be greatest near supports; but, resistance is also high due to closely-spaced stirrups. At midspan, shear demand is low; but stirrup-spacing and hence resistance is also low. Capacity is per *AASHTO* Eqn. 5.8.4.1-1, $V_n = cA_{cv}+\mu[A_{cf}f_y+P_c]$ where A_{cv}= area of concrete engaged in shear transfer, A_{vf}= area of shear reinforcement crossing the shear plane, c= cohesion factor, μ= friction factor, P_c= permanent net compressive force normal to the shear plane. Alternatively, either *AASHTO* Eqn. 5.8.4.1-4 may be satisfied in the case of beam-slab interfaces: $A_{vf} \geq \dfrac{0.05b_v}{f_y}$, where b_v is the width of interface; or, the requirement waived if V_n/A_{cv} is less than 0.100 ksi. In any event, check to see that *AASHTO* Eqns. 5.8.4.1-2,3 aren't exceeded: $V_n<0.2f'_cA_{cv}$, and $V_n<0.8A_{cv}$.

EXAMPLE: BENT CAP

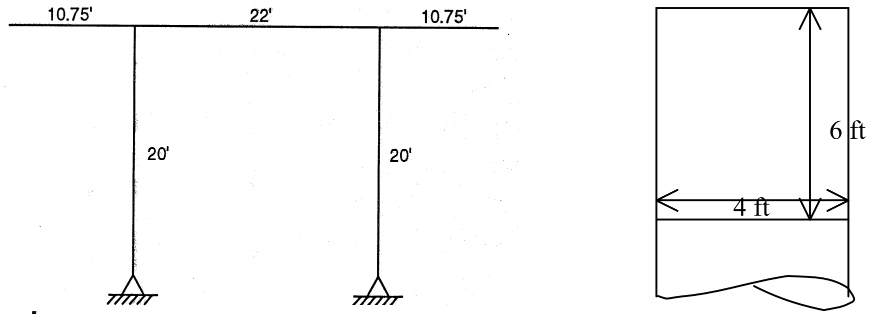


Fig. 2 Bent Cap Elevation and Section

Given:

- 4-ft diameter columns. Ignore joint-shear requirements for seismic.
- Live-loads, one lane including dynamic load allowance (*IM*): HL93--310 kips
- top reinforcing A_s, 10 in.^{2}/girder
- bottom reinforcing A_s, 6 in.^{2}/girder
- Table 1, below

Table 1 Force Effects, $V_u = 1.25V_{DL}+1.75V_{LL}$

Location	V_{DL} *(kips)*	Max V_{LL}, (controlling case)	$\lvert V_u \rvert$ *(kips)*	*M* *assoc-DL (ft-k)*	*M* *assoc-LL (ft-k)*	$\lvert M \rvert$ *assoc-ult (ft-k)*
d_v ft from left face of column	26	155 (1 lane on cantilever)	304	-62	-159	356
Left face of column	307	155 (1 lane on cantilever)	655	-730	-770	2260
Centerline of column	452	310 (2 lanes centered)	1108	1407	167	2051
Right face of column	435	310 (2 lanes centered)	1086	-536	-500	1545
d_v ft from right face of column	166	254 (2 lanes straddling over the column)	652	378	-17	502
Midspan	140	56 (2 lanes straddling over the column)	273	1125	534	2340

1. Determine shear depth, d_v (*AASHTO* 5.8.2.9)

   The effective shear depth is taken as the distance, measured perpendicular to the neutral axis, between the resultants of the tensile and compressive forces due to flexure. It need not be taken to be less than the greater of $0.9d_e$ or $0.72h$. Alternatively, $d_v=M_n/A_sf_y$. Here,

   - $d_e = h - \frac{a}{2} - cover - 0.5*(bar\ diam.) = 72 - \frac{3.43}{2} - 0.5*1.41 = 67.6\ in.$
   - d_v need not be less than the greater of
     0.9*67.6 *in.*=60.6 *in.*, and
     0.72*72 *in.*=51.8 *in.*
   - Alternatively,

$$d_v = \frac{M_n}{A_s f_y} = \frac{3016/0.9}{10*60} = 67.0\ in.\text{(top)}$$

$$d_v = \frac{M_n}{A_s f_y} = \frac{1843/0.9}{6*60} = 68.2\ in.\text{(bottom)}$$

Proceed using d_v=60.6 in. (top and bottom) to be conservative.

2. Calculate V_p (no prestressing in this member; step 2 doesn't apply).

3. Calculate shear width, b_v .

   Check that $V_u \le \phi V_n$ where $V_n = 0.25f'_c b_v d_v + V_p$ (*AASHTO* Eqn. 5.8.3.3-2)
   ϕV_n= 0.9 * 0.25 * 4 * 48 * 60.6 = *2618 kips*
   Since V_u = 1108 *kips* (centerline of column), which is < 2618 *kips*, b_v is adequate; proceed with design.

4. Calculate shear stress.

   Initially assume ε is 0.25x10 $^{-3}$. Calculate shear stress, normalized for its strength:
   $\frac{v}{f_c^{'}} = \frac{V_u}{\varphi f_c^{'} b_v d_v}$. (Shown in table, below.)

