Ductile Connectors for a Precast Concrete Frame

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A precast concrete ductile frame system is presented that takes advantage of the inherent discrete nature of precast concrete by providing ductile links in the connections. These ductile connectors contain a rod that yields at a well-defined strength, effectively limiting the load that can be transferred to less ductile components of the frame. The ductile connectors also transfer all vertical shear forces, eliminating the need for corbels. High strength [150 ksi (1034 MPa)] reinforcing steel is used in the beams to reduce congestion. A building designed for Uniform Building Code Seismic Zone 2A is presented; however, the system is versatile and can be used in all seismic zones.

The use of structural precast concrete in seismic regions of the United States has decreased over the years due to prescriptive code requirements that effectively prohibit creative structural solutions using non-traditional construction techniques. This has not affected the majority of the precast concrete producers across the country, but this too is changing. Seismic hazard awareness has become a popular topic in the central and eastern parts of the United States as the model codes and local jurisdictions begin to incorporate and implement seismic design regulations with the objective of improving the seismic safety of new building stock.

The past several decades have taught structural engineers that seismic structural systems perform best when a ductile link is introduced somewhere in the system. These ductile links limit the inertial loads that can be generated within the system, thereby limiting forces to other, less ductile, struc-
Monolithic Emulation

In a monolithic emulation system, the connections between precast concrete elements are designed to be stronger than the ductile link (beam hinge). Yielding is then forced to occur within the concrete element itself. Many different strong beam-to-column connections have been proposed for use in seismic regions. Ochs and Ehsani (1993) proposed two welded connections that relocate the plastic hinges (ductile links) away from the column face. French et al. (1989) developed connectors using post-tensioned bars to connect the beam to the column. The post-tensioned bars were designed to relocate yielding away from the interface.

A "drop in" beam system, built by Rockwin Corporation in the 1980s (Englekirk, 1990); used monolithic concrete technology by building precast concrete frames with member splices away from anticipated regions of inelastic action. While these types of details have been shown to behave similarly to monolithic concrete frames, they are often difficult and expensive to implement in the field.

Many of the proposed details require a mixing of trades (i.e., welding, grouting, post-tensioning or cast-in-place concrete). This slows the progress of the project, and as a consequence, eliminates one of the major benefits of precast concrete: its ability to be erected quickly. The "drop-in" beam system requires awkward precast concrete members (cruciforms, "H" shapes or trees) that increase transportation costs.

In addition to the cost normally associated with "strong" connectors, the overstrength required in the connector becomes quite large as the hinge location is moved away from the column face. Fig. 1 shows the moment diagram for a frame beam subject to lateral loads only. If the hinge is located a distance $e$ from the beam-column interface, the maximum shear that can be developed in the beam is:

$$ M_{	ext{interface}} $$

$$ M_{	ext{hinge}} $$

Fig. 1. Moment diagram.

CURRENT PRACTICE

Unlike other model codes, the Uniform Building Code (UBC) severely limits the opportunities for a design engineer to develop a seismic system that may perform quite well while not meeting the prescriptive requirements of one of the predefined systems. For example, the UBC (ICBO, 1994) requires that:

"Structural systems shall be classified as one of the types listed in Table 16-N,"

Table 16-N, unfortunately, does not list a single precast concrete system. This has forced the precast concrete industry in regions of the United States governed by the UBC to approach the construction of buildings in seismic zones using either mixed construction or monolithic emulation.

