READER COMMENTS

Shear and Torsion Design of Prestressed and Non-Prestressed Concrete Beams*

by Michael P. Collins and Denis Mitchell

Comments by Krister Cederwall and Lennart Elfgren, W. H. Dilger, Peter Gergely, Karl Kordina and Manfred Teutsch, Peter Mueller, K. S. Rajagopalan, Himat T. Solanki, Hrista Stamenkovic, and Authors

KRISTER CEDERWALL† and LENNART ELFGREN‡

The authors have presented an excellent and comprehensive paper and thus we only wish to make a few comments.

When using the truss model to simulate torsion behavior one decisive step is the determination of the angle of inclination of the concrete diagonals. Several suggestions have been presented in the literature to accomplish this. For example:

1. The angle θ corresponds to the angle of direction of the maximum compressive stresses just before cracking.
2. The angle θ is chosen so that a minimum of interior energy is obtained for a cracked stage (see Kupfer Petersson[36]).
3. The angle θ is chosen so that yield is obtained in both the transverse and longitudinal reinforcement.[37–41]

To this list can thus be added the assumption used by the authors, namely that θ corresponds to the direction of the maximum compressive strain for a cracked stage. It would be interesting if the authors could comment upon this choice somewhat more.

According to the experience of the writers the cracks start according to Hypothesis 1 and get close to Hypothesis 3 in a failure state if the reinforcement is not too heavy. Eventually, there will be a second crack pattern according to Hypothesis 3 crossing the first generation following Hypothesis 1.

It would also be interesting to know if the authors are prepared to use this value of θ for the purpose of calculating torsional stiffness or if they have any other suggestions. We are aware of the fact that the procedure presented by the authors is intended for design purposes but despite this a general discussion of the topic could be valuable.

In calculating the torsional resistance, it is assumed that the concrete cover outside the

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hoops is spalled corresponding to zero tension strength of the concrete. This seems to be a very conservative approach for normally reinforced sections and the risk of spalling could preferably be taken care of by detailing regulations and/or restrictions of the amount of reinforcement.

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W. H. DILGER*

The authors are to be congratulated for this comprehensive paper which can be considered one of the milestones in this complex field of design for shear and torsion. Compared with other existing methods of shear and torsion design, the new approach is based on a *rational* theory which allows a check of both strength and serviceability.

The approach clearly reflects all the relevant features which are important for the design of shear and torsion, particularly the effect of the amount of transverse steel on the magnitude of the longitudinal forces, a fact which is not clearly reflected in the present ACI Code. The new method also leaves more freedom to the designer with regard to the design of the transverse reinforcement in that the angle θ can be chosen between certain limits.

This freedom is particularly desirable for members in which sufficient longitudinal steel is available to resist the additional force ΔN_u. Also, longitudinal steel is much easier to fabricate and place than transverse steel so that a substantial economical advantage may result when selecting a small angle θ.

There is, however, one detail which needs some more study, namely, the crack control in reinforced concrete members subjected high shear stress. In the absence of prestress, Eq. (28) simplifies to:

$$V_{ocr} = b_w d\, 4\sqrt{f'_c}$$

This expression clearly overestimates the load at formation of shear cracks, and therefore the limiting angle θ defined by Eq. (31) becomes too small. This means that the strain in the transverse steel may exceed the allowable limit $\epsilon_{te} = 0.001$ under service load, and in turn result in excessive width of shear cracks.

In addition, the maximum allowable strain of 0.001 recommended in Reference 28 may also be too high since crack widths of 0.02 in. (0.50 mm) and more have been recorded under service loads in several experiments (see for

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example Reference 42) and recently on a reinforced concrete pedestrian bridge by this writer.

A value of ϵ_{te} = 0.0008 for the maximum strain in the transverse reinforcement, combined with a more appropriate value for the cracking stress for reinforced concrete, would result in a better safeguard against excessive width of shear cracks in reinforced concrete members.

