Creep Analysis of Prestressed Concrete Structures Using Creep-Transformed Section Properties

Walter H. Dilger
Professor of Civil Engineering
The University of Calgary
Calgary, Alberta
Canada

In the analysis of prestressed members the presence of more than one layer of prestressed or non-prestressed steel complicates the computation of prestress loss and deformation.[1,2,3] This is particularly true in the case of a combination of prestressed and non-prestressed reinforcement in one or more layers, or in the case of composite beams. In these cases the analysis is greatly simplified by using the so-called "creep-transformed" section properties in a quasi-elastic stress analysis.

The proposed approach makes use of well-known methods of stress analysis and is, in principle, similar to the elastic stress analysis of a member consisting of two materials in which one component (concrete) changes its temperature while the temperature of the other (reinforcement) remains constant. The easiest way to determine the temperature-induced stresses in the two component materials is to apply the forces corresponding to the free temperature strain of the one component, to the transformed section which takes account of the different material properties of the two components.

In this proposed time-dependent analysis of reinforced or prestressed concrete members, the strain due to free shrinkage and creep corresponds to the temperature strain, and because of the time-dependent nature of the problem at hand, the "creep-transformed" cross section properties include the effect of concrete creep.

General Description of Proposed Method

The method described in this paper deals only with *uncracked* reinforced or prestressed members.

Because of the gradual development of the strains due to creep and shrinkage, the time-dependent forces developed by creep and shrinkage in the steel and in the concrete also develop gradually. The response of the concrete to gradually changing stress is best calculated by Bazant's[4] age-adjusted effective modulus formula:

$$E_c^* = E_c(t_o)/[1 + \chi\phi(t,t_o)] \qquad (1)$$

where

$E_c(t_o)$ = modulus of elasticity of concrete loaded at age t_o

$\phi(t,t_o)$ = creep coefficient at time t for concrete loaded at age t_0

χ = aging coefficient

The concept of the aging coefficient was first introduced by Trost[5] and further developed by Bazant.[4] The aging coefficient expresses the aging effect on creep of concrete loaded gradually and it depends on the magnitude of the creep coefficient, the age of the concrete at first loading, and the time under load. Strictly speaking, the argument (t,t_o) should be added to χ but since this argument is always the same as that of the creep coefficient χ with which it is associated, it is omitted.

The aging coefficients presented in Figs. 1 and 2 were established according to the procedure reported by Bazant in Reference 4, but instead of the ACI creep function, the 1978 CEB-FIP[6] creep function was used. Additional graphs are given in Reference 7. The aging coefficients based on the ACI creep functions are tabulated in Reference 4.

To arrive at the time-dependent stresses and deformations in the member, the forces in the steel corresponding to unrestrained creep (i.e., not restrained by steel) to free shrinkage of the concrete (Fig. 3) and to "reduced" relaxation of the prestressing steel (if any) are applied to the creep-transformed section in which the steel is included with the modular ratio:

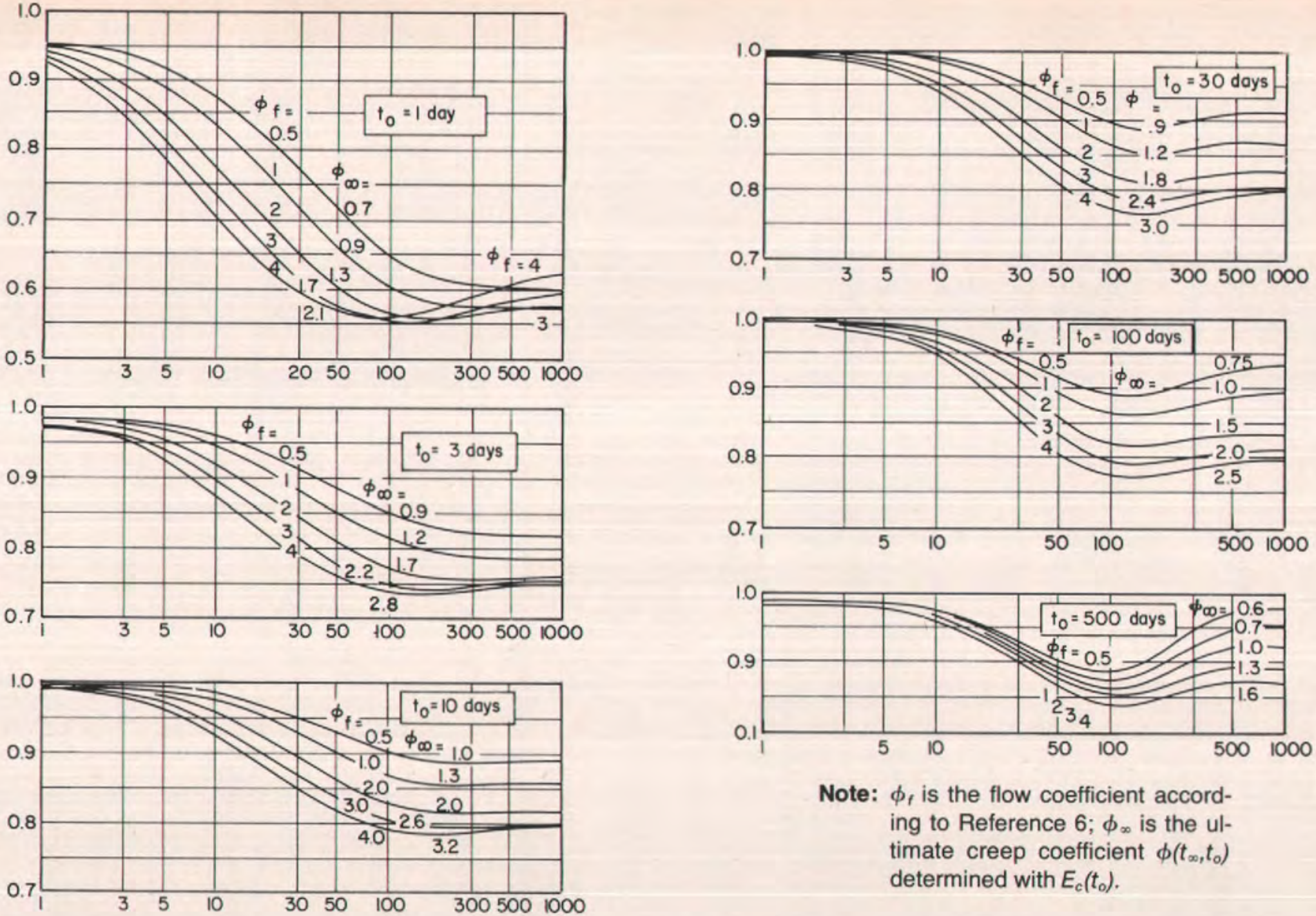
$$n^* = E_s/E_c^* = n_o[1 + \chi\phi(t,t_o)] \qquad (2)$$

where E_s is the modulus of elasticity of the steel and n_o is the elastic modular ratio.

The term "reduced" relaxation will be explained shortly. For reasons of internal equilibrium the forces change sign when applied to the creep-transformed section. The concrete stresses resulting from this analysis are due to all the time-dependent effects, and the corresponding steel stresses (obtained with the modular ratio n^*) are added to the stresses due to unrestrained creep, free shrinkage and (reduced) relaxation to obtain the time-dependent steel stress. The method is entirely general and rigorous if it is assumed that shrinkage develops at the same rate as creep. It can be applied to any cross

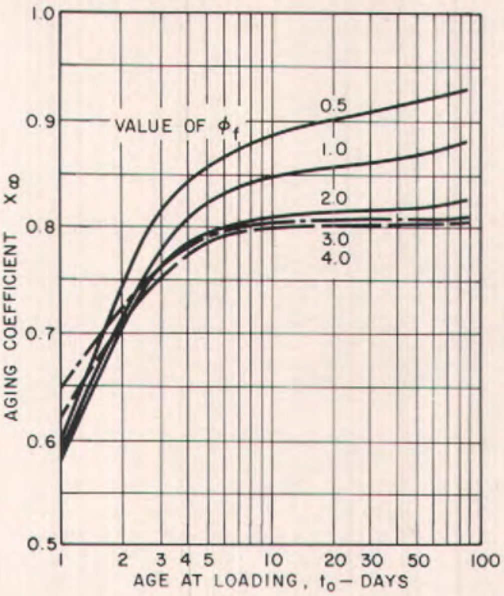
Synopsis

A simple yet accurate method of analyzing creep in uncracked reinforced and prestressed concrete members is presented which makes use of the aging coefficient to calculate so-called "creep-transformed" cross-sectional properties. With these properties time-dependent stresses, deformations and statically indeterminate forces are calculated in a quasi-elastic analysis. This method is particularly advantageous for members with multiple layers of prestressed and/or non-prestressed steel and for composite beams.



