RESEARCH PROJECT NO. 6

STRENGTH OF MEMBERS WITH DAPPED ENDS

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University of Washington
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**PCI SPECIALLY FUNDED R & D PROGRAM**
*Phase I-1982-1985*

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Research Project No. 6

STRENGTH OF MEMBERS WITH DAPPED ENDS

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Five dapped end reinforcement schemes, suitable for thin stemmed prestressed concrete members such as the double tee, were studied. (The five reinforcement schemes are illustrated in Figs. 3 through 7.) The studies involved subjecting full scale specimens to a combination of shear and outward tension at the bearing plate. Based on these studies, design procedures suitable for use in practice were developed for each of the reinforcement schemes. The design procedures are detailed in Appendices B, C and D, and examples of the use of these design procedures are set out in Appendix E.

The first two reinforcement schemes studied are currently used in industry. The experimental study has resulted in a better understanding of their behavior and refinement of their design.

The three other reinforcement schemes were developed from the first two reinforcement schemes and also from that utilizing vertical hanger reinforcement contained in Sec. 6.13 of the Third Edition of the PCI Design Handbook (7). These reinforcement schemes simplify fabrication through reduction in reinforcement congestion in the nib of the dapped end and so facilitate the placing and compaction of the concrete.

The following conclusions concerning dapped ends in thin stemmed prestressed concrete members derive from this research, and are of general significance to the precast industry:
1. Anchorage of hanger reinforcement is crucial if it is to develop its yield strength at ultimate, as assumed in design.

2. If the lower end of the hanger reinforcement is to be anchored by extending it horizontally in the bottom of the beam web, it must extend a distance from the point of tangency not less than 1.7 times the bar development length specified in the ACI Code (10).

3. It is possible to develop the yield strength of #3 and #4 reinforcing bars used as hanger reinforcement, by anchoring them at the upper end with a 180 degree loop transverse to the axis of the member, with a minimum bend diameter of 6 bar diameters. (The orientation of the loop is critical. It is only effective as an anchorage if compression stresses in the concrete act across the plane containing the loop, and so prevent a concrete splitting failure inside the loop.)

4. Serviceability and fatigue resistance are greatly improved through lower reinforcement stresses and finer and less extensive cracking by, –
   a. Using inclined hanger reinforcement rather than vertical hanger reinforcement.
   b. Passing about half the prestressing strands through the nib of the dapped end.

5. A concentric or near concentric arrangement of hanger reinforcement in the beam web, relative to the web centerline, leads to the best behavior.

6. Terminating all the strands below the level of the nib bearing plate, (i.e. no strands passing through the nib,) results in inferior performance of a dapped end, both at service load and at ultimate. With this
arrangement of strands, transfer of prestress causes tensile stresses in the reinforcement near the re-entrant corner. This results in earlier, more extensive and wider cracks than if some of the prestressing strands pass through the nib.

7. In a pretensioned prestressed concrete beam with a dapped end, it does not appear possible to develop a shear strength in the full depth web which is greater than the diagonal tension cracking shear, through the provision of web reinforcement. This is because a flexural bond failure occurs immediately a diagonal tension crack passes through or near to the bottom corner of the web. It is therefore proposed that for that part of the full depth web adjacent to a dapped end, the nominal shear strength be taken as equal to the lesser of $V_{cw}$ and $V_{ci}$ calculated for the section distance $h/2$ from the end of the full depth web.
ACKNOWLEDGEMENTS

This research was carried out in the Structural Research Laboratory of the Department of Civil Engineering, University of Washington. Contributions were also made by Messrs. J. H. Frazier, L. N. Gmeiner and K. B. Knowlen of the staff, and by graduate students M. Aden and S. Morikawa.

The research was sponsored by the Prestressed Concrete Institute through its Specially Funded Research and Development program. The project steering committee was chaired by Mr. F. J. Jacques of Stanley Structures, Denver, whose counsel has been greatly appreciated.
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The objectives of this research project were:

(1) To attain a better understanding of the behavior of dapped ends in thin stemmed, precast-prestressed concrete members such as the "double tee."

(2) To develop reinforcing schemes and associated methods of design, which combine simplicity of application with economy of fabrication, while providing the margin of safety required by present building codes.

1 - INTRODUCTION

The dapped end beam enables the depth of a precast floor or roof structure to be reduced, by recessing the supporting corbel or ledge into the supported beam. It also permits members to be supported by pockets formed in walls. However, the "dapped" or cut away section at the end of the beam results in a severe stress concentration at the re-entrant corner. This can initiate diagonal tension cracking at a lower shear than would otherwise be expected. If suitable reinforcement is not provided close to the re-entrant corner, the diagonal tension crack can grow rapidly and failure can occur with little or no warning. Dapped ends are also sensitive to horizontal tension forces caused by restraint of shrinkage or creep shortening of members. These effects were discussed by Mast (1).

Previous studies (2,3,4,5) of dapped end beams primarily involved rectangular section members, which were either
reinforced concrete beams (2,5) or were post-tensioned (4). Hamoudi et al. (4) tested eight pretensioned T-beams, but four failed in flexure. Those that did fail by diagonal tension cracking at the re-entrant corner of the dapped end were reinforced with post-tensioned inclined bars. Also the nominal shear stresses in the nib at failure of the dapped ends were less than 300 psi (2.1 MPa). This contrasts with nominal shear stresses at ultimate of up to 700 psi (4.8 MPa), which can occur in the dapped ends of double tees in practice. Of the foregoing studies, only that of Mattock and Chan (5) included horizontal tension as well as shear.

Martin and Korkosz (6) have discussed current design practice. They reported that the most commonly used reinforcement scheme is that utilizing an orthogonal arrangement of reinforcement. In this scheme, vertical hanger reinforcement, placed as close as possible to the front face of the dap, is designed to carry the total shear.

Progressively refined design procedures for this arrangement of reinforcement have appeared in successive editions of the PCI Design Handbook. Those appearing in the third edition (7) are essentially similar to the design recommendations contained in reference (5). The principal disadvantage of this reinforcement scheme, is that it necessitates threading the prestressing strand through the closed stirrups generally used as the hanger reinforcement.

An alternative scheme proposed in the PCI Design Handbook (7) is the use of an inclined bar with hooked ends as the hanger reinforcement. The bar is sized so that the vertical component of its yield strength is equal to the shear at ultimate. The problem with this scheme is the questionable effectiveness of the hooks as anchorage for the hanger reinforcement.
In both the foregoing reinforcement schemes it is also difficult in practice to ensure accurate location of the local reinforcement in the end of the member. Accurate location of this reinforcement is essential if the dapped end is to be able to develop its design strength. To overcome this problem some precast concrete producers use welded assemblies of reinforcing bars and steel plates, which can be located positively in the end of the member.
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2 - DEVELOPMENT OF RESEARCH PROGRAM

The research program was developed in consultation with the project steering committee, who established the following initial requirements:

1) The tests to be of "standard daps" in thin stemmed prestressed double tee type members. (The steering committee to assist in selecting a geometry representative of a "standard dap").

2) Specimens to be prestressed, full size cross-sections. The dimensions to be similar to those shown in Chapter 2, Product Information, of the PCI Design Handbook, 2nd Edition (8).

3) A minimum of two dap reinforcement schemes to be developed. One scheme to utilize a nib consisting primarily of reinforced concrete, the other to utilize a nib supplemented with structural steel shapes or plates. The load carrying capacity of each scheme to be demonstrated by a rational mechanical model.

4) Consideration to be given to practical production considerations: reinforcing congestion, concrete placement, requirements for threading prestressing strand, welding problems, etc.

5) Material properties used in test program to be representative of those used throughout the precast concrete industry; i.e., 1/2 in. (12.7 mm) dia. 7 wire 270K strand and concrete with a strength at test of 5000 psi (34.5 MPa) or more.
Testing to utilize static loading and to include horizontal tension as well as shear.

On the basis of the above requirements it was decided to make the test beams represent one half of an 18 in. (460 mm) deep, 8 ft. (2.4 m) wide double tee. Details of a typical test beam are shown in Fig. 1. A dapped end specimen was formed on each end of the test beam, and these specimens were tested separately. The slope of the front face of the dap was either 45, 60 or 90 degrees to the horizontal.

The beams were prestressed by four 1/2 in. (12.7 mm) dia. 270K strands, typically arranged as shown in Fig. 1, so that two strands passed through the nib and two strands terminated at the front face of the dap. However, in two test beams the strands were carried straight from end to end of the beam, with the strands arranged as shown in Section A-A of Fig. 1. In these beams none of the strand passed through the nib and all the strands terminated at the front face of the dap. The strand stress-strain curve is shown in Fig. 2.

Each strand was tensioned so as to have a stress of 189 ksi (1300 MPa), (0.7f_pu), after anchorage in the prestressing bed. For purposes of evaluating the test results, the loss of prestress up to the time of test was calculated in each case, using the "General Method for Computing Prestress Losses," contained in the report (9) of the PCI Committee on Prestress Losses. The average effective prestress at the time of test was 156 ksi (1080 MPa).

All beams were provided with ACI Code (10) minimum web reinforcement over their whole full depth length. This was in the form of a single sheet of welded wire fabric. The 15 in. (380 mm.) long W2.9 vertical wires were at 7.5 in. (190 mm) centers and two W2 longitudinal wires were provided for anchorage.
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Distance of first leg of WWF web reinforcement from bottom corner of web, 1.5 in. for right-angle dap, 1 in. for eloping web end face.

Fig. 1 - Typical test beam

Fig. 2 - Stress-strain curves for prestressing strand and for wire used as web reinforcement.
at 0.5 in. (12.7 mm) and 2.5 in. (64 mm) from each end of the vertical wires. At each end of the beam, the first vertical wire was located 1 in. (25 mm) from the end of the bottom face when the front face of the dap was sloped, and 1.5 in. (38 mm) from the end of the bottom face when the front face of the dap was vertical. The actual diameters of the vertical and longitudinal wires were 0.187 in. (4.75 mm) and 0.150 in. (3.81 mm) respectively. The stress-strain curve for the vertical wires is shown in Fig. 2.

The flanges of all the beams were reinforced with W2.9 x W2.9 welded wire fabric. It was placed so that the transverse wires on 6 in. (150 mm) centers were at mid-thickness of the flange and the longitudinal wires on approximately 12 in. (300 mm) centers were below them. The top longitudinal wire of the web reinforcement was tied below the center longitudinal wire of the flange reinforcement.

2.1. Survey of current practice

Concurrently with the design of the basic test beam, and the design and construction of the formwork, a survey was made of PCI Producer Members to obtain information on currently used reinforcing details and design procedures for dapped ends in double tees. The most used detail and design procedure is that contained in the PCI Design Handbook (8), which utilizes vertical and horizontal reinforcement. Variations on this scheme are in use, which are designed to overcome the problem of how to fit closed stirrup hanger reinforcement into the slender web of the double tee. For example, one producer uses single hanger bars with a small steel plate welded to the lower end and a 90 degree hook at the upper end.

Two other schemes utilize sloping hanger reinforcement. In one case the sloping bars are positively anchored at their upper end by welding them to a short length of steel angle.
Their lower ends are anchored by extending the bars horizontally in the bottom of the beam web. The nib flexural reinforcement is welded to the bearing plate. Design is based on assumed truss-like behavior of the reinforcement and the cracked concrete in the beam end. The nominal shear strength is taken equal to the vertical component of the yield strength of the inclined bars. The anchor angle and the nib bearing plate are connected by welding them to opposite ends of small diameter vertical bars. This enables the total reinforcement assembly to be placed in the form as a unit.

In the other scheme utilizing sloping hanger reinforcement, the hanger bar is anchored by welding it to a vertical steel plate, which itself is welded to the bearing plate. The lower end of the bar is anchored by extending it horizontally in the bottom of the beam web. The nib flexural reinforcement is also welded to the bearing plate, creating a reinforcing assembly which is readily placed in the form and which positively locates the reinforcement. Design is based on satisfying statics for a "free body" consisting of that part of the end of the beam cut off by a crack assumed to run upwards into the beam web from the re-entrant corner of the dap, at an angle of 38 degrees. It is assumed that the nominal shear strength is equal to the sum of the vertical component of the yield strength of the inclined bar and the shear which would cause cracking of the concrete at the re-entrant corner.

2.2. Common specimen design and test requirements

'The project steering committee decided that for all specimens the dap should be half the total section depth, i.e., 9 in. (229 mm) and that the flange should be cut away for the 6 in. (152 mm) length of the nib. They further decided that the centerline of the reaction should be 1.5 in. (38 mm) from the end of the nib, i.e., 4.5 in. (114 mm) from the re-entrant corner of the dap, and that the specimens should be subjected to an outward

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force parallel to the beam axis, equal to 20 percent of the reaction perpendicular to the beam axis.

The target nominal shear strength, \( V_n \), of the specimens was based on a review of the values of the end shear per web corresponding to the loads and spans tabulated in the PCI Design Handbook (8), for an untopped 8DT18 section. It was found that for these loads and spans, the required nominal shear strength per web ranged between 8.2 kips (36 kN) and 23.2 kips (103 kN). It was decided that the specimens should be designed to have a nominal shear strength as close to the larger of these two values as was possible, taking into account the actual strengths of the available reinforcing bars.

It was proposed that each specimen be subject to several applications of service load and moderate overload, before being loaded to failure. Hence it was necessary to decide what fraction of the nominal strength should be regarded as representative of service load, i.e., unfactored dead load (D) plus unfactored live load (L).

Using the ACI Code (10) load factors and the strength reduction factor for shear, the ratio of service load to nominal strength varies between 0.51 and 0.55 as the ratio of \( L \) to \( D \) varies between 10 and 1. For the 8DT18 section, the service load to nominal shear strength ratio is close to 0.53 when the end shear is a maximum. The service load shear, \( V_s \), was therefore taken equal to 0.53 \( V_n \).

For moderate overload tests it was decided to subject the specimens to a shear corresponding to the load prescribed in the ACI Code (10), Sec. 20.4 - Load tests of flexural members, i.e., \( 0.85(1.4D + 1.7L) \). The overload shear is therefore \( 0.85V_u \) or \( 0.85(0.85V_n) \), i.e., \( 0.72V_n \). This corresponds to a 36 percent overload.
2.3. Program of tests

Based on the foregoing discussion, all specimens were subjected to the following series of tests:

Test (a) - Incremental load test to service load shear
\[ V_s = 0.53V_n. \]

Test (b) - Nine cycles of loading to service load.

Test (c) - Incremental load test to moderate overload, maximum shear \[ 0.72V_n, (1.36V_s). \]

Test (d) - Ten cycles of loading to service load.

Test (e) - Incremental load test to failure.

Reinforcement strains and crack growth were monitored at each load increment in tests (a), (c) and (e), and at maximum and minimum loads in alternate loading cycles in tests (b) and (d). In all tests, the specimens were subjected at all times to an outward force parallel to the axis of the beam, equal to 20 percent of the shear force acting on the dapped end.
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3 - EXPERIMENTAL PROGRAM

The experimental program was developed in consultation with the steering committee as the research proceeded. It is summarized in Table 1, which shows the reinforcement schemes used and the special characteristics of each specimen. The specimens are identified by a number and a letter. The number reflects the reinforcement scheme and the letter the type of specimen.

Table I - Summary of Test Program

<table>
<thead>
<tr>
<th>Reinforcement Scheme</th>
<th>Specimen Type</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L=I_d Draped strand</td>
<td>L=1.7I_d Draped strand</td>
<td>L=1.7I_d strand in nib</td>
<td>L=1.7I_d Draped strand</td>
<td>L=1.7I_d Draped strand</td>
<td></td>
</tr>
<tr>
<td>1A</td>
<td>1B</td>
<td>1C</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2A</td>
<td>2B</td>
<td>2C</td>
<td>2D</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bar B omitted</td>
<td>Bar B omitted</td>
<td>Cover to A increased.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3A</td>
<td>3B</td>
<td>3C</td>
<td>3D</td>
<td>3E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>β=45°</td>
<td>β=45°</td>
<td>β=60°</td>
<td></td>
<td></td>
<td>β=90°</td>
<td></td>
</tr>
<tr>
<td>4A</td>
<td>4B</td>
<td>4C</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5A</td>
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<td></td>
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</tbody>
</table>

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3.1. The test specimens

It was decided that this study should emphasize schemes of reinforcement that use inclined hanger reinforcement, since most previous tests had been of dapped ends having vertical hanger reinforcement. The first two specimens, 1A and 2A therefore used reinforcement schemes 1 and 2. Details of specimens using reinforcement scheme 1 are shown in Fig. 3. In these specimens the upper ends of the inclined bars are anchored by welding to opposite faces of a vertical plate, which is itself welded to a short length of steel angle. The lower ends of the inclined bars are anchored by extending them horizontally in the bottom of the beam web. The nib flexural reinforcement is welded to the bearing plate, which is in turn attached to the anchorage at the upper end of the inclined bars. The nominal shear strength is taken equal to the vertical component of the yield strength of the inclined bars, assuming a truss-like behavior. The assumed behavior model and a design procedure for dapped ends incorporating the first reinforcement scheme are set out in Appendix B.

Details of specimens using reinforcement scheme 2 are shown in Fig. 4. In these specimens the upper end of the single inclined bar is anchored by welding to a vertical plate, which is in turn welded to the bearing plate. The nib flexural reinforcement is also welded to the bearing plate. The lower end of the inclined bar is anchored by extending it horizontally in the bottom of the beam web. The nominal shear strength of this type of dapped end is taken equal to the sum of the vertical component of the yield strength of the inclined bar and the calculated shear to cause cracking at the re-entrant corner of the dap. The assumed behavior model and a design procedure for dapped ends incorporating the second reinforcement scheme are set out in Appendix C.
Fig. 3 - Details of specimens using reinforcement scheme 1.

Fig. 4 - Details of specimens using reinforcement scheme 2.
Both of the first two reinforcement schemes have the advantage of providing positive anchorage in the nib for the inclined reinforcement, and positive location of the reinforcement. The resulting reinforcement assemblies can also be placed in the form as a unit. However, both reinforcement schemes involve a significant amount of welding, with the possibility of associated quality control problems, and both also use a fair additional weight of structural steel. Scheme 1 has the further disadvantages that prestressing strands passing through the nib must be threaded between the inclined bars and that in design, attention must be paid to possible interference between these strands and the steel plate and angle anchorage. The top leg of the angle can also make the placing of concrete in the nib of the dapped end more difficult.

Specimens 1A and 2A were provided with horizontal extensions of the inclined reinforcement equal to the development length for these bars specified in the ACI Code (10). Although these specimens respectively developed 96 and 100 percent of their calculated nominal strengths, in both cases the inclined reinforcement did not develop its yield strength. A combined flexural bond and diagonal tension failure occurred in the beam web adjacent to the dapped end. All subsequent specimens were therefore provided with a horizontal extension of the hanger reinforcement equal to 1.7 times the development length specified in the ACI Code (10), i.e. the length of a Class C reinforcement splice.

The third and fourth reinforcement schemes were devised as simplifications of reinforcement scheme 1 and of that using vertical hanger reinforcement described in the PCI Design Handbook (8). In both cases it was assumed that adequate anchorage could be provided at the upper end of the hanger reinforcement by looping it through 180 degrees, with a bend diameter equal to six times the bar diameter. The details of specimens utilizing reinforcement scheme 3 are shown in Fig. 5.
Fig. 5 - Details of specimens using reinforcement scheme 3.

Fig. 6 - Details of specimens using reinforcement scheme 4.
As for specimen 1A, a truss-like behavior was assumed and the nominal shear strength taken equal to the vertical component of the yield strength of the inclined bars. The assumed behavior model and a design procedure for dapped ends using the third reinforcement scheme are set out in Appendix D.

The details of specimens utilizing reinforcement scheme 4 are shown in Fig. 6. The closed stirrup used for vertical hanger reinforcement in the PCI Design Handbook (8) is replaced by the looped bar, which is anchored at its lower end by bending through 90 degrees and extending along the bottom of the beam web. Shear reinforcement in the nib is provided in the form of a single hairpin bar, tied to one leg of the hanger reinforcement and to a vertical #3 bar welded to the bearing plate. The nib of the dapped end is assumed to act as an inverted corbel and the hanger reinforcement is assumed to carry the entire shear acting on the dapped end. The model for behavior and the design procedure followed were as detailed in reference (5). However, because the flange is cut back over the length of the nib, it is necessary to check the **flexural** strength both at the centerline of the hanger reinforcement and at the vertical face of the dap.

The looped bar anchorage used for the hanger reinforcement in the third and fourth reinforcement schemes was satisfactory. In both cases the yield strength of the reinforcement was developed before failure. The loop anchorage is probably effective because compression stresses act in the concrete across the plane containing the loop, preventing a splitting failure of the concrete inside the loop. The advantage of reinforcement schemes 3 and 4 over schemes 1 and 2 are that the amounts of welding and structural steel used are reduced. However, the reinforcement is not located as positively in schemes 3 and 4, although it was found possible to tie it together into fairly rigid reinforcement cages which could be placed in the form as a unit.
Details of the fifth reinforcement scheme are shown in Fig. 7. This scheme was designed to combine the advantages of the first two schemes, whilst eliminating some of their disadvantages. This reinforcement scheme provides positive location of the reinforcement and enables a symmetrical (or near symmetrical) arrangement of hanger reinforcement to be used. It also eliminates the threading of prestressing strands through the reinforcement in the nib and reduces reinforcement congestion in the nib, thus making placing and compaction of concrete in the nib easier. A design procedure for reinforcement scheme 5 is set out in Appendix C.

3.2. Materials and fabrication

The specimens were made from Type 3 Portland cement, sand, 3/4 in. (20 mm) maximum size glacial outwash gravel and a water reducing admixture. The concrete was obtained from a local ready-mixed concrete supplier. It was designed to have a strength of 4750 psi (33 MPa) at age 7 days, i.e., at time of prestress transfer. The mix proportions were 1:2.39:3.55, with a 3 in. (75 mm) slump. However, as delivered, the slump varied considerably. It was therefore necessary to monitor the concrete strength from day to day, and to transfer prestress and test the specimens when the strength was judged to be sufficient. The actual concrete strengths at prestress transfer and at the time of test for each specimen, are listed in Table 2.

The deformed reinforcement used conformed to ASTM Specifications A706 or A615, as indicated on Figs. 3 through 7. Coupons were taken from each bar used and stress-strain curves obtained for use in interpreting the reinforcement strains measured in the tests of the dapped end specimens. The yield strengths of the reinforcing bars used in each specimen are listed in Table 2. (The stresses tabulated are based on the nominal cross-sectional areas of the bars.) These actual yield strengths were used in the design of the specimens and in the

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Bearing plate 6 $\times \frac{3}{8} \times 4\frac{7}{8}$

All dim. in in.

Fig. 7 - Details of specimen 5B.

Fig. 8 - Examples of reinforcing bar stress-strain curves.
Table 2 - Material Strengths

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Concrete Strength (psi)</th>
<th>Rebar Size</th>
<th>Yield Strength (ksi)</th>
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<td>5345</td>
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<td>4815</td>
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<td></td>
<td>#4 65.3</td>
</tr>
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</table>

(1 psi = 6.895 kPa, 1 ksi = 6.895 MPa)
evaluation of the test results. Examples of reinforcing bar stress-strain curves are shown in Fig. 8.

The 0.5 in. (12.7 mm) diameter 270K seven wire strand conformed to ASTM Specification A416. Its stress-strain curve and physical properties are shown in Fig. 2. Its ultimate elongation in 24 in. (610 mm) was 7.2 percent.

A mono-strand prestressing jack was used to stress the strands. The prestressing force in each strand was measured by load cells beneath the strand anchors. The strands were stressed a second time, and appropriately sized shims were placed under the anchors to compensate for draw-in of the anchors.

The specimens were cast in a form of plastic coated plywood, braced by steel angles. Variously shaped end block-outs were used, depending on the specimens being fabricated. The specimens were cured in the form at room temperature, under a polyethylene sheet, until transfer of prestress. About twenty 6 x 12 in. (150 x 300 mm) cylinders were cast with the test beam and stored near the beam under a polyethylene sheet. The cylinders were stripped when the beam was removed from its form after prestress transfer. Both test beam and cylinders were then stored in the laboratory until test, usually 5 to 7 days later.

3.3. Testing arrangements and instrumentation

Each of the two dapped ends on a test beam was tested separately, using the testing arrangement shown in Fig. 9. This testing arrangement results in the dapped ends being subjected to a combination of shear perpendicular to the beam axis and a tension force parallel to the beam axis equal to 20 percent of the shear. The loads were applied to the beam through 4 x 1.5 x 24 in. (100 x 40 x 610 mm) bearing plates. These were supported in a horizontal position on tapered pads of high strength plaster.
Fig. 9 - Arrangements for testing specimens.
The points of application of the loads were chosen so that the distribution of shear and moment in the end of the beam near the dapped end under test should approximate that which would be caused by uniformly distributed loading in practice. It was also felt that the load nearest the dapped end should not be so close as to influence the cracking pattern which would develop near the dapped end, and hence perhaps artificially increase the failure load. The load was applied by a hydraulic testing machine, through a load cell. Foil type electrical resistance gages of 1/8 in. (3.2 mm) gage length were used to measure the strains in the dapped end reinforcement. These gages were located as follows:

(1) On both the hanger reinforcement and the nib flexural reinforcement, where intersected by a plane passing through the dap re-entrant corner at 45 degrees to the horizontal.

(2) On the nib flexural reinforcement, where intersected by a plane passing through the end of the bottom face of the beam at 45 degrees to the horizontal.

(3) On the horizontal extension of the hanger reinforcement, at the point of tangency.

(4) In specimen 2A, on the compression reinforcement, just beyond the front edge of the vertical plate.

The strain gages were monitored at prestress transfer, before application of load, and along with the load cell at each load increment, using a Vishay digital recorder.

Draw-in of the bottom two prestressing strands was measured using 0.0001 in. (0.0025 mm)/division dial gages, mounted on the end of the beam as shown in Fig. 10. Each dial gage probe rested against the end of a steel bar clamped to the
strand and pivoting on a bearing mounted on the opposite face of the beam to the dial gage. The change in dial gage reading was twice the draw-in of the strand.

'The reaction due to the weight of the test beam, the load distribution system and the load cell, was measured before carrying out the tests, using a small load cell placed below the fixed roller support.

3.4. *Testing procedures*

Before testing, the beam was supported temporarily on the crib of wooden blocks seen in Fig. 10, with the dapped end bearing out of contact with its support. Zero load readings were taken at this time on the strain gages and the dial gages. The beam end was then lifted, the tapered wooden wedge removed from below the beam web and the dapped end bearing brought down into contact with its support. Initial dead load readings of the gages were taken at this time. The extension from the head of the testing machine was then brought into contact with the load cell and the first test commenced. Fig. 10 shows a typical specimen at this point.

At high loads the tapered wooden wedge seen in Fig. 10 was placed on top of the wooden crib, so that its top face was slightly below the bottom face of the web. This was to catch the beam, should an abrupt failure of the dapped end occur, and so prevent possible injury to the test personnel and extensive damage to the beam.

The program of tests (a) through (e) previously outlined was followed. The full set of tests for one dapped end specimen took a day to carry out. In tests (a) and (c) load increments of 1.5 kips (6.7 kN) were used: this resulted in
Fig. 10 - Typical specimen before loading.
increments of shear of about 1 kip (4.5 kN). In the final test, the load increments were reduced to 1 kip (4.5 kN) or 0.5 kip (2.2 kN) at high loads, as distress became evident.

On completion of the tests of the first dapped end, the test beam was turned end-for-end and the same procedure followed for the second dapped end.
**4 - BEHAVIOR OF TEST SPECIMENS**

The following is a summary of the behavior of the specimens and a discussion of the implications of the results obtained. The specimens are grouped by reinforcement scheme. Complete descriptions of each test, including relevant data plots, are contained in a supplemental report which is appended to this report. A summary of the specimen strengths and modes of failure is given in Table 3.