Mixed Construction

A mixed construction solution uses precast concrete gravity load carrying members in combination with a cast-in-place or masonry seismic bracing system. While this solution is more than adequate from a behavior perspective, the inefficiencies caused by mixing construction trades often result in a total cost exceeding a single trade solution (i.e., a totally cast-in-place concrete system). This effectively eliminates opportunities for precast concrete buildings.
Fig. 2. Hinge rotations: (a) hinge at interface and (b) relocated hinge.

\[ V_p = \frac{\lambda_o \cdot M_{hinge}}{\phi \cdot \left( \frac{L_{clear} - e}{2} \right)} \]  

where:
- \( M_{hinge} \) = nominal strength of beam hinge
- \( L_{clear} \) = clear span of beam
- \( e \) = distance of hinge from beam-column interface
- \( \lambda_o \) = overstrength factor, related to the maximum possible hinge strength that can be developed; normally, \( \lambda_o = 1.25 \)
- \( \phi \) = flexural reduction factor of section; for a flexural hinge, \( \phi = 0.9 \)

The required nominal moment strength at the interface is then:

\[ \phi M_{interface} \geq V_p \cdot \frac{L_{clear}}{2} \]  

\[ \phi M_{interface} \geq \lambda_o \cdot e \cdot M_{hinge} \cdot \frac{L_{clear}}{2} \cdot \frac{1}{L_{clear} - e} \]  

For a typical 28 ft (8.53 m) bay, with 3 ft (0.91 m) wide columns and a hinge relocated 3 ft (0.91 m) from the column face, the required interface strength is:

\[ M_{interface} \geq \frac{1.25}{0.9 \cdot 0.9} \cdot M_{hinge} \cdot \frac{12.5}{12.5 - 3} \]

\[ \geq 2.03 \cdot M_{hinge} \]

Thus, a connection relocated 3 ft (0.91 m) from the column face must be at least twice as strong as the hinge. This significantly increases the cost of the connection.

A third problem with the hinge relocation approach is that relocating the hinge away from the column face increases the rotational ductility demand to the hinge for a given story drift. Good seismic performance requires that a system be able to sustain a large lateral deformation without significant loss of strength.

Currently, the UBC requires a story drift demand of 1.1 percent for buildings with a fundamental elastic period greater than 0.7 seconds, and 1.5 percent for shorter period buildings. Actual deformations experienced in recent earthquakes may have been even higher.

While some of this deformation is elastic, the majority of the story drift is provided by inelastic yielding of the beam hinge. Fig. 2a shows the beam hinge rotation required for a hinge that forms at the face of the column in the system described above. The plastic hinge rotation, \( \theta_p \), is equal to the post-elastic story drift, \( \theta_c \). Fig. 2b shows the rotation required for a hinge that is relocated a distance \( e \) from the column face. This hinge is required to sustain significantly more plastic rotation for the same post-elastic story drift, \( \theta_c \). For the relocated hinge, the required rotation is:

\[ \theta_b = \theta_c \left(1 + \frac{e}{L_{clear} - e} \right) \]  

For the example discussed above, the required plastic hinge rotation is:

\[ \theta_b = \theta_c \left(1 + \frac{3}{12.5 - 3}\right) = 1.32 \theta_c \]

which is 32 percent greater than that required for a hinge located at the column face.

Martin and Korkosz (1982) describe several other connections that can be used for beam-to-column moment connections; however, these connections are generally limited to non-seismic applications because they do not generate enough overstrength in the connection to ensure yielding in the beam itself. These types of connections should not be used in seismic applications unless their ductility can be assured.

As discussed by Yee (1991), mechanical splices, such as NMB Splice Sleeves, ERICO Interlock, and similar splices are a very effective devices for providing monolithic emulation in precast concrete frames in seismic zones. These connectors emulate the behavior of a cast-in-place frame and allow connections of sufficient strength to be practical in building applications. This type of connector appears to be the only practical application on the market today. Unfortunately, these connectors cannot be used to connect precast concrete beams to precast concrete columns without a pour strip.