Note: ϕ_f is the flow coefficient according to Reference 6; ϕ_∞ is the ultimate creep coefficient $\phi(t_\infty,t_o)$ determined with $E_c(t_o)$.

Fig. 1. Aging coefficent versus time under load for different creep coefficients and different ages t_o at first application of load.



(a) For different values of the flow coefficient ϕ_f (according to Reference 6).

(b) For different values of the ultimate creep coefficient ϕ_∞ [based on $E(t_o)$].

Fig. 2. Ultimate values of aging coefficient χ as a function of the age at loading.

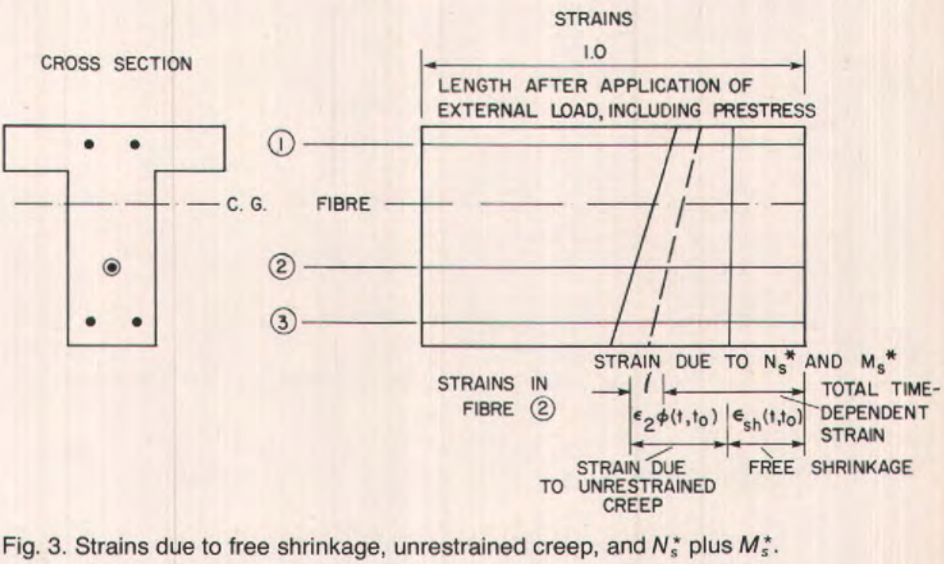


Fig. 3. Strains due to free shrinkage, unrestrained creep, and N_s^* plus M_s^*.

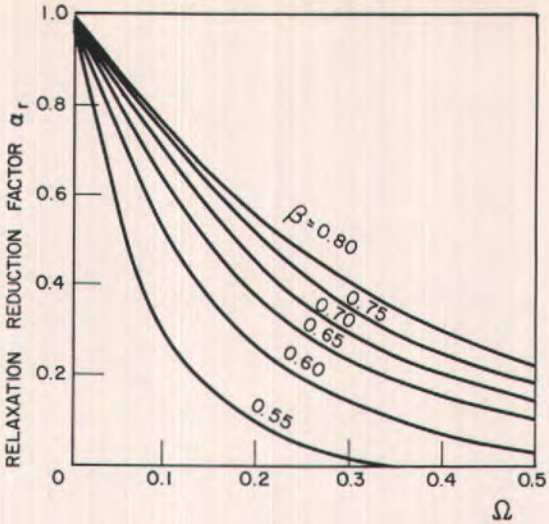


Fig. 4 Relaxation reduction factor α_r as a function of parameter Ω for different values of β (from Reference 8).

section (even a composite one) containing any number of layers of nonprestressed or prestressed steel.

Reduced Relaxation

Before proceeding with the detailed discussion of the new method the term "reduced relaxation" is explained. It is well known that creep and shrinkage reduce the intrinsic relaxation of prestressing steel. The inter-relationship between the loss in prestress due to creep and shrinkage of concrete and the relaxation of steel can be taken into account accurately by the procedure developed by Tadros et al.[2,8]

Based on a step-by-step numerical procedure and the relaxation-time function developed by Magura et al,[9] a chart has been developed (Fig. 4) which gives the relaxation reduction coefficient α_r as a function of the ratio:

$$\Omega = \frac{\text{Loss due to creep and shrinkage}}{\text{Initial prestress}} = \frac{\Delta f_{s(cp+sh)}}{f_{so}}$$

For different ratios:

$$\beta = \frac{\text{Initial prestress}}{\text{Ultimate strength}} = \frac{f_{so}}{f_{pu}}$$

The "reduced relaxtion" is:

$$f_r'(t) = \alpha_r f_r(t) \quad (3)$$

where $f_r(t)$ is the intrinsic relaxation developed from the time of prestressing until time t.

Since the losses due to creep and shrinkage alone must be evaluated before the coefficient α_r can be determined, the calculation of the total loss due to prestress will require two steps as indicated in the examples to follow.

Non-Composite Members

The new method is now explained in detail for the simple case of a prestressed concrete beam with one layer of prestressed steel.

The change in strain due to unrestrained creep (i.e., the restraining effect of steel on creep is not considered) and due to free shrinkage at the tendon level is:

$$\Delta\epsilon_c^*(t) = \epsilon_{c1}\,\phi(t,t_o) + \epsilon_{sh}(t,t_o) \quad (4)$$

where

$\epsilon_{c1} = f_{c1}/E_c(t_o)$ elastic concrete strain at the level of the tendon (fiber 1) due to load applied at age t_o, producing the stress f_{c1}

$\epsilon_{sh}(t,t_o)$ = free shrinkage since time of prestressing

The corresponding steel stress, including reduced relaxation, is:

$$f_s^*(t) = n_o f_{c1}\,\phi(t,t_o) + \epsilon_{sh}(t,t_o)\,E_s + f_r'(t) \quad (5)$$

and the corresponding normal force is found by multiplying this stress by the steel area A_s:

$$N_s^* = A_s f_s^*(t) \quad (6)$$

The subscript p which is normally used to indicate that we are dealing with prestressing steel is not used because these equations (without the relaxation term) are also applicable to non-prestressed steel.

The normal force N_s^* is normally acting eccentrically on the creep-transformed section and generates a bending moment:

$$M_s^* = N_s^* y_1^* \quad (7)$$

where y_1^* is the distance between the centroid of the steel and the centroid of the creep-transformed section.

Stresses

The concrete stress corresponding to the forces N_s^* and M_s^*:

$$\Delta f_c(t) = -\left(\frac{N_s^*}{A_c^*} + \frac{M_s^*}{I_c^*}\,y^*\right) \quad (8)$$

is the actual time-dependent stress in concrete. As mentioned before, for reasons of internal equilibrium the steel forces change signs when applied to the concrete section, hence the minus sign in this equation. The terms in Eq. (8) not previously defined are:

A_c^* = cross-sectional area and

I_c^* = moment of inertia.