4.1. **General behavior**

Although details of behavior varied between specimens, certain aspects of behavior were common to all or most of the specimens. Without exception, the first crack occurred at the re-entrant corner of the dap. This is to be expected, because of the stress concentration caused by the corner. However, the shear at which a crack became visible at this location varied with arrangement of reinforcement, and in particular was affected by the arrangement of the prestressing strands at the end of the beam.

In Fig. 11 are shown the various types of crack which occurred in most specimens. As mentioned above, the re-entrant corner crack was the first to form and after multiple cycles of service load was usually 2 to 5 in. (50 to 125 mm) long. This was always the widest crack, and its maximum width was close to the re-entrant corner.

Under the moderate overload test (c), the re-entrant corner crack usually extended to within 1 or 2 in. (25 or 50 mm) of the web-flange junction, and a short branch crack would form approximately over the hanger reinforcement. In most cases the branch crack only extended upwards as shown in Fig. 11, but in a few cases it also extended downwards a little way also. This
Table 3 - Summary of Specimen Strengths (kips) and Modes of Failure

<table>
<thead>
<tr>
<th>Spec.</th>
<th>V(test) (1)</th>
<th>Vn (2)</th>
<th>V(test)/Vn</th>
<th>Failure Mode</th>
<th>Hanger Keinft. Yield?</th>
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<td>0.96*</td>
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</tr>
<tr>
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<td>0.72*</td>
<td>S &amp; A</td>
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<td>1.00&quot;</td>
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</tr>
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<td>1.08&quot;</td>
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(1) Maximum shear attained in test.
(2) Calculated nominal strength of dapped end: * , - using initial or only method of calculation; + , - using modified method of calculation.
(3) Faulty beam, see text for discussion.

A = Diagonal tension failure in beam web.
B = Flexural bond failure in beam web.
C = Inclined bar burst sideways out of web at bottom corner.
D = Diagonal tension failure in nib.
kN) a flexure crack occurred 8.5 in (60 mm) from the bottom corner of the web and a diagonal tension crack occurred above it, extending over two thirds of the depth of the web.

It was possible to increase the shear to its maximum value of 22.75 kips (101.2 kN) when a flexure crack occurred at the end of the horizontal extension of the hanger reinforcement. This was followed immediately by propagation of a second diagonal tension crack from this point, and the extension of the first diagonal tension crack over the whole depth of the beam.

As in specimen 1A the dapped end itself did not fail, the maximum shear attained being governed by a combined diagonal tension and flexural bond failure of the beam adjacent to the dapped end. Because of this, the horizontal extension of the hanger reinforcement was increased for subsequent specimens.

The nib of the specimen remained uncracked throughout the test. Also, the stress in the compression reinforcement just ahead of the front edge of the vertical plate was small at all loads, reaching a maximum value of 6.8 ksi (47 MPa). The stresses at maximum load in the hanger reinforcement and the nib flexural reinforcement near the re-entrant corner were respectively 60.0 and 58.0 ksi (414 and 400 MPa).

4.3.2. Specimen 2B: The difference between this specimen and specimen 2A was that the compression reinforcement was omitted, (since it carried such small stresses in specimen 2A), and the horizontal extension of the hanger reinforcement was increased to 1.7 times the development length specified in the ACI Code (10).

Before testing, a hairline crack was visible on the face of the web closest to the inclined bar. It extended from 2.5 in. (64 mm) up the sloping end face, to a point 11.5 in. (290 mm) from the bottom corner of the web and just above the location of the horizontal extension of the hanger reinforcement. This
type of crack is presumably caused by high local bond stresses on the surface of the hanger reinforcement, close to where it is crossed by the re-entrant corner crack.

Also under the moderate overload a fine crack usually occurred in the nib at the inner edge of the bearing plate, and in some cases a short inclined crack occurred in the nib. These nib inclined cracks tended to center themselves beyond the tip of the branch crack running up into the nib.

In all cases in which two prestressing strands passed through the nib and two terminated in the face of the dap below the nib, cracking behavior under service load was very good. On the last application of service load in test (d), the maximum crack width in these specimens was between 0.005 and 0.010 in. (0.13 and 0.25 mm), unless noted otherwise. On removal of load the crack widths reduced to half or less than half of their maximum value.

In the final test, under increasing shear, secondary inclined cracks propagated from the face of the dap. These cracks formed progressively further from the re-entrant corner as the tension stress in the hanger reinforcement increased. The secondary inclined cracks had the general shape shown in Fig. 11. These cracks on opposite faces of the web were often connected together by cracks across the end face of the web. In some cases the secondary inclined cracks linked together and/or linked with the re-entrant corner crack, which spread along the web-flange junction as the shear increased.

Vertical cracks formed on the end face of the web approximately over the hanger reinforcement in some of the specimens. They were usually fine enough to be of no concern. Additional fine inclined cracks usually occurred in the nib at higher loads.
Approaching the failure load, a succession of short flexural cracks occurred in the bottom of the web in most cases. The first crack formed near the bottom corner of the web, as the tensile stress at this location in the horizontal extension of the hanger reinforcement started to become significant.

In almost every case, failure occurred as a result of a major diagonal tension crack extending from close to the bottom corner of the web, up to the web-flange junction, 30 to 40 in. (760 to 1015 mm) from the end of the flange. This critical diagonal tension crack was accompanied by slip of the lower prestressing strands as a flexural bond failure occurred. In some cases it was difficult to decide which came first, the diagonal tension failure or the flexural bond failure. In Table 3 the order in which it is believed these two types of failure occurred is indicated by the order of the letters A and B, signifying diagonal tension failure and flexural bond failure respectively.

The dapped end reinforcement did not always yield before failure of the full depth web, even in cases where the maximum shear carried was in excess of the nominal shear calculated assuming that yield would occur. Whether or not the hanger reinforcement yielded before failure is indicated in the last column of Table 3.

In the following sections of the report, exceptions to the general behavior described above are discussed for each specimen.

4.2. Reinforcement scheme 1

4.2.1. Specimen 1A: Failure occurred before yield of the dapped end reinforcement, at a shear of 21.82 kips (97.1 kN), 96 percent of the calculated strength. The nib of the dapped end had only fine cracks in it at maximum load.
Failure appeared to be a combined flexural bond failure and diagonal tension failure of the web of the beam adjacent to the dapped end, triggered by a series of flexural cracks in the bottom face of the web. The maximum shear was reached when the first flexure crack occurred.

Because flexural bond failure occurred before the dapped end reinforcement could develop its yield strength in both this specimen and specimen 2A, it was decided that in subsequent specimens the horizontal extension of the hanger reinforcement should be 1.7 times the development length of the reinforcing bar specified in the ACI Code (10).

4.2.2. Specimen 1B: At the failure shear of 21.82 kips (97.1 kN) in specimen 1A, the hanger reinforcement was only stressed to 49.2 ksi (339 MPa) and hence carried a shear of 17.04 kips (75.8 kN). The calculated vertical component of the force in the prestressing steel at the interface of the nib and the beam web was 0.37 kips (1.6 kN). The concrete in the plane of the interface was therefore carrying a shear of 4.41 kips (19.6 kN), at failure of the beam web, but the nib was still intact. This shear corresponds to a nominal shear stress in the nib of \(1.8\sqrt{f'_c}\) psi (0.15\(\sqrt{f'_c}\) MPa).

It was therefore decided that the nominal shear strength of specimen 1B should be taken as the sum of the vertical component of the yield strength of the inclined reinforcement and a shear corresponding to a nominal shear stress of \(2\sqrt{f'_c}\) psi (0.17 \(\sqrt{f'_c}\) MPa) in the nib. The amount of hanger reinforcement in specimen 1B was reduced, so that the nominal strength calculated in this way should be near the target strength. The required nib flexural reinforcement was calculated taking into account the effect of the shear force in the concrete on the moment equilibrium of the nib. [See Appendix B, part (d).]
The behavior of this specimen in the service load range was very satisfactory. The variation with shear of stresses in the inclined bars at the re-entrant corner and near the bottom corner of the web are shown in Fig. 12. The maximum stress and the range of stress in the hanger reinforcement in the last load cycle of test (d) were 25.6 ksi (177 MPa) and 18.7 ksi (129 MPa) respectively. The stresses in the flexural reinforcement near the re-entrant corner were very similar in magnitude. The hanger reinforcement near the bottom corner of the beam web remained in compression during these first four tests.

The hanger reinforcement near the re-entrant corner developed its yield strength at a shear of 23.8 kips (106 kN) and continued to develop this or a slightly greater force up to ultimate, as may be seen in Fig. 12.

It can also be seen in Fig. 12 that the stress in the horizontal extension of the hanger reinforcement near the bottom corner of the web started to increase significantly at shears above about 22.5 kips (100 kN). At a shear of 26.4 kips (117 kN) a vertical crack about 4 in. (100 mm) long occurred 0.75 in. (19 mm) horizontally from the bottom corner.

At a shear of 27 kips (120 kN) a major diagonal tension crack occurred in the beam web. It extended upwards at about 40 degrees, from a point 13 in. (330 mm) from the bottom corner, up to the web-flange junction. The shear dropped to 26.4 kips (117 kN) after diagonal tension cracking. However, it was possible to increase the shear to the maximum value of 27.93 kips (124 kN), when a second major diagonal tension crack occurred, passing near the bottom corner of the web.

Although failure finally occurred as a result of diagonal tension cracking in the beam web, the behavior of the specimen is considered satisfactory. Both the hanger
3. Shear (hips)  

$$V_{\text{max}} = 27.93 \text{ k} = 1.16V_n(\text{calc})$$

- Gages 5 & 6 (ave.)
- Gages 3 & 4 (ave.)
- $0.72V_n(\text{calc}) = 17.10 \text{ k}$
- Service load, $V_s = 12.58 \text{ k}$
- $V_d = 1.66 \text{ k}$

**Fig. 12** - Variation with shear of stress in inclined bars, specimen 1B.
reinforcement and the nib flexural reinforcement yielded before maximum load, this resulted in the re-entrant corner crack becoming very wide before failure, so giving warning of impending failure. Also, the failure was much less abrupt than in the case of specimen 1A. The cracks in the nib and on the sloping face remained fine at all stages of loading.

4.2.3. Specimen 1C: This specimen was intended to be identical to specimen 1B, except that no prestressing strand passed through the nib. All the prestressing strand ran straight from end to end of the beam, and terminated at the front faces of the daps. Unfortunately, problems were encountered with the concrete used in this specimen and specimen 2C. Although the slump was near the target value of 3 in. (75 mm), the concrete gained strength much more slowly than in the case of the other specimens. As a consequence, the prestress was not transferred until age 28 days, at a concrete strength of 4300 psi (29.7 MPa). Testing was carried out at age 61 days, at a concrete strength of 4815 psi (33.2 MPa). Although the prestress was transferred gradually using hydraulic rams, cracking of the concrete below the bottom strand was observed after the beam was removed from the form. This cracking became more extensive with time. The strain measurements at transfer indicated that the bottom strand must have slipped. As a result of this cracking and the slip of the bottom strand, both specimens 1C and 2C failed prematurely. Or more correctly, the full depth sections adjacent to the dapped ends failed prematurely in shear and flexural bond.

Because of the premature failures, the tests of specimens 1C and 2C did not provide any information as to the effect on the strength of dapped ends of the absence of prestressing strands from the nib. They did however provide information on service load behavior.

In the specimens with strands passing through the nib, both the nib flexural reinforcement and the hanger reinforcement
near the re-entrant corner were compressed by the transfer of prestress. However, in specimen 1C and in the other specimens in which the strands did not pass through the nib, tension was produced at these same locations at prestress transfer. As a result, cracking occurred at the re-entrant corner almost as soon as the nib carried any shear, and developed much more rapidly than in specimens in which strands passed through the nib. The maximum crack width at service load in the tenth loading cycle was 0.012 in. (0.30 mm) compared with 0.005 in. (0.13 mm) in specimen 1B at the same load. Also the stresses at service load in both the hanger reinforcement and the nib flexural reinforcement near the re-entrant corner, were about double those occurring in similar specimens but with strands passing through the nib. Draping strands through the nib is clearly very beneficial from the point of view of serviceability.

4.3. **Reinforcement scheme 2**

4.3.1. **Specimen 2A**: Behavior in the service load range was very satisfactory. However, under overload, on the face nearest the vertical steel plate, an almost vertical crack branched from the re-entrant corner crack in the vicinity of the front edge of the plate, travelling upwards about 3 in. (75 mm). (Similar cracking occurred in all specimens using this reinforcement scheme.)

In the final test, at a shear of 16.42 kips (73.0 kN), a fine crack occurred on the end face approximately over the inclined bar, extending downwards 4 in. (100 mm) from the re-entrant corner. At about this same shear, the stress in the hanger reinforcement near the bottom of the web started to increase more rapidly. As the shear was increased to 21.92 kips (97.5 kN), the crack over the reinforcing bar extended to 0.5 in. (123 mm) from the bottom of the sloping end face and the stress in the hanger reinforcement near the bottom corner of the web had increased to 19.2 ksi (132 MPa). At a shear of 22.55 kips (100.3 kN)
a flexure crack occurred 8.5 in (60 mm) from the bottom corner of the web and a diagonal tension crack occurred above it, extending over two thirds of the depth of the web.

It was possible to increase the shear to its maximum value of 22.75 kips (101.2 kN) when a flexure crack occurred at the end of the horizontal extension of the hanger reinforcement. This was followed immediately by propagation of a second diagonal tension crack from this point, and the extension of the first diagonal tension crack over the whole depth of the beam.

As in specimen 1A the dapped end itself did not fail, the maximum shear attained being governed by a combined diagonal tension and flexural bond failure of the beam Web adjacent to the dapped end. Because of this, the horizontal extension of the hanger reinforcement was increased for subsequent specimens.

The nib of the specimen remained uncracked throughout the test. Also, the stress in the compression reinforcement just ahead of the front edge of the vertical plate was small at all loads, reaching a maximum value of 6.8 ksi (47 MPa). The stresses at maximum load in the hanger reinforcement and the nib flexural reinforcement near the re-entrant corner were respectively 60.0 and 58.0 ksi (414 and 400 MPa).

4.3.2. Specimen 2B: The difference between this specimen and specimen 2A was that the compression reinforcement was omitted, (since it carried such small stresses in specimen 2A), and the horizontal extension of the hanger reinforcement was increased to 1.7 times the development length specified in the ACI Code (10).

Before testing, a hairline crack was visible on the face of the web closest to the inclined bar. It extended from 2.5 in. (64 mm) up the sloping end face, to a point 11.5 in. (290 mm) from the bottom corner of the web and just above the location of the horizontal extension of the hanger reinforcement. This
crack was extremely fine and did not elongate or widen under the service load or moderate overload tests. It probably resulted from restraint of shrinkage of the concrete by the adjacent #5 reinforcing bar. Measurements made after failure indicated that the side cover to this bar near the bottom corner of the web was only 0.625 in. (15.9 mm), instead of the intended 0.75 in. (19.1 mm). This difference is less than the tolerance on cover to reinforcement specified in the ACI Code (10).

The service load behavior was very satisfactory, except that in the second cycle of service load after the overload test a vertical crack occurred on the sloping end face, close to the location of the hanger reinforcement and extending 4 in. (100 mm) from the re-entrant corner. In the final test this crack gradually extended and at a shear of 19.38 kips (86.2 kN) had reached a point 1.25 in. (32 mm) from the bottom edge. At this shear, the crack was about 1/16 in. (1.5 mm) wide over the top 4 in. (100 mm) of its length.

Failure occurred at a shear of 20.05 kips (89.2 kN), when this crack extended to the bottom edge and widened to about 0.1 in. (2.5 mm) in the upper half of its length. The failure occurred suddenly, when the inclined bar pulled out sideways at the bend. The concrete inside the bend failed by splitting approximately in the plane of the bar, and by crushing further up the web. Simultaneously, the crack which had existed before loading widened, as the piece of concrete below this crack tended to rotate outwards. The appearance of specimen 2B after failure can be seen in Fig. 13. (The inclined crack in the nib occurred during handling after completion of the tests.)

In the test of specimen 2A, only the average strain in the inclined bar was measured. In the test of specimen 2B, strains were measured on both sides of the inclined #5 bar. During the first ten cycles of service load these strains were approximately equal. However, when the load was increased above
Fig. 13 - Appearance of specimen 2B after test.

Fig. 14 - Variation with shear of stress in inclined bar, specimen 2B.
the service load in test (c) and cracking of the concrete became more extensive, the tensile strain on the outer side of the bar, i.e. that nearest the face of the beam web, became progressively larger than the tensile strain on the inner side of the bar. This can be seen in Fig. 14. This indicated bending of the bar toward the centerline of the beam due to lateral pressure from the surrounding concrete. This is probably due to the concrete in the end part of the web resisting the torsional moment caused by the 0.81 in. (21 mm) eccentricity of the upward acting inclined bar force with respect to the downward load applied at the centerline of the beam.

Since the beam was loaded vertically on its centerline and did not rotate, the resultant reaction on the bearing plate must also have acted at the beam centerline. For this to occur, an upward reaction force must have acted on the concrete on the opposite side of the beam from that containing the inclined reinforcing bar. This would provide the torsional moment in the concrete which tended to bend the lower part of the inclined bar toward the beam centerline. In turn, the bar was pressing out against the concrete cover, as well as pulling upward. It can be seen in Fig. 14 that as the maximum load was approached and the vertical crack on the sloping end face penetrated closer to the bottom face, the lateral bending of the inclined bar became less.

On the basis of the results of the tests of this specimen, it appears that when using #5 bars as hanger reinforcement a specified cover greater than 0.75 in. (19 mm) should be used. Also, that if a single hanger bar is used, its eccentricity with respect to the centerline of the beam should be made as small as is practicable. A symmetrical or near symmetrical arrangement of hanger reinforcement appears to be preferable.

4.3.3. Specimen 2C: This specimen was intended to be identical to specimen 2B, except that no prestressing strand
passed through the nib. It was part of the same test beam as specimen 1C and so experienced the same problems as a result of the low concrete strength, i.e., cracking below the bottom strand and slip of the bottom strand at prestress transfer.

As with specimen 1C, a premature flexural bond and diagonal tension failure of the full depth web occurred, hence no information was obtained as to the effect on the ultimate strength of the nib of omitting strand from the nib. However, as in all other specimens in which the strand did not pass through the nib, the tests of specimen 2C showed that the maximum stress and the range of stress in the reinforcing bars near the re-entrant corner, are about twice those measured in the similar specimens but having strand passing through the nib. As in specimen 1C, tensile stresses occurred at prestress transfer in the dapped end reinforcement near the re-entrant corner. As a result, cracking occurred at a very low value of shear, and more extensive and wider cracking occurred at service load than in specimen 2B, which had strand passing through the nib.

4.3.4. Specimen 2D: This specimen was similar to specimen 2B, but the cover to the hanger reinforcement was increased to 1 in. (25 mm) from 0.75 in. (19 mm). Also, the eccentricity of the hanger reinforcement relative to the beam centerline was reduced from 0.81 in. (21 mm) to 0.69 in. (18 mm).

It appeared from the strains measured in the dapped end reinforcement near the re-entrant corner, at transfer of prestress, that a shorter transfer length was obtained for the strands in this specimen than in specimen 2B. The compressive strains in these bars due to prestress transfer were about 35 percent higher in 2D than in 2B. This was reflected in the service load range behavior of specimen 2D. Cracking was delayed and less extensive, and the maximum stress and the range of stress in the dapped end reinforcement were both less than in specimen 2B. This can be seen in Fig. 15.
In the final test, at a shear of about 20 kips (89 kN) a fine vertical crack 1 in. (25 mm) long occurred on the sloping end face, located approximately over the inclined reinforcing bar. This crack progressively extended down to the bottom of the end face, but remained fine up to maximum load.

At the maximum shear of 23.82 kips (106.0 kN), the vertical branch of the re-entrant corner crack (near the front edge of the vertical plate), extended 2 in. (50 mm) horizontally into the nib, about 2.5 in. (65 mm) below the top face of the nib. It also extended into the beam web at about 30 degrees to the horizontal, to a point 0.5 in. (13 mm) below the web-flange junction and 3.5 in. (90 mm) from the end of the flange. After 10 minutes the shear reduced to 22.9 kips (101.9 kN). When the shear was being brought back to its original value, a sudden diagonal tension failure of the nib occurred at a shear of 23.7 kips (105.4 kN). If the shear had been maintained at its maximum value, it is likely that this diagonal tension failure would have occurred after a few minutes at that load.

The diagonal tension cracks causing failure extended from the outer bottom corner of the nib, across the nib and into the end 5 or 6 in. (125 or 150 mm) of the beam web. At maximum shear the average stress in the inclined bar was 62.9 ksi (433 MPa), 95 percent of yield, and the stress in the nib flexural reinforcement was 59.6 ksi (401 MPa), 81 percent of yield. The nominal shear stress in the nib at failure was 693 psi (4.8 MPa), $9.6\sqrt{\frac{f_c}{f_t}}$ psi ($0.80\sqrt{\frac{f_c}{f_t}}$ MPa).

It can be seen in Fig. 15 that as in specimen 2B, the strain on the face of the inclined bar nearest the web face was greater than the average strain in the bar, indicating that the bar was subject to bending as well as to direct tension. However, in this case the difference in stress was much less. The maximum difference in stress was 4.2 ksi (29.0 MPa). The
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

![Graph: Shear (kips) vs Stress (ksi)](image)

**Fig. 15** - Variation with shear of stress in inclined bar, specimen 2D.

![Diagram: Cracking pattern](image)

**Fig. 16** - Cracking pattern after failure in specimen 3B.
reduction in cracking over the reinforcement on the end face and in bending occurring in the inclined bar, can reasonably be attributed to the greater thickness of cover over the inclined bar and to the reduction in the eccentricity of the inclined bar in this specimen.

4.4. Reinforcement scheme 3

The primary purpose of testing the specimens utilizing this reinforcement scheme was to check whether a simple 180 degree loop at the upper end of the inclined bar could anchor the bar in the nib, so that it could develop its yield strength.

4.4.1. Specimens 3B and 3D: These two specimens were similar, except that the angle of inclination of the hanger reinforcement, and of the web end face, was 45 and 60 degrees to the horizontal in 3B and 3D respectively. Both specimens were designed assuming truss like behavior of the nib at ultimate, with the compression chord provided by the concrete being horizontal. Any contribution from the prestressing strand was neglected. The calculated nominal shear strength was therefore equal to the vertical component of the yield strength of the inclined bars.

In the final test the cracking behavior of both specimens was similar. Between shears of 16 and 21 kips (71 and 93 kN), secondary inclined cracks originated at progressively increasing distances from the re-entrant corner. As the last of these inclined cracks formed, the stress in the horizontal extension of the inclined bars started to increase more rapidly, eventually reaching 37 ksi (255 MPa) in 3B at failure. (In 3D these gages became inactive before failure.)

As the shear was increased, the branch crack over the inclined bar gradually extended further into the nib and was crossed by inclined cracks in the nib, as seen in Fig. 16.
At shears of 24.40 and 25.24 kips (108.5 and 112.1 kN) respectively, major diagonal tension cracks occurred in specimens 3B and 3D. In the case of specimen 3D this was the maximum shear attained. The diagonal tension failure was followed by a flexural bond failure, after flexural cracking at the end of the horizontal extension of the inclined bars. The maximum shear in specimen 3D was 1.50 times the calculated nominal strength of the nib. The inclined reinforcement developed its yield strength at failure, but the nib flexural reinforcement was 6 percent short of yield.

In the case of specimen 3B it was possible to increase the load further after diagonal tension cracking. Failure eventually occurred at a shear of 27.93 kips (124.4 kN). A flexure crack formed close to the end of the horizontal extension of the inclined bars. This caused a flexural bond failure and the crack developed into a diagonal tension crack.

Although the final failure was brittle in character, there was warning of failure by extensive cracking starting at about 87 percent of ultimate, and the yield strength of both the inclined reinforcement and the nib flexural reinforcement was developed before failure. The maximum load was 1.51 times the calculated nominal strength of the nib.

The variation with shear of the stress in the inclined bars was similar to that which occurred in specimen 1B, shown in Fig. 12.

It is clear from the strengths obtained in specimens 3B and 3D that the concrete must be contributing to the shear strength. This contribution probably occurs because the top chord of the analogous truss is not horizontal, as assumed in calculating the strength, but is in fact inclined. This hypothesis is supported by the orientation of the cracks in the...
nib above the inclined bars in both specimens. This can be seen in Fig. 16 for the case of specimen 3B.

The increase in shear strength is only possible if the flexural strength increases also. In the original design, the nib flexural reinforcement was designed to provide all the tension force required for flexure and to carry the outward horizontal force, (see Appendix D). The forces in the prestressing strands passing through the nib were neglected. It is clear that to provide a nib flexural strength to match the increased shear strength, the prestressing strands must have contributed to flexural strength.

An estimate was made of the contribution of the concrete to nib shear strength in specimen 3B. It was assumed that the force in the compression chord of the analogous truss acted along a line joining the center of action of the flexural compression force at the nib-web interface, to the truss force intersection point 0, in Fig. D1. The stress in the prestressing strands was calculated assuming a 36 in. transfer length. The contribution of the concrete calculated in this way is 7.34 kips (32.7 kN). The vertical component of the strand forces at the nib-beam interface is 0.37 kips (1.7 kN). Hence the nominal shear strength of the nib calculated in this way is 26.18 kips (116.5 kN), much closer to the test value of 27.93 kips (124.2 kN). Taking the prestressing strand into account, the shear corresponding to the nominal flexural strength of the nib-web interface is 27.4 kips (121.9 kN).

4.4.2. Specimen 3C: This specimen was similar to specimen 3B except that no prestressing strands passed through the nib. As previously mentioned, in those specimens in which all the strands terminated at the face of the dap, tensile stresses were caused in the dapped end reinforcement near the re-entrant corner at the time of prestress transfer. As a result, cracking occurred at
the re-entrant corner on first application of shear, and
developed more rapidly than in the case of specimen 3B.

On the first application of service load, the cracking
in specimen 3C was as extensive as in specimen 3B after twenty
cycles of service load and 36 percent overload. In addition, a
second inclined crack originated at the sloping end face, 4 in.
(100 mm) from the re-entrant corner, and extended upwards for
about 9 in. (230 mm). The variation with shear of the stress in
the inclined bars of specimen 3C is shown in Fig. 17.

During the overload test, the inclined crack
originating at the sloping end face extended to within 1 in. (25
mm) of the web-flange junction, and another inclined crack
initiated about 2 in. (50 mm) below it. A crack also originated
at the inner edge of the bearing plate and extended to within 1
in. (25 mm) of the top face of the nib, close to the end of the
flange.

Only minor additional cracking occurred during the 10
cycles of service load after overload and the maximum crack width
did not increase. In Fig. 17 it can be seen that there was only
about a 4 ksi (27.6 MPa) increase in maximum stress in the
inclined bars between the end of test (a) and the end of test
(d). This was a reflection of the fact that most of the cracking
occurred on first application of service load.

In the final test, as the shear was increased, addi-
tional cracks initiated along the sloping end face, spreading
upwards to join one another. The branch crack from the re-
entrant corner crack gradually extended further into the nib
along the inclined bar. It was crossed by a series of inclined
cracks in the nib, in a manner similar to that observed in
specimen 3B, (see Fig. 16).
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

Fig. 17 - Variation with shear of stress in inclined bars, specimen 3C.

Fig. 18 - Appearance of specimen 3E after test.
At a shear of 19.7 kips (87.6 kN) a shallowly inclined crack about 3 in. (75 mm) long occurred in the web about at the level of the re-entrant corner. At a shear of 21.16 kips (94.1 kN) this crack extended more steeply for about 3.5 in. (90 mm). This load was sustained for 7 minutes, then a major diagonal tension crack occurred a little below the existing inclined crack. It extended from close to the bottom corner of the web to the web-flange junction about 28 in. (710 mm) from the end of the flange. Immediately following the occurrence of this crack, a flexural bond failure occurred.