4. Then read θ, the angle of inclination of the diagonal compressive stresses, from Table 5.8.3.4.2-1.

Table 2 Data for Arriving at Crack Angle, θ

Location	V_u (kips)	$v/f'c$	M_u (ft-k)	θ (degrees)
d_v from face of column	304	0.029	356	26.6
Left face of column	655	0.063	2260	26.6
Right face of column	1086	0.104	1545	27.1
d_v from face of column	652	0.062	502	26.6
Midspan	273	0.026	2340	26.6

5. Calculate strain (*AASHTO* Eqn. 5.8.3.4.2-2), crack angle, coefficient β, concrete capacity.

$$\varepsilon_x = \left[\frac{\frac{M_u}{d_v} + 0.5N_u + 0.5(V_u - V_p)\cot\theta - A_{ps} f_{po}}{2*(E_s A_s + E_p A_{ps})} \right]$$

At d_v from the right face of column: $\varepsilon_x = \left[\frac{\frac{502}{60.6/12} + 0.5*652*\cot 27.1}{2*29{,}000*10.0} \right]$. Since this is higher than the original value of 0.00025 assumed for ε, re-enter the table, see that θ reads 36^{o}, revise the value for θ to 36^{o} in the previous equation, and recalculate strain.

Table 3 Data for Arriving at Strain, ε

Location	V_u (kips)	M_u (ft-k)	ε_ξ x10^{-3} (in./in.)	θ (degrees)	rev. ε (in./in.)
d_v from face of column	304	356	.0006	34	0.0005
Left face of column	655	2260	.0019	36	0.0015
Right face of column	1086	1545	.0024	36	0.0018
d_v from face of column	652	502	.0013	36	0.0009
midspan	273	2340	.0013	36	0.0011

Notes:

- M_u is always taken as positive in *AASHTO* Eqns. 5.8.3.3-1,2,3, however near the point of inflection both maximum and minimum M_u can be checked to know the crack angle from both the top and bottom faces.
- The values in Table 5.8.3.4.2-1 were revised in the '00 Interims to be less conservative than those in the 2nd Edition.
- The ε=0.0015, 0.0020 columns of values are being deleted in the '03 interims.

To be conservative, select a value for β from the next highest ε-column of *AASHTO* Table 5.8.3.4.2-1, rather than the next lowest or interpolating. A higher value for strain means a lower value for β, which means more stirrups-- which means conservatism. Finally, $V_c=0.0316\beta\sqrt{f'_c}b_vd_v$ (*AASHTO* Eqn. 5.8.3.3-3) is calculated in the table below, where ϕ=0.90 for shear. (The last column in the table below will be used in the next step.)

Table 4 Data for Arriving at V_c and V_s

Location	β	V_u/φ (kips)	V_c (kips)	V_s reqd. (kips)
d_v from face of column	2.59	338	476	--
Face of column	2.23	728	410	318
Face of column	2.23	1210	410	800
d_v from face of column	2.23	558	410	148
Midspan	2.23	303	410	--

7. Deterimine shear reinforcing

Minimum reinforcement, $A_v = 0.0316\sqrt{f'_c}\dfrac{b_v s}{f_y} = 0.0316 * 2.0 * \dfrac{48*12}{60} = 0.61\ in.^2$

(*AASHTO* Eqn. 5.8.2.5-1).

Solve $V_n=V_c+V_s+V_p$ (*AASHTO* Eqn. 5.8.3.3-1) and $V_u \leq \phi V_n$ for V_s. This is done in the table above.

Rearrange *AASHTO* Eqn. 5.8.3.3-4, $V_s = \dfrac{A_v f_y d_v(\cot\theta + \cot\alpha)\sin\alpha}{s}$, and substitute in $\alpha=90^o$ for a vertical angle of stirrup inclination.

Try #5 @12 U. Then, A_v/s=0.62 $in.^2/ft$ = 0.052 $in.^2/in.$

At d_v from the face-of-column, V_s= 0.052 $in.^2/in.$ * 60ksi * 60.6 $in.$ * $cot36^o$ = <u>259 *kips*</u>, which is greater than the 148 kips required, so OK.

Maximum spacing depends on whether the shear stress demand is greater than or less than $0.125f'_c$ Here, 0.125 * 4 =0.050 ksi (*AASHTO* 5.8.2.7).

- At midspan, $\dfrac{V_u}{b_v d_v} = \dfrac{273}{48 * 60.6} = 0.094\ ksi$ > 0.050 ksi

$\Rightarrow s_{max}$= 0.4d_v =0.4*60.6 $\leq$ <u>12.0 in</u>.

- At d_v from rt face of column, $\dfrac{V_u}{b_v d_v} = \dfrac{652}{48 * 60.6} = 0.224\ ksi$ > 0.050 ksi

$\Rightarrow s_{max}$= 0.4d_v $\leq$ 0.4*60.6 = <u>12.0 in</u>

$\therefore$Use #5@12 U's.

8. Check longitudinal steel.

The previously designed flexural steel must be checked to see that it can also carry the required horizontal component of the diagonal compressive stresses for the V_c previously calculated. At d_v to the right face-of-column, the top steel is in tension when live load lanes are placed to cause maximum shear.

