OPEN FORUM
PROBLEMS AND SOLUTIONS

Design of Hanger Steel in Ledger Beams

Q1: *When designing the hanger steel in ledger beams, how much of the ledge force is carried by the stem?*

A1: To determine the requirements for hanger steel A_{sh}, it is necessary to calculate the portion of the shelf load V_u that is carried by the stem of the ledger or inverted T beam. This portion is calculated by subtracting the shear carried by the ledge, V_l, and the shear carried in torsion from the total shear, V_u. This article deals with the calculation of the shear carried by the ledge, V_l (see Fig. 1). The shear carried by torsion will be the subject of a separate article.

The shear carried by the ledge is determined by integrating the shear stress in the ledge based on the equation:

$$V_l = \frac{V_u \int_{x=0}^{h_l} Q(x)dx}{I} \tag{1}$$

Thus, the portion carried by the stem (ignoring the torsion effect) is:

$$V_s = V_u - V_l \tag{2}$$

The moment of inertia, I, is calculated as follows:

A	x	Ax	Ax^2	I_o
$h \times b$	$h/2$	$h^2b/2$	$h^3b/4$	$h^3b/12$
$h_l \times w_l$	$h_l/2$	$h_l^2w_l/2$	$h_l^3w_l/4$	$h_l^3w_l/12$

The neutral axis is at:

$$\bar{x} = \frac{\sum Ax}{\sum A} = \frac{1}{2}\frac{h^2b + h_l^2w_l^2}{hb + h_lw_l}$$

The moment of inertia is:

$$I = \sum I_o + \sum Ax^2 - x^{-2}\sum A$$

Therefore:

$$I = \frac{h^3b}{12} + \frac{h_l^3w_l}{12} + \frac{h^3b}{4} + \frac{h_l^3w_l}{4} - \frac{1}{4}\left(\frac{h^2b + h_l^2w_l}{hb + h_lw_l}\right)^2 (hb + h_lw_l) \tag{3}$$

The moment of the cross section above position x is:

$$Q(x) = x(b + w_l)\left(\bar{x} - \frac{x}{2}\right)$$

Integrating the moment of the cross section over the depth of the ledge yields the following equation:

$$\int_{x=0}^{h_l} Q(x)dx = (b + w_l)\left(\frac{\bar{x}h_l^2}{2} - \frac{h_l^3}{6}\right) \tag{4}$$

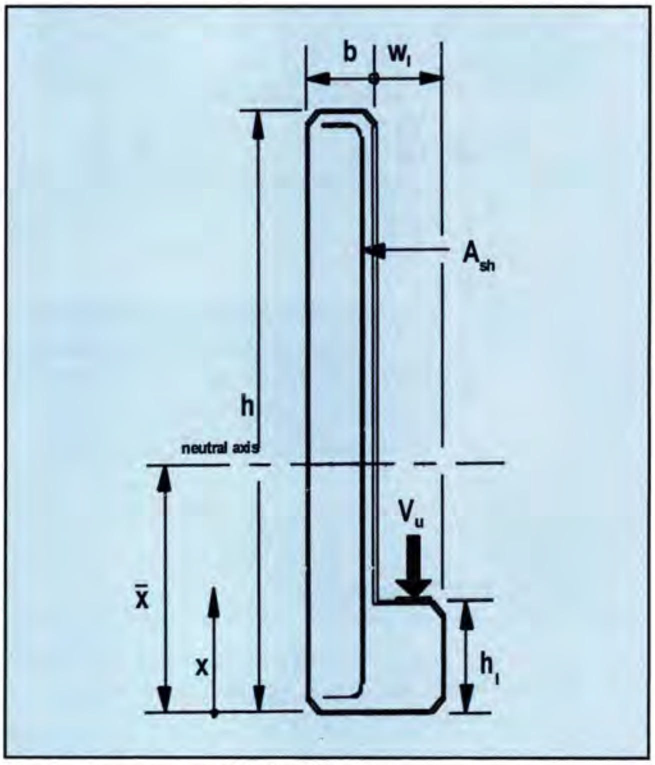

Fig. 1. Cross section of ledger beam.

Finally, the portion of load carried by the stem, V_s, is determined from Eqs. (1) through (4):

$$V_s = V_u\left(1 - \left\{\frac{(b + w_l)\left[\frac{h_l^2}{4}\frac{(h^2b + h_l^2w_l)}{(hb + h_lw_l)} - \frac{h_l^3}{6}\right]}{\frac{h^3b + h_l^3w_l}{3} - \frac{(h^2b + h_l^2w_l)^2}{4(hb + h_lw_l)}}\right\}\right) \tag{5}$$

Eq. (5) can be converted into dimensionless variables by setting $\alpha = w_l/b$ and $\beta = h_l/h$, resulting in the following:

$$V_s = V_u\left(1 - \left\{\frac{(1 + \alpha)\left[\frac{\beta^2(1 + \alpha\beta^2)}{4(1 + \alpha\beta)} - \frac{\beta^3}{6}\right]}{\frac{(1 + \alpha\beta^3)}{3} - \frac{(1 + \alpha\beta^2)^2}{4(1 + \alpha\beta)}}\right\}\right) \tag{6}$$

Eq. (6) is plotted in Fig. 2 for values of α ranging from 0 to 1.0 and values of β ranging from 0 to 1.0. Obviously, the deeper the ledge (i.e., the closer β is to 1.0), the less shear is carried by the stem.