The maximum shear was 1.14 times the calculated nominal strength based on the inclined bars carrying all the shear. However, the hanger reinforcement was only stressed to 61.3 ksi (423 MPa), (93 percent of yield), at maximum shear. This indicates that this shear the concrete in the nib was carrying 3.8 kips (16.9 kN) shear. This is consistent with the direction of the cracks in the upper part of the nib.

Although the nib had not actually failed at maximum load, it is felt that if no prestressing strand passes through the nib, as in this case, the calculated nominal strength should be based on the assumption that all the shear is carried by the hanger reinforcement. This, because of the extensive cracking and high dapped end reinforcement stresses which occurred at service load. It is clear that, from the point of view of serviceability, it is preferable to carry some of the prestressing strand through the nib of a dapped end.

4.4.3. Specimen 3E: This specimen was similar to specimen 3B except that the end face of the web was made vertical and the welded wire fabric web reinforcement was carried through to that vertical face, i.e., overlapping the inclined bars in the lower part of the web. The purpose of testing this specimen was to see whether placing the lap of the prestressing strand and the horizontal extension of the inclined bars further from the end of
the strand, would improve **flexural** bond behavior at ultimate.

Specimens 3B and 3D both developed strengths 50 percent greater than their nominal strengths calculated assuming that all the shear was carried by the inclined bars alone. Therefore, the test loads applied to specimen 3E were based on a nominal shear strength taken equal to the sum of the vertical component of the yield strength of the inclined bars, and a contribution from the concrete corresponding to a nominal shear stress of $2\sqrt{f'_c}$ in the nib. This was done in order to check whether behavior at the increased service load and moderate overload levels would be satisfactory.

The behavior of specimen 3E at service load and moderate overload was similar to that of specimen 3B. The pattern of cracking was essentially the same at the end of the 20 cycles of service load and the 36 percent overload.

In the final test, under increasing load a similar pattern of cracking developed in the nib and the upper part of the beam web, as had occurred in specimen 3B. But below the level of the re-entrant corner, the cracking developed in a somewhat different manner. Only one inclined crack initiated at the end face of the web, as may be seen in Fig. 18. However, two inclined cracks initiated in the vicinity of the inclined bars, at increasing distances from the re-entrant corner, at shears of 24.5 kips (109 kN) and 27.0 kips (120 kN). When they first appeared, these cracks were about 4 in. (100 mm) long, centered on the location of the inclined bars and approximately normal to them. These cracks evidently occurred as tension stresses in the bars spread downwards from the re-entrant corner. Branch cracks extended downwards in the web, along the alignment of the inclined bars, from the re-entrant corner crack and from the inclined crack originating at the web end face.
At shears greater than 25 kips (111 kN), the draw-in of the bottom strand increased by increasing amounts at each load increment, and at the maximum shear of 29.5 kips (131 kN) the slip increased rapidly and the shear dropped to 28.75 kips (128 kN). At this point a major diagonal tension crack occurred in the web, together with two flexure cracks. (These are indicated by dashed lines in Fig. 18.)

Both the nib flexural reinforcement and the inclined bars yielded before failure. The maximum load coincided with a rapid increase in strain to about 10,000 millionths in both the inclined bars and the nib flexural reinforcement.

The maximum shear was 26 percent greater than the nominal strength calculated as indicated above. This strength is only attainable because of the contribution of the prestressing strands in the nib, to the flexural and shear strengths of the nib. However, it is felt that in practice the prestressing strands passing through the nib should not be taken into account when calculating the flexural strength of the nib, in case some slip of the strands should occur near their ends at transfer. The presence of the strands in the nib should only be relied upon to improve serviceability. The nib flexural reinforcement must therefore be increased if a contribution from the concrete is included in the calculation of the nib shear strength. See part (d) of Appendix D, "Modified Method of Calculation."

The use of the vertical end face to the web, instead of sloping end face close to the inclined bars, did result in a 6 percent increase in a maximum shear. It also resulted in less extensive cracking in the lower part of the web at high loads. However, the eventual failure still occurred in the beam web.

If the modified method of strength calculation in part (d) of Appendix D is applied to specimens 3B and 3D, the ratio $V_{n(test)}/V_{n(calc.)}$ is equal to 1.18 and 1.16 respectively. (On
this occasion the contribution of the prestressing strand to the flexural strength of the nib was taken into account.) This method of calculation could not be applied to specimen 3C, because the specimen was found to be deficient in flexural strength, when a contribution from the concrete was included in the calculation of the shear strength.

4.5. Reinforcement scheme 4

This is an adaptation of the reinforcement scheme using vertical hanger reinforcement contained in the PCI Design Handbook (8), suitable for slender webbed double tee members. The only difference between specimens 4B and 4C is that in specimen 4B two prestressing strands pass through the nib, whilst in specimen 4C all the strands are terminated at the vertical face of the dap. In design, the hanger reinforcement is assumed to carry the entire shear.

4.5.1. Specimen 4B: In this specimen the branch crack was almost vertical. At the end of the overload test and the additional 10 cycles of load in test (d), this crack extended to 2 in. (50 mm) from the web-flange junction. At this point the maximum crack width at service load was 0.009 in. (0.23 mm), and the service load stresses in the hanger and nib flexural reinforcement were respectively 13.3 and 16.5 ksi (91.7 and 73.4 MPa). It can be seen in Fig. 19 that a noticeable increase in stress had also occurred in the nib flexural reinforcement at a distance from the re-entrant corner equal to the depth of the dap.

In the final test at a shear of 19.4 kips (86.3 kN), a 2 in. (50 mm) long inclined crack occurred in the nib. This crack started near the inner edge of the bearing plate and extended upward at 60 degrees. As the shear was increased, this crack extended to and along the web-flange junction.
30

$V_{max} = 27.45 \text{k} = 1.37V_n(\text{calc})$

Fig. 19 - Variation with shear of stress in nib flexural reinforcement, specimen 4B.

Spec. 4B

$V_{max} = 19.54 \text{k}$

Spec. 4C

$0.72V_n(\text{calc})$

Fig. 20 - Comparison of hanger reinforcement stresses in specimens 4B and 4C.
The vertical branch of the re-entrant corner crack extended upward until it almost linked up with the nib inclined crack. Cracks inclined at between 30 and 45 degrees to the horizontal extended downward from the re-entrant corner crack toward the vertical end face. They were joined by a crack across this face at a shear of 21.9 kips (97.4 kN). While they were forming, the stress in the nib flexural reinforcement 9 in. (230 mm) from the re-entrant corner increased more rapidly.

At a shear of 24.5 kips (109.0 kN), a 6 in. (150 mm) long independent diagonal tension crack occurred, inclined at 30 degrees to the horizontal and starting 2 in. (50 mm) below the re-entrant corner and 3.5 in. (90 mm) from the end face of the web.

The maximum shear was 27.45 kips (122.1 kN), when a major diagonal tension crack traversed the web at a slope of about 35 degrees. It extended from the bottom corner of the web up to and into the flange. This crack occurred so abruptly that it was not possible to pin-point its origin.

The diagonal tension cracking was accompanied by some diagonal compression crushing of concrete in the lower part of the web. It was followed by inclined cracking in the vicinity of the horizontal extension of the hanger reinforcement and by a draw-in of the bottom prestressing strand of 0.1 in. (2.5 mm).

The diagonal tension failure of the beam web was preceded by yield of the hanger reinforcement at a shear of 26 kips (116 kN). The nib flexural reinforcement stress at maximum load was very close to its yield strength of 67.74 ksi (467 MPa) at the re-entrant corner and also 9 in. (230 mm) in from that corner. This observation verifies the need to provide anchorage for the nib flexural reinforcement for a development length beyond the point at which a 45 degree line from the bottom corner of the web crosses this reinforcement.
Although the final failure was very abrupt, there was warning of failure by extensive cracking in the web of the beam near the dapped end, above about 85 percent of ultimate. The maximum shear was 1.37 times the nominal shear strength, calculated assuming that the entire shear is carried by the hanger reinforcement. It appears that this reinforcement scheme is suitable for dapped ends in double tees, but that it does not control cracking under service load quite as well as schemes using inclined reinforcement which crosses the re-entrant corner crack more nearly at right angles.

4.5.2. Specimen 4C: As in the other specimens in which all the prestressing strands terminated in the end face of the web below the re-entrant corner, transfer of prestress produced tensile stresses in the dapped end reinforcement near that corner. This resulted in cracks developing there immediately that load was applied.

On first application of service load, the cracking was more extensive than that observed in specimen 4B after twenty cycles of service load plus 36 percent overload. An additional inclined crack occurred, running from about 3.5 in. (90 mm) below the re-entrant corner, to within 1.5 in. (40 mm) of the web-flange junction about 12 in. (300 mm) from the end of the flange. The maximum crack width on this first application of service load was 0.015 in. (0.38 mm), closing to 0.007 in. (0.18 mm) on removal of load. However, on the twentieth cycle of service load the maximum crack width had increased to 0.021 in. (0.53 mm).

The stresses in the dapped end reinforcement at comparable stages of loading were much greater in specimen 4C than in specimen 4B. This can be seen in Fig. 20, where the stresses in the hanger reinforcement near the re-entrant corner in these two specimens are compared. To simplify the figure, only stresses for tests (a), (c) and (e) are shown.
In the final test several inclined cracks occurred in the faces of the nib, starting at a shear of 16.22 kips (72.2 kN), whereas only one such crack occurred in specimen 4B.

Failure occurred after a shear of 19.54 kips (86.9 kN) had been sustained for 3 minutes. Failure initiated when short flexure cracks occurred in the end 12 in. (300 mm) of the full depth web. Seconds later major diagonal tension cracks occurred, extending from the bottom corner of the web, up to the web-flange junction 25 in. (635 mm) from the end of the flange. Immediately following the diagonal tension cracking, flexural bond cracking occurred over the horizontal extension of the hanger reinforcement.

Draw-in of the bottom prestressing strand was 0.004 in. (0.10 mm) before the diagonal tension failure, and 0.065 in. (1.65 mm) after it.

The maximum shear was 0.98 times the nominal strength calculated assuming that the hanger reinforcement carried all the shear as it developed its yield strength. However, the stress in the reinforcement at maximum shear was only 88 percent of yield, indicating that 1.86 kips (8.3 kN) of shear was being carried by the concrete.

The maximum shear carried by specimen 4C was significantly less than that carried by specimen 4B and its service load behavior was much inferior. Both differences in behavior are attributable to the different arrangement of the prestressing strands in the two specimens.

4.6. **Reinforcement scheme**

This arrangement of reinforcement was designed to provide positive location and anchorage of the reinforcing bars, whilst reducing reinforcement congestion in the nib and so
facilitating the placing and compaction of the concrete. The design of specimen 5B followed the same procedure as was used for specimen 2B, i.e., the nominal shear strength is taken as the sum of the vertical component of the yield strength of the inclined bars and the calculated shear at which cracking of the concrete occurs at the re-entrant corner, see Appendix C. In addition, the flexural strength of the interface between the nib and the full depth web was checked.

The vertical plates have a height approximately $\frac{1}{8}$th of the height of the beam web. The cross-section of each plate is chosen such that its shear yield strength is not less than the yield strength of the inclined bar which is welded to it. The weld attaching each inclined bar to its vertical plate is sized to carry the yield strength of the inclined bar. The weld between the tail of the hook and the vertical plate is nominal in size. The bend diameter of the hook is a minimum of six bar diameters. The purpose of the hook is to control diagonal tension cracking in the nib. Therefore the proportions of the nib and the hook must be such that a straight line drawn from the center of action $V_n$ on the bearing plate, to the center of action of the resultant concrete compression force at the nib-web interface, crosses the hook not higher than the point of tangency.

The maximum crack width on the twentieth cycle of service load was 0.009 in. (0.23 mm), about the same as that in specimen 1B and 40 percent less than that in specimen 2B at this point.

In the final test the re-entrant corner crack extended to the web-flange junction about 13 in. (330 mm) from the end of the flange at a shear of 17.5 kips (77.8 kN). Also at this same shear a secondary inclined crack occurred and the stress in the horizontal extension of the inclined bars started to increase much more rapidly.
At a shear of 21.3 kips (94.8 kN) the re-entrant corner crack had extended along the web-flange. Also a branch crack had travelled backwards into the nib at about 45 degrees, reaching 3.5 in. above the re-entrant corner and about 1 in. (25 mm) into the nib. At this same shear the first flexure crack occurred in the bottom face of the web, about 7 in. (180 mm) from the end of the bottom face. This crack was very fine and about 0.8 in. (20 mm) long.

At a shear of 22.2 kips (98.8 kN) a 2 in. (50 mm) long inclined crack formed in one face of the nib, centered about 0.5 in. (13 mm) beyond the end of the branch crack running into the nib.

At shears above 20 kips (89.0 kN) the draw-in of the bottom strand increased more rapidly, but did not exceed 0.006 in. (0.15 mm) until a major diagonal tension crack occurred at a shear of 24.76 kips (110.1 kN), at which time draw-in increased to 0.058 in. (1.5 mm).

At the same time as the major diagonal tension crack occurred, three additional flexure cracks occurred in the end 18 in. (450 mm) of the web. The diagonal tension crack originated the web and spread downwards to a point about 2 in. (50 mm) from the bottom corner of the web, and upward to the web-flange junction 34 in. (865 mm) from the end of the flange. When an attempt was made to recover the load, a flexure crack formed at about the end of the horizontal extension of the inclined reinforcement, and a flexural bond failure occurred, together with another diagonal tension crack. The appearance of the specimen after failure can be seen in Fig. 21.

The variation of stress in the inclined bars with increase in shear in specimen 5B is shown in Fig. 22. The maximum shear reached was 24.76 kips (110.1 kN), when the first major diagonal tension crack occurred. This shear is 1.08 times
Fig. 21 - Appearance of specimen 5B after test.

Fig. 22 - Variation with shear of stress in inclined bars, specimen 5B.
the calculated nominal shear strength of the dapped end, $V_n$, and the behavior of the specimen was considered satisfactory. The inclined reinforcement developed 87 percent of its yield strength when diagonal tension failure of the web occurred.

4.7. **Diagonal tension cracking in the beam web**

Most of the specimens suffered diagonal tension cracking of the web, and many of them failed when this occurred. This despite the fact that all the beams were provided with the minimum web reinforcement required by the ACI Code (10), in the form of welded wire fabric. The vertical W2.9 wires on 7.5 in. (190 mm) centers should have provided an increase in shear after diagonal tension cracking of 3.34 kips (14.9 kN).

In Table 4 is listed the increase in shear between that at which the first diagonal tension crack occurred and that at which the specimen failed. (Specimens 1C, 2B and 2C are not included in the table because they failed prematurely, see previous discussion.) In seven specimens there was no increase in shear after the first diagonal tension crack occurred. Only in two specimens was the increase in shear after diagonal tension cracking about equal to the 3.34 kips (14.9 kN) that the minimum web reinforcement should have provided. The average increase in shear after diagonal tension cracking was only 0.75 kips (3.3 kN).

Also shown in Table 4 is the difference between the failure shear and the calculated diagonal tension cracking shear, which was taken as the lesser of $V_{ci}$ and $V_{cw}$. It can be seen that although the difference between the failure shear and the calculated diagonal tension cracking shear was greater than or equal to the theoretical contribution of the minimum web reinforcement in three specimens, it was actually a negative quantity for four other specimens. The average increase was only 1.47 kips (6.5 kN).
Table 4 - Increase in Shear After Diagonal Tension Cracking (kips)

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$V_{(test)} - V_{c(test)}$</th>
<th>$V_{(test)} - V_{c(calc)}$</th>
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<tbody>
<tr>
<td>1A</td>
<td>0.00</td>
<td>-1.28</td>
</tr>
<tr>
<td>1B</td>
<td>0.95</td>
<td>2.34</td>
</tr>
<tr>
<td>2A</td>
<td>0.00</td>
<td>-0.30</td>
</tr>
<tr>
<td>2D</td>
<td>(l)</td>
<td>0.72</td>
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<tr>
<td>3B</td>
<td>3.53</td>
<td>4.48</td>
</tr>
<tr>
<td>3c</td>
<td>0.00</td>
<td>-1.36</td>
</tr>
<tr>
<td>3D</td>
<td>0.00</td>
<td>1.88</td>
</tr>
<tr>
<td>3E</td>
<td>0.00</td>
<td>3.42</td>
</tr>
<tr>
<td>4B</td>
<td>3.00</td>
<td>5.84</td>
</tr>
<tr>
<td>4c</td>
<td>0.00</td>
<td>-0.99</td>
</tr>
<tr>
<td>5B</td>
<td>0.00</td>
<td>1.40</td>
</tr>
</tbody>
</table>

Average 0.75 1.47
Standard Deviation 1.37 2.41

(1 kip = 4.45 kN)

$V_{(test)} =$ maximum shear attained in test.

$V_{c(test)} =$ shear at first diagonal tension crack in web.

$V_{c(calc)} =$ calculated shear at diagonal tension cracking.

(l) No diagonal tension cracking occurred in web in this specimen.
In all specimens in which the first diagonal tension crack extended downwards to a point near the bottom corner of the web, the specimen failed when this crack occurred. It was also accompanied by slip of the lower prestressing strands. Only if the diagonal tension crack was not in this critical location was any further increase in shear possible. In these specimens, failure eventually occurred either when another diagonal tension crack formed at this critical location, or a flexural bond failure occurred.

It appears that if a diagonal tension crack occurs in this critical location, a flexural bond failure follows, due to an increase in the flexural reinforcement stress at the bottom end of the diagonal tension crack. This prevents the development of any truss action by the web reinforcement, because the tension chord of the truss is effectively disabled. In these tests, the first diagonal tension crack formed at this critical location in the majority of specimens. Also, it may be seen in Fig. 23 that the calculated shear at diagonal tension cracking typically tends to be smallest near the bottom corner of the web and so it is reasonable to expect that diagonal tension cracking is most likely to occur first in or near this location.

It is therefore proposed that in pretensioned, prestressed concrete tee or double tee members with dapped ends, the nominal shear strength of the full depth web adjacent to the dapped end should be limited to the lesser of Vci and Vcw'. Note that minimum web reinforcement should still be provided to prevent a disruptive failure, even though it is not taken into account when calculating the shear strength.

In Table 5 the shear at first diagonal tension cracking is compared with the calculated diagonal tension cracking shear. This is taken as the lesser of Vci and Vcw', which were calculated using equations (11-11), (11-12) and (11-13) of ACI 318-77 (10).
Fig. 23 - Diagonal tension cracking in specimen 3C.
Table 5 - Diagonal Tension Cracking Shears (kips)

<table>
<thead>
<tr>
<th>Specimen</th>
<th>V&lt;sub&gt;C&lt;/sub&gt;(test)</th>
<th>V&lt;sub&gt;cw&lt;/sub&gt;</th>
<th>V&lt;sub&gt;ci&lt;/sub&gt;</th>
<th>V&lt;sub&gt;C&lt;/sub&gt;(test)/V&lt;sub&gt;C&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A</td>
<td>21.82</td>
<td>23.10</td>
<td>24.61</td>
<td>0.94</td>
</tr>
<tr>
<td>1B</td>
<td>26.98</td>
<td>25.59</td>
<td>26.24</td>
<td>1.05</td>
</tr>
<tr>
<td>2A</td>
<td>22.55</td>
<td>23.10</td>
<td>24.61</td>
<td>0.98</td>
</tr>
<tr>
<td>2D</td>
<td>No D.T. crack</td>
<td>23.19</td>
<td>24.67</td>
<td>&gt;1.00</td>
</tr>
<tr>
<td>3B</td>
<td>24.40</td>
<td>23.45</td>
<td>24.86</td>
<td>1.04</td>
</tr>
<tr>
<td>3c</td>
<td>21.16</td>
<td>22.52</td>
<td>25.68</td>
<td>0.94</td>
</tr>
<tr>
<td>3D</td>
<td>25.24</td>
<td>23.36</td>
<td>24.67</td>
<td>1.08</td>
</tr>
<tr>
<td>3E</td>
<td>29.51</td>
<td>26.09</td>
<td>26.86</td>
<td>1.13</td>
</tr>
<tr>
<td>4B</td>
<td>24.45</td>
<td>21.61</td>
<td>27.88</td>
<td>1.13</td>
</tr>
<tr>
<td>4c</td>
<td>19.54</td>
<td>20.53</td>
<td>31.32</td>
<td>0.95</td>
</tr>
<tr>
<td>5B</td>
<td>24.76</td>
<td>23.36</td>
<td>24.67</td>
<td>1.06</td>
</tr>
</tbody>
</table>

Average (not incl. 2D) = 1.03
Standard Deviation = 0.07

(1 kip = 4.45 kN)

(1) Shear at first diagonal tension crack in web.
(2) Calculated at h/2 from end of full depth section, except for 3E where calculation was made at h/2 from center of bend in inclined bars.
(3) V<sub>C</sub> is the lesser of V<sub>ci</sub> and V<sub>cw</sub>.
It can be seen that the calculated diagonal tension cracking shears are in close agreement with the observed values.

The values listed in Table 5 were calculated for a section h/2 from the end of the full depth section. ("h" is the overall depth of the tee section.) An exception was specimen 3E, for which the calculation was made at h/2 from the center of the bend in the inclined reinforcement. This seemed more appropriate for this specimen in which the end face was vertical, but the hanger reinforcement extended into the web at 45 degrees.

When making such calculations, the effect of the horizontal tension force must be taken into account when calculating the values of $f_{pe}$ and $f_{pc}$ used in equations (11-12) and (11-13). This force reduces both of these stresses, and so reduces both $V_{ci}$ and $V_{cw}$.

Also when making such calculations, the actual build up of the prestress force in each strand must be taken into account when calculating $f_{pe}$ and $f_{pc}$. The values of shear listed in Table 5 were calculated assuming that the prestress force in each strand built up linearly from zero at the end of that strand, to the effective prestress force after losses at the end of the transfer length. This was assumed equal to 36 in. (914 mm) for the clean 0.5 in. (13 mm) diameter strand used in these specimens. This value was taken from the PCA study of the influence of concrete strength on strand transfer length, by Kaar, LaFraugh and Mass (11). It was found to lead to consistently close estimates of the diagonal tension cracking strength.

$V_{ci}$ and $V_{cw}$ were calculated for several points near the end of the full depth section in each specimen. An example of the variation in $V_{ci}$ and $V_{cw}$ with distance from the bottom corner is shown in Fig. 23. In this figure the calculated values of $V_{ci}$ and $V_{cw}$ for specimen 3C are compared with the actual diagonal
tension cracking shear and the location of the crack. Similar results were obtained for other specimens and support the choice of the critical section proposed for use in design when calculating $V_{ci}$ and $V_{cw}$, i.e., $h/2$ from the end of the full depth section.
5 - CONCLUSIONS FOR DESIGN

On the basis of the results obtained in this study, the following conclusions and recommendations appear reasonable, relative to dapped ends on pretensioned, prestressed concrete tee and double tee members:

(1) All five of the reinforcement schemes investigated are suitable for use in practice.

(2) In all cases the horizontal extension of the hanger reinforcement in the bottom of the web must be not less than 1.7 times the development length specified in the ACI Code (10), in order that the yield strength of the hanger reinforcement can be developed.

(3) The design procedures for reinforcement schemes 1, 2, 3 and 5 set out in Appendices B, C and D, and for reinforcement scheme 4 set out in references (5) and (7), yield safe estimates of the strength of dapped ends using these reinforcement schemes.

The "truss analogy" used in the design of reinforcement schemes 1 and 3, and the "free body" equilibrium approach used in the design of reinforcement schemes 2 and 5 are statically equivalent. The truss analogy is used in the design of reinforcement schemes 1 and 3 to control stresses within the nib, since it models the actual structural behavior closely in these cases. In reinforcement schemes 2 and 5 the free body equilibrium approach is appropriate, since the truss analogy does not model structural behavior within the nib in these cases where a special plate reinforcement detail is provided to carry the shear.
The proportions of the specimens in the test program were not varied. When dapped ends are designed with different proportions, care must be taken to ensure that the location of the reinforcement reflects the assumptions made in the design procedure for the reinforcement scheme being used. Also, that all reinforcement is securely anchored in the nib so as to be able to develop its yield strength.

(4) In the design procedure for reinforcement schemes 2 and 5 set out in Appendix C, it is implicitly assumed that not less than approximately half the prestressing strands pass through the nib. If this is not so, then the term $V_{cr}$ in such design calculations should be taken equal to zero.

(5) The modified methods of strength calculation, taking into account a contribution to nib shear strength from the concrete, (set out in part (d) of Appendices B and D,) should only be used if not less than approximately half the prestressing strands are passed through the nib. This condition is to ensure satisfactory serviceability.

(6) Application of the design procedures for reinforcement schemes 1, 2, 3 and 5, set out in Appendices B, C and D, should be limited to dapped ends meeting the following conditions:

(a) For normal weight concrete, the nominal shear stress $V_{n}/b_{dd}$ must not exceed $0.2f'_{c}$ nor 800 psi, (5.5 Mpa). For "all-lightweight concrete" the nominal shear stress $V_{n}/b_{dd}$ must not exceed $0.2 - 0.07a/d)f'_{c}$ nor $(800 - 280a/d)$ psi, [(5.5 - 1.9 a/d) MPa]. For "sand-lightweight concrete" the nominal shear stress $V_{n}/b_{dd}$ must not exceed
(0.2 - 0.07a/d)f'_c nor (1000 - 350a/d) psi, 
[(6.9 - 2.4a/d) MPa]. [See Ref. (14).]

(b) The ratio of the shear span to the effective depth of the nib must not exceed unity, where the shear span is the distance from the line of action of the support reaction, to the intersection of the centerlines of the hanger reinforcement and the nib flexural reinforcement.

(c) The inclination of the hanger reinforcement to the vertical must be between 20 and 45 degrees.

For "all-lightweight" or "sand-lightweight" concrete dapped ends, the term v_{cr} in all strength calculations should be multiplied by 0.75 or 0.85 respectively.

When using the fifth reinforcement scheme, the proportions of the nib and the hook must be such that a straight line drawn from the center of action of the reaction on the bearing plate, to the center of action of the resultant concrete flexural compression force at the nib-web interface, crosses the hook not higher than the point of tangency.

The reinforcement schemes using inclined hanger reinforcement provide better control of cracking than scheme 4 which uses vertical hanger reinforcement, particularly if all the prestressing strands are terminated at the face of the dap.

From the point of view of strength in service, it does not appear necessary to provide compression reinforcement in reinforcement scheme 2. However, it may be considered that nominal reinforcement in that location...
is desirable to resist any accidental downward load on the nib during handling.

(11) Since nib flexure cracks initiated at the inner edge of those bearing plates which did not cover the entire length of the nib, it is recommended that bearing plates should extend to the re-entrant corner. (The jog formed in the concrete by the inner edge of a short bearing plate evidently acts as a "stress raiser" in the nib concrete.)

(12) Passing half the prestressing strands through the nib of the dapped end significantly improved serviceability for all of the reinforcement schemes, by reducing the maximum stress under service load in the reinforcement near the re-entrant corner. This reduced both the extent of cracking and the width of the cracks. It is therefore strongly recommended that, if at all possible, not less than about half the prestressing strands should be passed through the nib.

(13) Terminating all the prestressing strands at the face of the dap, (i.e., no strands pass through the nib,) resulted in inferior performance under service load, due to the transfer of prestress causing tensile stresses in the dapped end reinforcement near the re-entrant corner. This resulted in earlier, more extensive and wider cracks than occurred when half of the prestressing strands passed through the nib.

(14) It is possible to develop the yield strength of #3 and #4 reinforcing bars by anchoring them with a 180 degree loop transverse to the axis of the member, and having a minimum bend of diameter of 6 bar diameters. (The orientation of the loop is critical, since it can only act as an effective anchorage if compression stresses
in the concrete act across the plane containing the loop and so prevent a splitting failure.)

(15) Cracking of the end face of the web over the hanger reinforcement is non-existent or acceptably fine if 0.75 in. (19 mm) concrete cover is provided for #3 and #4 bars, or 1.0 in. (25 mm) cover is provided for #5 bars.