The writer applied the new approach to the design of a prestressed curved box girder bridge subjected to relatively high shear and torsion. It was found that the proposed method is less cumbersome than the existing approach. The transverse steel according to the new method was less than that required according to the Canadian Bridge Code,[43] in which it is specified that the transverse steel for torsion be designed according to the space truss model with $\theta = 45$ deg.

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PETER GERGELY*

It is well known that the subject of shear and torsion in reinforced concrete has had a rather frustrating and controversial history. A tremendous amount of testing has been ineffectual but it was necessary because of the lack of a reliable theory. A number of "solutions" were advanced over the years, but none could answer such fundamental questions as the amount of shear or torsion transmitted by diagonally cracked concrete. Some experts in the field even suggested several years ago that the finding of a solution is hopeless and that research in shear and torsion should be abandoned. Fortunately, most humans, especially engineers, are generally optimistic.

The paper by Collins and Mitchell is undoubtedly a landmark among the hundreds of articles on the subject. It shows that a satisfactory and rational solution is possible and indeed is within reach. The approach advanced by the authors is rational but it is not simple. Considering the complex state of indeterminate stresses and strains in a cracked member, it is not surprising that the method involves many steps. However, the greatest asset of the theory is that each step in the development is clearly identifiable and can be checked or improved.

Some of the components of the approach required a more detailed understanding of the constitutive relationship of cracked concrete than was available and this led to further research. This indirectly indicates that a reliable prediction for a wide variety of geometries and loading would not be possible without this increased understanding and without the sophistication and complexity of the theory.

What are the potentials of the approach? The assumptions can be clearly stated and, as the authors say, could be cast in a simple form as is currently the case for simple flexure. However, it is unlikely that this will be acceptable for code use until the theory gets long and detailed exposure in textbooks and technical papers, or until it can be considerably simplified at least for the prediction of strength.

The theory traces the behavior of a member from the beginning to failure; none of the other theories can do that for the general case of loading. Therefore, the approach can be used for parametric studies and to explain the effects of many variables on shear and torsion behavior. In fact, this writer has performed such an evaluation using an earlier version of the theory a couple of years ago. As a result of detailed parametric analyses, it will be possible to establish which variables are important, the knowledge of which is essential in the development of a simplified approach if that is at all feasible.

It would be highly useful to have more comparisons of the theory with experimental results. It is likely that for typical beams the differences between the calculated and experimental results would not be significantly smaller than using other methods which are almost entirely empirical. The reason is that this general theory reflects many variables, such as the position of the concentrated load and the effect of supports which affect the results; it is necessary to have carefully performed and well documented tests to allow meaningful comparisons. For unusual geomet-

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ries or loadings the theory should be superior to most other predictions.

The writer cannot do much better than to use the customary and hackneyed phrase that the authors are to be congratulated for an outstanding piece of work extending over a number of years and for a clearly written and stimulating paper. It surely represents a significant jump in our quest for the elusive solution of the shear/torsion/flexure/axial load riddle. ACI-ASCE Committee 445, Shear and Torsion, has discussed this paper during their last two meetings and will continue the study and evaluation of the compression field theory.

KARL KORDINA* and MANFRED TEUTSCH†

In contrast to the usual design procedures, e.g., following the ACI Code requirements, the authors have offered a consistent, compatible, and relatively simple solution to the shear and torsion problem with the added feature that the theory does not contain complicated modifying equations. Nevertheless, the authors have not reached their goal in a completely satisfactory manner, neither with respect to the determination of the effective geometrical values nor in the prediction of compression failure of the concrete diagonal struts. These aspects are discussed in greater detail below.

Members in Torsion

(a) According to Fig. 5 the authors assume that similar to flexure the actual stress distribution in the diagonals may be replaced by a uniform stress block. The value of the stress block ordinate $\alpha_1 \bullet f_c'$ corresponds with $\alpha_1 = 0.85$ to the intensity of the compressive strength of concrete subject to bending.