**Ductile Link Connectors**

Fortunately, a logical alternative to monolithic emulation does exist. Precast concrete, like structural steel, is most cost effective to produce, transport and erect when columns and beams can be fabricated independently, then joined at the column face. However, as has recently been discovered in steel frame construction [Englehardt (1993), AISC (1994)], joining of the beam and column at the column face normally creates a brittle weak link in the system unless this region is specifically strengthened.
In steel frame construction, new design methods are being proposed that strengthen the beam at the interface, moving the hinge region away from the beam-to-column interface and into the body of the beam. This solution is similar to the solutions proposed for relocated hinges in precast concrete construction, as discussed previously.

Unlike welded steel frames, however, the joining of precast concrete elements at the beam-to-column interface provides a unique opportunity to provide excellent seismic performance because the connector can be made to behave in a ductile manner, thereby creating a link that will control the inertial loads that are generated in the rest of the building system.

**PROPOSED PRECAST CONCRETE FRAME**

A ductile precast concrete frame (DPCF) system, using a ductile link connector, has been developed by Englekirk & Nakaki, Inc., Irvine, California, and Dywidag Systems International, USA, Inc., Long Beach, California. This connector allows the beams and columns to be cast independently and joined at the column face by bolting. A sample designed building is detailed here to discuss the design and construction of the DPCF.

Fig. 3 depicts a typical floor plan of a six-story building. This building was developed to study various Precast Seismic Structural Systems (PRESSS)...
in a United States-Japan coordinated research program [Nakaki and Engekirk (1991)]. The gravity load system is typical of that found in many parts of the country—it uses precast concrete columns, beams and hollow-core slabs. A 2.5 in. (63.5 mm) topping slab is assumed in the design, though this is not a requirement of the bracing system. A perimeter non-loadbearing precast concrete cladding system is assumed to contribute no resistance to lateral loads.

The sample building described here is designed in accordance with the UBC (ICBO, 1994). While this system is also appropriate for high seismic zones (Zones 3 and 4), the sample building is designed to a UBC Zone 2A strength criterion ($Z = 0.15 \, g$), and assumes a site factor, $S$, of 1.2.

Experimental work, discussed below, has shown that the DPCF will perform better than a monolithic concrete special moment-resisting space frame (SMRSF) because it can withstand story drifts in excess of 4 percent without loss of strength at repeated post-yield cycles of drift. Therefore, an $R_v$ factor of 12 is appropriate for design.

**Frame Description**

Six single bay frames are used in each direction in the example framing scheme described in Fig. 3. Alternative framing schemes are possible. A typical transverse building section is shown in Fig. 4. The frame, located in the center bay, supports gravity load from the precast concrete slabs. This distributed system allows the building to be constructed using up and out construction, without the need for temporary bracing beyond that required to align the building.

The frame columns, at 24 x 28 in. (610 x 711 mm), are only slightly larger than the non-frame columns. Note that corbels are only required on, at most, two opposite faces of any column. Frame beams are 20 x 44 in. (508 x 1118 mm) deep and allow a 9 ft (2.74 m) ceiling for the adopted 13 ft (3.96 m) story height. This beam width is required primarily to support the precast concrete slabs, as shown in Fig. 5. While six frames are required at the ground floor, the number of frames can be reduced in the upper levels of the building.

**Connector Design**

For the design base shear of 455 kips (2033 kN), an elastic analysis of the building indicates a maximum seismic design moment of 419 kip-ft (568 kN-m) in the second floor beam. Using the construction procedure described below, the dead load of the beam and slabs themselves is resisted by a simply supported beam. The bolts are tightened after the hollow-core slabs are in place; thus, the only dead load moment applied to the connector is that caused by the topping slab, partitions, and other miscellaneous loads. For the configuration shown, the dead and live load moments on the connector are 49 and 26 kip-ft (66.4 and 35.2 kN-m), respectively.