Both properties are calculated for the concrete cross section in which the steel is transformed with:

$$n^* = n_o[1 + \chi\phi(t,t_o)]$$

The steel stress obtained from the relation:

$$\Delta f_s^*(t) = -\left(\frac{N_s^*}{A_c^*} + \frac{M_s^*}{I_c^*}\,y_1^*\right) n^* \quad (9)$$

is added to the stress f_s^* expressed by Eq. (5) in order to obtain the time-dependent change in stress. Thus:

$$\Delta f_s(t) = f_s^*(t) + \Delta f_s^*(t) \quad (10)$$

For more than one layer of steel, the steel stress $f_s^*(t)$ has to be found for each individual layer, and the normal force and bending moment due to the stresses in all layers have to be determined. For m layers:

$$N_s^* = \sum_{i=1}^{m} f_{s,i}^*(t) A_{s,i} \quad (11)$$

and

$$M_s^* = \sum_{i=1}^{m} f_{s,i}^*(t) y_i^* A_{s,i} \quad (12)$$

Deformation

The time-dependent deformations are calculated by multiplying the initial elastic deformations by the creep coefficient $\phi(t,t_o)$, adding the deformations due to free shrinkage, and then deducting the deformations due to the moments N^* and M^*, which are:

$$\Delta\epsilon^*(t) = -\frac{N_s^*}{A_c^* E_c^*} \quad (13)$$

and

$$\Delta\psi^*(t) = -\frac{M_s^*}{I_c^* E_c^*} \quad (14)$$

The time-dependent axial strain at

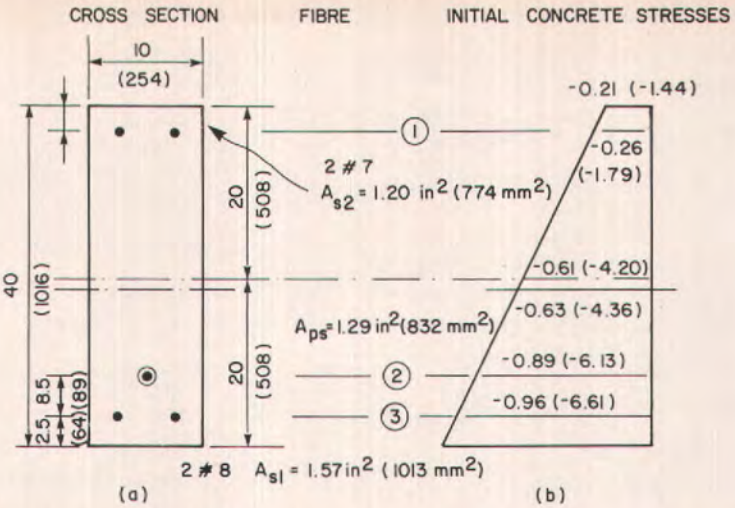


Fig. 5. Cross section and initial concrete stress distribution of member analyzed in Example 1. [Note: Dimensions are in in. (mm) and stresses in ksi (MPa).]

the level of the centroid of the creep-transformed section is thus given by the expression:

$$\epsilon(t) = \epsilon_{co}\phi(t,t_o) + \epsilon_{sh}(t,t_o) - \frac{N_s^*}{A_c^* E_c^*} \quad (15)$$

where ϵ_{co} is the initial strain at the level of the centroid of the creep-transformed section.

The time-dependent curvature is:

$$\psi(t) = \psi_o \, \phi(t,t_o) - \frac{M_c^*}{I_c^* E_c^*} \quad (16)$$

where ψ_o is the initial curvature of the section due to external load and prestressing, both applied at age t_o.

Knowing the time-dependent curvature the time-dependent deflection at a given point can be determined using the well-known relationship:

$$\Delta a(t) = \int_l \Delta\psi(t,x) M_{u1}(x) dx \quad (17)$$

where

$M_{u1}(x)$ = moment at point x due to unit load applied at the given point

$\Delta\psi(t,x)$ = time-dependent curvature at point x

l = length of the span

For the special case of a parabolic variation of the time-dependent curvature along the beam, with a maximum value $\Delta\psi(t)_{max}$ at midspan, the time-dependent deflection at midspan is:

$$\Delta a(t) = \frac{5}{48}\left[\Delta\psi(t)\right]_{max} l^2 \quad (18)$$

For a simply supported member with time-dependent curvature $\left[\Delta\psi(t)\right]_{x=0}$ at the supports and $\left[\Delta\psi(t)\right]_{max}$ at midspan, and assuming a parabolic variation between these two points, the time-dependent deflection at midspan is:

$$\Delta a(t) = \frac{l^2}{48}\left[5\Delta\psi(t)_{max} + \Delta\psi(t)_{x=0}\right] \quad (19)$$

EXAMPLE 1 — MEMBERS WITH MULTIPLE LAYERS OF STEEL

For members containing multiple layers of steel, the calculation of creep-transformed section properties and time-dependent stresses (prestress losses) is best performed in tabular form.

The loss of prestress and the deflection of the beam are calculated with the cross section of Fig. 5a subjected at age t_o = 3 days to the initial stresses de-

Table 1. Properties of creep-transformed section.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Area in.^{2}	Multiplier	$\frac{A^*}{(1)\times(2)}$ in.^{2}	y in.	A^*y (3) × (4) in.^{3}	$y^* = (y - \bar{y}^*)$ in.	$A^*(y^*)^2$ (3) × (6) in.^{4}	I_g in.^{4}
A_g = 400	1.0	400	0	0	−1.11	493	53,333
A_{s1} = 1.20	$(n^* - 1)$	26.6	−17.5	−465.5	−18.61	9,212	—
A_{ps} = 1.29	= (23.2 − 1)	28.6	14.0	400.4	12.89	4,752	—
A_{s2} = 1.57	= 22.2	34.8	17.5	609.0	16.39	9,348	—
Σ		A_c^* = 490		543.9		23,805	53,333

$$\bar{y}^* = \frac{543.9}{490} = 1.11 \text{ in.} \qquad I_c^* = 77{,}138 \text{ in.}^4$$

picted in Fig. 5b. The span is l = 50 ft (15.25 m). The sign convention adopted is: tension and elongation positive and compression and shortening negative. The data given are:

Free shrinkage: $\epsilon_{sh\infty} = -400 \times 10^{-6}$
Creep coefficient: $\phi(t_\infty,3) = 2.5$
Aging coefficient: $\chi(t_\infty,3) = 0.75$ (Fig. 2)
Intrinsic relaxation: $f_{r\infty} = -20$ ksi (138 MPa)
Initial prestress: f_{so} = 189 ksi (1302 MPa)
Tensile strength of prestressing steel: f_{pu} = 270 ksi (1860 MPa)

$E_c(3) = 3.6 \times 10^3$ ksi (24.8×10^3 MPa)
$E_s = E_{ps} = 29 \times 10^3$ ksi (200×10^3 MPa)
$n_o = E_s/E_c(3) = 9.0$

With this information we calculate:

$$n^* = 8.0(1 + 0.75 \times 2.5) = 23.2$$
$$E_c^* = 3.6 \times 10^3/(1 + 0.75 \times 2.5) = 1.25 \times 10^3 \text{ ksi } (8.63 \times 10^3 \text{ MPa})$$

The creep-transformed section properties are calculated in Table 1 and the time-dependent stresses in the three layers of steel are computed in Table 2. In order to include the reduced relaxation of the prestressing steel, the loss of prestress due to creep and shrinkage is calculated first. With a loss due to creep and shrinkage of −20.8 ksi (see value in bracket in Column 11 of Table 2), it is found that $\beta = 20.8/189 = 0.110$, $\Omega = 189/270 = 0.70$, so that Fig. 4 yields: $\alpha_r = 0.71$. With this, $f_r' = 0.71(-20) = -14.2$ ksi.