(16) A concentric or near concentric arrangement of hanger reinforcement is preferable. If the hanger reinforcement is eccentric relative to the centerline of the web, then that eccentricity should be minimized and special care taken to provide adequate side cover to the lower part of the hanger reinforcement and its horizontal extension.

(17) Even when the yield strength of the dapped end reinforcement was developed, and the strength obtained was equal to or greater than the calculated nominal strength, the maximum shear was reached when a diagonal tension failure of the full depth web occurred immediately adjacent to the dapped end.

(18) In most of these pretensioned, prestressed concrete beams with dapped ends, it was not possible to develop a full depth beam shear strength greater than the diagonal tension cracking shear, through the provision of web reinforcement. Once a critical diagonal tension crack occurred, passing through or near to the bottom corner of the web, a flexural bond failure followed at once. The web reinforcement was unable to contribute to shear strength through truss action, since the tension chord of the truss was effectively disabled. It is therefore proposed that, for that part of the full depth web adjacent to a dapped end, the nominal shear
strength $V_n$ be taken equal to $V_c$, where $V_c$ is equal to the lesser of $V_{ci}$ and $V_{cw}$, calculated for the section distance $h/2$ from the end of the full depth web.

In the future some reinforcement detail may be developed which can overcome this problem, but at present no test data validating such a detail is available.

When calculating $V_{ci}$ and $V_{cw}$ for that portion of the full depth web adjacent to a dapped end, the following factors must be taken into account in design.

(a) Any horizontal tension force acting at the bearing will reduce both $f_{pe}$ and $f_{pc}$. The tension force and its moment about the section centroid must be taken into account when calculating $f_{pe}$ and $f_{pc}$ for use in the ACI Code (10) equations for $V_{ci}$ and $V_{cw}$.

(b) The actual build up of the prestressing force in each strand must be calculated, and appropriate values used when calculating $V_{ci}$ and $V_{cw}$. The results obtained in this study indicate that it is reasonable to assume a linear increase in prestress force over the transfer length of the strand, taken as 36 in. (915 mm) for clean 0.5 in. (13 mm) seven wire strand.
6 REFERENCES


7. PCI DESIGN HANDBOOK (Third Edition), Prestressed Concrete Institute, Chicago, 1985.

8. PCI DESIGN HANDBOOK (Second Edition), Prestressed Concrete Institute, Chicago, 1978.


APPENDIX A

NOTATION

= shear span, defined as distance from support reaction to intersection of hanger and nib flexural reinforcement

\(a\) = depth of equivalent rectangular compressive stress block

\(a_1 \rightarrow a_5\) = distance defined in Fig. Cl

\(A_s\) = area of nib flexural tension reinforcement

\(A_{sh}\) = area of hanger reinforcement

\(b_a\) = length of anchorage angle

\(b_d\) = average width of nib

\(b_w\) = width of nib at depth \(x/2\) from the top face

\(b_1, b_2\) = distances defined in Fig. C1

\(c\) = cover

\(C\) = resultant compression force, Fig. C1

\(C_1, C_2\) = compression strut forces, Figs. Bl and D1

\(d_b\) = reinforcing bar diameter

\(d_{bc}\) = cross bar diameter

\(d_c'\) = distance between nib flexural tension and compression reinforcement

\(d_d\) = effective depth of nib

\(D\) = unfactored dead load

\(e\) = distance defined in Figs. Bl and D1

\(E_s\) = modulus of elasticity of reinforcement

\(f'_c\) = compressive strength of concrete measured on 6 x 12 in. (150 x 300 mm) cylinders, psi (MPa)

\(f_{pu}\) = ultimate tensile strength of wire and strand

Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.
\( f_{py} \) = nominal yield strength of prestressing strand

\( f_y \) = yield strength of reinforcing bar

\( F \) = tensile force in nib flexural tension reinforcement, Fig. Cl

\( h \) = overall depth of precast member

\( h_d \) = overall depth of nib

\( h_t \) = distance from top face of member to bottom face of nib

\( d \) = reinforcing bar development length

\( d_v \) = distance from re-entrant corner to line of action of vertical reaction acting on nib

\( L \) = unfactored live load

\( L_{bc} \) = length of cross bar

\( N_n \) = nominal tension force resisted at ultimate (acting at bottom face of nib, parallel to axis of member)

\( t \) = thickness of flange of member

\( T \) = tensile force of hanger reinforcement, Fig. Cl

\( V_{ci} \) = shear at flexure-shear diagonal tension cracking

\( V_{cr} \) = contribution of concrete to shear strength of dapped end

\( V_{cw} \) = shear at web-shear diagonal tension cracking

\( V_d \) = shear due to unfactored dead load

\( V_{\text{max}} \) = maximum shear resisted in test

\( V_n \) = nominal shear force resisted at ultimate

\( V_s \) = shear at service load

\( w_u \) = total factored load per unit length

\( x \) = distance from top face of nib to truss force intersection point 0 in Figs. Bl and Dl

\( x'_1, x'_2 \) = distance from top face of nib to intersection of line of action of force C2 and vertical plane through re-entrant corner
\( y \) = distance from centerline of hangar reinforcement to end face of beam web, Figs. Bl and D1

\( \alpha \) = inclination of end face of beam web to vertical

\( \beta \) = inclination of endface of beam web to the horizontal

\( \gamma \) = inclination to vertical of line of action of compression strut force C1

\( \phi \) = ACI Code strength reduction factor

\( \phi \) = symbol for nominal diameter of seven wire strand
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.
APPENDIX B

CALCULATION OF DAPPED END STRENGTH

FOR REINFORCEMENT SCHEME 1

(a) Initial Method of Calculation

It is assumed that at ultimate the force system within the nib may be idealized as a truss, as shown in Fig. B1. The compression strut forces \( C_1 \) and \( C_2 \) are compression forces in the concrete. The tension members \( A_s \) and \( A_{sh} \) are assumed to be developing their yield strength. The forces in any prestressing strands present in the nib are neglected.

For equilibrium of the nib the following conditions must be satisfied:

1. For vertical equilibrium
   \[
   V_n = A_s f_y \cos \alpha \quad (1)
   \]

2. For moment equilibrium, (taking moments about truss force intersection point \( O \).)
   \[
   V_n e + N_n (d_e - x) = A_s f_y (d_e - x) \quad (2)
   \]

3. For horizontal equilibrium
   \[
   N_n + C_2 = A_s f_y + A_{sh} f_y \sin \alpha \quad (3)
   \]

   Hence,
   \[
   C_2 = A_s f_y + A_{sh} f_y \sin \alpha - N_n
   \]

   It is assumed that the horizontal compression strut has a width \( b_a \) equal to the length of the anchorage angle. Its depth is \( 2x \). Assuming a stress of \( 0.85 f'_c \) in the strut at ultimate:
   \[
   C_2 = 0.85 f'_c b_a (2x) = 1.7 f'_c b_a x
   \]
Fig. B1 - Assumed "Truss-Action" in nib for first reinforcement scheme.
hence,

\[ x = \frac{A_s f + A_{sh} \sin \alpha - N_n}{1.7 f' b c} \]  

\[ x = \frac{A_s f + A_{sh} \sin \alpha - N_n}{1.7 f' b c} \]  

also

\[ e = \ell_v + y / \cos \alpha - 'h d - x) \tan \alpha \]  

(b) Design Procedure

(1) Calculate \( A_{sh} \) using Eq. (1).

(2) Choose rebar for \( A_{sh} \) and calculate "y."

(3) Assume a value for "x" and calculate "e" using Eq. (5).

(4) Calculate \( A_s \) using Eq. (2).

(5) Calculate "x" using Eq. (4) and compare with value assumed in step (3). If values of x are sufficiently close, \( A_s \) obtained in step (4) is correct. If not, modify value of x assumed in step (3) and recalculate \( A_s \) and x, continuing iteration until agreement is satisfactory.

(6) Check that \( \tan \gamma = e / (d_d - x) \) is not more than 0.15, i.e. about half the static coefficient of friction for concrete against steel. If \( \tan \gamma \) is greater than 0.15, provide a cross bar, diameter \( d_{bc} \), length \( L_{bc} \), welded to bearing plate, so that

\[ 0.85 f' d_{bc} L_{bc} \text{ is } V_n e / (d_d - x) \]  

(c) Check on Equilibrium of Nib if a Crack Propagates from Re-entrant Corner to Underside of Flange at 45 Degrees to Horizontal.

Compression force in flange is assumed to act at mid-thickness.

Let V be shear force which would just cause rotation about point Y in Fig. Bl.
The:

\[ V(\ell_v + h_d) + N_n(h_d + t/2) \]
\[ = [A_{sf}f_y(d_d + t/2) + A_{shf}\cos \alpha (\ell_v + h_d - e) + A_{shf}\sin \alpha (x + t/2)] \]
\[ = [A_{sf}f_y(d_d - x + (x + t/2)) + V_n(\ell_v + h_d - e) + V_n \tan \alpha (x + t/2)] \]
\[ = [V_v + N_n(h_d - x) + A_{sf}f_y(x + t/2) + V_n(h_d - x) \tan \alpha] \]
\[ = y/\cos \alpha + h_d \}
\[ + V_n \tan \alpha (x + t/2)] \]
\[ = [V_v + V_y/\cos \alpha - V_{ntan} (h_d - x) + N_n(h_d - x) + A_{sf}f_y(x + t/2) + V_{ntan}(x + t/2)] \]
\[ = [V_v + h_d) + N_n(h_d - x) + A_{sf}f_y(x + t/2) + V_{ntan}(x + t/2)] \]
\[ = [V_v + h_d) + N_n(h_d + t/2) - N_n(x + t/2) \]
\[ \} \quad \{A_{sf}f_y + V_n \tan \alpha \}(x + t/2)] \]

\[ V = V" + \{A_{sf}f_y - N" + V_n \tan \alpha \}(x + t/2)/(\ell_v + h_d) \]

Since \( A_{sf}f_y \) is greater than \( N" \), the shear \( V \) which would result in failure due to rotation about \( Y \) will be greater than \( V_n \), i.e. this failure mode is not critical for dapped end designed as above.

(d) Modified Method of Calculation

It is assumed that instead of acting horizontally as in Fig. B1, force \( C_2 \) acts through the truss intersection point \( 0 \) at a\(^n\) angle, such that it has a vertical component \( V_{cr} \) equal to \( 2b_d d \sqrt{E_f} \) pounds, (0.17b_d d \sqrt{E_f} Newtons).

The equilibrium equations then become:

(1A) For vertical equilibrium of nib

\[ V_n = A_{shf}\cos \alpha + V_{cr} \]
For moment equilibrium of nib about truss force intersection point 0.

\[ V_{ne} + N_{n}(h_d - x) = A_s f_y(d_d - x) \]  

\[ (2A) \]

For horizontal equilibrium of nib

\[ N_n + c_2 = A_s f_y + A_s f_s \sin \alpha \]  

\[ (3A) \]

hence

\[ c_2 = A_s f_y + A_s f_s \sin \alpha - N_n \]  

\[ (4A) \]

As before, \( x \), the depth of point 0 from top face of nib is given by:

\[ x = \frac{A_s f_y + A_s f_s \sin \alpha - N_n}{1.7 f_c b_a} = \frac{c_2}{1.7 f_c b_a} \]  

\[ (5A) \]

where

\[ b_a = \text{length of anchorage angle.} \]

and

\[ e = \beta_v + \frac{y}{ \cos \alpha - (h_d - x) \tan \alpha} \]  

\[ (6A) \]

Now the line of action of \( C_2 \) will intersect the vertical plane through the re-entrant corner at a distance \( x_1' \) below the top face of the nib, where

\[ x_1' = x - (\beta_v - e)(V_{cr}/C_2) \]  

\[ (7A) \]

and \( x_1' \) cannot be less than \( x_2' \) given by

\[ x_2' = C_2/(1.7 f_c b_w) \]  

\[ (8A) \]

where \( b_w \) is width of nib at depth \( x_2' \) given by Eq. (8A).

**Design Procedure**

1. Calculate \( A_{sh} \) using Eq. (1A).
2. Choose rebar for \( A_{sh} \) and calculate \( y \).
3. Assume value for \( x \) and calculate \( e \) using Eq. (6A).
4. Calculate \( A_s \) using Eq. (2A).
5. Calculate \( C_2 \) using Eq. (4A).
(6) Calculate $x$ using Eq. (5A), compare with value assumed in step (3). If $x$ from Eq. (5A) is close to and less than assumed value, continue to step (7); if not, repeat steps (3) through (6) until agreement is satisfactory, then continue with step (7).

(7) Calculate $x_1$ using Eq. (7A) and check that it is not less than $x_2$ given by Eq. (8A). If it is not less, the value of $A_s$ is that calculated in step (4) above. If it is less, assume a larger value of $x$ and repeat steps (3), (4), (5) and (7); repeating the process until $x_1$ calculated using Eq. (7A) is not less than $x_2$ calculated using Eq. (8A). Corresponding value of $A_s$ is correct value.

(8) If $e/(d - x) \geq 0.15$, provide cross-bar of diameter $d_{bc}$ and length $L_{bc}$, welded to bearing plate such that

$$0.85f'_c L_{bc} \geq \frac{V_n e/(d - x)}{d_{bc}}.$$

(9) Design upper anchorage for hanger reinforcement.
APPENDIX C

CALCULATION OF DAPPED END STRENGTH

FOR REINFORCEMENT SCHEMES 2 AND 5

(a) Method of Calculation, Reinforcement Scheme 2

This method of calculation closely follows the method set out in Stanley Structures in-house Engineering Standard ES-61 - Double Tee Dap Design. However, this presentation of the method is in terms of nominal strength, so as to be consistent with the presentation of the calculation methods for the other reinforcement schemes.

It is assumed that an inclined crack has propagated from the re-entrant corner up to the underside of the flange, at 38 degrees to the horizontal. A "free body" diagram for that part of the end of the beam cut off by the assumed crack is shown in Fig. Cl(a).

It is assumed that the concrete compression zone makes a contribution $V_{cr}$ to the shear strength, where $V_{cr}$ is the shear which will cause cracking at the re-entrant corner. Based on finite element analysis and test results, $V_{cr}$ is calculated using:

$$V_{cr} = 1.95 b_d d_d \sqrt{F_c^T}$$

in which $d_d$ is taken equal to $0.9 h_d$, and the equation bounds are

$$0.5 < h_t/h < 0.8.$$  

For equilibrium of the "free body" the following equations must be satisfied:

(1) For moment equilibrium about point 0 in Fig. Cl(a)

$$0 = V_n (b_v - b_1) + N_n (h_t - a_5) + (w_u/\phi)(b_2)(b_1 + b_2/2) + V_{cr}(b_1 + b_2) - F(h_t - a_5 - a_1)$$

(2)
(a) "Free-Body" used to calculate required \( T, F \) and \( C \).

(b) "Free-Body" used to calculate location of compression reinforcement.

**Figure C1** - "Free Bodies" used in strength calculations for second reinforcement scheme.
(2) For vertical equilibrium

\[ 0 = V_n - (w_u/\phi)(b_2) - V_{cr} = T \cos \alpha \]  

(3) For horizontal equilibrium

\[ 0 = -N_n + T \sin \alpha + F - C \]

Solving for required forces \( F, T \) and \( C \) we have:

From Eq. (2)

\[ F = [V_n (\lambda_v - b_1) + N_n (h_t = a_5) + (w_u/\phi)(b_2)(b_1 + b_2/2) \\
    + V_{cr} (b_1 + b_2)]/(h_t - a_5 - a_1) \]  

From Eq. (3)

\[ T = [V_n - (w_u/\phi)(b_2) - V_{cr}] / \cos \alpha \]

From Eq. (4)

\[ C = -N_n + T \sin \alpha + F \]

\[ b_1 = (h_t - a_5) \tan \alpha = a_4 / \cos \alpha \]

\( a_5 \) is assumed to equal \( t/2 \)

\[ b_2 = (h_t - t) / \tan 38^\circ \]

The location of the nib compression reinforcement is calculated by considering the equilibrium of the nib if cut off vertically, as shown in Fig. Cl(b).

Taking moments about point 0 in Fig. Cl(b):

\[ 0 = V_n b_v + T a_4 + F a_l - C(d_c' + a_l) \]

From which:

\[ d_c' = -a_l + [V_n b_v + T a_4 + F a_l] / C \]

(b) Design Procedure, Reinforcement Scheme 2

(1) Calculate \( V_{cr} \) using Eq. (1)
(2) Calculate required forces \( F, T \) and \( C \), using Eq. (5), (6), and (7).

(3) Calculate required bar sizes by dividing \( F, T \) and \( C \) by \( f_y \).

(4) Calculate distance \( d'_c \) using Eq. (9).

(c) **Modified Design Procedure, Reinforcement Scheme 2**

(1) As in part (b) above, but omit compression reinforcement.

(2) Height of vertical plate in nib to be not less than \( (d'_c + a) \).

(3) Inclined bar \( A_{sh} \) must lap the vertical plate an amount not less than 80 percent of the height of the plate.

(d) **Design Procedure, Reinforcement Scheme 5**

(1) As in part (b) above, but omit calculation of \( C \) and \( d'_c \).

(2) Check flexural strength at interface of nib and full depth web.

Depth of flexural compression stress block,

\[
a = \frac{A_f y + A_{sh} f \sin \alpha - N}{0.85 f'_c b} (10)
\]

\( b = \) width of flexural compression zone at top of web.

\[
M_n = (A_f y - N_n)(d_d = a/2) + A_{sh} f \sin \alpha [h_n - (a_4 / \sin \alpha) - a/2]
\]

\[
(11)
\]

Required \( M_n \)

\[
= V n v + N_n (h_n - d_d) = V n v + N a_{sh} \]

\[
(12)
\]

(3) Check that a straight line drawn from center of action of \( V_n \) on bearing plate, to point \( a/2 \) below top of nib on nib-web interface, crosses \( A_{sh} \) below point of tangency of hook.

(4) (a) Height of vertical plates to be not less than \( 1/8 \)th the height of the beam web.
APPENDIX D

CALCULATION OF DAPPED END STRENGTH
FOR REINFORCEMENT SCHEME 3

(a) Initial Method of Calculation

It is assumed that at ultimate the force system within the nib can be idealized as a truss, as shown in Fig. D1(a). The compression strut forces $C_1$ and $C_2$ are compression forces in the concrete. The tension members $A_s$ and $A_{sh}$ are assumed to be developing their yield strength. The forces in any prestressing strands present in the nib are neglected.

The compression force $C_2$ is initially assumed to act at the centroid of the semi-circular area bounded by the centerline of the 180 degree bend in the hanger reinforcement $A_{sh}$, see Fig. Cl(b). For an interior bend diameter of $6d_b$, the distance to the centroid from the centerline of the bar at the middle of the bend is $2.0d_b$.

$C_2$ will also act not higher than the center of action of the flexural compression stress block in the vertical plane through the reentrant corner.

For equilibrium of the nib the following conditions must be satisfied:

1. For vertical equilibrium
   \[ V_n = A_{sh} f_y \cos \alpha \]  \hspace{1cm} (1)

2. For moment equilibrium, (taking moments about truss force intersection point O)
   \[ V_n e + N_n (h_d - x) = A_s f_y (d - x) \] \hspace{1cm} (2)

3. For horizontal equilibrium
   \[ N_n + C_2 = A_s f_y + A_{sh} f_y \sin \alpha \] \hspace{1cm} (3)

If vertical concrete cover to top of bend is "$c$" (as in Fig. D1(a))
(a) Assumed "Truss-Action" in the nib.

(b) Location of centroid of rebar loop.

Fig. D1 - Assumed "Truss-Action" in nib and rebar loop geometry, for third reinforcement scheme.
\[ x = c + \frac{d_b}{2} + 2d_b \cos \alpha \]  
\[ e = \frac{L_v}{v} + \frac{y}{\cos \alpha} = (h_d - x) \tan \alpha \]  

From (3),
\[ C_2 = \frac{A_{sy}}{y} + \frac{A_{sh} f \sin \alpha}{s} - \frac{N}{n} \]

Also, \( C_2 \) cannot be more than \( 0.85f'_{bw}(2x) \).
\[ \therefore x \text{ must be } \geq \frac{A_{sy}}{y} + \frac{A_{sh} f \sin \alpha}{s} - \frac{N}{n} \]
\[ \frac{1.7f'_{bw}}{c_{bw}} \]  

where \( b_w \) is width of web at the depth \( x \) given by Eq. (6).

**Design Procedure**

1. Calculate \( A_{sh} \) using Eq. (1).
2. Choose rebar for \( A_{sh} \) and calculate "y."
3. Calculate value of "x" using Eq. (4).
4. Calculate \( A_s \) using Eq. (2).
5. Calculate "x" using Eq. (6) and check that it is not greater than \( x \) calculated using Eq. (4). If it is not greater, value of \( A_s \) is that already calculated. If it is greater, assume a larger value for \( x \) in Eq. (2), recalculate \( A_s \) and \( x \), repeating until value of \( x \) used in Eq. (2) is in agreement with \( x \) calculated using Eq. (6).
6. If \( e/(d_d - x) \) is \( \geq 0.15 \), provide cross bar of diameter \( d_{bc} \) and length \( L_{bc}' \) welded to bearing plate, such that
\[ 0.85f'_{c bw} L_{bc}' \geq \frac{V e/(d_d - x)}{n} \]

(c) **Equilibrium Check if 45° Crack Propagates from Reentrant Corner.**

This will be the same as in part (c) of Appendix B. This mode of failure is not critical for a dapped end designed as above.
(d) Modified Method of Calculation

It is assumed that instead of acting horizontally, as in Fig. D1(a), force \( C_2 \) acts through the truss intersection point 0 at an angle, such that it has a vertical component \( V_{cr} \) equal to \( 2b_d d \sqrt{f_c} \) pounds, \( (0.17b_d d \sqrt{f_c} \text{ Newtons}) \).

The equilibrium equations for the nib then become:

(1A) For vertical equilibrium of nib

\[
V_n = A_h f_y \cos \alpha + V_{cr}
\]

(2A) For moment equilibrium of nib about truss force intersection point 0

\[
V_n e + N_n (h - x) = A_s f_y (d_d - x)
\]

(3A) For horizontal equilibrium of nib

\[
N_n + C_2 = A_s f_y + A_h f_y \sin \alpha
\]

as before,

\[
x = c + d_b / 2 + 2d_b \cos \alpha
\]

and

\[
e = \lambda_v + y / \cos \alpha - (h_d - x) \tan \alpha
\]

From (3)

\[
C_2 = A_s f_y + A_h f_y \sin \alpha - N_n
\]

Now the line of action of \( C_2 \) will intersect the vertical plane through the m-entrant corner at a distance \( x_1' \) below the top face of the nib,

where

\[
x_1' = x - (\lambda_v - e) (V_{cr} / C_2)
\]

and

\[
x_2' = C_2 / (1.7 f_y b)
\]
where \( w \) is width of web at depth \( x' \) given by Eq. (8A).

**Design Procedure**

1. Calculate \( A_{sh} \) using Eq. (1A).
2. Choose rebar for \( A_{sh} \) and calculate "y."
3. Calculate value of \( x \) using Eq. (4A).
4. Calculate \( A_s \) using Eq. (2A).
5. Calculate \( C_2 \) using Eq. (6A).
6. Calculate \( x_1' \) using Eq. (7A) and check that it is not less than \( x_2' \) calculated using Eq. (8A). If it is not less, value of \( A_s \) is as calculated in step (4) above. If it is less, assume a larger value for \( x \) and repeat steps (4,5) and (6), repeating process until \( x_1' \) calculated using Eq. (7A) is not less than \( x_2' \) calculated using Eq. (8A). Corresponding value of \( A_s \) is correct value.
7. If \( e/(d_d - x) \) is \( \geq 0.15 \), provide cross-bar of diameter \( d_{bc} \), and length \( L_{bc} \) welded to bearing plate such that,

\[
0.85f'c_{bc}L_{bc} \geq V_n e/(d_d - x).
\]

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APPENDIX E

DESIGN EXAMPLES

Design the reinforcement for the dapped end shown in Fig. El. The 8DT20 section is taken from page 2-14 of the PCI Design Handbook (7). The vertical reaction is assumed to act at 3/4 the length of the nib from the re-entrant corner, i.e. 4.5 in. from the re-entrant corner. The length of the flange of the double-tee is 50 ft. i.e. the span is 50 ft-9 in. Normal weight concrete, compressive strength $f'_c = 5000$ psi. Effective prestress in 1/2 in. diameter 270 K strand, $f_{se} = 150$ ksi.

Live load = 55 psf; additional dead load = 8 psf.

Tension acting on each web due to restraint of shortening is 2.5 kips. (This force to be treated as a live load, as required for corbels by ACI Code.)

Dead load of 8DT20 is 47 psf.

Live load end shear/web = $25 \times 4 \times 55 / 1000 = 5.5k$.

Dead load end shear/web = $25 \times 4 \times (47+8) / 1000 = 5.5k$.

$$V_u = [1.4D+1.7L] = [1.4(5.5) + 1.7(5.5)] = 17.1 \text{ kips}$$

$$N_u = 1.7(2.5) = 4.25 \text{ kips}$$

$$N_u = (4.25/17.1)V_u = 0.25 \text{ } V_u > 0.2 \text{ } V_u \text{ o.k.}$$

($N_u$ should not be taken as less than 0.2$V_u$, as required for corbels in the ACI Code (10).)

It is recommended that the values of $V_u$ and $N_u$, calculated using the ACI Code load factors, should be multiplied

97
First leg of WWF web reinforcement to be lin. from bottom corner of full depth web.

Fig. El - Dapped end of an 8DT20 double tee.
by 1.15 to reduce the possibility of failure initiating at a connection, and so ensuring overall ductility of the member; i.e. for each dapped end, 

\[ u = 1.15(17.1) = 19.7 \text{ kips} \]

\[ N_u = 1.15(4.25) = 4.89 \text{ kips} \]

hence required values of nominal strength are,

\[ V_n = \frac{V_u}{\phi} = \frac{19.7}{0.85} = 23.2 \text{ kips} \]

\[ N_n = \frac{N_u}{\phi} = \frac{4.89}{0.85} = 5.75 \text{ kips} \]

a). Check that \( V_n/b_{d_d} \) is not more than \( 0.2f_c' \) nor 800 psi. \( 0.2f_c' = 1000 \text{ psi} \), hence limit is 800 psi.

Assuming \( 3/8 \) in. thick bearing plate and \#4 bar for A, \( d_d \) will be 7.38 in. Average nib width will be 5.30 in.

\[ V_n/(b_{d_d}) = \frac{23,200}{[(5.30)(7.38)]} = 593 \text{ psi} < 800 \text{ psi} \text{ o.k.} \]

b). Check shear span to depth ratio \( a/d_d \) is not greater than unity.

Assuming distance from centerline of hanger reinforcement to end face of web is 1.25 in.

\[ a = 4.5 = 0.625(\tan30 + 1.25/(\cos30)) = 5.58 \text{ in.} \]
\[ \frac{a/d_d}{d_d} = \frac{5.58}{7.38} = 0.76 < 1.0, \text{ o.k.} \]

c). Check that nominal shear strength of full depth web at \( h/2 \) from bottom corner of web is not less than required \( V_n \) for dapped end, i.e. that \( V_c \) for beam web is \( \geq V_n \) for dapped end.

Locations of prestressing strands are shown in Fig. E-1. They correspond to values of \( e_s \) of 4.59 in. and 11.59 in. at support and midspan respectively.

Table El relates to a section at \( h/2 = 10 \) in. from bottom corner of web, and assumes a transfer length of 36 in. for \( l/2 \) in., 270K strand, as determined in PCA studies (11). "e" is strand eccentricity relative to centroid of section. Forces and moments relate to one web, i.e. half total section.