The ultimate design moment is then:

$$M_u = 0.75(1.4M_D + 1.7M_L + 1.7 \times 1.1M_E) \quad (5)$$

Substituting numbers:

$$M_u = 0.75[1.4(49) + 1.7(26) + 1.7 \times 1.1(419)] = 672 \text{ kip-ft (911 kN-m)}$$

The Dywidag ductile connector (DDC) is currently available in one configuration, shown in Figs. 6a and 6b. With this configuration, the required moment strength is provided by adjusting the beam depth and number of DDCs. For one DDC (consisting of two ductile rods top and bottom), the nominal moment strength of the connection shown in Fig. 6 is:

$$F = 240 \text{ kips (1068 kN)} \quad \text{(strength of one DDC)}$$

$$d = h - 5 = 44 - 5 = 39 \text{ in. (991 mm)}$$

$$\phi M_e = \phi Fd = 0.9 \times 240(39/12) = 702 \text{ kip-ft (950 kN-m)}$$

which is greater than the required demand.

Fig. 5. Typical beam section.
Ductile Rod Material

The critical quality component of the DPCF is the ductile rod itself. It is made from a very high quality steel that has well-defined strength characteristics and high elongation capacity. Currently, the UBC requires that yielding elements in a concrete system be made from A706 (or equivalent) material. Table 1 compares the material properties of the ductile rod with that of A706 reinforcing steel. Since the design yield strength of the two materials is different, the comparisons have been made in terms of $F_y$.

The yield strength of the ductile rod material does not vary much, which will limit the possibility of unintentional overstrength. This will provide even more reliability in the system performance than was apparent during the prototype testing, which used a different material.

Joint Design

For the design of a SMRSF, the UBC uses an empirical description of the shear capacity of the joint. While the code equations relating $F_c$ to the joint shear capacity have been shown to provide acceptable performance, they do not help in the understanding of the behavior mechanisms that transfer forces through the joint.

In a traditional monolithic ductile frame, shear is transferred through the joint using two mechanisms. First, compression developed at the face of the joint is transferred diagonally through the joint by means of a compression strut in the concrete. Once the concrete cracks, a truss mechanism resists additional load using the joint ties and concrete core. These mechanisms are described in detail by Paulay and Priestley (1992).10

Unlike SMRSF design, the load transfer mechanism through the DPCF joint must be evaluated explicitly. Fig. 8 describes the load path through the joint. Since the ductile rod is anchored in the center of the joint, the diagonal compression strut is unable to form. Therefore, all of the joint shear must be transferred using the truss mechanism. The required joint reinforcement must be calculated explicitly.
Table 1. Material properties.

<table>
<thead>
<tr>
<th>Property</th>
<th>Ductile rod</th>
<th>A706 reinforcing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield strength, minimum</td>
<td>50 ksi</td>
<td>60 ksi</td>
</tr>
<tr>
<td>Yield strength, maximum</td>
<td>$1.10F_y$</td>
<td>$1.36F_y$</td>
</tr>
<tr>
<td>Tensile strength</td>
<td>$1.50F_y$</td>
<td>$1.30F_y$</td>
</tr>
<tr>
<td>Elongation</td>
<td>35 percent in 2 in.</td>
<td>10 percent in 8 in.</td>
</tr>
</tbody>
</table>

Note: 1 in. = 25.4 mm; 1 ksi = 6.895 MPa.

Beam and Column Design

Capacity design of the DDC components eliminates the possibility that the longitudinal reinforcement in the beam will yield. Since inelastic behavior is prevented in the beam, the contribution of aggregate interlock, friction and dowel action ($\phi V$) can be relied on to resist both seismic and gravity load shear within the beam. The required beam ties are significantly reduced from those required for a SMRSF.

Column design, however, follows the current philosophy contained in the code for SMRSFs. Confinement reinforcing steel is required in the column unless a true capacity design is performed for the frame. The only difference is that the capacity of the connector, not the girder, is used to determine the required strength of the column.

Construction Sequence

Ductile rods, threaded to receive a high strength bolt, are cast within the column using a template to ensure alignment with the beam connector plate assembly. The connector plate is also threaded to receive high strength reinforcing bars (Threadbars) (150 ksi (1034 MPa)) that then become the main reinforcement in the beam. The Threadbars are threaded into the connector plate in the shop, then the connector plate/Threadbar assembly is cast into the beam.