Time-Dependent Deformation

The time-dependent axial strain at the level of the centroid of the creep-transformed section is obtained from Eq. (15) with the values $\epsilon_{sh\infty} = -400 \times 10^{-6}$, $\epsilon_{co} = -0.63/(3.6 \times 10^3) = -175 \times 10^{-6}$ and the value of N_s^* from Table 2:

$$\Delta\epsilon(t_\infty) = (-175 \times 10^{-6})2.5 - 400 \times 10^{-6} - \frac{-124.8}{1.25 \times 10^3} = -738 \times 10^{-6}$$

Assuming the tendon profile to be parabolic, its eccentricity at the support equal to zero, and the applied load to be uniformly distributed, the time-dependent deflection is obtained from Eq. (18) with:

$$\psi_o = \frac{-0.96 - (-0.26)}{(40 - 2 \times 2.5)3.6 \times 10^3} = -5.56 \times 10^{-6} \text{ in.}^{-1} (-141 \times 10^{-6} \text{ mm}^{-1})$$

and

Table 2. Calculation of losses.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Steel Fiber i	Steel Area $A_{s,i}$ in.^{2}	$n_0 f_{c,i}\phi$ ksi	$\epsilon_{sh}E_s$ ksi	$f'_{r,i}$ ksi	$f^*_{s,i}$ = (3)+(4)+(5) ksi	Normal force N^*_{si} [Eq. (7)] (2)×(6) kip	y^*_i (see Table 1) in.	Moment M^*_{si} [Eq. (8)] (7)×(8) kip-in.	$\Delta f_{c,i}$ [Eq. (9)] ksi	$\Delta f_{s,i}$ [Eq. (11)] ksi
1	1.20	−5.2	−11.6	—	−16.8	−20.2	−18.61	375.9	−0.021	−17.3
2	1.29	−17.8	−11.6	—	(−29.4)	(−37.9)	12.89	(−488.5)	(0.369)	(−20.8)
				−14.2	−43.6	−56.2		−725.0	0.446	−33.3
3	1.57	−19.2	−11.6	—	−30.8	−48.4	16.39	−793.3	0.497	−19.3

NOTES: (a) Subscript i in caption denotes fiber i.

(b) Values in brackets are without relaxation of steel.

(c) Argument of all time-dependent terms are omitted for brevity.

$N^*_s = \Sigma N^*_{si}$ = (−106.5) kip −124.8 kip $M^*_s = \Sigma M^*_{si}$ = (− 905.9) kip-in. −1142.4 kip-in.

Conversion Factors: For in. to obtain m: 0.0254.
For ksi to obtain MPa: 6.89.
For kip to obtain kN: 4.448.
For kip-in. to obtain kNm: 0.1130.

$$\Delta\psi(t_\infty) = (-5.56 \times 10^{-6})2.5 - \frac{-1142.4}{77{,}138 \times 1.25 \times 10^3}$$

$$= -2.05 \times 10^{-6}$$
$$(-52.7 \times 10^{-6}\,\text{mm}^{-1})$$

to be

$$\Delta a(t_\infty) = \frac{5}{48}(-2.05 \times 10^{-6})(65 \times 12)^2$$
$$= -0.13 \text{ in. } (3.3 \text{ mm})$$

Actually, because of the different areas of A_{s1}, A_{s2}, the time-dependent curvature at the support is not exactly equal to zero, but this is neglected.

Composite Members

The use of the creep-transformed section is also useful in solving the complex problem of time-dependent stresses and deformations in a composite member. The approach is, in principle, the same as for non-composite members. However, the following additional points have to be considered: the force and moment corresponding to the difference in time-dependent free strains between the girder and the deck have to be added to N^*_s and M^*_s, repsectively, and the concrete deck has to be included in the creep-transformed section.

The difference in time-dependent strains between the precast girder and the cast-in-place deck is calculated at the level of the centroid of the concrete deck under the assumption that girder and deck are separated and that the steel does not influence the development of the concrete strains (i.e., we are dealing with "free" or "unrestrained" creep and "free" shrinkage). The free time-dependent strain in the girder is due to creep caused by grider weight and prestress, to creep caused by slab weight, and to shrinkage of the girder concrete. The free time-dependent strain in the deck is caused primarily by the shrinkage of the deck concrete and by the load applied to the composite section.

All strains are determined for the time after the beginning of the composite action. Referring to Fig. 6, the initial elastic strain ϵ_1 in any fiber of the precast section (subscript 1) due to the weight of the girder (moment $M^{(1)}$ and to prestressing (both applied at age t_o) will increase due to unrestrained creep and free shrinkage from the beginning of the composite action (time t_1) until time t by:

$$\Delta\epsilon_1(t) = \epsilon_1\left[\phi_1(t,t_o) - \phi_1(t_1,t_o)\right] + \epsilon_{sh1}(t,t_1) \quad (20)$$

At the level of the centroid of the cast-in-place deck (fiber 2), this increase is:

$$\Delta\epsilon_{1,2}(t) = \epsilon_{1,2}\left[\phi_1(t,t_o) - \phi_1(t_1,t_o)\right] + \epsilon_{sh1}(t,t_1) \quad (21)$$

Because of the loss of prestress occurring before the beginning of the composite action, the above expressions for the creep strain due to girder weight and prestressing need some discussion. If the initial elastic strain is separated into two parts, one due to girder weight $\epsilon_1^{(1)}$ and one due to prestressing, $\epsilon_1^{(P)}$, then the term $\epsilon_1^{(1)}\left[\phi_1(t,t_o) - \phi(t_1,t_o)\right]$ is the correct expression for creep due to girder weight, developing after the beginning of the composite action. But if $\epsilon_1^{(P)}$ is determined for the initial prestressing force the term $\epsilon_1^{(P)}\left[\phi_1(t,t_o) - \phi(t_1,t_o)\right]$ overestimates the creep strain caused by prestressing because it includes the effect of the loss of prestress occurring before time t_1. However, if it is assumed that the strain due to prestress is found by multiplying the elastic strain due to the prestress force at age t_1, $P(t_1) = \left[P_o + \Delta P(t_1)\right]$ (where P_o is the initial prestressing force and $\Delta P(t_1)$ the loss occurring between t_o and t_1) by $\left[\phi_1(t,t_o) - \phi_1(t,t_o)\right]$, a fairly good approximation is obtained for the time-dependent strain due to prestressing, developing after time t_1 because the loss of prestress is normally small and the time-dependent strain due to the predominant term P_o is expressed correctly by the multiplier $[\phi_1(t,t_o) - \phi_1(t_1,t_o)]$. A more rigorous expression can be formulated, but the extra work required for such analysis is not warranted in view of the fact that the predominant parameter of this analysis is the differential shrinkage between the two concretes.

While the girder concrete develops the strain expressed by Eq. (21), the deck shrinks by an amount $\epsilon_{sh2}(t,t_1)$ where the ages t and t_1 are counted from the moment at which the composite action begins, which normally is 1 to 3 days after the casting of the deck concrete.

In *unshored* construction where the weight of the slab is carried by the precast girder, the time-dependent strain in fiber 2 is increased by $\epsilon_{1,2}^{(2)}\phi_1(t,t_1)$ where $\epsilon_{1,2}^{(2)}$ is the elastic strain in fiber 2 due to the moment $M^{(2)}$ (caused by the weight of the slab) in the precast girder. Moments due to the superimposed loads applied after the commencement of the composite action are treated in the same way as the moment due to slab weight in shored construction (to be discussed later).

With the free strains, developing in the precast girder the steel stresses $f_s^*(t)$ are calculated for each fiber containing steel, and the corresponding normal forces and bending moments are determined in the way described for non-composite members. The relaxation of the steel is allowed for by adding the reduced relaxation $f_r'(t)$ to the stresses of the prestressed layer(s), if any.

In addition to the steel forces, the deck generates a normal force and a bending moment. The normal force corresponds to the difference between the free shrinkage of the deck, $\epsilon_{sh2}(t,t_1)$, and the strain due to unrestrained creep and free shrinkage in fiber 2 which develops after time t_1 due to the forces acting on the girder. This difference in strain is:

$$\Delta\epsilon_2^*(t,t_1) = \epsilon_{1,2}[\phi_1(t,t_o) - \phi_1(t,t_o)] + \epsilon_{1,2}^{(2)}\epsilon_1(t,t_1) + \epsilon_{sh1}(t,t_1) - \epsilon_{sh2}(t,t_1) \tag{22}$$

In *shored* construction where the weight of the slab is carried by the composite section, the time-dependent difference in strain due to $M^{(2)}$ at the level of the centroid of the deck is equal to $\epsilon_2^{(2)}[\phi_1(t,t_1) - \phi_2(t,t_1)]$. In this expression $\epsilon_2^{(2)}$ is the elastic strain in fiber 2 due to slab moment $M^{(2)}$, $\phi_1(t,t_1)$ is the creep coefficient of the girder concrete (after time t_1), and $\phi_2(t,t_1)$ is the creep coefficient of the deck concrete (also after time t_1). Thus, for shored construction, $\epsilon_{1,2}^{(2)}\phi_1(t,t_1)$ in Eq. (22) is replaced by $\epsilon_w^{(2)}[\phi_1(t,t_1) - \phi_2(t,t_1)]$.