$T_{provided}$ (10 #8) = $A_s f_y$ = 10.0 * 60 = <u>600 k</u>. (top)

$T_{provided}$ (6 #8) = $A_s f_y$ = 6.0 * 60 = <u>360 k</u>. (bottom)

$$T_{reqd} = \frac{M_u}{\phi d_v} + 0.5\frac{N_u}{\phi} + \left(\frac{V_u}{\phi} - 0.5V_s\right)\cot\theta$$

For <u>top bars</u> at d_v from the face of the column,

$$= \frac{502}{0.9 * 5.05} + 0.5\frac{0}{\phi} + \left(\frac{652}{0.9} - 0.5 * 259\right)\cot 36$$

=110+819=929 kips NOT OK

Change to #5 @ 4 in. Then 110+473=583 kips OK

For <u>bottom bars</u> at midspan, T_{reqd} doesn't apply because loads are applied "directly" at this maximum moment location. Any cracking is vertical and due to flexure alone; diagonal tension is not an issue. Notes:

- This process must be repeated where ever attempting to increase stirrup spacing or discontinue flexural reinforcement, such as near the point of inflection.
- Near the point of inflection, this check should be run twice, once for M_u and θ for on top, and again for M_u and θ for on bottom.

- The increase in steel requirements near the point of inflection shows how deep members behave differently than typical girder elements. In this example, strut-and-tie methodology could have been used, as directed in *AASTHO* 5.8.1.1 for components where the distance from the face-of-support to the point of 0.0 shear, is less than twice the depth. However, steel requirements would still be in excess of those based on *Standard Specifications* due to consideration of the tensile component of V_c.

EXAMPLE: COLUMN

Given column loads $\underline{V_u=309\ kips,\ M_u=1790\ ft\text{-}kips.}$

1. Check effective shear depth: Use $0.72h$ because of difficulties in using $0.9d_e$ with circular section. (*AASHTO* 5.8.2.9) 0.72 * 4 = 2.88 = <u>34.56 in</u>.

2. Calculate V_p (no prestressing; step 2 doesn't apply).

3. Check that $V_u < \phi V_n$ when $\phi V_n = \phi 0.25 f'_c b_v d_v$ (*AASHTO* 5.8.3.3-2) Here,
   0.9*0.25*4 * say24in.*34.56 = 746 >>309 kips required so OK.

4. Evaluate shear stress ratio: $\dfrac{\upsilon_u}{f'_c} = \dfrac{V_u / A}{f'_c} = \dfrac{309/65{,}111}{4} = 0.0012$

5. Pick θ off of Table 5.8.3.4.2-1, assuming ε=0. $\Rightarrow \theta$=21.8°

6. Calculate strain using *AASHTO* Eqn. 5.8.3.4.2-1. Calculate V_c using *AASHTO* Eqn. 5.8.3.3-3

$$\varepsilon_x = \left[\frac{\dfrac{M_u}{d_v} + 0.5N_u + 0.5(V_u - V_p)\cot\theta - A_{ps} f_{po}}{2*(E_s A_s + E_p A_{ps})} \right]$$

$$= \left[\frac{\dfrac{1790}{3} + 0.5(309)\cot 21.8}{2*(29{,}000*10*1.27)} \right] = 0.0005$$

Revise estimate for θ to 30.5°. Then ε =0.0002 and β = 2.94.

$V_c = 0.0316\beta\sqrt{f'_c}\,b_v d_v$=0.0316*2.94*2*24*35= <u>*156 kips.*</u>

Table 5 - Values of θ and β for Sections with Transverse Reinforcement--From *AASHTO LRFD Specifications, Table 5.8.3.4.2-1*

v_u/f'_c	$\varepsilon_x x10^3$								
	$\leq$-0.20	$\leq$-0.10	$\leq$-0.05	$\leq$0	$\leq$0.125	$\leq$0.25	$\leq$0.50	$\leq$0.75	$\leq$1.00
$\leq$0.075	22.3 6.32	20.4 4.75	21.0 4.10	21.8 3.75	24.3 3.24	26.6 2.94	30.5 2.59	33.7 2.38	36.4 2.23
$\leq$0.100	18.1 3.79	20.4 3.38	21.4 3.24	22.5 3.14	24.9 2.91	27.1 2.75	30.8 2.50	34.0 2.32	36.7 2.18
$\leq$0.125	19.9 3.18	21.9 2.99	22.8 2.94	23.7 2.87	25.9 2.74	27.9 2.62	31.4 2.42	34.4 2.26	37.0 2.13
$\leq$0.150	21.6 2.88	23.3 2.79	24.2 2.78	25.0 2.72	26.9 2.60	28.8 2.52	32.1 2.36	34.9 2.21	37.3 2.08
$\leq$0.175	23.2 2.73	24.7 2.66	25.5 2.65	26.2 2.60	28.0 2.52	29.7 2.44	32.7 2.28	35.2 2.14	36.8 1.96
$\leq$0.200	24.7 2.63	26.1 2.59	26.7 2.52	27.4 2.51	29.0 2.43	30.6 2.37	32.8 2.14	34.5 1.94	36.1 1.79
$\leq$0.225	26.1 2.53	27.3 2.45	27.9 2.42	28.5 2.40	30.0 2.34	30.8 2.14	32.3 1.86	34.0 1.73	35.7 1.64
$\leq$0.250	27.5 2.39	28.6 2.39	29.1 2.33	29.7 2.33	30.6 2.12	31.3 1.93	32.8 1.70	34.3 1.58	35.8 1.50