Once the column is erected in the field, the frame beam is erected on temporary erection angles to allow the crane to immediately continue with erection. The temporary corbels can be designed to carry the weight of the hollow-core slabs and beams, or the bottom bolts can be tightened to resist the dead load shear caused by the hollow-core slabs.

Permanent corbels are not required to carry the permanent vertical or seismic shears, although the performance of the system would not be affected by the use of permanent corbels designed to support the construction loads. Design shear forces are reliably resisted by friction between the steel surfaces of the special connector. This friction is developed by pretensioned bolts (to
Fig. 9a. DPCF tested configuration.
Fig. 9b. DPCF tested configuration.
resist gravity loads) and flexure (to resist seismic loads).

The only field activity required to complete the structural system, once the system is aligned, is to feed the high strength bolts through oversize holes in the connector plate and tighten them into the ductile rods. The oversize holes allow for misalignment of the ductile rods, and shims are used to make up for slightly short or long beams.

The DPCF construction sequence promotes very quick erection, similar to a steel building. Once the bolts are tightened, the connection is complete, for no welding or structural grouting is required. Grout is placed between the beam and column, but this grout has no structural purpose and serves primarily to protect the steel pieces. The load transfer path through the interface is solely steel to steel.

**PRECAST CONCRETE FRAME BEHAVIOR**

The UBC specifies elastic-based design loads known to be significantly below those that are expected to occur during the design level earthquake. The safety of this under-strength approach relies on system ductility that is attained through the use of prescriptive detailing requirements, none of which apply to precast concrete. The code does allow other concrete systems to be used, though, only if they meet specific requirements. Section 1921.2.1.5 states:

"A reinforced concrete structural system not satisfying the requirements of this section may be used if it is demonstrated by experimental evidence and analysis that the proposed system will have strength and toughness equal to or exceeding those provided by a comparable monolithic reinforced concrete structure satisfying this section." (ICBO, 1994)

The six-story DPCF design was planned to have a strength equal to a SMRSF system with an $R_w$ equal to 12. Satisfying the strength requirements of the code is straightforward and is done through the design process.

**Load-Displacement History**

System toughness is less easily compared through calculation. For these purposes, a prototype DPCF beam and column subassembly was

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**Fig. 10. Elevation of the test setup. (Courtesy SEQAD, Inc.)**

**Fig. 11. Connector subassembly load-displacement history.**
tested under cyclic loading to failure. The specimen design is described in Figs. 9a and 9b, and an elevation of the test setup is shown in Fig. 10.

The subassembly was tested using the PRESSS loading sequence [Priestley (1992)]. The PRESSS criteria suggest that a precast concrete frame should be able to withstand a lateral deformation of at least 2 percent without significant loss of strength. This follows from the UBC requirement that a frame must be able to withstand an average drift of 1.1 to 1.5 percent, depending on the fundamental period of the building [ICBO (1994)].

The load-displacement history resulting from the test is presented in Fig. 11. The DPCF was able to sustain cycles up to a drift of 4.5 percent calculated as the lateral displacement of the load point divided by the distance between the bottom pivot and the load point — see Fig. 10).

Two interior beam-to-column monolithic concrete joints were tested at the National Institute of Standards and Technology [Cheok and Lew (1991)]. These joints were designed for Seismic Zone 4 and were detailed in accordance with the requirements for a SMRSF contained in the 1988 UBC [ICBO (1988)]. These requirements have not significantly changed in later versions of the UBC. The load-displacement history for one of the SMRSF subassemblies is shown in Fig. 12.

A comparison of Figs. 11 and 12 indicates that the DDC subassembly was able to achieve significantly higher drift than the SMRSF, with considerably less strength degradation.