<table>
<thead>
<tr>
<th>Strand No.</th>
<th>Embedded Length (in.)</th>
<th>Embeded Distance (in.)</th>
<th>( f_{se} ) at X-X (ksi)</th>
<th>F (kips)</th>
<th>Fe (k.in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.59</td>
<td>11.16</td>
<td>( 150(11.16)/36 = 46.5 )</td>
<td>7.12</td>
<td>89.64</td>
</tr>
<tr>
<td>2</td>
<td>7.42</td>
<td>14.33</td>
<td>( 150(14.33)/36 = 59.7 )</td>
<td>9.14</td>
<td>67.85</td>
</tr>
<tr>
<td>3</td>
<td>3.66</td>
<td>21.77</td>
<td>( 150(21.77)/36 = 90.7 )</td>
<td>13.88</td>
<td>50.75</td>
</tr>
<tr>
<td>4</td>
<td>1.76</td>
<td>21.77</td>
<td>( 150(21.77)/36 = 90.7 )</td>
<td>13.88</td>
<td>24.37</td>
</tr>
<tr>
<td>5</td>
<td>-0.14</td>
<td>21.77</td>
<td>( 150(21.77)/36 = 90.7 )</td>
<td>13.88</td>
<td>-2.00</td>
</tr>
</tbody>
</table>

Hence, \( \Sigma F = 57.90 \) kips and \( \Sigma Fe = 230.6 \) k. in.
Taking into account tension force $N_n = 5750$ lb acting at bottom of nib.

$$f_{pc} = \frac{(57,900 - 5750)}{(363/2)} = 287 \text{ psi}$$

$$f_{pe} = 287 + \frac{230,600}{(860/2)} - 5750\frac{(10.0 - 5.41)}{(860/2)} = 762 \text{ psi}$$

$$V_p = 9.14\frac{(12.09 - 7.09)}{(304.5)} + 13.88\frac{(11.59 - 3.09)}{(304.5)} + 13.88\frac{(10.59 - 0.91)}{(304.5)} = 1.52 \text{ kips}$$

$$b_w = \text{average web width of full depth section} = 4.15 \text{ in.}$$

$$d = \text{greater of effective depth and 0.8h.}$$

$$= \text{greater of (5.41 + 5.06) and 0.8(20)}$$

$$= 16 \text{ in.}$$

$$V_{cw} = (3.5\sqrt{f_c} + 0.3f_{pc})b_wd + V_p$$

$$= [3.5\sqrt{5000} + 0.3(287)](4.75)(16.0)/1000 + 1.52$$

$$\therefore V_{cw} = 26.87 \text{ kips}$$

At X-X,

$$M_d = 113,176 \text{ lb. in. and } V_d = 5.40 \text{ kips}$$

$$f_d = \frac{113,176}{(860/2)} = 263 \text{ psi}$$

$$M_{cr} = Z_{ten} [6\sqrt{f_c} + f_{pe} - f_d]$$

$$= (860/2)[6\sqrt{5000} + 762 - 263]/1000$$

$$= 397.0 \text{ k.in.}$$

Since both DL and LL are uniformly distributed,

$$V_e/M_{max} = \frac{V_d}{M_d} = \frac{5.40}{113.2}$$

$$= 0.0477 \text{ in.}^{-1}$$
\[ V_{ci} = 0.6\sqrt{f_{w}b_{w}d} + V_d + \left(\frac{V_{c}/V_{max}}{M_{cr}}\right) \]
\[ = 0.6\sqrt{5000(4.75)(16.0)/1000} + 5.40 + 0.0477(397.0) \]
\[ \therefore V_{ci} = 27.57 \text{ kips} > V_{cw} \]

\[ V_{c} = V_{cw} = 26.87 \text{ kips} > V_n = 23.2 \text{ kips} \]

Therefore section adjacent to dapped end is strong enough in shear. Provide minimum web reinforcement in full depth web as required by Sec. 11.5.5 of ACI Code (10).

**Design of dapped end reinforcement**

It is assumed that the reinforcing bars are either type A706 or type A615, and that preheating appropriate to the carbon equivalent for the bar chemistry is used. [See Table 5.2 of the American Welding Society "Structural Welding Code - Reinforcing Steel" (13).]

The plates are assumed to be of A36 steel and E70 electrodes are appropriate for the welding of reinforcing bars to these plates.

**Design for reinforcement scheme 1**

1. \[ V_{cr} = 2b_{d}d\sqrt{f_{c}} = 2(5.30)(7.38)\sqrt{5000}/1000 \]
   \[ = 5.53 \text{ kips} \]

\[ \text{Req'd} \ V_n = 23.2 \text{ kips} \]

\[ \text{and} \ V_n = A_{sh}f_{y}\cos\alpha + V_{cr} \] (1A)
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Fig. E2 - Example design using reinforcement scheme 1.
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

![Graph showing shear vs. strand end slip](image)

*Fig. 3 - End slip of bottom strands in final tests of 3B & 4B*
7. \[ x'_1 = x - (f_v - e)(V_{cr}/C_2) \]
\[ = 0.69 - (4.5 - 1.44)(5.53/17.7) \]
\[ = 0.26 \text{ in} \]

\[ x'_2 = C_2/(1.7f_{cr}b_w) \]
\[ = 17/7.[1.7(5.00)(5.7)] \]
\[ = 0.37 \text{ in.} > x'_1 \text{ N.G.} \]

Assuming \( x = 1.2 \text{ in.} \) in step (3) yields \( e = 1.73 \text{ in.} \),
\( A_s = 0.214 \text{ in.}^2 \) and \( C_2 = 19.09 \text{ kips} \).

then \( x'_1 = 1.2 - (4.5 - 1.73)(5.53/19.09) \)
\[ = 0.40 \text{ in.} \]

and \( x'_2 = 19.09/[1.7(5.00)(5.7)] \)
\[ = 0.40 \text{ in.} = x'_1 \text{ o.k.} \]

Req’d. \( A_s = 0.214 \text{ in.}^2 \)

USE 2 #3 grade 60 bars for \( A_s \),

\( A_s \) should project into the beam web the further of,

a. \( 1.7d \) beyond re-entrant corner

b. to a distance from end face of nib equal to transfer length of strand.

a. For #3, 60 grade bars in 5000 psi concrete \( d = 12 \text{ in.} \);

\[ 1.7d = 20.4 \text{ in.} \]

b. Transfer length of 1/2 in. dia., 270 K strand assumed to be 36 in., as determined in PCA studies (11), and nib length is 6.0 in.

Projection beyond re-entrant corner will be 30 in. (>20.4 in.)
Provide 1-1/4 in. of 3/16 in. E70 flare bevel groove weld to attach each #3 bar to 6 x 3/8 x 4-3/4 in. bearing plate.

8. Check whether cross-bar welded to bearing plate is necessary, to take horizontal component of compression force Cl in Fig. Bl.

\[ \frac{e}{d_d-x} = 1.73/[(8 - 0.375 - 0.375/2) - 1.21 = 0.277 > 0.15 \]

Cross-bar needed.

Minimum \( L_{bc} d_{bc} \)

\[ = \frac{V_n e/(d_d-x)}{(0.85f'_c)} = [23.2(0.277)]/[(0.85)(5.0)] = 1.51 \text{ in.}^2 \]

USE #4 cross-bar, 3 in. long, \( L_{bc} d_{bc} = 1.50 \text{ in.}^2 \)

Provide lin. total of 1/4 in. E70 flare bevel groove weld to attach cross-bar to bearing plate. (Weld to carry \( (0.277)(23.2) = 6.42 \text{ kips force} \)).

9. Upper anchorage for hanger reinforcement.

If anchor angle has 2 in. vertical leg, required length,

\[ b_a = C_2/\left[0.85f'_c(2.0)\right] = 19.1/[0.85(5.0)2.0)] = 2.25 \text{ in.} \]

Allowing for 5/8 in. plate, angle length = 2.87".

USE L2 x 4 x 1/4 in., length 3 in.
Weld #4 hanger bars to opposite faces of 1-3/4 x 3-3/4 x 5/8 in. plate welded inside anchor angle. Length of 1/4 in. E70 flare bevel groove weld between each bar and 5/8 in. plate = 1-3/4 in. (Table 6.20.3 of Ref. 7)

Length of 1/4 in. E70 fillet weld between plate and angle

\[ = \frac{C_2}{(\text{weld strength/in.})} \]

\[ = 19.1/6.19 \] (Table 6.20.2 of Ref. 7)

\[ = 3.09 \text{ in.} \]

Provide 2 in. total of 1/4 in. fillet weld each side of plate.

See Fig. E2 for details.

Design for reinforcement scheme 2

1. \[ V_{cr} = 1.95 \frac{b_d}{d} \sqrt{f_c} \]

where \( d_d = 0.9h_d = 0.9(8) = 7.20 \text{ in.} \)

\[ V_{cr} = 1.95(5.30)(7.2)\sqrt{5000}/1000 \]

\[ = 5.26 \text{ kips} \]

2. \[ F = \left\{ \frac{V_n(\alpha_v - b_1) + N_n(h_t - a_5) + (w_u/\phi)(b_2)(b_1 + b_2/2)}{V_{cr}(b_1 + b_2)} \right\}/(h_t - a_5 - a_1) \] (5)

\[ v = 4.5 \text{ in.} \text{.: } a_5 = 1.0 \text{ in.} \]

\[ a_1 = 0.375 + 0.5/2 = 0.63 \text{ in.} \]

(Assuming 3/8 in. bearing plate and that \( A_s \) is #4 bar.)

\[ a_4 = 1.00 + 0.75/2 = 1.375 \text{ in.} \]

(Assuming 1 in. cover to #6 bar)
Fig. E3 - Example design using reinforcement scheme 2.
\[ b_1 = (ht - a_5)\tan\alpha - a_4/\cos\alpha \]
\[ = (10 - 1)\tan30 - 1.375/\cos30 \]
\[ = 3.61 \text{ in.} \]

\[ b_2 = (ht - t)/\tan38 \]
\[ = (10 - 2)/\tan38 \]
\[ = 10.24 \text{ in.} \]

\[ w_u = [1.4(47 + 8)4 + 1.7(55)4]/(1000 \times 12) \text{ k/in.} \]
\[ = 0.057 \text{k/in.} \]

\[ V_n = 23.2 \text{ kips} ; N_n = 5.75 \text{ kips} \]

\[ F = [23.2(4.5 - 3.61) + 5.75(10 - 1) + (0.057/0.85)(10.24)(3.61 + 10.24/2) + 5.26(3.61 + 10.24)]/(10 - 1 - 0.63) \]
\[ = [20.648 + 51.750 + 5.995 + 72.851]/(8.370) \]
\[ = 18.07 \text{ kips} \]

\[ T = [V_n - (w_u/\phi)(b_2) - V_c]/\cos\alpha \quad (6) \]
\[ = [23.2 - (0.057/0.85)(10.24) - 5.26] \cos30 \]
\[ = 19.92 \text{ kips} \]

\[ A_{sh} = T/f_y = 19.92/60 = 0.33 \text{ in.}^2 \]

**USE 1 #6 grade 60 bar for** \( A_{sh} \) **(1 in. cover)\)**

\[ A_s = F/f_y = 18.07/60 = 0.30 \text{ in.}^2 \]

**USE 1 #5 grade 60 bar for** \( A_s \)

Extend Ash horizontally distance \( L \) in bottom of web, beyond point of tangency, where \( L = 1.7d \).
For #6, grade 60 bars in 5000 psi concrete, \( \lambda_d = 18 \text{ in.} \)

\[ L = 1.7(18) = 30.6 \text{ in.} \]

say \( L = 31 \text{ in.} \).

As for reinforcement scheme 1, \( A_s \) will extend 30 in. beyond the **re-entrant corner**. \( (1.7\lambda_d = 26 \text{ in.}) \)

Provide 6 x 3/8 x 4 3/4 in. bearing plate.

Height of **vertical** plate to be not less than \( (d_C' + a_L) \).

\[
C = -N_n + T \sin \alpha + F \\
= -5.75 + 19.92 \sin 30 + 18.07 \\
= 22.28 \text{ kips}
\]

\[
d_C' = -a_L + V_n \lambda_v + T_a + \frac{F_{al}}{C} \\
(\frac{d_C' + a_L}{2}) = \frac{[(23.2)(4.5) + (19.92)(1.375) + (18.07)(0.63)]}{22.28} \\
= 6.43 \text{ in.}
\]

Provide 6 x 3/8 x 6-1/2 in. **"vertical"** plate, welded to bearing plate with 1/4 in. fillet weld both sides.

"**Vertical"** plate must be inclined slightly so as to provide 1 in. side cover to horizontal extension of hanger reinforcement, \( A_{sh} \).

Inclined **bar must** lap vertical plate at least 80 % of plate height, i.e. 5.2 in., say 5-1/4 in.

Weld \( A_{sh} \) to vertical plate with 2-1/2 in. of 3/8 in. E70 flare bevel groove weld.

Weld \( A_s \) to bearing plate with 2-1/4 in. of 5/16 in. E70 flare bevel groove weld.
See Fig. E3 for details.

Design for reinforcement scheme 3

1. \[ V_{cr} = 2b_d d_d f_c^r = 2(5.30)(7.38)\sqrt{5000}/1000 \]
   \[ = 5.53 \text{ kips} \]

   \[ A_{Sh} = (V_n - V_{cr})/f_y \cos \alpha = (23.2 - 5.53)/(60\cos 30) \]
   \[ = 0.34 \text{ in.}^2 \]

   USE #4 grade 60 bar, looped through 180° at upper end, for \( A_{Sh} \).

As in reinforcement scheme 1, extend \( A_{Sh} \) horizontally for 21 in. in bottom of web, beyond point of tangency.

2. With \( 3/4 \) in. cover to #4 bars, distance \( y \) in Fig. D1 is 1.00 in.

3. \[ x = c + d_b/2 + 2d_b \cos \alpha \]
   \[ = 0.75 + 0.5/2 + 2(0.5)\cos 30 \]
   \[ = 1.87 \text{ in.} \]

   \[ e = \lambda_y + y/\cos \alpha = (h_d - x) \tan \alpha \]
   \[ = 4.5 + 1/\cos 30 - (8 - 1.87)\tan 30 \]
   \[ = 2.12 \text{ in.} \]

4. \[ A_s = [V_n e + N_n (h-x)]/[f_y (d_d - x)] \]
   \[ = [23.2(2.12) + 5.75(8 - 1.87)]/[60(7.38 - 1.87)] \]
   \[ = 0.255 \text{ in}^2 \]
Fig. E4 - Example design using reinforcement scheme 3.
5. \[ C_2 = A_{sfy} + A_{shfy} \sin \theta - N_r \quad (6A) \]
\[ = 0.255(60) + 0.40(60) \sin 30 = 5.75 \]
\[ = 21.55 \text{ kips} \]

6. \[ x'_1 = x - (\gamma_v - e) \left( \frac{V_{cr}}{C_2} \right) \quad (7A) \]
\[ = 1.87 - (4.5 - 2.12)(5.53/21.55) \]
\[ = 1.26 \text{ in.} \]

\[ x'_2 = \frac{C_2}{1.7f'c'w} \quad (8A) \]
\[ = \frac{21.55}{1.7(5)(5.7)} \]
\[ = 0.45 \text{ in.} < x'_1 \quad \text{o.k.} \]

\[ \therefore \text{Req'd A,} = 0.255 \text{ in.}^2 \]

USE 1 #3 + 1 #4 grade 60 bars for A,.

As for Reinforcement Scheme 1, A, will extend 30 in. beyond the re-entrant corner.

Provide 6 x 3/8 x 4-3/4 in. bearing plate.

Weld #4 bar of A, to bearing plate with 1 3/4 in. of 1/4 in. E70 flare bevel groove weld.

Weld #3 bar of A, to bearing plate with 1-1/4 in. of 3/16 in. E70 flare bevel groove weld.

7. Check whether cross-bar welded to bearing plate is necessary to take horizontal component of compression force C1 in Fig. D1.

\[ \frac{e}{d_d - x} = 2.12/(7.38 - 1.87) = 0.385 > 0.15 \]

\[ \therefore \text{Cross-bar needed} \]
Minimum \( L_{bc} d_{bc} \)
\[ = \frac{V_n e}{(d_4 - x)} \times 0.85f_c^l \]
\[ = \frac{23.2(0.385)}{0.85(5)} \]
\[ = 2.10 \text{ in.}^2 \]

USE \#4 cross bar 4-1/4 in. long, \( (L_{bc} d_{bc} = 2.13 \text{ in.}^2) \)

Provide 1-1/2 in. total of \( 1/4 \text{ in.} \) E70 flare bevel groove weld to attach cross-bar to bearing plate. (Weld to carry \( (0.385)(23.2) = 8.9 \text{ kips force.} \))

See Fig. E4 for details

Design for reinforcement scheme 5

1. As for reinforcement scheme 2,

\[ V_{cr} = 5.26 \text{ kips} \]

2. \[ F = [V_n(k_v - b_1) + N_n(h_t - a_5) + (w_u/f)(b_2)(b_1 + b_2/2)
\]
\[ + V_{cr}(b_1 + b_2)]/(h_t - a_5 - a_1) \]
\[ q_v = 4.5 \text{ in.} \ a_5 = 1.0 \text{ in.} \]

\[ a_1 = 0.375 + 0.5/2 = 0.63 \text{ in.} \]

(Assuming 3/8 in. bearing plate and that As is \#4 bar)

\[ a_4 = 0.75 + 0.5/2 = 1.00 \text{ in.} \]

(Assuming 3/4 in. cover to \#4 bar.)

\[ b_1 = (h_t - a_5)\tan\alpha - a_4/\cos\alpha \]
\[ = (10 - 1)\tan30 - 1.00/\cos30 \]
\[ = 4.04 \text{ in.} \]
Fig. E5 - Example design using reinforcement scheme 5.
b2 = (ht - t)/\tan 38
= (10 - 2)/\tan 38
= 10.24 \text{ in.}

As for reinforcement scheme 2, \( w_u = 0.057 \text{ kips/in.} \)

\[
\therefore F = [23.2(4.5 - 4.04) + 5.75(10 - 1) + (0.057/0.85)(10.24)\{(4.04 + (10.24/2)\} + 5.26(4.04 + 10.24)]/(10 - 1 - 0.63)
= [10.672 + 51.750 + 6.290 + 75.113]/(8.370)
= 17.18 \text{ kips}
\]

\[
\therefore A_S = F/f_y = 17.18/60 = 0.29 \text{ in.}^2
\]

USE 1 #3 + 1 #4 grade 60 bars for \( A \)

3. \( T = [V_n = (w_u/ b_2) = V_{cr}]/\cos \alpha \) \hspace{1cm} (6)
= [23.2 = (0.057/0.85)(10.24) - 5.26]/\cos 30
= 19.92 kips

\[
\therefore A_{Sh} = T/f_y = 19.92/60 = 0.33 \text{ in.}^2
\]

USE 2 #4 grade 60 bars for \( A_{Sh} \) (3/4 in. cover)

As for reinforcement schemes 1 and 3, extend \( A_{Sh} \) horizontally for 21 in. in bottom of web, beyond point of tangency, and extend \( A \), 30 in. beyond the re-entrant corner.

Provide a 6 \( \times \) 3/8 \( \times \) 4-3/4 in. bearing plate.

4. Check flexural strength at interface of nib and full depth web.

\[
dd = h_d - a_1 = 8.00 - 0.63 = 7.37 \text{ in.}
\]
Depth of flexural compression stress block
\[ a = (A_{f_y} + A_{sh} f_y \sin \alpha - N_n) / (0.85 f_{cb}) \]
\[ = [(0.31)(60) + (0.40)(60) \sin 30^\circ - 5.75] / 0.85(5)(5.7) \]
\[ = 1.03 \text{ in.} \]

\[ M_n = (A_{f_y} - N_n)(d_d - a/2) + A_{sh} f_y \sin \alpha [h_d = (a_4 / \sin \alpha - a/2) \]
\[ = [(0.31)(60) - 5.75](7.37 - 1.03/2) + (0.40)(60)[8.00 - (1.00 / \sin 30^\circ - (1.03/2))] \]
\[ = 153.9 \text{ k.in.} \]

Moment acting on nib-web interface
\[ = V_n b + N_n a_1 \]
\[ = [23.2(4.5) + 5.75(0.63)] \]
\[ = 108.0 \text{ k.in.} < M_n \text{ o.k.} \]

5. Check that a straight line drawn from center of action of \( V_n \) on bearing plate, to point \( a/2 \) (0.52 in.) below top of nib on nib-web interface, crosses \( A_{sh} \) below point of tangency of hook. This line is drawn as a chain-dash line in Fig. E5 and it can be seen that the above requirement is satisfied.

6. Height of "vertical" plates \( > \) (web height)/8 = 2.25 in.

Say use \( 2-1/2 \times 3/8 \times 6 \) in. "vertical" plates.

Shear yield strength of one plate
\[ = (0.58 f_y) bh = [0.58(60)(0.375)(2.5)] \]
\[ = 19.58 \text{ kips.} \]

Yield strength of one #4 bar
\[ = (0.20)(60) = 12.00 \text{ kips} < 19.58 \text{ kips o.k.} \]

USE \( 2-1/2 \times 3/8 \times 6-1/4 \) in. "vertical" plates.

These plates are welded to bearing plate with outer
faces 3/4 in. from edges of bearing plates. They are tilted outward slightly, so that side cover of 7/8 in. is provided for horizontal extension of Ash.

Weld each #4 inclined bar to "vertical" plate with 1-3/4 in. of 1/4 in. E70 flare bevel groove weld.

Weld #4 bar of As to bearing plate with 1-3/4 in. of 1/4 in. E70 flare bevel groove weld.

Weld #3 bar of As to bearing plate with 1-1/4 in. of 3/16 in. E70 flare bevel groove weld.

See Fig. E5 for details.

Design for reinforcement scheme 4

In this case the front face of the dap is vertical, but all other dimensions and location of strands are the same as in Fig. El. It can be shown that \( V_c \) for a vertical section \( h/2 = 10 \) in. from the bottom corner of the web is 25.1 k., i.e. greater than the required \( V_n \), (23.2k).

Using the procedure proposed in reference (5)

1. \[ V_n/b_{dd} = 23,200/(4.86 \times 7.38) \]
   \[ = 647 \text{ psi}, \quad < 800 \text{ psi}. \quad \text{o.k.} \]

2. Hanger reinforcement

\[ A_{sh} = V_n/f_y = 23.3/60 \]
\[ = 0.39 \text{ in.}^2 \]
Fig. E6 - Example design using reinforcement scheme 4.
USE a looped #4 grade 60 bar for Ash

Extend Ash horizontally distance $L$ in bottom of web beyond point of tangency, where $L = 1.7\lambda_d$

For #4, 60 grade bars in 5000 psi concrete, $\lambda_d = 12$ in.

\[
\therefore L = 1.7(12) = 20.4''
\]

say $L = 21$ in.

3. Reinforcement to resist flexure.

Since flange is cut away over nib, must check both at nib/web interface and at centerline of hanger reinforcement.

a. At nib/web interface $a = \lambda_v = 4.5$ in.

\[
a/d = 4.5/7.38 = 0.61 < 1.0 \quad \text{o.k.}
\]

Req'd $M_n = V_n a + N_n (h_d - dd)$

\[
= 23.2(4.5) + 5.75(8.00 - 7.38)
\]

\[
= 108.0 \text{ k.in.}
\]

\[
R = M_n / (bd^2 f_c')
\]

\[
= 108.0 / [(5.7)(7.38)^2(5.0)]
\]

\[
= 0.070
\]

\[
\rho = (f_c' / f_y) (0.85 - \sqrt{0.72 - 1.7R})
\]

\[
= (5/60)(0.85 - \sqrt{0.72 - 1.7(0.070)})
\]

\[
= 0.0062 \text{ for } f_c' = 5000 \text{ psi and } f_y = 60 \text{ ksi}
\]

\[
\therefore Af = 0.0062(5.7)(7.38)
\]

\[
= 0.26 \text{ in.}^2
\]
b. At centerline of hanger reinforcement, \( a = 5.5 \) in.
\[
\frac{a}{d} = \frac{5.5}{7.38} = 0.75 < 1.0 \text{ o.k.}
\]

Req'd \( M_n = 23.2(5.5) + 5.75(8 - 7.38) \)
\[
= 131.2 \text{ k.in.}
\]

Assume concrete compression force acts at centroid of area bounded by centerline of 180 degree loop in hanger reinforcement.

For a bend diameter of \( 6d_b \), the centroid is \( 2.5d_b \) below the outside of the top of the loop
\[
= 2.5(0.5) = 1.25 \text{ in.}
\]

If \( 3/4 \) in. cover is provided to top of loop, distance from top of flange to centroid
\[
= 1.25 + 0.75 = 2.00 \text{ in.}
\]

Assuming \( A_f \) to be made up of \#4 bars and using a \( 3/8 \) in. thick bearing plate, internal lever arm is given by,

\[
j_d = 10.0 -2.0 -0.375 -0.25
\]
\[
= 7.38 \text{ in.}
\]

Req'd \( A_f = \frac{M_n}{f_yj_d} = \frac{131.2}{60 \times 7.38} \)
\[
= 0.30 \text{ in.}^2 > 0.26 \text{ in.}^2 \text{ (controls)}
\]

4. Reinforcement to resist horizontal force

\[
A_n = \frac{N_n}{f_y} = \frac{5.75}{60}
\]
\[
= 0.10 \text{ in.}^2
\]

5. Shear transfer reinforcement using Modified Shear Friction
\[ A_{vf} = [(V_n/0.8) - 0.5b_d d_d]/f_y \]
\[ = [(23.2/0.8) - 0.5(4.86)(7.38)]/60 \]
\[ = 0.18 \text{ in.}^2 \]

\[ 0.2b_d d_d/f_y = 0.2(4.86)(7.38)/60 \text{ (Min. shear friction reinf.)} \]
\[ = 0.12 \text{ in.}^2 \text{ (does not control.)} \]

6. Main reinforcement in nib

\[ 2/3 A_{vf} = 0.12 \text{ in.}^2 < A_f = 0.30 \text{ in.}^2 \]

\[ \therefore A_S = A_f + A_v = 0.30 + 0.10 \]
\[ = 0.40 \text{ in.}^2 \]

USE 2 #4 grade 60 bars for \( A_v \),

7. Anchor these bars in nib by welding to bearing plate, requires 1-3/4 in. of \( 1/4 \) in. flare bevel groove weld for each bar.

These bars must extend a distance beyond the re-entrant corner,

\[ L_1 = [(\text{distance from centerline of } A, \text{ to bottom of beam}) + \lambda_d], \text{ but } > 1.7 \lambda_d \]

\[ \lambda_d = 12 \text{ in. for } #4 \text{ grade 60 bars in } 5000 \text{ psi concrete} \]

\[ \therefore L_1 = (10 + 0.375 + 0.25) + 12, \text{ but } > 1.7(12) \]
\[ = 22.6 \text{ but } > 20.4 \text{ in.} \]

say \( L_1 = 23 \text{ in.} \)
Horizontal reinforcement $A_h$ in nib,

$$A_h = 0.5(A_s - A_n) = 0.5(0.40 - 0.10) = 0.15 \text{ in.}^2$$

USE #3 grade 60 bar hairpin.

Extend hairpin bar for $1.7\lambda_d$ beyond re-entrant corner

$$= 1.7(12) = 20.4 \text{ in.}, \text{ say 21 in.}$$

See Fig. E6 for details.
PCISFRAD
Project #6

Strength of Members with *Dapped* Ends

Supplement to
Final Report

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October, 1985

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Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.
Testing arrangements viewed from north end.
Testing arrangements viewed from south end.
Specimen 1A

SUMMARY OF BEHAVIOR

1. Prior to Loading.

A hairline crack was observed to run from about 1/2 in. below bottom strand to bottom of end face, thence approximately along center of bottom face for about 6 in.

2. Test (a) - to service load, \( V_S = 12.04 \) kips.

At between 75% and 80% of service load, some short, randomly distributed, horizontal hairline cracks were observed in upper half of west side of web. These cracks could only be seen readily using a magnifying glass and did not open further under service load. Similarly fine cracks were observed at service load at the web-flange junction on both east and west faces of web. Crack on bottom face did not increase in length. Max. bottom strand draw-in 0.0008 in.