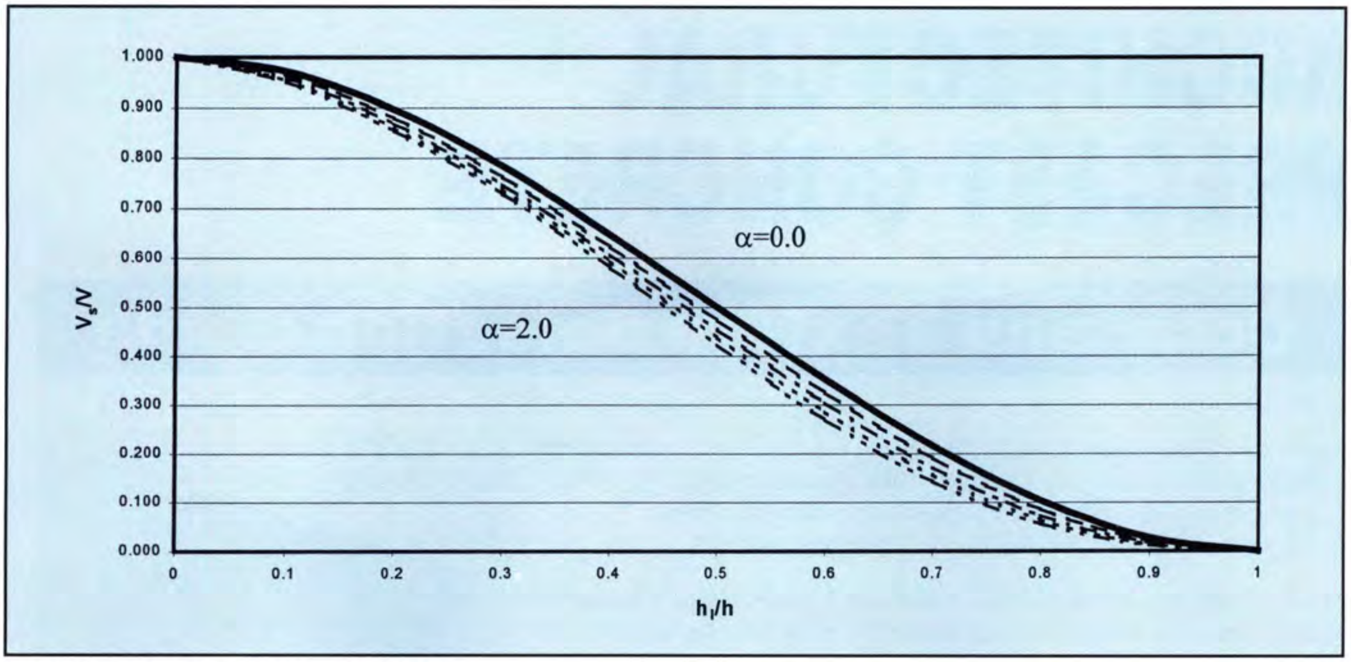

Fig. 2. Portion of applied load carried by stem.

The most conservative curve in Fig. 2 corresponds to $\alpha = 0$ (i.e., no ledge). Then Eq. (5) reduces to the following:

$$V_s = V_u\left[1-\left(3-2\frac{h_l}{h}\right)\left(\frac{h_l}{h}\right)^2\right] \qquad (7)$$

which appears in the equation for m in Table 6.14.1 of the PCI Handbook, Fourth Edition.[1]

DESIGN EXAMPLE

Consider the ledger beam (Example 6.14.1) in the PCI Design Handbook, Fourth Edition.[1]

Given:

V_u = 18 kips (80 kN)
h = 36 in. (914 mm)
h_l = 12 in. (305 mm)
b = 8 in. (203 mm)
w_l = 6 in. (152 mm)
α = 6/8 = 0.75
β = 12/36 = 0.33

By Eq. (7):

$$V_s = 1.8[1-(3-2\times 0.33)(0.33^2)] = 18(1-0.259)$$
$$= 13.3 \text{ kips (59 kN)}$$

By Eq. (6):

$$V_s = 18\left(1-\left\{\frac{(1+0.75)\left[\frac{0.33^2(1+0.75\cdot 0.33^2)}{4(1+0.75\cdot 0.33)}-\frac{0.33^2}{6}\right]}{\frac{(1+0.75\cdot 0.33^2)}{3}-\frac{(1+0.75\cdot 0.33^2)^2}{4(1+0.75\cdot 0.33)}}\right\}\right)$$

$$= 18\left[1-\left(\frac{0.0313}{0.1079}\right)\right] = 12.8 \text{ kips (57 kN)}$$

As seen in this example, the more complicated Eq. (6) results in a small (4 percent) decrease in shear force carried by the ledger beam stem.

REFERENCE

1. *PCI Design Handbook,* Fourth Edition, Precast/Prestressed Concrete Institute, Chicago, IL, 1992.

[Contributed by Andrew E. N. Osborn, Wiss, Janney, Elstner Associates, Inc., Princeton Junction, New Jersey.]