The pinching in the DPCF system is caused by the horizontal expansion of the system, shown in Fig. 13. Once the ductile rods yield in tension, the system must overcome the post-yield elongation on the return cycle by yielding the rod in compression before the beam comes into contact with the column again.

During this behavior stage, stiffness at the connection is provided solely by the steel couple because the concrete is not in contact. Once the beam contacts the column in the compression zone, the system stiffness again increases. For this system and loading history, contact occurs at approximately zero drift, as indicated in Fig. 11.

It is important to note that the
monolithic frames tested at NIST also became more pinched as the specimens reached higher drift levels. As the reinforcing bars yield and the concrete cracks, the flexibility of the SMRSF system increases significantly for identical reasons. Fig. 14 compares the third cycle at a drift of 3.5 percent for both the DPCF and one of the SMRSF specimens. While the monolithic system does dissipate more energy per cycle, more structural damage in the form of concrete cracking is experienced. Figs. 15a and 15b show the DPCF and one of the SMRSF specimens at a drift of approximately 3.5 percent, respectively. More appropriate is the comparison of ability to achieve deformation without loss of strength. As is apparent in Fig. 11, the DPCF system did not degrade, even after three cycles at a very large drift. The SMRSF system, however, showed a considerable loss of strength, particularly in the negative direction. Therefore, the displacement capacity of the precast concrete assembly produces a system whose behavior is better than that of a monolithic frame.

Overstrength

The maximum horizontal force developed during testing was 210 kips (936 kN). This force translates into a beam moment as follows:

\[ V_b = V_c \frac{L_e}{L_b} \]  

and

\[ M_b = V_b \frac{(L_b - h_c)}{2} = V_c \frac{L_e (L_b - h_c)}{2} \]

where
\[ L_c = \text{column height between load point and bottom pivot} \]
\[ L_b = \text{beam span between end tiedown pins} \]
\[ h_c = \text{column width in direction of loading} \]

For the test specimen, at maximum horizontal force, the beam moment was:

\[ M_b = \frac{(210.5)^9}{16} \cdot \frac{(16-2.67)}{2} \]
\[ = 789 \text{ kip-ft (1070 kN-m)} \]

This moment is 35 percent higher than the nominal beam moment of 585 kip-ft (793 kN-m), and was caused by excessive overstrength of the rods. It is important to note, however, that at the design drift limit of 2 percent story drift, the horizontal force was only 175 kips (778 kN), resulting in a beam moment equal to 656 kip-ft (890 kN-m) with an overstrength of 1.12. Therefore, for a design drift limit of 2 percent, the overstrength factor of 1.25 is appropriate, even for the original rod material. Since the test, however, the quality of the rod material has been improved so that the yield strength is more controlled (see description of rod material, previously).

**Joint Behavior**

Despite the fact that the ductile rod repeatedly elongated and shortened within the joint, there were no signs of horizontal cracking in the joint. More diagonal cracking was apparent in the DPCF joint region than in the NIST monolithic specimens, due to the exclusive activation of the truss mechanism and the much higher stress in the ties. (Note that the cracks in the SMRSF are exaggerated due to pen markings used during the test to follow the crack development.) However, the cracks in the DPCF were small and well distributed until very high drift levels had been achieved, and this behavior did not compromise the ability of the column to support load.

**Vertical Shear Transfer**

Vertical shear at the beam-column interface was resisted by a friction force created by the moment. Slippage did not occur until a drift of 3.5 percent was reached. At this drift level, the beam slipped vertically as the load changed from positive to negative. This was caused by the spalling of the concrete, below the bell of the ductile rod, that required the rod to bend before sufficient bearing surface came in contact with the column.

Even though the drift level where this occurred is much higher than the anticipated demand drift level, the bell of the rod has been increased, reducing the bearing stress on the concrete below the bell to mitigate this failure mechanism in future designs.