7. Evaluate V_s

- $V_s = V_u - \phi V_c$ (rearrangement of AASHTO Eqn. 5.8.3.3-1): 309-0.9*156k = 169 kips
- Solve for V_s using *AASHTO* Eqn. 5.8.3.3-4:

$$V_s = \frac{A_v f_y d_v (\cot\theta + \cot\alpha)\sin\alpha}{s}$$

$$\frac{A_v}{s} = \frac{V_s}{f_y d_v \cot\theta} = \frac{169}{60*35*\cot 26.6} = 0.04 in.^2 / in = 0.48 in.^2 / ft$$

- Could use #5 spiral at 15 in., but check maximum spacing. If $v_u \geq 0.125f'_c$, then s_{max}= $0.4d_v$ or 12 in. maximum(*AASHTO* Eqn. 5.8.2.7-2). Here, 309kips/1809in.^{2}=0.171 ksi > 0.125*4ksi = 0.05ksi, so 12-in. spacing applies.
- Use #5 spiral at 12 in., unless extreme event load combinations require closer transverse spacing for confinement.

8. The longitudinal steel is not checked for *AASHTO* Eqn. 5.8.3.5-1 because longitudinal column bars are assumed to be adequate for flexure, and will all be continuous.

EXAMPLE: PRE-CAST, PRE-TENSIONED I-GIRDERS

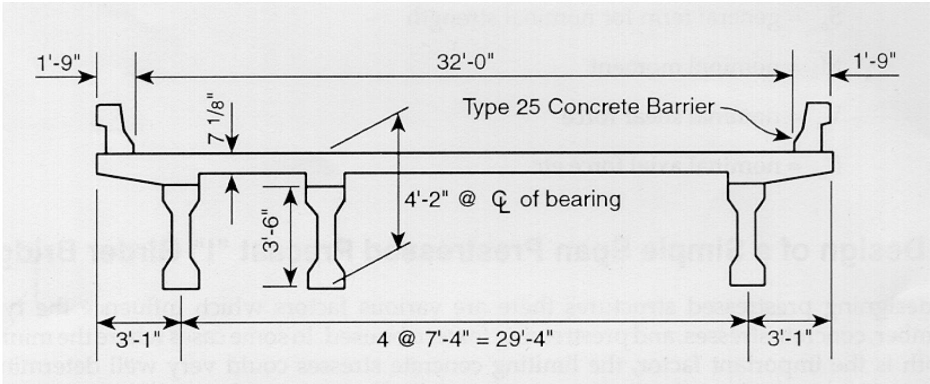


Fig. 3 Typical Section
(3-span, continuous for live loads; span length L, 68 ft)

Given (interior girder):

- initial prestressing force P_{jack}, 518 kips
- harping at third-points
- V_p, component of prestressing force in direction of the shear force, 33.8 kips (face-of-cap)
- girder web-width, 7 in.
- prestressing steel A_{ps}, 3.67 in.^{2}/girder
- mild reinforcing A_s, 12 in.^{2}/girder
- shear reinforcing is vertical, i.e. α=90^o
- Table 6, below

Table 6 Factored Force Effects

($0.9L_1$, int. gdr.)	Strength I	Strength II
V_{DL} (kips)	42	42
1.25*V_{DL} (kips)	53	53
V_{ADL} (kips)	13	13
1.5*V_{ADL} (kips)	20	20
V_{LL} (kips; with IM)	80	156
γ_{LL}*m_{gdf} # of lanes	1.75*0.77	1.35*0.77
V_{LL}* (kips; factored)	108	162
$V_u=\Sigma\gamma_i V_i$ (kips)	181	**235**

1. Check $V_n = 0.25 f'_c b_v d_v + V_p$, and $V_u = \phi V_n$ (*AASHTO* Eqn. 5.8.3.3-2)

   Rearranging, $b_v \geq \dfrac{V_u / \phi - V_p}{0.25 f'_c d_v}$, where

   - d_v =0.9*39.5 in. =35.6 in. vs. 0.72*50 in. = 36 in. Use <u>35.6 in.</u>
   - V_u from the previous table for a typical interior girder at $0.9L_1$.

$$b_v = \frac{\frac{235}{0.9} - 35.9}{0.25 * 5 * 35.6} = 5.06 \text{ in.,}$$

which is > 7 in. provided, so OK.

2. V_p = 33.8 kips (given)

3. Calculate concrete shear stress,

$$v = \frac{V_u - \varphi V_p}{\varphi b_v d_v} = \frac{235 - 0.90 * 33.8}{0.90 * 7 * 35.6} = \frac{205}{224} = 0.91\ ksi$$

$$v / f'_c = \underline{0.18}$$

4. Estimate shear strain.

   Enter *AASHTO* Table 5.8.3.4.2-1 using this value for v/f'_c and an assumed value for ε_x, longitudinal strain in the web reinforcement (flexural tension side of the member). Try ε_x = 0.0.