The force in the deck (subscript 2) corresponding to the difference in strains expressed by Eq. (22) is:

$$N_{c2}^* = \Delta\epsilon_2^*(t,t_1)E_{c2}^*A_{c2} \tag{23}$$

where

$$E_{c2}^* = E_{c2}(t_1)/[1 + \chi_2\phi_2(t,t_1)] \tag{24}$$

and

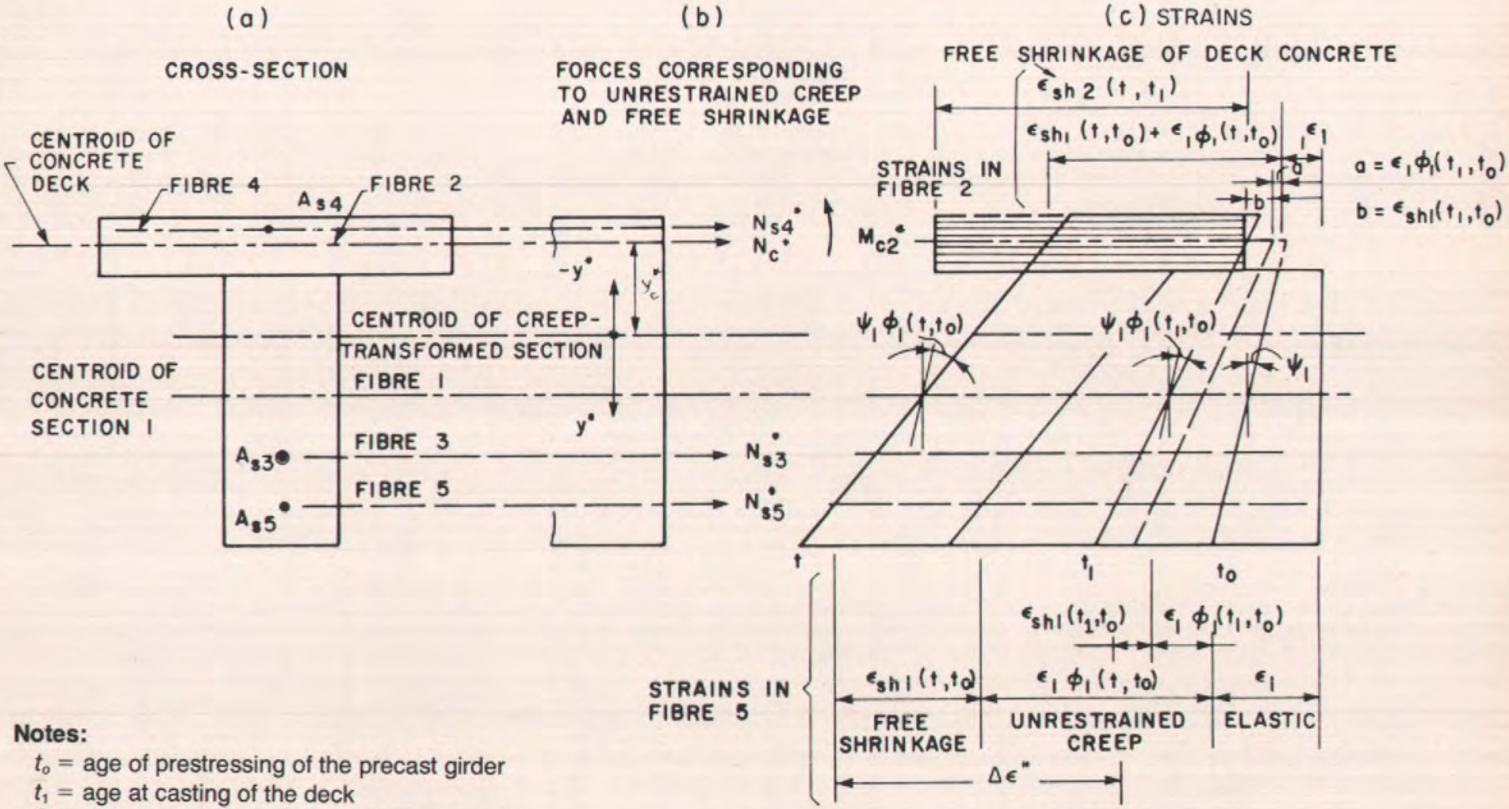
A_{c2} = cross-sectional area of the concrete deck

χ_2 = aging coefficient for the deck concrete

$\phi_2(t,t_1)$ = creep coefficient for the deck concrete at time t from the beginning of the composite action (time t_1).

The age-adjusted effective modulus E_{c2}^* is used because of the gradual development of the normal force N_{c2}^*.

The moment about the centroid of the creep-transformed section is $N_{c2}^*y_c^*$, y_c^* being defined in Fig. 6. In addition to this moment, a moment is generated in the concrete deck by the time-dependent curvature which develops in the precast girder after the beginning of the composite action. This moment is



Notes:
t_o = age of prestressing of the precast girder
t_1 = age at casting of the deck
Subscripts 1 and 2 in Fig. (c) refer respectively to precast girder and cast-in-place deck.

Fig. 6. Strains in composite girder due to initial forces on girder, unrestrained creep and free shrinkage.

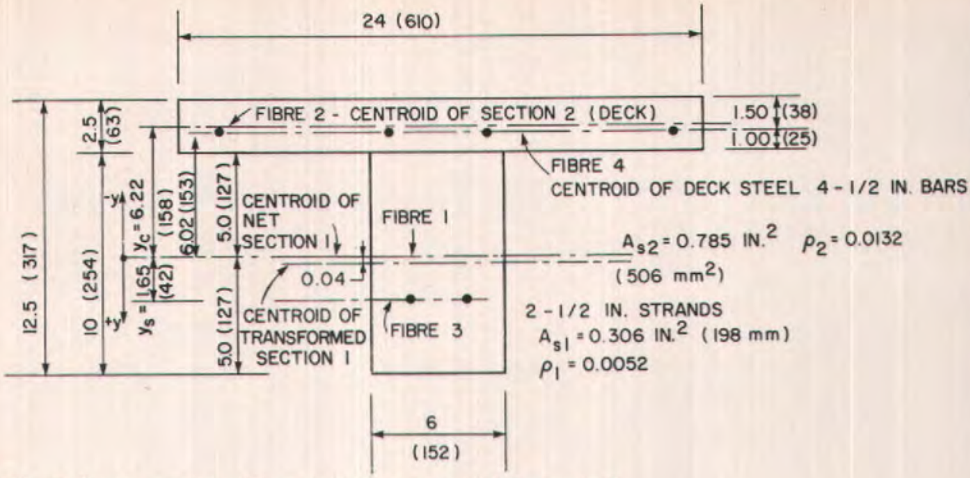


Fig. 7. Cross section of the composite girder of Example 2.

found by multiplying the change in time-dependent curvature in Section 1, by the time-dependent flexural rigidity of the deck:

$$M^*_{c2} = \Delta\psi_1(t,t_1)\, I_{c2} E^*_{c2} = \left\{ \frac{M^{(1)}}{I'_c E_{c1}(t_o)} \left[\phi_1(t,t_o) - \phi_1(t_1,t_o)\right] + \frac{M^{(2)}}{I'_c E_{c1}(t_1)}\, \phi_1(t,t_1) \right\} I_{c2} E^*_{c2} \qquad (25)$$

Note that in Eq. (25) I_{c2} is the moment of inertia of the concrete deck. The total moment is thus:

$$M^*_c = N^*_{c2}\, y^*_c + M^*_{c2} \qquad (26)$$

It should be noted, however, that the contribution of the moment M^*_{c2} to the total moment M^*_c is normally so small that it may be neglected in most practical cases (see Example 2).