Primary cracking due to load was an inclined crack propagating from re-entrant corner for about 5 in. at about 45 degrees, initiated at a shear of 6.41 kips. Max. crack widths at service load occurred near re-entrant corner, 0.004 in. on west side, 0.003 in. on east side.

3. Test (b) = 9 cycles of service load, \( V_S = 12.04 \) kips.

Major crack extended about 1 in. Max. crack widths at service load at 9th cycle, 0.006 in. west side, 0.005 in. east side. Crack on bottom face extended 2 in., still very fine. Bottom strand draw-in after 9th cycle, 0.0011 in.

4. Test (c) - overload test to \( 0.72V_S(\text{calc}), (1.36V_S), 16.35 \) kips.

Major crack extended about 4 in. at about 1:4 slope to the horizontal. Inclined crack initiated in nib at inside edge of bearing plate, penetrated about 4 in. in a direction toward end face of flange at top of web. Max. crack width at 0.72V_S(calc), 0.011 in. on both east and west sides, close to re-entrant corner.

5. Test (d) = 10 cycles of service load, \( V_S = 12.04 \) kips.

Major crack did not extend. Crack in nib extended about 1-1/2 in. Additional short hairline cracks became visible on side faces of web - seemingly randomly distributed. Hairline crack on bottom face extended 1 in. Max. crack width at service load on 10th cycle, 0.009 in. on east side.

Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.
6. Test (e) - incremental loading to failure.

Major crack extended about 4 in. to and along the web-flange junction by shear of 21.28 kips, \((1.77V_{s})\). At shear of 18.43 kips, \((1.53V_{s})\) an additional inclined crack developed in the beam web, starting about 3-1/2 in. below the re-entrant corner, with an initial length of about 6 in. This subsequently increased in length about 1 in. This crack intersected the next to bottom strand on the sloping end face. At a shear of 20.33 kips, \((1.69V_{s})\) a fine vertical crack extended downward about 2-1/2 in. from this crack, starting about 1/2 in. to one side of the strand on the sloping face.

The maximum shear reached was 21.82 kips, \((1.81V_{s} \text{ or } 0.96V_{n}^{\text{calc}})\), at which point a flexural crack initiated about 4-1/2 in. from the bottom corner of the web. This crack extended upward about 4-3/4 in. from the bottom of the web, at which point a diagonal tension crack occurred. This diagonal tension crack extended in a flat arc to the web-flange junction and also backward to within about 1 in. of the bottom face at about 2 in. from the bottom corner of the web, with an average slope of about 45 degrees. Within seconds, two other flexural cracks occurred at about 7-1/2 in. and 11 in. from the bottom corner of the web. These extended upward 2 to 3 in.

When the initial crack occurred, the slip of the bottom strand increased from 0.0030 in. to 0.0111 in. As the other cracks formed, the slip increased to 0.0205 in. and the shear decreased to 20.84 kips.

Attempts were made to increase the load above that causing the initial flexural tension crack, but each time this load was approached additional flexural and diagonal tension cracks formed further along the beam, the bottom two strands slipped further and the load fell away. Loading was discontinued when cracking extended over the end 4 ft of the beam and the bottom and next to bottom strands had slipped 0.095 in. and 0.072 in. respectively. At this point horizontal cracks were propagating from the flexure cracks at the level of the bottom strand.

It is clear that if the beam had been subjected to dead weight loading, (rather than hydraulic loading,) a brittle and complete failure would have occurred immediately following the flexural cracking 4-1/2 in. from the bottom corner of the web. All the cracks observed after formation of that flexural crack would have occurred in rapid succession, probably resulting in complete lose of bond between the bottom two strands and the surrounding concrete over the end two to three feet of the beam.

The failure appeared to be a combined flexural bond and diagonal tension failure in the web of the beam adjacent to the beam-flange junction.
depped end. The nib of the dapped end had only fine cracks in it at maximum load and the inclined and flexural dapped end reinforcement did not yield.

The maximum stresses reached were 49.2 ksi (0.75f_y) in the inclined reinforcement and 56.2 ksi (0.76f_y) in the flexural reinforcement near the re-entrant corner. As the diagonal tension cracks occurred in the beam web, the stress in the flexural reinforcement on the line extending at 45 degrees upward from the bottom corner of the web increased rapidly, reaching a maximum value of 52.0 ksi.
Fig. 1 - Variation with shear of stress in flexural reinforcement, Spec IA.

Fig. 2 - Variation with shear of stress in inclined bars, Spec IA.
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

Fig. 3 - End slip of bottom strands in final test. (Specimens IA & IA)
Specimen 1A after test, west face.

Specimen 1A after test, east face.

Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.
Specimen 10

SUMMARY OF BEHAVIOR

1. Prior to loading.

A hairline crack at the center of the bottom face extended about 1.5 in. from the end, but no continuation of it could be traced on the sloping end face.

2. Test (a) - to service load. $V_s = 12.58$ kips.

Cracking initiated at the re-entrant corner at a shear of 9.26 kips on both East (E) and West (W) faces. At service load the crack on the E side extended about 4 in. at about 60 degrees to the horizontal. On the W side the original crack initially travelled at 45 degrees for 2 in. and then at a shallower inclination to a point 3 in. above and 6 in. into the beam from the re-entrant corner. In addition, a second fine inclined crack occurred on the W face, initiating 4 in. above the re-entrant corner and extending to the web-flange junction 4.5 in. from the end of the flange. The max. crack width at service load occurred near the re-entrant corner, 0.005 in. on the W side and 0.004 in. on the E side. (The No. 3 inclined bar was nearest the W side and the No. 4 inclined bar was nearest the E side.) On removal of the load the cracks closed to 0.001 in. at the re-entrant corner.

On the center-line of the bottom face a series of disconnected very fine short cracks became visible at the service load, extending about 11 in. from the end of the bottom face. On the sloping end face a very fine crack extended about 1.25 in. up from the bottom face at service load. Max. draw-in of the bottom strand was 0.0006 in.

3. Test (b) - 9 cycles of service load, $V_s = 12.58$ kips.

On the W face the crack originating at the re-entrant corner extended nearly horizontally to a point 9 in. from the re-entrant corner. The second inclined crack on this face extended downward in the nib to the inner edge of the bearing plate. (This crack had a max. width of 0.002 in.)

On the E face the crack extended at a smaller angle to the horizontal, to a point 4 in. above the re-entrant corner and 4.5 in. horizontally from the re-entrant corner.

Max. crack widths on the last loading cycle were 0.005 in. on both faces. The cracks closed to 0.001 in. on removal of the load. Max. draw-in of bottom strand 0.0008 in.
4. Test (c) - overload test to $0.72V_n(\text{calc})$, $(1.36V_s)$, 17.10 kips.

On W face, the re-entrant corner crack extended to a point 2 in. below the web-flange junction and 15 in. from the re-entrant corner. The secondary inclined crack on this face did not increase in length and only had a maximum width of 0.003 in. at $0.72V_n(\text{calc})$.

On E face, the re-entrant corner crack extended to a point 1 in. below the web-flange junction and 11 in. from the re-entrant corner.

On both faces an additional inclined crack initiated at a shear of 16 kips, originating at a point on the sloping end face about 2.5 in. below the re-entrant corner. It extended upwards about 3 in. at about 60 degrees to the horizontal. The ends of these inclined cracks were joined by a crack across the sloping end face. The maximum crack width at $0.72V_n(\text{calc})$ was 0.008 in., closing to 0.003 in. on removal of the load. Max. draw-in of bottom strand was 0.0011 in.

5. Test (d) - 10 cycles of service load, $V_s = 12.58$ kips.

There was no extension of cracking on the W side under this loading. On the E side the inclined crack originating on the sloping end face extended about 1.5 in. There was no additional cracking on the bottom face or on the sloping end face of the beam due to these load cycles.

Max. crack width on last cycle of load was 0.008 in. on the W side and 0.006 in. on the E side. On removal of load these cracks closed to 0.004 in. on the W side and 0.002 in. on the E side. Max. draw-in of bottom strand was 0.0013 in.

6. Test (e) - incremental loading to failure.

As the load was increased, the cracks originating at the re-entrant corner gradually extended upward and along the beam, reaching the web-flange junction at a shear of 21.28 kips, $(1.69V_s)$. 0" the W side this occurred 17 in. from the end of the flange and on the E side 12 in. from the end of the flange. 0" both faces these cracks eventually travelled along the web-flange junction to a point about 20 in. from the end of the flange. At this point the cracks were commencing to spread onto the underside of the flange, at about 45 degrees to the axis of the beam.

Additional inclined cracks initiated at the sloping end face on both E and W faces of the web at shears of 20 kips and upwards. These cracks extended steeply upwards, eventually almost joining the cracks originating at the x-e-entrant corner...
at a shear of about 26 kips. These cracks were joined across the sloping end face, on which two fine more or less vertical cracks also occurred. One of these cracks ran between the two strands and the other, extending over the middle half of the depth, was approximately aligned with one of the inclined reinforcing bars. An inclined crack occurred in the nib on the E side at a shear of about 23 kips, approximately mirroring that which had occurred earlier on the W side.

At a shear of 26.35 kips a vertical crack occurred on the W face of the web, starting about 0.75 in. both horizontally and vertically from the bottom corner, and extending 4 in. upwards.

At a shear of 27 kips a major diagonal tension crack occurred on both faces of the web of the beam, extending upward at an angle of about 40 degrees, from a point about 2 in. above the bottom face and about 13 in. from the bottom corner, up to the web-flange junction. The crack became visible almost simultaneously over its entire length. At the same load, flexural cracks occurred at about 11 in. and 16 in. from the bottom corner and extended upwards about 2 in. The shear dropped to 26.35 kips on formation of the diagonal tension crack, but it was possible to increase the load further until additional flexure cracks and a" additional major diagonal tension crack occurred at a shear of 27.93 kips. The load could not be brought back up to its maximum value, but a shear of about 23 kips was supported in a stable manner. When the load was removed some additional cracking occurred, as beam deflection was recovered under the action of the prestressing force. The cracks on the sloping end face remained fine at all stages of loading.

Although failure finally occurred as a result of diagonal tension cracking of the beam web, the specimen behavior is considered to have been satisfactory. This is because both the inclined reinforcing bars and the nib flexural reinforcement yielded before maximum load, resulting in the re-entrant corner crack becoming very wide before failure and so giving warning of impending failure. Also, the failure was much less abrupt than in the case of companion specimen 1A. The increase in length of the horizontal prolongation of the inclined reinforcement effectively prevented a bond failure on formation of the first major diagonal tension cack and enabled the full strength of the inclined reinforcement to be developed. The cracks in the nib and on the sloping end face remained small even at failure.

Draw-in of the bottom strand was 0.025 in. just before failure, increasing to 0.071 in. after failure when the shear had dropped to 26.1 kips.

The maximum shear sustained was 1.16 times $V^*$ calculated using the actual concrete strength at the time of test.
Fig. 1 - Variation with shear of stress in flexural reinforcement, Spec. 1B.

Fig. 2 - Variation with shear of stress in inclined bars, Spec. 1B
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

Fig. 3 - End slip of bottom strands in final tests of IB and 2B
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

Specimen 1B after test. west face.

Specimen 1B after test. east face.
1. Prior to loading.

On removal of the beam from the form after transfer of prestress, a fine crack was seen to run down the sloping end face of the web from the bottom strand to the bottom face. It then ran along the bottom face for 12 in., about 0.75 in. to one side of the center-line. This crack remained of hairline width at the strand, but increased in width at the bottom face during the 34 days between prestress transfer and the day of test. At test this crack was about 0.01 in. wide at the bottom face.

The strains at transfer in the gages on the horizontal extension of the hanger reinforcement, 2 in. from the end of the bottom face, were only about one sixth as large as similar strains measured in Spec. 3C. This indicates that some slip of the bottom strand occurred at transfer. (In Spec. 3C a similar crack was visible before loading, however it was of only hairline width on the bottom face and only extended for 6 in. along the bottom face.)

Inclined hairline cracks 2 in. long were visible at the re-entrant corner on both East (E) and West (W) web faces, after setting the beam up for test. These were presumably caused by the weight of the beam during handling.

2. Test (a) - to service load, $V_s = 11.60$ kips.

Under increasing load, the cracks originating at the re-entrant corner extended and also branched about 2 in. from the re-entrant corner. The branch cracks initially travelled approximately parallel to the sloping end face for about 1.5 in. and then vertically. These cracks reached within 1.5 in. of the web-flange Junction at service load.

At a shear of 9.26 kips inclined cracks occurred on both web faces, starting 3.5 in. below the re-entrant corner and travelling at about 45 degrees to the horizontal for about 5 in. As the load was increased, these cracks extended in the same direction, reaching 1.5 in. below the web-flange Junction at service load.

The crack running along the bottom face did not increase in width or length during this test.

The max. width of crack at service load was adjacent to the re-entrant corner on both web faces. These crack widths being 0.011 in. and 0.012 in. respectively on the E and W faces. The width of these cracks reduced to 0.003 in. on removal of the load. The max. draw in of the bottom strand was 0.0008 in.
3. Test (b) - 9 cycles of service load, $V_S = 11.60$ kips.

On the W face the only increase in cracking was an 0.75 in. extension of the inclined crack which originated 3.5 in. below the re-entrant corner.

On the E face a similar extension of the corresponding crack occurred. In addition, 5 more inclined cracks occurred, each about 1.5 in. long. One of these was near the sloping end face and about 0.75 in. below the lower major inclined crack. The four other inclined cracks were just below the level of the re-entrant corner and from 7 to 11 in. away from it horizontally. A 3 in. long horizontal crack also occurred 5 in. from the bottom face of the beam, starting 9.5 in. horizontally from the re-entrant corner.

The crack on the bottom face did not increase in width or length during this test.

The max. width of crack after 10 applications of service load was 0.012 in. and 0.014 in. respectively on the E and W web faces. These cracks closed to 0.003 in. and 0.004 in. respectively on removal of the load.

The max. draw-in of the bottom strand was 0.0018 in.

4. Test (c) - overload test to $0.72V_n (\text{calc}) , (1.36V_S), 15.74$ kips.

No significant increase in cracking occurred until the shear was increased to 15.14 kips, $(1.31V_S)$. Shortly after this shear was reached, a major diagonal tension crack occurred in both faces. This crack extended upwards from the end of the bottom face at about 45 degrees and reached the web-flange junction. Two fine flexure cracks became visible after formation of the diagonal tension crack. They were located 4.5 in. and 9.5 in. from the end of the bottom face. These cracks extended vertically 3.5 in. and 5.5 in. respectively. At this stage of loading the longitudinal crack in the bottom face still had not increased in width or length. However, on formation of the diagonal tension crack, the draw-in of the bottom strand increased suddenly from 0.002 in. to 0.0135 in.

When the diagonal tension crack occurred, the shear fell to 12.93 kips before the beam stabilized. It was then possible to increase the shear to 14.64 kips, when another major diagonal tension crack occurred. This was approximately parallel to that which had just recently occurred and 5 in. further into the beam. When this diagonal tension crack occurred, the longitudinal crack in the bottom face of the beam extended 6 in., and two additional flexure cracks occurred 13 in. and 17 in. from the end of the bottom face. Also, more or less horizontal cracks occur-
red 2 in. from the bottom of the beam and centered about 16 in. from the end of the web, being 3 in. and 2 in. long respectively on the E and W faces.

As the diagonal tension crack occurred, the draw-in of the bottom strand increased from 0.019 in. to 0.0375 in. and the shear fell to 12.80 kips.

It was subsequently possible to increase the load once more, until the shear reached the maximum value attained of 15.74 kips, \((0.72V_n)_{\text{calc}}\). At this shear a third major diagonal tension crack occurred, parallel to the previous cracks and about 6 in. further into the beam. This crack was accompanied by extensive flexural and flexural bond type cracking, extending to 25 in. from the end of the bottom face of the beam, and by large slip of the bottom two prestressing strands. The shear carried fell to 8.75 kips and the test was terminated.
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

Fig. 1 - Variation with shear of stress in flexural rein ft., Spec. IC

Fig. 2 - Variation with shear of stress in inclined bars, Spec. IC
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

Fig. 3 - End slip of bottom strands, Spec. 1C & 2C
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

Specimen 1C after test, west face.

Specimen 1C after test, east face.
Specimen 2A

SUMMARY OF BEHAVIOR

1. Prior to loading.
   No cracks observed.

2. Test (a) - to service load, $V_S = 12.08$ kips.

   Crack initiated at re-entrant corner at shear of 9.26 kips, extended about 2-1/2 in., at about 40 degrees to horizontal, with max. width 0.001 in. At service load this crack extended to a point 2 in. above and 3 in. from re-entrant corner on web face closest to inclined reinforcing bar. On opposite face it extended to a point 5 in. above and 8-1/2 in. from the re-entrant corner. Max. crack width at service load 0.003 in. on both faces. No other cracks occurred. Max. bottom strand draw-in 0.002 in.

3. Test (b) - 9 cycles of service load, $V_S = 12.08$ kips.

   Crack on face nearest inclined reinforcing bar (west face) increased in length in a discontinuous fashion in first two load cycles, to become as long as the crack on the east face. The crack on the east face extended a further 1 in. after 8 cycles of load. Max. crack width after 9 cycles, 0.006 in. Bottom strand draw-in after 9 cycles, 0.003 in.

4. Test (c) - overload test to $0.72V_n(calc), (1.36V_S), 16.42$ kips.

   Inclined crack on both faces propagated to web - flange junction and then along the junction to a point 24 in. from the re-entrant corner on the east face and 12 in. from the re-entrant corner on the west face. Additional short hairline cracks occurred in the vicinity of the primary cracks. Max. crack width at $0.72V_n(calc)$, 0.011 in. on west face, 0.013 in. on east face. On removal of applied load these cracks closed to 0.003 in. on west face and 0.005 in. on east face. Bottom strand draw-in at $0.72V_n(calc)$, 0.0043 in.

5. Test (d) - 10 cycles of service load, $V_S = 12.08$ kips.

   Major crack on west face extended along web - flange junction to a point 24 in. from the re-entrant corner after 6 cycles. Additional short fine cracks occurred in the upper part of the web faces. Max. crack width at cycle 10, 0.008 in. on west face, 0.010 in. on east face. Bottom strand draw-in after 10 cycles, 0.0050 in.
6. Test (e) - incremental loading to failure.

The original inclined crack starting at the z-e-entrant corner did not extend further, but some additional short cracks formed adjacent to it "ear the web - flange Junction.

At a shear of \(0.72V_{n}(\text{calc})\), (16.42 kips,) an inclined crack occurred on the east face (i.e. nearest the inclined bar,) starting 4 in. below the re-entrant corner and travelling at about 60 degrees to the horizontal to join the original inclined crack. Simultaneously a fine crack appeared on the sloping end face, in the plane of the #5 inclined reinforcing bar. This crack extended from the re-entrant corner downwards to the new inclined crack, to which it was joined by a horizontal extension of the inclined crack on the sloping face. At this same load the stress in the horizontal extension of the inclined bar commenced to increase more rapidly, but was still quite low at 1.4 ksi. There was no increase in the bottom strand draw-in. up to this load.

The max. crack widths at service load and at \(0.72V_{n}(\text{calc})\) were 0.007 in. and 0.009 in. respectively on the east face and 0.012 in. and 0.020 in. respectively on the west face.

A matching inclined crack occurred on the west face at a shear of 19.38 kips (1.60V_{s},) but this crack only travelled 2-1/2 in. toward the original inclined crack. Under subsequent load increments it extended to within about 1 in. of the original inclined crack, but did not join it. At this same shear the vertical crack on the sloping face extended to within 2 in. of the bottom face.

At a shear of 21.28 kips (1.76V_{s},) additional short cracks inclined at about 70 degrees to the horizontal occurred on the west face approximately at the point of tangency of the horizontal extension of the inclined reinforcing bar. about 2-1/2 in. from the bottom corner of the web. The stress measured at the point of tangency of the horizontal extension of the inclined reinforcing bar increased more rapidly with each increment of load above the shear of \(0.72V_{n}(\text{calc})\) and at a shear of 21.92 kips had reached 19.2 ksi.

Draw-in of the bottom strand increased slowly at shears above \(0.72V_{n}(\text{calc})\). At shears of from 20.33 kips to 21.92 kips. the draw-in increased by about 0.0025 in. whilst the shear was held constant at each increment of load. The draw-in at a shear of 21.92 kips was 0.0190 in.

At a shear of 22.55 kips a flexure crack occurred 8-1/2 in. from the bottom corner of the web. This crack had extended about 4 in. vertically, when a diagonal tension crack occurred above it, travelling both toward the top and bottom of the web, over about 2/3 the depth of the web. Simultaneously with the...
OCCURRENCE of these cracks, the draw-in of the bottom strand increased from 0.0195 in. to 0.0350 in. and the stress in the horizontal extension of the inclined bar at the point of tangency reached 23.3 ksi.

A second attempt was made to increase the shear to $V_n^{(\text{calc})}$ (22.80 kips,) but at a shear of 22.75 kips a second flexure crack occurred 19 in. from the bottom corner of the web, (approximately at the end of the horizontal extension of the inclined reinforcing bar). Simultaneously the first diagonal tension crack widened and extended to the web - flange Junction at about 31 in. from the re-entrant corner. A second diagonal tension crack propagated from the second flexure crack and "ear horizontal cracks extended from the flexure cracks in the vicinity of the bottom strand and the horizontal extension of the inclined reinforcing bar. As these cracks developed, the load fell to a shear of 18.76 kips and the draw-in of the bottom strand increased to 0.0955 in. The stress in the horizontal extension of the inclined reinforcing bar reached 27.3 ksi.

At this point the load was removed from the beam. It was clear that had the beam been subject to dead weight loading, complete collapse would have occurred at a shear of 22.75 kips, in a combined flexural bond and diagonal tension failure of the beam web adjacent to the dapped end.

The nib of the dapped end remained uncracked throughout the test. However, some fine vertical cracks occurred in the web Just before failure, approximately at the location of the front edge of the vertical plate.

The stress in the compression reinforcement Just ahead of the edge of the vertical plate was small throughout all the tests, reaching a maximum value of 6.8 ksi at maximum load.

The maximum stresses reached were 60.4 ksi ($0.89f_y$) in the inclined reinforcement and 58.0 ksi in the flexural reinforcement "ear the re-entrant corner.
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

Fig. 1 - Variation with shear of stress in flexural reinf., Spec 2A

Fig. 2 - Variation with shear of stress in inclined bar, Spec 2A

(4)
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

Fig. 3 - End slip of bottom strands in final test. (Specimens 1A & 2A)
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

Specimen 2A after test, west face.

Specimen 2A after test, east face.
Specimen 2B

SUMMARY OF BEHAVIOR

1. Prior to loading.

A very fine crack was visible on the West (W) face of the web, extending from 2.5 in. up the sloping end face to a point about 0.875 in. from the bottom face at about 7.5 in. from the bottom corner. Then, after about a half inch vertical break, it continued in a" undulating path to a point 1.5 in. from the bottom face and 11.5 in. from the bottom corner. The crack also extended horizontally about 1 in. on the sloping end face. This crack was extremely fine and probably occurred as a result of restraint of shrinkage of the concrete by the adjacent #5 reinforcing bar. (Measurements made after failure indicated that the side cover to this bar "ear the bottom corner of the beam was only 0.625 in., rather than the intended 0.75 in.)

2. Test (a) - to service load, Vs = 12.36 kips.

A very fine inclined crack became visible on the W face of the nib at a shear of 6.4 kips. It extended from a point 2 in. above the bottom face of the nib and 1 in. into the nib from the re-entrant corner, on a" arc to a point 4.5 in. above the bottom face of the nib and 1 in. into the beam web from the x-e-entrant corner.

A crack initiated at the re-entrant corner on the W face at a shear of 10.2 kips and extended 2 in. at about 60 degrees to the horizontal. This crack subsequently propagated at a more shallow angle and at service load it extended to a point 9 in. from the re-entrant corner and 3.5 in. below the web-flange junction.

A similar crack initiated on the E face at a shear of 11.2 kips. By service load this crack extended, (with short breaks,) to about 2 in. below the web-flange Junction and 9 in. from the re-entrant corner.

On the W face a short vertical crack extended downward from the major inclined crack at service load, about 1 in. from the re-entrant corner.

Some other short fine cracks occurred on both faces at about service load, located between the primary inclined crack and the top of the web. No additional cracks occurred on the sloping end face. The max. crack width was "ear the ra-entrant corner and was only 0.001 in. at service load. Max. draw-in of the bottom strand was 0.0012 in.
3. Test \((b)\) - 9 cycles of service load, \(V_S = 12.36\) kips.

On the \(W\) face, the short vertical crack extending downward from the major inclined crack propagated to the sloping end face about 1 in. from the re-entrant corner, on the third load cycle. Three additional short fine cracks became visible in the beam web during this test, approximately at the level of the re-entrant corner and about 11 in. into the beam. These cracks did not propagate under subsequent loading. No additional cracking occurred on the \(E\) face during these cycles of service load. Max. crack width after 9 load cycles was 0.005 in. Bottom strand draw-in after 9 load cycles was 0.0016 in.

4. Test \((c)\) - overload test to \(0.72V_n(calc), (1.36V_S), 16.79\) kips.

At maximum load, the re-entrant corner inclined cracks on both faces propagated to points close to the web-flange junction and about 10 in. from the end of the flange. Other short branching cracks occurred close to, or connected to, the major inclined cracks.

The max. crack width at \(0.72V_n(calc)\) was 0.016 in. on the \(E\) face and 0.017 in. on the \(W\) face. On removal of the load the max. crack width was 0.005 in. on both faces. Max. bottom strand draw-in was 0.003 in.

5. Test \((d)\) - 10 cycles of service load, \(V_S = 12.36\) kips.

On the second load cycle a vertical crack occurred on the sloping end face, extending 4 in. downward from the re-entrant corner and located about 0.6 in. to the West of the beam centerline, (i.e. toward the location of the inclined reinforcing bar.) About 1 in. from its bottom end the crack branched, the branch travelling to a point about 1 in. from the \(W\) face and 4 in. from the re-entrant corner. Also on the second cycle, two additional short inclined cracks occurred above the major crack, extending about 1 in. each side of the re-entrant corner. On this same load cycle, the major inclined crack extended the remaining short distance to the web-flange junction.

On the fourth load cycle the major inclined crack on the \(E\) face extended along the web-flange junction a further 4.5 in. Max. crack width on the tenth load cycle was 0.013 in. on the \(E\) face and 0.015 in. on the \(W\) face. Bottom strand draw-in after the tenth load cycle was 0.0043 in.

6. Test \((e)\) - incremental loading to failure.

At a shear of 14.32 kips, \((1.16V_S)\), the branch crack on the sloping end face extended about 1.4 in. vertically downwards, and a half inch long very fine crack propagated at 45 degrees.
downward from the bottom strand and toward the W face.

At a shear of 17.49 kips, \(1.42V_s\), an inclined crack originated at the sloping end face, 5 in. from the re-entrant corner. It traveled slightly downward for 1 in. and then upward at 45 degrees to the horizontal for 2 in. This crack also extended horizontally on the sloping end face about 0.75 in.

At a shear of 17.49 kips, an inclined crack originated at the sloping end face, 5 in. from the re-entrant corner. It traveled slightly downward for 1 in. and then upward at 45 degrees to the horizontal for 2 in. This crack also extended horizontally on the sloping end face about 0.75 in.

At a shear of 18.43 kips, the major inclined crack on the W face extended 3.5 in. along the web-flange junction. Also an additional short inclined crack occurred about 0.75 in. below the 45 degree sloping part of the inclined crack which initiated at a shear of 17.49 kips.