5. Note θ, the angle of inclination of diagonal compressive stresses, from the corresponding cell.

6. Calculate strain (*AASHTO* Eqn. 5.8.3.4.2-1, 2, 3), assuming numerator will be positive:

$$\varepsilon_x = \left[\frac{\frac{M_u}{d_v} + 0.5N_u + 0.5(V_u - V_p)\cot\theta - A_{ps} f_{po}}{2 * (E_s A_s + E_p A_{ps})} \right]$$

   where A_s is the area of nonprestressed steel on the flexural tension side of the member at the section under consideration. Bars which are terminated at a distance less than their development length from the section under consideration are to be ignored. M_u is taken as the bending moment associated with the maximum shear at the location in question.

$$\varepsilon = \left[\frac{\frac{1864}{35.6} + 0.5(235 - 35.9)\cot(26.2) - 3.36 * 189}{2 * (29{,}000 * 12 + 28{,}500 * 3.36)} \right] = \left[\frac{52.4 + 202 - 635}{887{,}520} \right] = -0.000429$$

   Since the above numerator is negative, *AASHTO* Eqn. 5.8.3.4.2-3 is required, which amounts to multiplying *AASHTO* Eqn. 5.8.3.4.2-1, above, by $\dfrac{E_s A_s + E_p A_{ps}}{E_c A_c + E_s A_s + E_{ps} A_{ps}}$

   where A_c is the area of concrete on the flexural tension side of the member. Use the area of the slab (7.125in.* 7.33ft * 12in./ft), plus $(c\text{-}t_s)*b_f$ for the area of the girder in compression (10.3-7.125)*19in. Get A_c = 695 in.^{2}

$$\frac{29{,}000 * 12 + 3.36 * 28{,}500}{3834 * (695) + 29{,}000 * 12 + 3.36 * 28{,}500} = 0.143$$

   The value for θ is also revised to 24.7, as implied by *AASHTO* Table 5.8.3.4.2-1 for the value for ε_x calculated above.

$$\varepsilon = \left[\frac{\frac{1864}{35.6} + 0.5(235 - 35.9)\cot(24.7) - 3.36 * 189}{2(29{,}000 * 12 + 28{,}500 * 3.36)} \right] * 0.143$$

=[(52.4+216-635)/887,520]*0.143= <u>-0.000059</u>

This is close to the previously calculated value for strain, so continue using θ=25°, and β=2.6. Note that the Designer could have assumed 0.5cotθ=1.0, thereby eliminating the iteration. [Commentary on this option is being added to Article 5.8.3.4 in the '03 Interims. The results do not change significantly.]

Finally, calculate the nominal shear resistance of concrete (V_c , *AASHTO* Eqn 5.8.3.3-3)

$V_c = 0.0316\beta\sqrt{f'_c}b_v d_v$=0.0316*2.6*√5 *7 * 35.6 = <u>46.8 kips</u>

[Note: In the case of post-tensioned girders, or pretensioned girders spliced together by post-tensioning, *AASHTO* 5.8.2.9 states that one-half the diameter of ungrouted ducts or one-quarter the diameter of grouted ducts shall be deducted from b_v. The author points out that this provision can significantly affect V_c and the resulting amount of shear reinforcing provided.]

7. Calculate shear reinforcing.

Stirrups required only if $V_u > 0.5\phi(V_c + V_p)$ (*AASHTO* Eqn 5.8.2.4-1), but better to provide minimum shear reinforcement, regardless. Solve $V_n = V_c + V_s + V_p$ (*AASHTO* Eqn 5.8.3.3-1) for V_s. In other words, the required contribution to shear capacity from the stirrups is: $\varphi V_s \geq V_u - \varphi(V_p + V_c)$

φV_s> 235-0.9(35.9+46.8) = 161 kips

Substitute into the given formula for capacity (*AASHTO* Eqn 5.8.3.3-1):

$$V_s = \frac{A_v f_y d_v (\cot\theta + \cot\alpha)\sin\alpha}{s}$$

and rearrange:

$$\frac{A_v}{s} \geq \frac{V_u - \varphi(V_p + V_c)}{\varphi f_y d_v (\cot\theta + \cot\alpha)\sin\alpha}$$

$$\frac{A_v}{s} \geq \frac{161}{0.9 * 60 * 35.6 * (\cot 24.7 + \cot 90)\sin 90}$$

=0.04 in.^{2} /in. = 0.46 in.^{2} /ft.

Minimum transverse reinforcement (*AASHTO* Eqn 5.8.2.5-1):

$$A_V = 0.0316\sqrt{f'_c}\frac{b_v s}{f_y}$$

Rearranging, minimum A_v/s =0.0316*√5 *7in./60ksi=0.008 in.^{2} /ft (OK)

Maximum spacing (*AASHTO* Eqn 5.8.2.7-1,2):

- If $V_u < 0.1 f_c^{'} b_v d_v$, then s=0.8d_v; 24.0 in. max.
- If $V_u \geq 0.1 f_c^{'} b_v d_v$, then s=0.4d_v; 12.0 in. max.