The total forces acting on the creep-transformed concrete section are those developed in the steel and in the deck concrete. Adding the normal forces defined by Eqs. (11) and (23), we find:

$$N^* = N^*_s + N^*_{c2}. \qquad (27)$$

The total moment is obtained by adding Eqs. (12) and (26):

$$M^* = M^*_s + M^*_c \qquad (28)$$

The time-dependent stresses in the steel and in the concrete of the girder are calculated by Eqs. (8), (9), and (10) replacing N^*_s and M^*_s by N^* and M^* of Eq. (27) and (28), respectively. The concrete stresses at the centroid of the deck slab is:

$$\Delta f_{c2}(t) = \Delta\epsilon^*_2(t,t_1)\, E^*_{c2} - \left(\frac{N^*}{A^*_c} + \frac{M^*}{I^*_c}\, y^*_c \right) \frac{E^*_{c2}}{E^*_{c1}} \qquad (29)$$

The creep-transformed section properties of the composite section are determined by multiplying the steel areas by $(n^*_1 - 1)$, where:

$$n^*_1 = E_s / E^*_{c1} \qquad (30)$$

and

$$E^*_{c1} = E_{c1}(t_1) / [1 + \chi_1 \phi_1(t,t_1)] \qquad (31)$$

and by multiplying the area of the deck concrete by the ratio:

$$\lambda^* = E^*_{c2} / E^*_{c1} \qquad (32)$$

In order to check the results, all the changes in normal force are summed, the requirement being $\Sigma\, \Delta N = 0$:

$$\Sigma\, \Delta f_{si} A_{si} + \Delta f_{c1}(t)\, A_{c1} + \Delta f_{c2}(t)\, A_{c2} = 0$$

The time-dependent deformations

Table 3. Material properties for Example 2.

Free shrinkage:	Concrete 1:	$\epsilon_{sh1}(48,7) = -365 \times 10^{-6}$
		$\epsilon_{sh1}(150,48) = -200 \times 10^{-6}$
	Concrete 2:	$\epsilon_{sh2}(7,119) = -560 \times 10^{-6}$
Creep coefficients:	Concrete 1:	$\phi_1(48,7) = 1.05$
		$\phi_1(150,7) = 1.45$
		$\phi_1(150,48) = 1.08$
	Concrete 2:	$\phi_2(119,7) = 1.54$
Modulus of elasticity:	Concrete 1:	$E_{c1}(7) = 4{,}090$ ksi (28.2×10^3 MPa)
		$E_{c1}(48) = 4{,}760$ ksi (32.8×10^3 MPa)
	Concrete 2:	$E_{c2}(7) = 3{,}020$ ksi (20.8×10^3 MPa)
	Steel:	$E_{ps} = 27{,}400$ ksi (189×10^3 MPa)
		$E_s = 29{,}000$ ksi (200×10^3 MPa)
Aging coefficient:	Concrete 1:	$\chi_1(150,48) = 0.82$
		$\chi_2(119,7) = 0.82$
Age adjusted effective modulus:		
	Concrete 1:	$E^*_{c1} = E_{c1}(48)/[1 + \chi_1\phi_1(150/48) = 2{,}530$ ksi (17.4×10^3 MPa)
	Concrete 2:	$E^*_{c2} = E_{c2}(7)/[1 + \chi_2\phi_2(119/7) = 1{,}330$ ksi (9.16×10^{-3} MPa)
Relaxation of steel:		$f_r(150,48) = -7.8$ ksi (-53.7 MPa)
Stress in prestressing steel at beginning of composite action:		
		$f_{ps} = 185$ ksi (1275 MPa)
Tensile strength of prestressing steel:		$f_{pu} = 270$ ksi (1860 MPa)

(strains, curvatures and deflections) are obtained by subtracting from those due to unrestrained creep and free shrinkage the values due to the forces N^* and M^*, calculated with the age-adjusted effective modulus of the girder concrete, E^*_{c1}, defined above. The time-dependent change in axial strain in Girder 1 is:

$$\Delta\epsilon_1(t) = \epsilon_1\left[\phi_1(t,t_o) - \phi_1(t_1,t_o)\right] + \epsilon_1^{(2)}\phi_1(t,t_1) + \epsilon_{sh1}(t,t_1) - \left(\frac{N^*}{A^*_c E^*_{c1}} + \frac{M^*}{I^*_c E^*_{c1}}\,y^*_1\right) \quad (33)$$

where ϵ_1 and $\epsilon_1^{(2)}$ are the strains at the centroid of Girder 1 due to the prestress force and moment $M^{(2)}$, respectively.

The change in curvature is expressed by:

$$\Delta\psi(t) = \psi_1^{(1)}\left[\phi_1(t,t_o) - \phi_1(t_1,t_o)\right] + \psi^{(2)}\phi_1(t,t_1) - \frac{M^*}{I^*_c E^*_{c1}} \quad (34)$$

where $\psi_1^{(1)}$ is the initial curvature of the precast girder due to girder weight and prestressing and $\psi^{(2)}$ is the curvature due to moment $M^{(2)}$ which is either applied to the girder section (unshored construction) or to the composite section (shored construction).

EXAMPLE 2 — COMPOSITE BEAMS

An example will now be worked out in order to further explain the method presented. Two of the composite beams investigated by Rao and Dilger[10] are analyzed. The girders were prestressed by a force of 65.7 kips (292 kN) at age t_o = 7 days, and a reinforced deck was cast at age 41 days while the girder was shored. The formwork was removed 7 days later (age t_1 = 48 days) and additional load was applied to one of the girders (Beam B) at age t_2 = 53 days. The dimensions of the composite member, spanning 12 ft (3.66 m) are given in Fig. 7.

The deck was kept moist during the 7 days before the removal of the formwork so that shrinkage of the deck can be considered to have started at the age of 7 days. A prestress loss of 9.4 kips (42 kN) occurred till the age of 48 days. Axial strains, curvatures and deflections were observed till the age of t_3 = 150 days. The girder and the slab generated a moment $M^{(1)} = M^{(2)} = 13.5$ kip-in. (15.3 kN•m) and a moment $M^{(3)} = 280$ kip-in. (31.5 kN•m) was produced by two loads applied at third points at age 53 days. The time-dependent data of the two beams are presented in Table 3.

The complete analysis of Beam B involves five steps: (1) elastic analysis at the age of 7 days, (2) period 7 to 48 days during which the girder alone is subjected to the action of its self-weight and of the prestressing force; (3) period 48 to 53 days after beginning of the composite action; (4) elastic analysis of the superimposed load applied at age 53 days; (5) time-dependent effects due to composite action and due to moment $M^{(3)}$.

For Beam A without the superimposed load, Step (3) extends until 150 days and Steps (4) and (5) are not needed. In the following numerical example, Step (3) is presented for Beam A, but a comparison of computed and measured deflections is given for both beams.

The properties of the transformed and of the creep-transformed section of the composite beam are calculated in Tables 4 and 5, and the calculation of the time-dependent stresses are performed in Table 6. The properties of the transformed section are needed to calculate the stresses due to moment $M^{(2)} = 13.5$ kip-in. (1.53 kN•m) caused by the slab weight.

Note that in this particular case these stresses are very small and could be neglected, but in a real structure, the weight of the deck causes much higher stresses. As mentioned before, if the precast girder is not shored during casting of the deck, the slab has to be carried by the precast girder alone.