In the section "Members in Shear" it is shown, however, that the compressive strength of a cracked concrete element depends also on the relation γ_m/ϵ_d (in which γ_m is the maximum shear strain and ϵ_d is the diagonal compressive strain). The investigation by Collins (Reference 22, Fig. 10) mentioned in this connection shows that with $\gamma_m/\epsilon_d = 9.5$ (which is fairly typical under torsional stress), the maximum stress corresponds only to about 40 percent of the cylinder strength. Because of this realization, the authors apparently limit the compression capacity in the diagonals of members subject to shear according to Eq. (15).

Under torsional loading, however, such a limitation is not required. The effective thickness of the wall — a_o being involved in calculating the torsional moment capacity [see Eq. (20)] — is consequently smaller than when considering a reduced strength of the concrete diagonals.

(b) In their model the authors do not consider the concrete beyond the area surrounded by stirrups. This assumption is justified because there is spalling of concrete under high torsional stress. It is known (partly based on our own test experiences[44]) that this spalling process occurs only under ultimate bearing conditions as a secondary phenomenon due to excessive strains of the reinforcement and the large curvature of the diagonal struts.

Spalling of the concrete occurs below the ultimate load only if the strain resultant of the concrete diagonals is situated beyond the area surrounded by stirrups (A_o/A_{oh}). Our own investigations[44,45] (which also include strain measurements over the depth of the diagonal struts) show that by taking into account a decreased compressive strength of the cracked concrete element according to Reference 22, theoretical values of wall thickness result (including the concrete cover of the stirrups), which are in agreement with the experimental data. Both experimental as well as theoretical values have been shown to be significantly greater than the values proposed by the authors. Comparative calculations show that our results, which avoid an arbitrary reduction of the actual concrete area, do not significantly differ from the results of the authors calculated according to Eq. (8).

(c) In Eq. (8) the term a_o^2 is missing. This omission is acceptable for small values of a_o. It should, nevertheless, be mentioned that Eq. (8) is only an approximation.

Members in Shear

The neglect of the concrete area near the surface of the beam does not appear to be satisfactory even when considering this type of load. Thus, the area of compression under bending should be diminished similarly, which is neither customary nor necessary.

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Design for Torsional Strength

The authors refer to a thin-walled hollow-box cross section in which the torsional stress can be expressed by:

$$\tau_n = q/a_o = T_n/(2 A_o a_o)$$

According to the definition for torsional stress [see Eq. (19)], the wall thickness is given by:

$$a_o = A_{oh}^2/(2A_o p_h) \approx A_{oh}/(2p_h)$$

For a square shaped cross section the wall thickness becomes $a_o = b_{oh}/8$.

Note that in the CEB/FIP Model Code $a_o = b_{oh}/6$.

Also, in Reference 46 Collins and Rabbat propose a significantly greater wall thickness, namely, $a_o = (3A_o)/(4p_o)$.

Note that the factor (3/16) b_o implies a square shaped cross section.

Our own investigations[45] show that for square shaped cross sections under torsion, values of a_o between (3/20)b and (1/4)b are obtained depending on the percentage of torsional reinforcement.

It is surprising that the authors use the expression $a_o = A_{oh}/2p$ in Eq. (19) but then ask for a second evaluation of a_o according to Eq. (20) in order to find the value of A_o. With $\tan\theta = 1$ and the definition of τ_n represented by Eq. (19), Eq. (20) can be rewritten as follows:

$$a_o = A_{oh}/p_h[1 - \sqrt{1 - \tau_n/(f'_c\, 0.85)}] \quad (20a)$$

Fig. 12 shows that τ_n/f'_c cannot exceed 0.3. From Eq. (20a) this leads to an even smaller thickness of $a_o < A_{oh}/(p_h\, 5)$, respectively, where $b_{oh}/20$ is for a square shaped cross section as Eq. (19) shows. It must be emphasized that such a small wall thickness neither conforms with experimental measurements (see Beam PT 5 in Reference 47) nor with the authors' theoretical results found in Reference 46.

The actual thickness of the wall of a hollow box cross section must not exceed the value a_o according to Eq. (20). Unfortunately, this limitation is missing from the authors' paper.