Substituting d_v =35.6 in., a maximum permitted spacing of 12 in. applies at 0.9L_1.
Use #5 @12 U's.

8. Evaluate longitudinal reinforcement

Assume that the deck slab prevents torsion. Then, check that longitudinal reinforcement is proportioned such that (*AASHTO* Eqn 5.8.3.5-1):

$$A_s f_y + A_{ps} f_{ps} \geq [\frac{M_u}{d_v \phi} + 0.5\frac{N_u}{\phi} + (\frac{V_u}{\phi} - 0.5V_s - V_p)\cot\theta]$$

$$12.0*60 + 3.67*202.5 \geq \frac{1864*12}{35.6*0.9} + 0.5\frac{0}{0.9} + \left(\frac{235}{0.9} - 0.5*179 - 35.9\right)\cot 25$$

720 +743 $\geq$ 698+0+291 = 988 kips OK at 0.9L

Only 1/3 of the negative reinforcement need be continuous beyond the point of inflection, and 2/3 of the steel may be shorter in length. **However, this reduced amount must be rechecked** in Eqn. 5.8.3.5-1 using V_u and V_s at the point of inflection (adjusting location for l_d), and where ever stirrup spacing is adjusted.

This check need not be done where prospective cracking would be vertical i.e. maximum moment location due to direct loading, or over a point of direct support as is the case of girders bearing on a drop cap. However, if the ends are dapped and sit on an inverted-T bent cap, support is mid-height in the member and therefore indirect. Longitudinal reinforcement must be checked for additional shear demand.

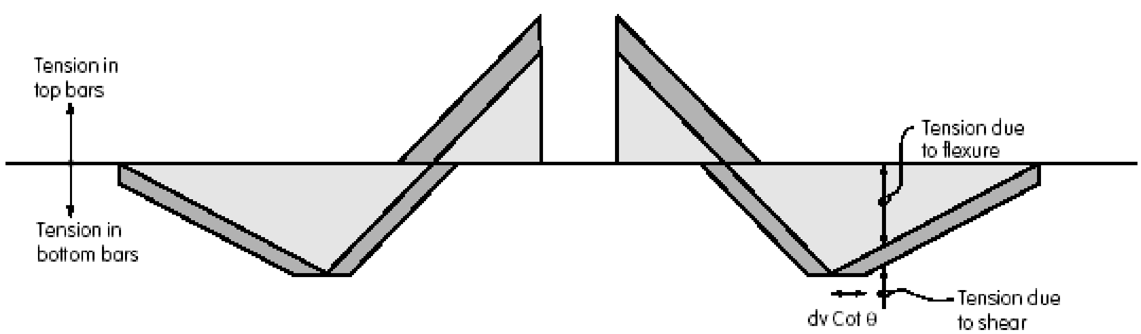


Fig. 4 Variation of Force in Longitudinal Steel of 2-Span Pre-cast Girder Bridge with an Inverted-T Bent Cap (Notes to Fig. 4: 1. Dapped ends at the bent cap mean that girders are indirectly supported, and that shear demand in longitudinal steel must be considered at the center support, as shown above. 2. Not to scale. 3. Enveloping of tension due to maximum positive and maximum negative flexure is not shown. In other words, the

length exhibiting "point of inflection behavior"--where tension due to shear exceeds that due to flexure, is actually wider than shown.)

10. Check horizontal shear

Demand, deducting shear due to girder (say 16 kips):

$$V_h = \frac{V_u Q}{I_g} = \frac{(235 - 1.25 * 16) * 7.33 * 12 * 7.125/2}{95{,}000} = 5.05\ kips/in.$$

Capacity (*AASHTO* Eqn. 5.8.4.1-1):

$$V_n = cA_{cv} + \mu\left[A_{vf} f_y + P_c\right]$$

Here, c= 0.10 ksi, μ= 1.0 for concrete placed against clean, hardened concrete roughened to an amplitude of 0.25 in. (*AASHTO* 5.8.4.2). Conservatively, P_c= 0.089 ksf*7.33ft = 0.65k/ft = 0.05k/in. due to slab, only. So,

V_n=0.10 * 21 + 1.0*[2*0.31 * 60/12 + 0.05] = <u>5.25 kips/in.</u>

Since $V_h < V_n$, stirrup spacing is OK.

Also, check to see that *AASHTO* Eqns. 5.8.4.1-2,3 aren't exceeded:

$V_n \leq 0.2 f'_c A_{cv}$= 0.2 * 5 * 21 = 21.0 kips/in. <u>OK</u>

$V_n < 0.8 A_{cv}$= 0.8 * 21 = 16.8 kips/in. <u>OK</u>

Note: When the factored torsional demand exceeds one-quarter of the cracking moment for torsion i.e. $T_u \geq 0.25\varphi T_{cr}$ (*AASHTO* Eqn 5.8.2.1-3), where $T_{cr} = 0.125\sqrt{f'_c}\,\frac{A_{cp}^{\;2}}{p_c}\sqrt{1 + \frac{f_{pc}}{0.125\sqrt{f'_c}}}$ k-in. (*AASHTO* Eqn 5.8.2.1-4), further analysis must be done. Here, the composite deck prevents members from twisting.