From Table 6 it is apparent that the differential shrinkage between the girder and the deck:

$$\Delta\epsilon_{sh}(t,t_1) = \epsilon_{sh1}(t,t_1) - \epsilon_{sh2}(t,t_1) = 360 \times 10^{-6}$$

is the main source of the time-dependent stresses in this member. If only the shrinkage induced stresses were of interest, they could be obtained simply by applying the force:

$$-N^* = -\Delta\epsilon_{sh}(t,t_1) A_{c2} E^*_{c2}$$

eccentric by y^*_c, to the creep-transformed section. The resulting stresses in the girder would be the shrinkage induced stresses. The stresses in the slab would be obtained from Eq. (29) with:

$$\Delta\epsilon^*_2(t,t_1) = \Delta\epsilon_{sh}(t,t_1)$$

The time-dependent curvature at midspan is calculated using Eq. (34) with:

$$\psi^{(1)}_1 = \frac{13.5 - 56.3\,(1.61)}{4090 \times 503.9} = -37.4 \times 10^{-6}\text{ in.}^{-1}\ (0.950 \times 10^{-3}\text{ mm}^{-1})$$

and

$$\psi^{(2)} = \frac{13.5}{4760 \times 1504} = 1.89 \times 10^{-6}\text{ in.}^{-1}\ (0.048 \times 10^{-3}\text{ mm}^{-1})$$

to yield:

$$\Delta\psi(150) = \left\{-37.4(1.45 - 1.09) + 1.89(1.08) - \frac{-113.8}{2530 \times 1491}\right\} \times 10^{-6} = 18.75 \times 10^{-6}\text{ in.}^{-1}\ (0.477 \times 10^{-3}\text{ mm}^{-1})$$

Repeating the procedure for the section over the support we find:

$$\Delta\psi(150) = 19.04 \times 10^{-6}\text{ in.}^{-1}\ (0.484 \times 10^{-3}\text{ mm}^{-1})$$

Assuming a parabolic variation of curvature along the beam, the time-dependent deflection can be calculated from Eq. (19):

Table 4. Section properties of composite beam.

1	2	3	4	5	6	7	8	9
Section	Area A_i	Multiplier	Transformed Area A'_i	y_i	$A'_i y_i$	y_i-$\bar{y}$	$A'_i(y_i-\bar{y})^2$	I
	in.^{2}		in.^{2}	in.	in.^{3}	in.	in.^{4}	
Precast Girder	60.0	1.0	60.0	0	0	2.50	375.0	500.0
Slab	60.0	0.634	38.04	−6.25	−237.8	−3.75	534.9	19.8
A_{ps}	0.306	(5.8-1)	1.47	1.65	2.4	4.15	25.3	—
A_s	0.785	(6.1-1)	4.00	−6.00	−24.0	−3.50	49.0	—
			A'_c = 103.5		−259.3		984.2	519.8

$E_{c2}(7)/E_{c1}(48) = 0.634$
$E_{ps}/E_{c1}(48) = 5.8$
$E_s/E_c(48) = 6.1$

$$\bar{y} = \frac{-259.3}{103.5} = -2.50 \text{ in.}$$

$$I'_c = 984.2 + 519.8 = 1{,}504 \text{ in.}^4$$

Table 5. creep-transformed section properties of composite beam for period t_1 = 48 days to t_3 = 150 days.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Fiber	Section	Area A_i	Multiplier	Transformed Area A^*_i	y_i	$A^*_i y_i$	$y^*_i = y_i - \bar{y}^*_i$	$A^*_i(y^*_i)^2$	I_i
		in.^{2}		in.^{2}	in.	in.^{3}	in.	in.^{4}	in.^{4}
1	Precast girder	60.0	1.0	60.00	—	—	2.36	334.2	500.0
2	Slab	60.0	0.532$^{(1)}$	31.92	−6.25	−199.5	−3.89	483.0	16.6
3	A_{ps}	0.306	10.8-1$^{(2)}$	3.00	1.65	5.0	4.01	48.2	
4	A_s	0.785	11.5-1$^{(3)}$	8.24	−6.00	−49.4	−3.64	109.2	
Σ				103.2		−243.9		974.6	516.6

(1) $E^*_{c2}/E^*_{c1} = 0.532$
(2) $E_{ps}/E^*_{c1} = 10.8$
(3) $E_s/E^*_{c1} = 11.5$

$$\bar{y}^* = \frac{-243.9}{103.2} = -2.36 \text{ in.}$$

$$I^* = 974.6 + 516.6 = 1491 \text{ in.}^4$$

Table 6. Computation of time-dependent stresses.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Fiber	Section	Concrete Stress†		Strains			E_i	$f_i^* =$ $[(5)+(6)+(7)] \times (8)$	f'_{r1}	$N_i^* =$ $A_i[(9)+(10)]$	$M_i^* =$ $N_i^* y_i^*$	Δf_{c1}^* Eq. (9)	$\frac{E_i}{E_{c1}^*}$	Δf (9)+(10)+(13)(14)
		$f_{c1}^{(1)}$	$f_{c1}^{(2)}$	$\frac{f_{c1}^{(1)}}{E_{c1}(7)}\phi_1^{(1)\S}$	$\frac{f_{c1}^{(2)}}{E_{c1}(48)}\phi_1^{(2)\S\S}$	ϵ_{sh}								
		ksi	ksi	$\times 10^{-6}$	$\times 10^{-6}$	$\times 10^{-6}$	ksi	ksi	ksi	kip	kip-in.	ksi	—	ksi
1	Precast girder	—	—	—	—	—	2,530	—	—	—	—	−0.010	1.0	−0.010
2	Slab	0.047	−0.034	4.1	3.3	360††	1,330	0.489	—	28.93××	−112.5	−0.487	0.532	0.230
3	A_{ps}	−1.161	0.037	−102.2	8.4	−200	27,400	−8.050	−6.94×	(−2.46) −4.59	(−9.88) −18.39	(0.072) 0.115	10.8	(−7.27) −13.74
4	A_s	0.009	−0.031	0.8	−7.1	−200	29,000	−5.983	—	−4.70	17.10	−0.468	11.5	−11.37
Σ										(21.77) 19.64	(−105.3) −113.8			

†Superscript (1) refers to stresses due to girder weight plus prestressing, and Subscript (2) to stresses due to slab weight calculated with the properties of Table 4.

†† $\epsilon_{sh1} - \epsilon_{sh2} = (-200 + 560) \times 10^{-6} = 360 \times 10^{-6}$

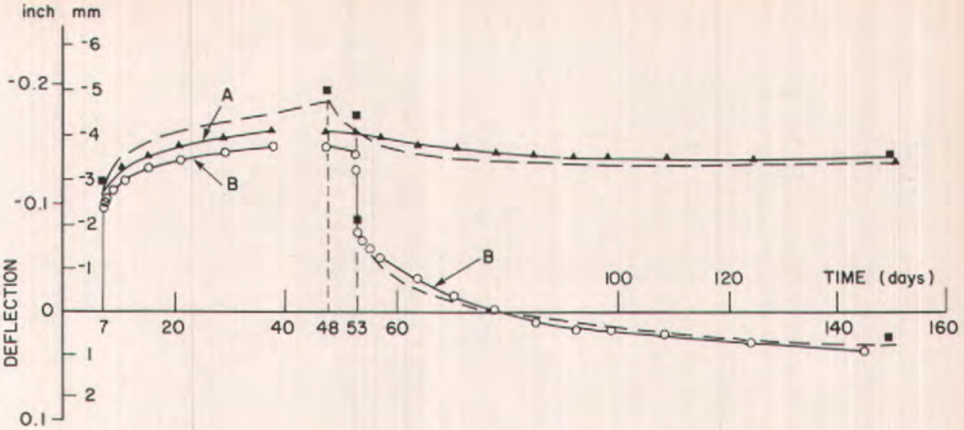
§ $\phi_1^{(1)} = \phi_1(150,7) - \phi_1(48,7) = 1.45 - 1.09 = 0.36$

§§ $\phi_1^{(2)} = \phi_1(150,48) = 1.08$, for slab $\phi_1^{(2)} - \phi_2^{(2)} = 1.08 - 1.54 = -0.46$

Notes: (a) The values in brackets in Columns (11), (12) and 15) are without relaxation of the prestressing steel.
(b) The areas A_i needed for calculation of N_i^* (Column 11) and y_i^* needed for calculation M_i^* (Column 12) are listed in Table 4.
(c) The numerical values of Column (14) are given in Footnotes (1) to (3) of Table 5.