At a shear of 19.38 kips, the crack on the sloping end face extended vertically to about 1.25 in. from the bottom edge. It was also about \(1/16\) in. wide over the top 3 or 4 in. of its length. (Because of the proximity of the reaction support. it was not possible to measure the width of the end face cracks accurately.) At this same shear the crack which initiated at a shear of 17.49 kips extended steeply upwards to a point at the level of the re-entrant corner and distant 6.5 in. from it. Also an independent crack occurred, extending from a point 3.5 in. vertically above the bottom corner. This crack first traveled 2 in. at 45 degrees toward the re-entrant corner, then 2.5 in. vertically, and finally at a steep slope to a point 3 in. above the level of the re-entrant corner and distant 7 in. from it. At this same shear, the crack visible on the W face before test extended 2 in. horizontally, but was still quite fine.

At a shear of 20.05 kips, the crack on the sloping end face extended to the bottom edge and widened to about 0.1 in. in the upper half of its length. Failure then occurred suddenly when the inclined bar pulled out sideways at the bend. The concrete inside the bend failed by splitting approximately in the plane of the bar and by crushing further up in the web. Simultaneously, the crack which had existed before loading widened, and some compression spalling appeared on the bottom face as the piece of concrete below this crack tended to rotate outwards.

Whilst the foregoing events were occurring, the inclined cracks propagated further along the web-flange junction, and a small amount of spalling occurred at the ends of these cracks on both sides of the beam. The max. draw-in of the bottom strand was 0.024 in. just before failure.

In the test of specimen 2A, only the average strain in the inclined bar was measured. In the test of specimen 2B, strains were measured on both the East and West sides of the inclined #5 bar. During the first 10 cycles of service load these strains were approximately equal. However, when the load was increased above the service load in the third test and cracking of the concrete became more extensive, the tensile strain on the West side.
of the bar became progressively larger than the tensile strain on the East side. This indicated bending of the bar toward the center-line of the beam due to lateral pressure from the surrounding concrete. This is probably due to the concrete in the end part of the web resisting the torsional moment caused by the 0.81 in. eccentricity of the upward acting inclined bar force with respect to the downward load applied at the center-line of the beam.

Since the beam was loaded vertically on its center-line and did not rotate, the resultant reaction on the bearing plate must also have acted at the beam center-line. For this to occur, an upward reaction force must have acted on the concrete on the opposite side of the beam from that containing the inclined reinforcing bar. This would provide the torsional moment in the concrete which tended to bend the lower part of the inclined bar toward the beam center-line. In turn, the bar was pressing outward on the concrete cover as well as pulling upward.
"Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

Fig. 1 - Variation with shear of stress in flexural reinforcement, Spec 2B

Fig. 2 - Variation with shear of stress in inclined bar, Spec 2B"
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

![Graph showing Strand End Slip vs Shear (kips)](image)

*Fig. 3* - End slip of bottom strands in final tests of 1B & 2B
Specimen 2B after tent, west face.

Specimen 2B after test, east face.
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.
Specimen 2C

SUMMARY OF BEHAVIOR

1. Prior to loading.

On removal of the beam from the form after transfer of prestress, a fine crack was seen to run down the sloping end face of the web from the bottom strand to the bottom face. It then ran along the bottom face for about 16 in., close to the center-line. This crack remained of hairline width at the bottom strand, but during the 36 days between prestress transfer and the day of test, it increased in width at the bottom face and extended 6 in. At time of test it was 0.015 in. wide at the bottom of the sloping end face, becoming less wide moving along the beam, so that the last 6 in. could only be traced with the aid of an illuminated magnifier.

The strains at transfer in the gages on the horizontal extension of the hanger reinforcement, 2 in. from the end of the beam were only about one third as large as similar strains measured in Spec 1C. This indicates that even greater slip of the bottom strand occurred at prestress transfer in this specimen than occurred in Spec 1C.

Randomly distributed, very fine shrinkage cracks were also visible on both web faces. In general these cracks did not widen or propagate under load.

2. Test (a) - to service load, \( V_s = 11.95 \) kips.

At a shear of 3.56 kips cracks propagated from the re-entrant corner on both East (E) and West (W) faces. 0" the E face the inclined crack travelled 3 in. vertically and 2 in. into the beam web. In addition a short inclined crack occurred almost vertically above the x-e-entrant corner and about 5.5 in. from it. On the W face the crack travelled 3.7 in. almost vertically, to link up with existing shrinkage cracks. One of the branches of the shrinkage cracks then extended 1 in. at 60 degrees to the horizontal in the web.

As the shear was increased to 7.36 kips, the inclined crack in the E face which originated at the re-entrant corner, extended to a point 2 in. below the web-flange junction.

At a shear of 10.21 kips an inclined crack initiated 4 in. below the re-entrant corner on the W face. This crack travelled to a point 2 in. above the re-entrant corner and 5.5 in. into the beam from it. Also on the W face a hairline flexure crack 1.2 in. long was observed 17 in. from the end of the bottom face. At this same shear, the re-entrant corner crack on the E face developed a horizontal branch crack 1 in. above the corner,
having a length of 1.5 in.

When the shear was increased to 11.16 kips an inclined crack occurred on the E face, originating below the re-entrant corner and running to a point 2 in. below the web-flange junction and 7 in. from the end of the flange. This crack was joined by a crack on the sloping end face to the inclined crack on the W web face, which had occurred at the preceding load increment. Also at this same shear, an additional inclined crack occurred on the W face approximately 3 in. below and parallel to the crack just referred to. The flexure crack on this face increased in length to 2.25 in., but remained very fine.

As the load was being increased to service load, (\(V_s = 11.95\) kips,) a major diagonal tension crack occurred at a shear of 11.79 kips. 0" both the E and W faces this crack extended from the level of the bottom strand at 11 in. from the end of the bottom face, to 1.5 in. below the web-flange junction at 36 in. from the end of the top flange. After diagonal tension cracking, two fine flexure cracks were found at about 8 in. and 14 in. from the end of the bottom face. The draw-in of the bottom strand increased abruptly from 0.0037 in. to 0.0250 in. when the diagonal tension crack occurred.

At diagonal tension cracking the load fell away, stabilizing at a shear of 11.03 kips, at which time the bottom strand draw-in had increased to 0.0335 in.

It was then possible to increase the load to service load (\(V_s = 11.95\) kips). Under this load, on both E and W faces the major diagonal tension crack extended more or less horizontally for about 3 in. at its upper end. At its lower end it extended downwards at a shallow angle, to a point 6 in. from the end of the bottom face. An additional flexure crack occurred on the E face 21 in. from the end of the bottom face.

At service load the max. crack width near the re-entrant corner was 0.011 in. and 0.015 in. respectively on the E and W faces. These cracks closed to 0.003 in. and 0.007 in. respectively on removal of the load.

Also at service load, the diagonal tension crack had a max. width of 0.034 in. and 0.036 in. respectively on the E and W faces. These cracks closed to 0.018 in. and 0.020 in. respectively on removal of the load. The max. draw-in of the bottom strand at service load was 0.0516 in., reducing to 0.0488 in. on removal of the load.

The longitudinal crack on the bottom face did not increase in length during this test, but was joined by extensions of the flexure cracks on the bottom face.
3. Test (b) - additional cycles of service load, $V_s = 11.95$ kips.

On the first application of service load, additional cracks of the flexural bond type occurred on the W face, approximately over the horizontal extension of the hanger reinforcement. This cracking extended to 27 in. from the end of the bottom face. On the E face the flexure crack 21 in. from the end of the bottom face, branched 1.3 in. from the bottom face. The branch crack travelled about 5 in. at 60 degrees to the horizontal, toward the end of the beam. The bottom strand draw-in increased to 0.0630 in.

No significant increase in cracking then occurred until the fourth load cycle, when a flexural bond failure apparently happened at a shear of 11.22 kips, i.e. just before service load was reached. The load fell away rapidly and could not be recovered. Extensive flexural bond type cracking occurred on both faces, extending to 32 in. from the end of the bottom face. These last cracks were further aggravated when the load was removed and the prestress force in the beam forced crack faces back into contact.
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

Max, stresses did not increase during the 4 load cycles of test (b).

Flexural bond failure at $V=11.22k$ in 4th load cycle of test (b).

Increase in max. stresses during test (b).

- $x$ = gages SC6 (ave.)
- $y$ = gages 3 & 4 (ave.)
- $z$ = gage 4

Flexural bond failure at $V=11.22k$ in 4th load cycle of test (b).

Diagonal tension crack, $(V=11.79k)$

Service load, $V_s=11.95k$

$V_a=1.66k$
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

Fig. 3 - End slip of bottom strands, Spec. 1C & 2C
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

Specimen 2C after test. west face.

Specimen 2C after test. east face.
Specimen 2D

SUMMARY OF BEHAVIOR

1. Prior to loading.

A hairline crack was observed to run from about 0.5 in. below the bottom strand to the bottom of the end face, thence approximately along the center line of the bottom face for 5.5 in.

2. Test (a) - to service load, $V_s = 11.88$ kips.

At a shear of 4.5 kips, the hairline crack on the bottom face could be traced to 7.5 in. from the end face, but was still too fine to measure. At a shear of 11.16 kips, cracks extended from the re-entrant corner on both East (E) and West (W) faces. On the E face the crack extended 0.6 in. almost vertically. On the W face the crack extended 0.6 in. at 45 degrees to the horizontal. At service load this crack extended vertically to 1 in. above the re-entrant corner.

The max. crack width was 0.001 in., becoming of hairline width on removal of load. The max. drew-in of the bottom strand was 0.0005 in.

3. Test (b) - 9 cycles of service load, $V_s = 11.88$ kips.

On both E and W faces the cracks extended about 0.2 in. The max. crack width on the last load cycle was 0.001 in. on both faces. The cracks closed to hairline width on removal of load. The max. draw-in of the bottom strand was 0.0010 in.

4. Test (c) - overload test to $0.72V_n$ (calc), $(1.36V_s)$, 16.16 kips.

At a shear of 14.64 kips the re-entrant corner cracks extended significantly on both faces. On the E face the crack extended to a point 3.8 in. above and 2.6 in. into the beam from the re-entrant corner. On the W face the re-entrant corner crack branched - one branch extending upward and backward to a point about 2.3 in. above the re-entrant corner, the other branch extended to a point 1 in. above the re-entrant corner and 1 in. into the beam. At maximum load the crack on the E face had extended in a shallow arc to a point 1.8 in. below the web-flange junction and 9 in. into the beam. Also on the E face a new crack initiated 2.25 in. down the sloping end face from the re-entrant corner. This crack extended to a point 1.2 in. below the re-entrant corner and 2.7 in. into the beam from the re-entrant corner. On the W face the upper branch crack extended to a point 3.4 in. above the re-entrant corner. The lower branch crack extended to a point 1.5 in. above the re-entrant corner and...
4.0 in. into the beam. The crack on the bottom face did not extend and remained of hairline width.

The max. crack widths at $0.72V_n(\text{calc})$ were 0.006 in. and 0.004 in. respectively on the E and W faces. The crack widths on removal of the load were respectively 0.003 in. and 0.001 in. The max. draw-in of the bottom strand was 0.0015 in.

5. Test (d) - 10 cycles of service load, $V_s = 11.88$ kips.

There was no increase in cracking during these 10 load cycles. The max. crack widths on the tenth load cycle were 0.003 in. and 0.006 in. respectively on the E and W faces. The crack widths on removal of load were respectively 0.003 in. and 0.001 in. The bottom strand draw-in after the tenth load cycle was 0.0018 in.

6. Test (e) - incremental loading to failure.

At a shear of 19.07 kips, $(1.61V_s)$, a crack initiated on the W face, 5 in. from the re-entrant corner at the sloping end face. This crack extended about 1.5 in. into the beam and also travelled across the sloping end face to a point 0.7 in. short of the centerline and 3.7 in. below the re-entrant corner. At this time the corresponding crack on the E face extended across the sloping end face toward the crack growing from the opposite face, but this crack also stopped about 0.5 in. short of the centerline. On the E face this crack now extended to a point 0.3 in. above the re-entrant corner and 3.6 in. into the beam from the re-entrant corner.

At a shear of 20.0 kips the cracks on the sloping end face joined one another and an approximately vertical branch crack extended 1.2 in. down the sloping end face and about 1 in. from the W face. i.e., near the location of the inclined rebar. At this same shear the re-entrant corner cracks on both faces had extended to the web-flange Junction, 15 in. and 12 in. from the end of the flange on the E and W faces respectively. In addition the crack on the E face which initiated 2.25 in. down the sloping end face from the re-entrant corner, extended to a point 2.2 in. above the re-entrant corner and 6.2 in. into the beam from the re-entrant corner, and the crack on the bottom face extended 0.3 in.

Also at a shear of 20.0 kips a crack initiated on the W face 6.5 in. down the sloping end face from the re-entrant corner. This crack initially travelled about 1.5 in. at 30 degrees, then almost vertically to a point 2.5 in. above the re-entrant corner and 6 in. into the beam.

At a shear of 21.92 kips a 1 in. high flexure crack occurred on the W face, 7 in. from the end of the bottom face.
Also, the longitudinal crack on the bottom face extended 0.7 in.

At a shear of 23.18 kips a 1.8 in. high flexure crack occurred on the E face, 9.5 in. from the end of the bottom face. By this same shear the re-entrant corner cracks had travelled along the web-flange junction to points 20.5 in. and 17.5 in. from the end of the flange on the E and W faces respectively.

The maximum shear reached was 23.82 kips. At this shear the upper branch of the re-entrant corner crack on the W face extended about 2 in. horizontally into the nib, about 2.5 in. below the nib top face, and also extended into the beam web at about 30 degrees to the horizontal, to a point 0.5 in. below the web-flange junction and 3.5 in. from the end of the flange. Other minor crack extensions occurred at this maximum shear. Over a period of about 10 minutes the shear dropped to 22.5 kips, when the shear was being brought back up to its original value. A sudden diagonal tension failure of the nib occurred at a shear of 23.69 kips. [$1.06V_n^{(calc)}$]. If the shear had been maintained at the maximum value of 23.82 kips, it is likely that the diagonal tension failure of the nib would have occurred after a few minutes.

The diagonal tension cracks causing failure extended from the outer bottom corner of the nib, across the nib and into the and 5 or 6 inches of the beam web. This failure was accompanied by yield of the nib flexural reinforcement and gross slip of the two prestressing strands passing through the nib. (Only relatively minor slip of the bottom two strands occurred.) At the maximum shear the average stress in the inclined bar was 62.8 ksi ($0.95f_v$) and the stress in the nib flexural reinforcement was 59.6 ksi ($0.81f_v$). The nominal nib shear stress at failure was 693 psi.

As in specimen 28, the strain on the face of the inclined bar nearest the web face was greater than the average strain in the bar. This indicates that the bar was subject to bending toward the beam centerline as well as to direct tension. However, in this case the difference in stress was much less. This was probably due to the fact that the eccentricity of the inclined bar was reduced and the concrete cover to the bar was increased from 0.75 in. to 1.0 in. The maximum difference in stress was 4.2 ksi. This occurred just before the inclined cracks originating on the sloping end face of the web joined together across the end face. The cracking of the beam through its full width evidently relaxed the restraint acting on the lower end of the inclined bar and the difference in stress subsequently decreased.
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

Fig. 1 - Variation with shear of stress in flexural reinforcement, Spec. 2D

\[ V_{\text{max}} = 23.82k = 1.06 V_n(\text{calc.}) \]
\[ V_d = 1.66k \]
\[ 0.72 V_n(\text{calc.}) = 16.16k \]

Service load:
\[ V_s = 11.88k \]

Fig. 2 - Variation with shear of stress in inclined bar, Spec. 2D

\[ V_{\text{max}} = 23.82k = 1.06 V_n(\text{calc.}) \]
\[ V_d = 1.66k \]
\[ 0.72 V_n(\text{calc.}) = 16.16k \]
Strand End Slip (in)

Shear (kips)

$3E; \ V_{\text{max}} = 29.51 \text{k}$

$2D; \ V_{\text{max}} = 23.82 \text{k}$

$\frac{3E}{2D} 0.72V_n(\text{calc.})$

$3E \text{ Service load}$

$2D \text{ Service load}$

$V_d = 1.66 \text{k}$

**Fig. 3** - End slip of bottom strands in final tests of $2D \& 3E$
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Specimen 2D after test. west face.

Specimen 2D after test. east face.
Specimen 3B

SUMMARY OF BEHAVIOR

1. Prior to loading.
   No cracks observed.

2. Test (a) - to service load, \( V_s = 10.11 \) kips.
   Hairline crack initiated at re-entrant corner at shear of 9.26 kips and extended about 1/2 in. at about 60 degrees to the horizontal. At service load the crack extended about 1 in. and had a max. width of 0.001 in. Max. bottom strand draw-in at service load 0.0007 in.

3. Test (b) - 9 cycles of service load, \( V_s = 10.11 \) kips.
   Crack from re-entrant corner increased to 2 in. in length. Also a hairline crack about 3 in. long occurred, more or less aligned with re-entrant corner crack but starting about 2 in. above it. Max. crack width 0.001 in.
   A hairline crack occurred on the centerline of the bottom face in the 7th load cycle, extending 4-1/2 in. from end of web and also 3/4 in. up the sloping end face. Max. draw-in of bottom strand, 0.0013 in.

4. Test (c) - overload test to \( 0.72V_n(\text{calc}), (1.36V_s), 13.72 \) kips.
   Cracks previously formed joined up and extended to within 1-1/2 in. of web - flange junction and about 5-1/2 in. from end of flange. Additional crack formed on east side, extending from re-entrant corner crack for about 2 in. upwards, approximately on centerline of inclined reinforcing bar. On the west side a crack occurred in the nib, extending about 2 in. upward from the bearing plate. Max. crack width at \( 0.72V_n(\text{calc}) \), 0.005 in. Max-draw-in of bottom strand, 0.0015 in.

5. Test (d) - 10 cycles of service load, \( V_s = 10.11 \) kips.
   No growth in length of major cracks. A few randomly scattered horizontal hairline cracks about 1 to 1-1/2 in. long became visible. These cracks did not develop subsequently. Max. crack width under service load 0.004 in., closing to 0.001 in. under dead load only. Max. draw-in of bottom strand 0.0018 in.
6. Test (e) - incremental loading to failure.

Inclined crack from se-entrant corner extended to web - flange junction about 10 in. from end of flange, at a shear of 17.5 kips. An additional inclined crack initiated at a shear of 16.6 kips, starting from the sloping end face about 4 in. from the re-entrant corner and extending about 4 in. at about 70 degrees to the horizontal. Under increasing load this crack extended vertically and joined the inclined crack from the re-entrant corner. Also at a shear of 16.6 kips. short inclined cracks commenced to form in the web, crossing the line of the inclined reinforcing bars approximately at right-angles. These cracks increased in number as the shear was increased further. On the east face of the web the crack aligned with the inclined reinforcing bar extended about 3 in. by a shear of 17.5 kips.

At a shear of 19.5 kips an additional crack occurred across the sloping end face, about 7 in. from the re-entrant CORNER. This crack extended vertically to link with the existing inclined cracks. Additional cracking of the end face occurred at slightly above this load and the stress in the horizontal extension of the inclined reinforcement started to increase much more rapidly.

At a shear of 22.2 kips a short hairline flexure crack occurred about 6 in. from the bottom corner of the web. At this shear the crack originating at the re-entrant corner had travelled along the web - flange Junction to a point 20 in. from the end of the flange.

Additional flexure cracks formed 2 in., 10 in. end 14 in. from the bottom corner of the web as the shear was increased to 24.4 kips. At this shear a diagonal tension crack occurred in the web, running from a point 2 in. above the bottom face of the beam and 9 in. from the bottom corner of the web. It ran at an angle of 35 - 40 degrees to the horizontal, extending to a point 5 in. below the web - flange Junction and 34 in. from the end of the flange. Also at this shear, horizontal cracks extended from the flexure cracks at about the level of the horizontal prolongation of the inclined reinforcement and further inclined cracking occurred in the nib.

At a shear of 27.6 kips a flexure crack occurred 18-1/2 in. from the bottom corner of the web. It travelled 7 in. upwards and 3 in. toward midspan.

Finally, at a shear of 27.93 kips, a flexure crack formed 23 in. from the bottom corner of the web, approximately at the end of the horizontal extension of the inclined reinforcement. This precipitated a flexural bond failure and the crack developed into a major diagonal tension crack. It also branched and extended toward the end of the beam, linking with previously existing cracks in a disruptive fashion. At failure the load fell away rapidly and could not be recovered.

Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.
Although the final failure was brittle in character, there was reasonable warning of failure by extensive cracking starting at about 87% of ultimate and the yield strength of both the inclined reinforcement and the flexural reinforcement was developed before failure. (The stress in the horizontal extension of the inclined reinforcement reached a stress of about 33 ksi at failure.)
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

Fig. 1 - Variation with shear of stress in flexural reinforcing Spec. 3B

Fig. 2 - Variation with shear of stress in inclined bars, Spec. 3B
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

Fig. 3 - End slip of bottom strands in final tests of 3B & 4B
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

Specimen 38 after test, west face.

Specimen 38 after test, east face.
Specimen 3c

SUMMARY OF BEHAVIOR

1. Prior to loading.

A hairline crack extended down the sloping end face of the web from the bottom strand to the bottom face, and thence approximately 6 in. along the bottom face. (This crack was not visible at transfer of prestress. It did not increase in length during any of the load tests.)

2. Test (a) - to service load, $V_S = 9.89$ kips.

Cracks initiated at the z-e-entrant corner on first application of the load ($V = 2.61$ kips), on both the East (E) and West (W) faces of the web. The cracks initially extended diagonally upwards 1.5 in. Under increasing load these cracks continued in the same direction to within 2 in. of the web-flange junction at service load. They also forked about 1.5 in. from the re-entrant corner, the branch crack initially travelling at right-angles to the original crack and then more or less vertically to 3 or 4 in. above the z-e-entrant corner. At service load a" additional crack occurred on both E and W faces of the web, starting about 4 in. down the sloping face from the re-entrant corner and extending diagonally upwards for 7 in. on the E face and 10 in. on the W face. These cracks were joined by a crack across the sloping end face.

The max. crack width of 0.008 in. occurred close to the re-entrant corner. This reduced to 0.005 in. on removal of the load. Max. bottom strand draw-in at service load was 0.0007 in.

3. Test (b) - 9 cycles of service load, $V_S = 9.89$ kips.

During these loading cycles the crack on the E face which formed on first reaching service load, extended to 10 in. from the sloping end face. A few short additional cracks occurred on the W face. The max. crack width at service load and the crack width on removal of load did not increase during these loading cycles. The bottom strand draw-in at the end of this test was 0.0012 in.

4. Test (c) - overload test to $0.72V_n(calc), (1.36V_S), 13.43$ kips.

The cracks originating 4 in. down the sloping end face from the re-entrant corner extended diagonally upwards to within 1 in. of the web-flange junction. An additional crack formed on each web face about 2 in. below and parallel to the cracks just described. These cracks extended 3 in. and 5 in. on the E and W faces respectively.
A crack also occurred on the W face of the nib, originating at the inner edge of the bearing plate. It travelled vertically for 2 in. and then inclined toward the web-flange junction at the end of the flange, reaching about 1 in. from the junction at maximum shear.

The max. crack width occurred near the re-entrant corner and was 0.008 in. and 0.01 in. on the E and W faces respectively. These cracks closed to 0.004 in. on removal of load. Max. draw-in of bottom strand was 0.0015 in.

5. Test (d) - 10 cycles of service load, $V_s = 9.89$ kips.

Some small extension of previously existing cracks occurred during these load cycles. The max. crack widths at service load and on removal of load were within 0.001 in. of the widths measured in test (a). The max. draw-in of the bottom strand was 0.0019 in.

6. Test (e) - incremental loading to failure.

The inclined cracks originating about 4 in. below the re-entrant corner. extended to the web-flange junction at a shear of 17.04 kips. As the shear was increased to 17.36 kips a crack occurred on the E face of the nib, originating at the inner edge of the bearing plate. It approximately mirrored the crack which occurred on the W face of the nib in test (c). At about the same shear, additional inclined cracks approximately 2 in. long propagated from the sloping end face about 9 in. below the re-entrant corner.

At a shear of 19.26 kips, short flexural cracks occurred on both web faces about 6 in. from the end face. These cracks were initially vertical, but then angled backward, approximately parallel to the sloping end face. At a shear of 20.52 kips they reached 2 in. and 3 in. above the bottom face on the E and W web faces respectively.

A short, fine, very shallowly inclined crack occurred on the E face of the web at the level of the re-entrant corner and about 14 in. from it, at a shear of 19.57 kips. At about this same shear additional inclined cracks occurred on both faces of the nib, above and approximately parallel to the original cracks in the nib faces.

As the shear was increased further, intermittent cracking occurred in the web faces. approximately over the inclined reinforcing bars. Presumably these were bond splitting cracks. The inclined cracks originating lower down the sloping end face became vertical as they extended, and tended to link with one another and with the inclined crack originating 4 in. below the
re-entrant corner.

At a shear of 21.16 kips, the shallowly inclined crack which originated at \( V = 19.57 \) kipa. extended upwards more steeply for about 3.5 in. This load was sustained for 7 minutes, then major diagonal tension cracks occurred on both web faces. These extended from close to the bottom of the end face, up to the web-flange junction about 28 in. from the end of the flange. Immediately following the occurrence of these diagonal tension cracks, a series of cracks occurred on both web faces approximately over the horizontal extension of the hanger reinforcement. These cracks extended from the previously existing flexure cracks to about 20 in. from the end face. Simultaneously with the formation of these flexural bond cracks, the bottom strand was drawn in about 0.10 in. As these cracks developed the load fell off to a shear of 17.42 kips, which was sustained. However, the load could not subsequently be increased.

The failure was a diagonal tension failure of the web, followed by flexural bond failure. The maximum shear was \( 1.14V_n\text{calc}\), however the hanger reinforcement did not reach its yield strength. At maximum load before diagonal tension cracking, the stress in the hanger reinforcement was 61.3 ksi, indicating that the reinforcement was carrying 17.34 kips shear and the concrete in the nib was carrying 3.82 kips. This division of the shear is consistent with the trajectory of the cracks in the upper part of the nib.

The draw-in of the bottom strand just before the diagonal tension failure was 0.015 in.
Fig. 1 - Variation with shear of stress in flexural reinfl., Spec 3C

Fig. 2 - Variation with shear of stress in inclined bars, Spec. 3C
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

Fig. 3 - End slip of bottom strands in final tests of 3C & 4C
Specimen 3C after test, west face.

Specimen 3C after test, east face.
Specimen 3D

SUMMARY OF BEHAVIOR

1. Prior to loading

A hairline crack was observed on the center-line of the bottom face, extending 2.2 in. from the end of the bottom face and also extending 1 in. up the sloping end face.

2. Test (a) - to service load, $V_s = 8.94$ kips.

Cracking initiated at the re-entrant corner at a shear of 7.36 kips. The cracks extended about 0.5 in. on both faces. At service load the crack on the East (E) face had not increased in length, but on the West (W) face the crack had extended to a point 1.3 in. above the re-entrant corner and 0.8 in. into the beam. The max. crack width was 0.001 in. on both faces at service load, becoming of hairline width on removal of the load. The max. draw-in of the bottom strand was 0.0002 in.

3. Test (b) - 9 cycles of service load, $V_s = 8.94$ kips.

On the E face the crack extended to a point 1.3 in. above the re-entrant corner and 0.4 in. into the beam. On the W face the crack extended to a point 2.7 in. above the z-e-entrant corner and 3.0 in. into the beam. Max. crack width on last load cycle was 0.001 in. and 0.002 in. respectively on the E and W faces. On both faces the cracks became of hairline width on removal of the load. The max. draw-in of the bottom strand was 0.0004 in.

4. Test (c) - overload test to $0.72V_n(\text{calc})$, $1.36V_s$, 12.17 kips.

No extension of cracks occurred on the E face. On the W face the crack originating at the z-e-entrant corner extended 0.60 in. horizontally. Also a flexure crack occurred in the nib at the inner edge of the bearing plate, extending to a point 4.0 in. above the bearing plate and 1.7 in. toward nispan. Max. width of crack at $0.72V_n(\text{calc})$ was 0.002 in. on both faces. The cracks closed to hairline width on removal of load. The max. draw-in of the bottom strand was 0.0005 in.