EXAMPLE: INVERTED-T BENT CAP

This example deviates from the 10-step Sectional Method because plane sections no longer remain. *AASHTO* 5.13.2.5, "Beam Ledges", applies. Given:

- f'_c = 4 ksi; f_y = 60 ksi
- Dead load (girder, slab)—130 kips/girder end
- Added dead load—30 kips /girder end
- HL93 w/dynamic load allowance—100 kips/girder end
- Ledge height, h = 30 in.
- Bearing pad width, W=19 in.; length, L=12 in.; thickness, 0.5 in.; modulus, 170 ksi; anticipated movement, 0.5 in.

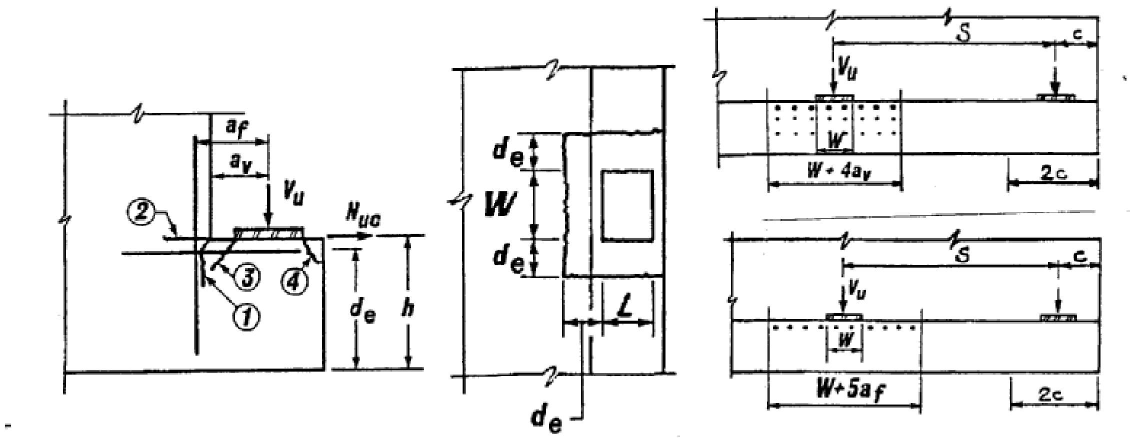


Fig. 5 Notation for Beam Ledges, from *AASHTO* 5.13.2.5.1,2

PUNCHING SHEAR

From the information provided, V_u=1.25*130+1.5*30+1.75*100 = 383 kips
Punching shear resistance at interior girders (*AASHTO* Eqn. 5.13.2.5.4-1):

$$V_n = 0.125\sqrt{f_c'}(W + 2L + 2d_e)d_e$$

= 0.125*2*(19+2*12+2*28.5)*28.5= 712 kips (>>383 kips/girder reqd.)

Punching shear resistance at exterior girders, where edge distance is 0.0 in.:

$$V_n = 0.125\sqrt{f_c'}(W + L + d_e)d_e$$

= 0.125*2*(19+12+28.5)*28.5=424 kips (383 kips/girder reqd. so OK)

SHEAR FRICTION

The "Beam Ledge" provisions in *AASHTO* 5.13.2.5.2 refer back to the general provisions for shear friction, using the effective width $W+4a_v$, where a_v is the distance from the centroid of the girder load to the face of support.

The equation for shear friction is shown below (*AASHTO* Eqn. 5.8.4.1-1). The first component is for cohesion, and the second is for friction. Solve for area of shear friction reinforcement, A_{vf}, required by substituting $V_u \leq \phi V_n$ and rearranging.

$$V_n = cA_{cv} + \mu[A_{vf} f_y + P_c]^*$$ where

c is the cohesion factor for monolithic concrete; =0.150 ksi

A_{cv} is the area of concrete engaged in shear transfer at girder;

$= d_e (W + 4a_v)$=28.5*(19+4*12) = 1910 in.^{2}

μ is the friction factor for monolithic concrete; =1.4

P_c is the compressive force; =0

$$V_u = \varphi V_n = \varphi\lfloor cA_{cv} + \mu[A_{vf} f_y + P_c]\rfloor =$$
$$= \varphi(0.15*1910) + \varphi(1.4*A_{vf}*60)$$
$$A_{vf} = \frac{V_u - 0.9*0.15*1910}{0.9*1.4*60} =$$
$$A_{vf} = \frac{383 - 257.8}{75.6} = 1.66\ in.^2$$

Assuming 6 #5's V_n=.15 * 1910 + 1.4 * 1.86 * 60 = <u>443 k</u>
*Author's note: this expression for V_n relies on a contribution to strength from the cohesion in concrete. Doing so results in less steel than past practice, which could be viewed as unconservative.

Check the upper limit for V_n (*AASHTO* Eqns. 5.8.4.1-2,3):
$0.2f'_cA_{cv}$=0.2*4*1910=1528 kips, and $0.8A_{cv}$=0.8*1910= 1528 kips. >> 443 kips OK
$0.8A_{cv}$=0.8*1910=1528 kips. >> 443 kips OK

FLEXURE

Here, the "Beam Ledge" provisions refer back to provisions for "Corbels and Brackets" in *AASHTO* 5.13.2.4.1.