×Reduced relaxation: $\beta = 7.27/185 = 0.039$, $\Omega = 185/270 = 069$ } $\alpha_r = 0.89$
$f'_r = 0.89 \times (-7.8) = -6.94$ ksi

××Net area of Slab $A_{c2} = 600.00 - 0.785 = 59.21$ in.²



(A) Beam A without superimposed load
(B) Beam B with superimposed load

— computer analysis (Reference 10)
■ analysis using creep-transformed section

Fig. 8. Comparison of theory and experiment

$$\Delta a(150) = \frac{144^2}{48} [5 \times 18.75 +$$

$$= 0.0487 \text{ in. } (1.24 \text{ mm})$$

A comparison between the calculated and measured deflections for Beams A and B is shown in Fig. 8. In addition, the results obtained by a step-by-step computer analysis are given for comparison.[10]

Continuous Composite Beams

Composite girders are frequently made continuous by cast-in-place joints and deck. The time-dependent moment developing at the cast-in-place joint can be determined by expanding the well-known compatibility conditions to include the time-dependent curvature. For a two-span continuous girder the following equation (see Fig. 9) must be satisfied:

$$D_1(t) + f_{11}(t)\, \Delta M_1(t) = 0 \qquad (35)$$

where

$$D_1(t) = \int_l \Delta\psi(t)\, M_{u1} dl \qquad (36)$$

which is equivalent to the time-dependent displacement developing at Coordinate 1 after the girder is made continuous.

From the energy relation:

$$f^*_{11}(t) = \int_l \frac{M^2_{u1}}{E^*_{c1} I^*_c}\, dl \qquad (37)$$

in which

M_{ul} = unit moment applied at Coordinate 1

$\Delta M_1(t)$ = unknown time-dependent moment developing at Coordinate 1

and E^*_{c1} and I^*_c are, respectively, the age-adjusted effective modulus of the girder concrete, and the moment of inertia for the creep-transformed section.

The time-dependent curvature $\Delta\psi(t)$ is defined by Eq. (34). The time-dependent flexibility coefficient $f^*_{11}(t)$ is the displacement due to the unit moment M_{ul} applied gradually to the composite beam.

If the beam analyzed in Example 2

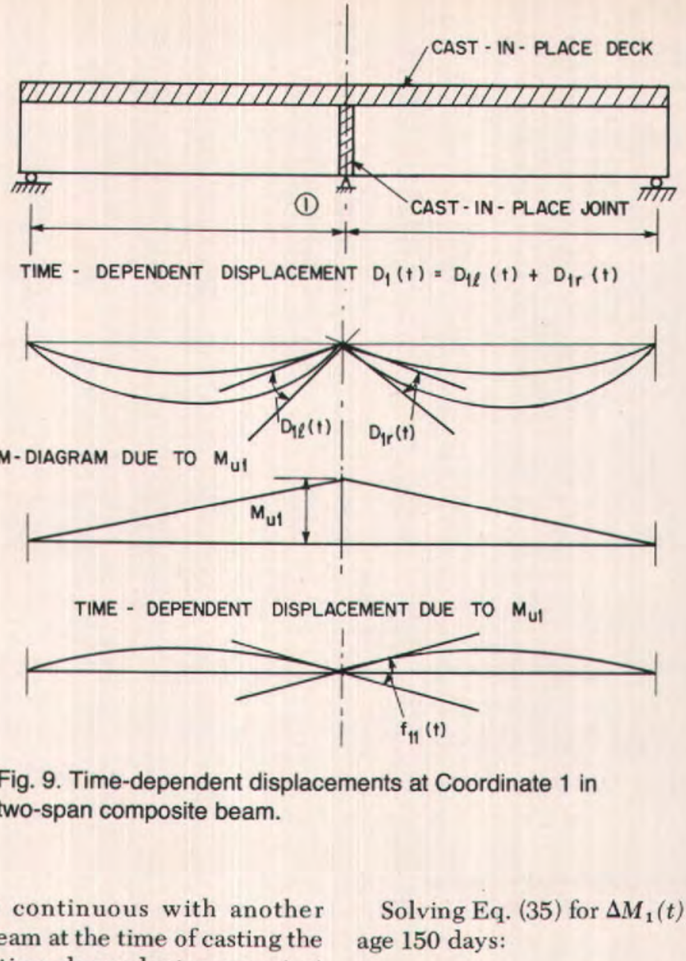


Fig. 9. Time-dependent displacements at Coordinate 1 in two-span composite beam.

was made continuous with another identical beam at the time of casting the deck, the time-dependent moment at the joint would be readily calculated. According to Eqs. (36) and (37):

$$D_1(t) = \int_l \Delta\psi(t)\, M_{ul}\, dl$$
$$= 2\left[\frac{1}{2} \times 19.04 - \frac{1}{3}(19.04 - 18.75)\right] \times 10^{-6} \times 144$$
$$= 2714 \times 10^{-6} \text{ rad.}$$

and

$$f_{11}^*(t) = \int_l \frac{M_{u1}^2}{E_{c1}^* I_c^*}\, dl$$
$$= \frac{2 \times 144}{3 \times 2530 \times 1491}$$
$$= 25.4 \times 10^{-6}\ (\text{kip-in.})^{-1}$$

Solving Eq. (35) for $\Delta M_1(t)$ we find at age 150 days:

$$\Delta M_1(150) = -\frac{2714 \times 10^{-6}}{25.4 \times 10^{-6}}$$
$$= -106.8 \text{ kip-in.} \ (-12.07 \text{ kN}\cdot\text{m})$$

The value calculated means that a negative moment is introduced at the joint. This is so because a positive time-dependent curvature is introduced in each simply supported beam by the predominant effect of shrinkage of the deck. Forces such as concentrated loads applied to the girder, or a prestressing force introduced after continuity has been provided will not induce time-dependent moments or reaction except those due to prestress losses.

CONCLUDING REMARKS

A simple method is presented for computing time-dependent effects in uncracked concrete members containing any number of layers of prestressed and/or non-prestressed steel. The method is also easily applied to composite beams and allows calculation of the time-dependent moments in composite beams made continuous by cast-in-place concrete.

The method can easily be expanded to the analysis of members where different layers of concrete exhibit different time-dependent properties as in the case of prestressed concrete pressure vessels and prestressed containment tanks where temperature differences lead to different creep and shrinkage behavior throughout the thickness of the concrete member.

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NOTE: A notation section appears on the following page.

APPENDIX — NOTATION

A = cross-sectional area
D_1 = displacement at Coordinate 1 in a statically determinate structure
E = modulus of elasticity
E_c^* = age-adjusted effective modulus
I = moment of inertia
M = moment
M_{ul} = unit moment applied at Coordinate 1
N = normal force
a = deflection
f = stress
F_{11} = displacement at Coordinate 1 due to unit moment M_{ul}
f_r' = reduced relaxation [see Eq. (3)]
f_{pu} = tensile strength of prestressing steel
l = span
n = modular ratio
t = time (in days) since casting of the concrete
y = distance from centroid
α_r = relaxation reduction coefficient
β = ratio of initial prestress to tensile strength of prestressing steel
ϵ = strain
$\phi(t,t_o)$ = creep coefficient at time t for concrete loaded at age t_o
χ = aging coefficent
Ω = ratio of loss due to creep and shrinkage to initial prestress
ψ = curvature

Subscripts

c = concrete
i = layer i containing steel
p = prestressing steel
r = relaxation of prestressing steel
s = steel
sh = shrinkage
o = at first application of load
1 = Section 1, Fiber 1 or Time 1
2 = Section 2, Fiber 2 or Time 2
∞ = at time infinity

Superscripts

$'$ = related to transformed section
$*$ = related to creep-transformed section method
(1) = Construction Stage 1
(2) = Construction Stage 2

Prefix

Δ = change in stress, strain, force, moment

Sign Convention

Elongation, tension: positive
Shortening, compression: negative

* * *

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