**Beam Behavior**

Although not a primary goal of the test, it is interesting to note that the beam itself experienced very little flexural cracking. Strain gauges were not placed on the Threadbars within the beam; however, based on the horizontal force developed by the system, the strain in the Threadbars can be calculated.

The maximum beam moment generated during the test was 789 kip-ft (1069 kN-m). Assuming that the two 1.25 in. (31.75 mm) diameter Threadbars take equal load, the force in the Threadbar, \( F \), is:

\[ F = \frac{789 \times 12}{(32-5)^2} = 175 \text{ kips (778 kN)} \]

and the stress, \( f \), is:

\[ f = \frac{175}{1.25} = 140 \text{ ksi (965 MPa)} \]

Although the force in the Threadbar was close to its specified ultimate strength [150 ksi (1034 MPa)], significant cracking did not occur. The strain in the steel at this high stress was more than twice the yield strain of Grade 60 reinforcing steel. The lack of cracking at these high strains should dispel concerns about the serviceability of precast concrete systems using high strength, unstrained reinforcing steel.

**CONCLUSION**

The most effective use of precast concrete in any seismic region requires the exploitation of the inherent attributes of precast concrete systems. Rather than forcing precast concrete to be “monolithic,” connectors can be enhanced to provide good ductile behavior within the connector itself. The DPCF achieves this without appreciably affecting the erection costs of the system. The balance of the design then follows the capacity design procedure, with the ductile link protecting the rest of the system from inelastic action. This allows considerable flexibility in the geometry and details of the beam, without compromising structural integrity in the event of a major earthquake.

**ACKNOWLEDGMENT**

The prototype test was performed by SEQAD Consulting Engineers, Inc., San Diego, California. The research performed at the National Institute of Standards and Technology (NIST) by Ms. Geraldine Cheok and Dr. H. S. Lew was indispensible in allowing us to perform the code required comparison to cast-in-place concrete. The use of the NIST data and photos does not imply a recommendation or endorsement by NIST nor does it imply that the product is necessarily the best available for the purpose.

**REFERENCES**

crete Institute, Chicago, IL, 1982.


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**APPENDIX — NOTATION**

\[
\begin{align*}
d &= \text{beam depth} \\
e &= \text{distance of hinge from beam-column interface} \\
f &= \text{stress in Threadbar} \\
F &= \text{force in Threadbar} \\
f' &= \text{specified compressive strength of concrete} \\
F_y &= \text{yield strength of reinforcing bar} \\
g &= \text{acceleration due to gravity; } g = 32 \text{ ft per sec}^2 (9.8 \text{ m/s}^2) \\
h_c &= \text{column width in direction of loading} \\
L_b &= \text{beam span between end tiedown pins} \\
L_c &= \text{column height between load point and bottom pivot} \\
L_{\text{clear}} &= \text{clear span of beam} \\
M_b &= \text{beam moment at maximum horizontal force} \\
M_D &= \text{dead load moment} \\
M_E &= \text{seismic design moment} \\
M_{\text{hinge}} &= \text{nominal strength of beam hinge} \\
M_{\text{interface}} &= \text{nominal moment strength at beam-column interface} \\
M_L &= \text{live load moment} \\
M_n &= \text{nominal moment strength} \\
M_u &= \text{ultimate design moment} \\
R_w &= \text{seismic force reduction factor} \\
S &= \text{site factor} \\
V_b &= \text{shear force in beam} \\
V_c &= \text{shear force in column} \\
V_p &= \text{maximum shear developed in beam} \\
Z &= \text{seismic zone factor} \\
\lambda_o &= \text{overstrength factor, related to the maximum possible hinge strength that can be developed; normally, } \lambda_o = 1.25 \\
\phi &= \text{flexural reduction factor of section; for a flexural hinge, } \phi = 0.9 \\
\theta_h &= \text{plastic hinge rotation} \\
\theta_c &= \text{post-elastic story drift}
\end{align*}
\]