$M_u = V_u*a_v+N_{uc}(h-d)$ = 383k*1ft+76.6k*(0.12ft)= <u>392ft-k</u>
where N_{uc} is horizontal pad shear, or a minimum of $0.2V_u$.

$$N_{uc} = \frac{\mathrm{mod}ulus * area * movement}{thickness}$$
$$= \frac{170*(12*19)*0.50}{0.5} = 39 \text{ kips.}$$

However, the minimum $0.2V_u$ =0.2*383=76.6k controls. Hence, <u>N_{uc}=76.6 k</u>.

Try 7#6's: A_s=3.08in.^{2}.

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{3.08*60}{0.85*4*43} = 1.26$$

$\phi M_n = 0.9*A_s f_y(d-a/2)$=0.9*3.08*60*(28.5-1.26/2)/12
=386 ft-kips. ≈392 ftk reqd. say OK

TENSION

Check that primary tension reinforcement, A_s calculated above, satisfies additional requirement for tension (*AASHTO* Eqn. 5.13.2.4.2-5, 7):

$$A_s \geq \frac{2A_{vf}}{3} + A_n$$
$$= 0.667*1.66 + \frac{76.6}{0.9*60}$$

= 1.10+1.42= 2.53 in.^{2} < 3.08in.^{2} assumed. 7-#6's OK

HANGER REINFORCEMENT

Design hanger reinforcement, A_{hr}, to satisfy *AASHTO* Eqn. 5.13.2.5.5-3:

$$V_n = (0.063\sqrt{f_c'}b_f d_f) + \frac{A_{hr} f_y}{s}(W + 2d_f)$$

=0.063*2*84*28.5+A_{hr}*60*(19+2*28.5)/s

$$\frac{V_u}{\varphi} = V_n = 302 + \frac{A_{hr}}{s} * 60 * 76$$

$$\frac{A_{hr}}{s} = \frac{\left(\frac{383}{0.9} - 302\right)}{60 * 76} = 0.03in.^2 / in. = 0.33in.^2 / ft$$

Must also check *AASHTO* Eqn. 5.13.2.5.5-2, $V_n = \frac{A_{hr} f_y}{s} S$, where S is the bearing spacing.

$$\frac{V_u}{\varphi} = V_n = \frac{A_{hr} f_y}{s} S$$

$$\frac{A_{hr}}{s} = \frac{V_u}{\varphi * f_y * S} = \frac{383}{0.9 * 60 * 84} = 0.08in.^2 / in. = 1.01in.^2 / ft$$

Torsion must be investigated if $T_u > 0.25\phi T_{cr}$, (*AASHTO* Eqn 5.8.2.1-3) where

$$T_{cr} = 0.125\sqrt{f'_c}\,\frac{A_{cp}^2}{p_c}\sqrt{1 + \frac{f_{pc}}{0.125\sqrt{f_c'}}} \quad (\textit{AASHTO}\text{ Eqn 5.8.2.1-4})$$

However, since the deck has been made continuous for live loads and tied into the inverted-T bent cap, torsion is prevented. $T_u = 1.25T_{DL} + 1.5T_{ADL} + 1.75T_{LL} = 0$

Provide #6 stirrups with 4 legs at 18 in. on center, in addition to shear reinforcement required for service loads between columns, wind, and any extreme event limit state requirements.

CONCLUSIONS

The design procedure for shear in non-disturbed regions based on the *AASHTO LRFD Specifications* has been presented along with examples. The first example, a rigid frame bent cap, illustrated steel requirements when the sectional method is applied to a deep member. The next example, a prestressed I-girder, used the sectional method for shear design and obtained more traditional results. The last example, and inverted-tee bent cap, showed application of the new ledge provisions, which now supplement the bracket/corbel provisions.

By working with values for longitudinal strain, crack angle, and stress in the longitudinal tensile steel as well as stress in the vertical stirrups, hints on the potential failure mechanism are available. Conceptual estimates are no more difficult than using the *Standard Specifications*, once designers are familiar with typical values for crack angle, longitudinal strain, horizontal tension, and the coefficient β in $V_c = \beta\sqrt{f'_c}$. The effort required is appropriate given technology available today, the maturity of modified compression field theory, and the increasing complexity of highway structures.

In some instances, small changes to the Specifications might assist designers:

- The terms direct loads, direct supports, indirect loads, indirect supports, are used but not defined.
- *AASHTO* Fig. C5.8.3.5-2 only shows a simple-span with a point load, rather than the more common case of a uniform dead load in combination with three axle (live) loads. Fanning of cracks in continuous members is not discussed or illustrated.
- Horizontal closed ties or stirrups as required for corbels, are perhaps unintentionally required for beam ledges when provisions for the latter refer to the prior.
- Punching shear provisions for exterior girders don't differentiate between bearing pads on the extreme end of a beam ledge, versus those that are further inward.

In the author's opinion:

- The critical section for shear should be simplified to d_v.
- Cohesion shouldn't be relied on when evaluating shear capacity of a beam ledge.
- Further study of effective b_v when grouted or ungrouted ducts are involved, is needed.

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The views expressed herein are solely those of the author, and not necessarily the California Department of Transportation.

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