5. Test (d) - 10 cycles of service load, $V_s = 8.94$ kips.

No additional cracking occurred during this test. Max. crack width at service load in last load cycle was 0.002 in. The cracks closed to hairline width on removal of load. The max. draw-in of the bottom strand was 0.0007 in.
6. Test (e) - incremental loading to failure.

As the shear was increased above the maximum previously applied, the cracks originating at the re-entrant corner progressively increased in length and reached the web-flange Junction 14 in. from the end of the flange, at a shear of 18.75 kips. They subsequently propagated an additional 4 in. along the web-flange Junction as the load was further increased.

At a shear of 13.05 kips a crack branched from the original crack on the E face 1.5 in. above the re-entrant corner. It travelled at 60 degrees to a point 3.2 in. above the re-entrant corner and 0.8 in. into the nib. At this same shear, the crack on the centerline of the bottom face extended 2.5 in. but remained very fine.

At a shear of 16.85 kips additional inclined cracks occurred on both faces. They originated at the sloping end face 4 in. from the re-entrant corner, and were joined by a crack across the sloping end face. On the E face the crack extended to a point 1 in. below the re-entrant corner and 5.2 in. into the beam web. On the W face the corresponding crack extended to a point 0.25 in. above the re-entrant corner and 7.2 in. into the beam web. As the shear was increased these cracks became progressively steeper, reaching to within 0.5 in. of the cracks originating at the re-entrant CORNER at a shear of about 22.5 kips.

At a shear of 17.80 kips additional short inclined cracks occurred on both faces, originating at the sloping end face, 2 in. from the re-entrant corner. These cracks were also joined by a crack ACROSS the sloping end face. At this same shear, the crack on the centerline of the bottom face extended to 6.5 in. from the end of the bottom face.

At a shear of 19.70 kips a further inclined crack occurred on both faces 3.5 in. from the bottom of the sloping end face. These cracks initially travelled at about 45 degrees, but then became almost vertical, eventually linking up with the inclined cracks which had originated further up the sloping end face at a shear of 16.85 kips. These cracks were linked by a crack ACROSS the end face.

Also at a shear of 19.70 kips, a diagonal tension crack occurred in the E face of the nib, centered 0.5 in. above the end of the branch crack, which by now extended to a point 4.0 in. above the re-entrant corner and 1.2 in. into the nib.

At a shear of 22.55 kips a fine flexural crack occurred on the E face, 8.5 in. from the end of the bottom face and extending 1.25 in. upwards. When the shear increased to 24.45 kips, the crack on the E face extended to 4.5 in. from the bottom face and a similar crack was observed on the W face. This crack was 6 in. from the end of the bottom face and was linked to the E face flexural crack by an S shaped crack on the bottom.
face. Also, on the W face a second flexure crack 2.5 in. long occurred 11.5 in. from the end of the bottom face. At this same shear, a second diagonal tension crack occurred in the E face of the nib, about 0.5 in. above the first nib diagonal tension crack.

At a shear of 25.24 kips a major diagonal tension crack occurred in both faces of the beam web, together with additional flexure cracks at 13.5 in. and 18 in. from the end of the bottom face. The diagonal tension crack extended from 8.5 in. from the end of the bottom face to about 40 in. from the end of the flange at the top of the web. After diagonal tension cracking the load fell away. When the shear was increased fractionally above 23.8 kips a flexural bond failure occurred, with cracking at the end of the horizontal extension of the inclined reinforcing bars.

Draw-in of the bottom strand occurred at an increasing rate at shears above 20 kips, but the draw-in did not exceed 0.008 in. until a shear of 24.45 kips. The draw-in did not exceed 0.016 in. until the diagonal tension failure occurred, at which time it increased suddenly to 0.046 in.

The inclined reinforcement developed its yield strength at failure, but the nib flexural reinforcement was just short of yield at failure (0.94fy). The failure load was 1.50Vn(calc), where Vn(calc) was taken equal to the vertical component of the yield strength of the inclined reinforcement only.
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

**Fig. 1** - Variation with shear of stress in flexural reinforced, Spec. 3D

\[
V_{\text{max}} = 25.24k = 1.50 V_p (\text{calc})
\]

Increase in max. stress during tests (b) & (d)

Service load; \( V_s = 8.94k \)

\[ V_d = 1.66k \]

**Fig. 2** - Variation with shear of stress in inclined bars, Spec. 3D

\[
V_{\text{max}} = 25.24k = 1.50 V_p (\text{calc})
\]

Increase in max. stress during tests (b) & (d)

Service load; \( V_s = 8.94k \)

\[ V_d = 1.66k \]
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

Fig. 3 - End slip of bottom strands in final tests of 3D & 5B
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

Specimen 3D after test, west face.

Specimen 3D after test, east face.
Specimen 3E

SUMMARY OF BEHAVIOR

1. Prior to loading.

   No cracks observed.

2. Test (a) - to service load, $V_s = 12.45$ kips.

   At a shear of 4.51 kips a hairline crack was observed on the East face of the nib, about 2 in. above the bearing plate and extending from 2.3 in. to 3.7 in. from the end face of the nib. This crack did not develop further and was probably a shrinkage crack missed in the initial inspection.

   Cracking initiated at the re-entrant corner on both East (E) and West (W) faces at a shear of 10.21 kips. On both faces the cracks extended for about 1 in. at 45 degrees.

   At a shear of 11.16 kips the crack on the W face extended to a point 1.5 in. above the re-entrant corner and 1.0 in. into the beam. On the E side the crack branched, the upper branch extending 0.5 in. at 45 degrees backward toward the nib, the lower branch extending 1.0 in. horizontally into the beam.

   At a shear of 12.10 kips an inclined crack occurred in the W face, starting near the end of the re-entrant corner crack and extending to a point 5.7 in. above the re-entrant corner and 3.5 in. into the beam.

   No further extension of cracking occurred and the max. crack widths were 0.002 in. and 0.004 in. respectively on the E and W faces. These cracks closed to hairline width and 0.002 in. respectively on removal of load. Max. draw-in of bottom strand was 0.0037 in.

3. Test (b) - 9 cycles of service load, $V_s = 12.45$ kips.

   On the E face the horizontal branch crack extended 1.0 in. in the same direction. The inclined crack on the W face also extended about an inch, to a point 2.0 in. below the web-flange Junction.

   The max. crack widths on the ninth load cycle were 0.002 in. and 0.004 in. respectively on the E and W faces. These cracks closed to, hairline width and 0.002 in. respectively on removal of load. The max. bottom strand draw-in was 0.0049 in.
4. Test (c) - overload test to $0.72V_n(\text{calc})$, $(1.36V_S)$, 16.92 kips.

At a shear of 14.95 kips the inclined crack on the W face extended to a point 1.7 in. below the web-flange junction and 5.7 in. from the end of the flange. Simultaneously the crack branched at a point 3.2 in. below the web-flange junction and 2.5 in. from the end of the flange. This branch extended 2.5 in. at a slightly shallower angle to the horizontal than the original crack.

At a shear of 15.30 kips the upper branch of the re-entrant corner crack on the E face changed direction, and extended to a point 3.4 in. above the re-entrant corner and 1.0 in. into the beam.

At $0.72V_n(\text{calc})$ the upper branch of the re-entrant corner crack on the E face extended a branch for 1.0 in. in its original direction, i.e., following the alignment of the inclined bar.

The max. crack widths at $0.72V_n(\text{calc})$ were 0.005 in. and 0.007 in. respectively on the E and W faces. These cracks closed to 0.001 in. and 0.002 in. respectively on removal of the load. The max. bottom strand draw-in was 0.0061 in.

5. Test (d) - 10 cycles of service load, $V_S = 12.45$ kips.

On the fourth load cycle a fine crack initiated at the front edge of the bearing plate on the W face of the nib. This crack extended to a point 3.0 in. above the bearing plate and 3.5 in. in from the end face of the nib. Subsequent cycles of load caused only minor extensions of pre-existing cracks.

The max. crack widths on the tenth cycle were 0.005 in. on both E and W faces. These cracks closed to 0.001 in. and 0.002 in. respectively on removal of the load. There was no increase in bottom strand draw-in during these cycles of load.

6. Test (e) - incremental loading to failure.

At a shear of 17.80 kips the crack in the W face of the nib extended to a point 3.5 in. above the bearing and 4.0 in. from the nib end face. Simultaneously an inclined crack formed above this crack. It extended from a point 3.3 in. above the bearing and 3.0 in. from the nib end face, to a point 5.1 in. above the bearing and 5.2 in. from the nib end face. At this same shear a crack branched from the re-entrant corner crack on the W face. The re-entrant corner, and travelled downward at about 45 degrees to a point 1.0 in. below the x-e-entrant corner.

At a shear of 19.70 kips a crack initiated 2.0 in. below the re-entrant corner and extended in a convex downward arc to a
point 0.5 in. below the re-entrant corner and 4.0 in. into the beam. Simultaneously, the inclined crack on the W face extended to a point 0.5 in. below the web-flange junction and 9.7 in. from the end of the flange. Also at this shear, the re-entrant corner crack on the E face extended as an inclined crack to a point 1.6 in. below the web-flange junction and 6.4 in. from the end of the flange. These last mentioned inclined cracks progressively increased in length with increase in shear, until they extended along the web-flange junction to points about 15 in. from the end of the flange at a shear of 28.9 kips.

As the shear was increased, a network of cracks extended into the nib on both faces. Some of these cracks approximately paralleled the inclined reinforcement, (apparently bond splitting cracks,) and others were approximately normal to the inclined reinforcement, (apparently due to direct tension.)

Also as the shear was increased, a series of cracks formed in the lower part of the web in the vicinity of the inclined reinforcement. These cracks formed progressively further from the re-entrant corner as the shear increased, presumably reflecting the progressive increase in tension in the inclined bars along their length. These cracks were approximately normal to the inclined bars "ear the bars, and then became more steeply inclined as they propagated into the beam.

At shears greater than 25 kips the draw-in of the bottom strand increased by increasing amounts at each load increment, and at the maximum shear of 29.51 kips the slip increased rapidly and the shear dropped to 28.75 kips. At this point a major diagonal tension crack occurred in the web, extending from about 8 in. from the end of the bottom face, to 32 in. from the end of the flange at the web-flange junction.

Both the nib flexural reinforcement and the inclined bars yielded before failure. Maximum load coincided with a rapid increase in strain to about 10,000 millionths in both inclined and nib flexural reinforcement. It is not clear whether the failure was initiated by the extensive yield of the dapped end reinforcement, or by the slip of the bottom prestressing strand.

The maximum shear was $1.26V_N(\text{calc})$, where $V_N(\text{calc})$ is equal to the sum of the vertical component of the yield strength of the inclined reinforcing bars and a contribution from the concrete taken to be $2\sqrt{f_c}(b_d d_d)$, ($b_d$ and $d_d$ being respectively the average width and the effective depth of the nib of the dapped end.)
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

Fig. 1 - Variation with shear of stress in flexural reinforcement, Spec. 3E

Fig. 2 - Variation with shear of stress in inclined bars, Spec. 3E
Fig. 3 - End slip of bottom strands in final tests of 24 $3E$
Specimen 3E after test, west face.

Specimen 3E after test, east face.
Specimen 4B

SUMMARY OF BEHAVIOR

1. Prior to loading.

No cracks observed.

2. Test (a) - to service load, \( V_s = 10.84 \) kips.

Hairline crack initiated at z-e-entrant corner at shear of 6.41 kips and extended about 1/8 in. at about 45 degrees to the horizontal. This crack extended to about 1 in. at a shear of 8.31 kips, with an overall inclination of about 30 degrees to the horizontal. At a shear of 9.26 kips this crack extended upwards to a point about 2 in. above and 1-1/2 in. into the beam from the re-entrant corner. Under service load this crack extended to a point 3 in. above and 2 in. into the beam from the re-entrant corner. In addition an almost horizontal hairline crack developed about 1-1/2 in. above the re-entrant corner and extended about 5 in. into the beam. Max. crack width at service load, 0.006 in. near the se-entrant corner. No draw-in of bottom strand occurred.

3. Test (b) - 9 cycles of service load, \( V_s = 10.84 \) kips.

The crack from the re-entrant corner extended upward to a point 4-1/2 in. above the re-entrant corner and 2 in. into the beam. This crack had a maximum width of 0.008 in. close to the re-entrant corner and 0.004 in. where the crack crossed the nib flexural reinforcement. The horizontal crack extended to about 6-1/2 in. into the beam and some fine horizontal cracks occurred at the web - flange junction. Max. draw-in of bottom strand 0.0001 in.

4. Test (c) - Overload test to \( 0.72V_n(\text{calc}), (1.36V_s), 14.70 \) kips.

The crack originating at the re-entrant corner and the horizontal crack 1-1/2 in. to 2 in. above the re-entrant corner joined together, and both extended further into the beam. The more or less horizontal crack now extended about 10 in. into the beam from the re-entrant corner. The max. crack width at \( 0.72V_n(\text{calc}) \) was 0.009 in. near the re-entrant corner and 0.007 in. where the crack crossed the nib flexural reinforcement. The max. draw-in of the bottom strand was 0.0001 in.

5. Test (d) - 10 cycles of service load, \( V_s = 10.84 \) kips.

Some small extension and further linking up of existing cracks occurred. Max. crack width was 0.011 in. near the re-
entrant corner and 0.008 in. at the level of the nib flexural reinforcement. The cracks closed to 0.004 in. under dead load only. Max. draw-in of bottom strand, 0.0001 in.

6. Test (e) - incremental loading to failure.

At a shear of 19.4 kips, a 2 in. long diagonal tension crack occurred in the nib of the dapped end. This crack extended to about 5-1/2 in. long when the shear was increased to 20.4 kips. It started "ear the inside edge of the bearing plate and extended upward at about 60 degrees to the horizontal. As the shear was increased further, this crack extended to and along the web - flange junction, linking up with pre-existing cracks.

The crack which originated at the re-entrant corner gradually extended upwards, until it almost linked with the nib diagonal tension crack. The more or less horizontal crack gradually extended at about 30 degrees to the horizontal, reaching the web - flange junction at a point 17 in. from the end of the flange, at a shear of 25.1 kips. Cracks inclined at about 30 degrees also extended downward from this crack toward the end face of the web. They joined with a crack across the vertical end face of the web at the level of the next to bottom strand, at a shear of 21.9 kips.

At a shear of 24.5 kips an independent diagonal tension crack occurred on the east face of the web, about 6 in. long, inclined at 30 degrees to the horizontal and starting 2 in. below the re-entrant corner and 3-1/2 in. from the end face of the web. This crack subsequently extended at both ends and linked up with pre-existing cracks.

At a shear of 25.7 kips a 1-1/2 in. long, almost vertical crack occurred across the horizontal extension of the hanger reinforcement, about 4-1/2 in. from the end face of the web. As the shear was increased, this crack extended both upwards and downwards, and at a shear of 26.9 kips it reached the bottom face of the beam. At this time the crack extended 4 in. upward from the bottom face of the beam. Also at this shear, the shallowly inclined crack had extended along the web - flange junction to a point 21 in. from the end of the flange.

The maximum shear reached was 27.45 kips, at which point a major diagonal tension crack traversed the web at an average slope of about 35 degrees, extending from "ear the bottom corner of the web up to and into the flange. This crack occurred so abruptly that it was not possible to pinpoint its point of origin, but it passed through the upper part of the flexure crack near the end of the web. It was accompanied by some diagonal compression crushing of the concrete in the lower part of the web and ravelled cracking along the horizontal extension of the hanger reinforcement. This failure was preceded by yield of the hanger reinforcement at a shear of about 26 kips.
The maximum crack width at service load was 0.011 in. near the se-entrant corner and 0.009 in. at the level of the nib flexural reinforcement. At 1.36Vs the maximum crack widths at the same locations were 0.015 in. and 0.014 in. respectively.

The draw-in of the bottom prestressing strand was less than 0.001 in. for shears up to 21.9 kips and then increased slowly to 0.0073 in. at a shear of 26.9 kips. After failure a slip of 0.175 in. had occurred, indicating that a flexural bond failure had accompanied the other modes of failure.

Although the final failure was very abrupt, there was considerable warning of failure by the extensive cracking in the web of the beam near the dapped end, above about 85% of ultimate.
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

Fig. 1 - Variation with shear of stress in flexural reinf., Spec. 4B

Fig. 2 - Variation with shear of stress in hunger reinf., Spec. 4B
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

**Fig 3** - Variation with shear of stress in hairpin reinforcement, Spec. 4B

**Fig 4** - End slip of bottom strand in final tests of 36445
Specimen 4B after test, west face.

Specimen 4B after test, east face.
Specimen 4C

SUMMARY OF BEHAVIOR

1. Prior to loading.
No cracks observed.

2. Test (a) - to service load, $V_s = 10.59$ kips.

   Cracks occurred on both faces at the re-entrant corner. on first application of load ($V = 2.61$ kips.) These cracks were inclined at about 60 degrees to the horizontal and extended about 2.5 in. As the load was increased further these cracks travelled roughly vertically, and at service load reached within 2.5 in. and 1 in. of the web-flange Junction on the East (E) and West (W) web faces respectively. On the W face this crack branched about 2.5 in. below the web-flange Junction, the branch travelling upwards at about 25 degrees to the horizontal for 4 in. On both faces these cracks also branched 2 in. above the re-entrant corner, the branch crack travelling slightly downward and into the beam for 3 in.

   At 87 percent of service load, inclined cracks occurred on both web faces. These cracks started about 1 in. from the end face and 3 in. below the re-entrant corner. They travelled upward at approximately 40 degrees to the horizontal, to about 3 in. above the level of the re-entrant corner. As the load was increased to service load, these cracks extended to within 1.5 in. of the web-flange Junction.

   At service load the maximum crack width was 0.009 in. and 0.015 in. respectively on the E and W faces, close to the re-entrant corner. On removal of the load these cracks closed to 0.004 in. and 0.007 in. respectively. The max. width of the inclined cracks which occurred close to service load was 0.005 in. and 0.008 in. respectively on the E and W faces.

   The max. draw-in of the bottom strand was 0.0005 in.

3. Test (b) - 9 cycles of service load, $V_s = 10.59$ kips.

   During this test there were minor extensions of existing cracks. In addition, short inclined cracks formed a little below the inclined cracks originating 3 in. below the re-entrant corner. The max. crack widths after 10 applications of service load were 0.011 in. and 0.017 in. respectively on the E and W web faces.

   The max. draw-in of the bottom strand was 0.0009 in.
4. **Test (c)** - Overload test to \(0.72V_{n}(\text{calc}), (1.36V_{S}), 14.38\) kips.

On the W face both of the major inclined cracks extended to the web-flange junction at maximum load. On the E face the cracks also extended, but in a more branching manner, not penetrating so far upwards.

The max. crack widths at \(0.72V_{n}(\text{calc})\) were 0.016 in. and 0.022 in. respectively on the E and W web faces. On removal of the load these cracks closed to 0.005 in. and 0.010 in. respectively.

The max. draw-in of the bottom strand was 0.0011 in.

5. **Test (d)** - 10 cycles of service load, \(V_{S} = 10.59\) kips.

Only minor extensions of existing cracks occurred during these loading cycles. Max. crack widths were 0.013 in. and 0.021 in. respectively on the E and W web faces. On removal of the load these cracks closed to 0.006 in. and 0.011 in. respectively.

The max. draw-in of the bottom strand was 0.0015 in.

6. **Test (e)** - Incremental loading to failure.

At shears of 16.22 kips and 17.17 kips respectively, inclined cracks occurred in the E and W faces of the nib. The cracks initially extended from the bearing plate to about 3.5 in. above it, but propagated rapidly toward the web-flange junction at the end of the flange. Additional cracks also occurred in the nib, roughly parallel to the initial cracks. The inclined cracks originating 3.5 in. below the re-entrant corner extended to the web-flange junction on both faces at a shear of 18.43 kips. The lower ends of these cracks were almost joined by discontinuous cracking across the web end face.

Failure occurred after a shear of 19.54 kips had been sustained for 3 minutes. Failure initiated when short flexure cracks occurred in the end 12 in. of the full depth web. Major diagonal tension cracks occurred within seconds. These cracks extended from about 2 in. from the bottom corner of the web up to the web-flange junction 25 in. from the end of the flange. Immediately following the diagonal tension cracking, flexural bond type cracking occurred over the horizontal extensions of the hanger reinforcement. This cracking originated near the lower end of the diagonal tension cracks and propagated away from the end of the beam. As the cracking progressed the load fell away, stabilizing at \(V = 16.35\) kips. It was not possible to increase the load. Any attempt to do so simply resulted in further propagation of the cracks, so loading was discontinued.
The draw-in of the bottom strand at maximum load was 0.0044 in. before the diagonal tension failure and 0.065 in. after failure.

The maximum shear resisted was \(0.98V_n(\text{calc})\), where \(V_n(\text{calc})\) was based on the hanger reinforcement carrying all the shear as it developed its yield strength. However, the average stress in the hanger reinforcement just before the diagonal tension failure was only \(0.88f_y\), indicating that 1.86 kips of shear was at that time being carried by the concrete.
Fig. 1 - Variation with shear of stress in flexural reinforce., Spec. 4C

Fig. 2 - Variation with shear of stress in hanger reinforce., Spec 4C
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

Fig. 3 - Variation with shear of stress in hairpin reinf., Spec. 4C

Fig. 4 - End slip of bottom strands in final tests of 3C & 4C
Specimen 4C after test, west face.
Specimen 5B

SUMMARY OF BEHAVIOR

1. Prior to loading.

   No cracks observed.

2. Test (a) - to service load, $V_s = 12.17$ kips.

   Cracking initiated at the se-entrant corner at a shear of 7.36 kips on the West (W) side and 8.31 kips on the East (E) side. At service load the crack on the E side extended at 45 degrees to a point 2.0 in. above the re-entrant corner. On the W side the corresponding crack extended at 45 degrees to a point 2.5 in. above the re-entrant corner. In addition, two branch cracks extended from this crack at a point 1.8 in. above the re-entrant corner. One of these cracks travelled approximately horizontally 1.5 in. toward the nib and the other about 1.5 in. downward and parallel to the sloping end face. Also at service load, a short, fine, diagonal tension crack occurred in the W web face about 1.5 in. below the web-flange Junction and near the end of the flange.

   The max. crack width at service load was 0.003 in. on both faces, near the re-entrant corner. These cracks closed to 0.001 in. on removal of the load. The max. draw-in of the bottom strand was 0.0006 in.

3. Test (b) - 9 cycles of service load, $V_s = 12.17$ kips.

   On the E face the re-entrant corner crack extended 0.2 in. and a short, fine, diagonal tension crack occurred near the web-flange Junction and the end of the flange, similar to that which had occurred on the W face in test (a). On the W face the re-entrant corner crack extended to a point 3.2 in. above and 5.3 in. horizontally from the re-entrant corner.

   The max crack width on the last load cycle was 0.004 in. on both faces. The cracks closed to 0.001 in. and 0.002 in. on the E and W faces respectively. The max. draw-in of the bottom strand was 0.0009 in.

4. Test (c) - overload test to $0.72V_n(calc), (1.36V_s), 16.50$ kips.

   On both faces the inclined crack originating at the re-entrant corner extended to about 0.75 in. below the web-flange Junction, at about 10 in. from the end of the flange. In addition, on the E face a branch crack occurred at a shear of 15.9 kips. This crack extended 1.5 in. horizontally toward the nib, from a point on the original inclined crack 2.3 in. above the
re-entrant corner.

The max. crack width at $0.72V_{\text{calc}}$ was 0.011 in. on the E face and 0.009 in. on the W face. Both these cracks closed to 0.003 in. on removal of the load. The max. draw-in. of the bottom strand was 0.0013 in.

5. Test (d) - 10 cycles of service load. $V_s = 12.17$ kips.

There was no additional cracking during these cycles of load. The max. crack width at service load on the tenth load cycle was 0.009 in. on the E side and 0.008 in. on the W side. Both of these cracks closed to 0.003 in. on removal of the load. The max. draw-in. of the bottom strand was 0.0016 in.

6. Test (e) - incremental loading to failure.

When the shear was increased above the previous maximum value to $V = 17.49$ kips, the cracks originating at the re-entrant corner extended to the web-flange Junction, at about 13 in. from the end of the flange. In addition, inclined cracks originated at the sloping end face, about 5 in. from the re-entrant corner on both faces and were joined together by a crack across the end face. On the E side this crack initially travelled at 45 degrees to the level of the re-entrant corner and then vertically 2.7 in. On the W side the corresponding crack initially travelled at about 60 degrees to a point 1.4 in. above the re-entrant corner. Under increasing load, both of these cracks extended upward to within 0.5 in. of the original inclined cracks. Also, at 17.49 kips the stress in the horizontal extension of the inclined bars started to increase much more rapidly.

At a shear of 21.28 kips, the cracks originating at the re-entrant corner had extended along the web-flange junction and were spreading onto the underside of the flange at about 15 in. from the end of the flange. In addition, the branch cracks extending toward the nib turned upward at 45 degrees, extending to points 3.5 in. above the re-entrant corner and 0.5 in. and 1.0 in. into the nib on the E and W faces respectively. At this same shear, a very fine flexural crack about 0.8 in. long, occurred at the bottom edge of both web faces. about 7 in. from the end of the bottom face.

At a shear of 22.23 kips a 2 in. long inclined crack occurred in the E face of the nib, centered about 0.5 in. beyond the end of the branch crack extending into the nib from the original inclined crack. At this same shear a fine crack was observed on the bottom face of the web, joining the flexural cracks which had occurred at the previous load stage.
As the shear was increased to 23.5 kips, additional cracking occurred on the eloping end face, more or less in the vicinity of the #4 inclined reinforcing bar. Also, the flexure crack on the E face extended upward to about 2.5 in. from the bottom edge.

At shears above 20 kipe the draw-in of the bottom strand increased more rapidly, but did not exceed 0.006 in. until a major diagonal tension crack occurred in both web faces at a shear of 24.76 kips, at which time the draw-in increased to 0.058 in.

At the same time as the major diagonal tension crack occurred, three additional flexure cracks extended upward from the bottom face at 10, 13 and 17 in. from the end of the bottom face. These cracks extended from 3.0 to 7.5 in. from the bottom face in both web faces. The diagonal tension crack originated at about midheight of the web and spread rapidly upward to the web-flange junction about 34 in. from the end of the flange, and downward to the bottom face at a point about 2 in. from the end of the bottom face.

As the diagonal tension crack occurred, the shear reduced to 22.5 kips. The shear was increased to 23.8 kips. at which time an additional flexure crack occurred 24 in. from the end of the bottom face, approximately at the end of the horizontal extension of the inclined bars. When the shear was increased to 23.94 kips a flexural bond failure occurred with extensive additional cracking, including an additional major diagonal tension crack running upward from the flexural crack at the end of the rebars.

The maximum shear reached was 24.76 kips, when the first major diagonal tension crack occurred. This shear is $1.06V_n(calc)$, however, neither the inclined reinforcement nor the nib flexural reinforcement had yielded at diagonal tension failure of the beam web. The max. average stress in the inclined reinforcement was $0.87f_y(ave.)$, and in the nib flexural reinforcement was $0.83f_y$. $V_n(calc)$ was taken equal to the vertical component of the yield strength of the inclined reinforcement plus a contribution from the concrete.
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

Fig. 1 - Variation with shear of stress in flexural reinforcement, Spec. 5'B

Fig. 2 - Variation with shear of stress in inclined bars, Spec. 5'B
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

Fig. 3 - End slip of bottom strands in final tests of 3D & 5B
Conclusions or recommendations in this report are the opinions of the authors. PCI assumes no responsibility for the interpretation or application of the information contained herein.

Specimen 5B after test, west face.

Specimen 5B after test, east face.
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