Designing for Combined Torsion, Shear and Flexure

For members under combined loading of flexure, shear and torsion, a verification check is needed. The authors show that a compression failure of the concrete diagonal struts must be taken into account if the semi-empirical Eq. (23) is not satisfied, e.g., if $\tau_n/f'_c > 0.30$. Note that $\tau_n/f'_c = 0.3$ results from Eq. (23) under the assumption that $\epsilon_l = \epsilon_t = 0.002$ (see Fig. 15).

The evaluation of τ_n according to Eq. (24) should be done using different wall thicknesses for each particular loading case, e.g., for torsion $a_o = A_{oh}/2p_h$ and for shear $a_o = b_v/2$. In practice, a uniform wall thickness can be expected for the actual loading combination.[45] Furthermore, the failure of the bending compression zone under combined loading does not need to be investigated although torsional stresses do increase the possibility of a failure of the compression zone since additional diagonal compression forces caused by torsion must be included. Lastly, the depth of the compression zone is decreased by torsion.

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PETER MUELLER*

The writer would like to congratulate the authors on their paper and on their research on torsion and shear in general. This research has closed a significant gap in the rational analysis procedures for torsion and shear developed in the last decade. While the plastic analysis procedures,[48-50] which often run for historical and

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promotional reasons under the somewhat ambiguous name "truss model," can successfully predict the strength of under-reinforced beams, they inherently cannot distinguish between under-reinforcement and over-reinforcement, because they do not predict total deformations. For over-reinforced beams, they can only accurately predict strength, if the strain softening of concrete is approximately considered by a reduced "effective concrete strength," which again depends on the deformations. To establish both the limit to over-reinforcement and the effective concrete strength, the deformations at failure must be predicted. In the diagonal compression field theory the authors have developed the tools for this analysis and in the proposed design recommendations, they have condensed their findings into a surprisingly simple set of rules.

Basically, two parts can be distinguished in these recommendations. A part relating to the determination of the reinforcement, which is essentially similar to the plasticity based codes of CEB[8] and SIA.[56] And a part that relates to the limits for τ_n and θ ensuring diagonal crack control and under-reinforcement, i.e., yielding of the reinforcement before diagonal crushing. In this truly innovative part the authors have replaced the empirical limits of the aforementioned codes with limits that rationally consider the deformations at the failure and service stage. Both the CEB and the SIA Codes specify a lower limit $\tan \theta = 0.6$, which is intended to ensure adequate cracking behavior at service load. This limit made it necessary to introduce a rather awkward transition region between uncracked and fully cracked behavior in order to avoid provisions requiring more reinforcement for low τ_n than previous editions. In addition to a more rational evaluation of these limits, the proposed design recommendations elegantly avoid this problem by allowing very low values of θ for low τ_n.

Within the practically important region of under-reinforced behavior, however, both plasticity based models for torsion and shear and diagonal compression field theory result in similar strength predictions. Here the powerful because simple plastic analysis allows the establishment of interaction relations for torsion, bending and shear with ease for complicated arrangements of reinforcement.[48] Moreover, it has been possible to show on the basis of the theory of plasticity that the static assumptions of uniform compression fields within each face of a beam and of a constant shear flow can directly be derived from the usual kinematic assumptions of beam theory, in particular the assumption of unrestrained warping.[50] Three-dimensional failure mechanisms for torsion, bending and shear paralleling in concept the skew bending mechanism have been developed that provide an intuitive understanding of the failure mechanism and that indicate when warping restraints may become significant.[51-53] Thus, plastic analysis and diagonal compression field theory complement each other. They differ in the method and the effort of analysis (plastic vs. step-wise inelastic analysis) rather than in the concept of shear resistance.

The model of shear resistance is basically the same in both approaches. At the failure stage a perpendicularly reinforced concrete beam is resisting shear by a diagonal compressive field only. The purpose of shear reinforcement is to balance and thus enable this compression field. Torsional and shear strength interact with flexural strength over the longitudinal resultant of the diagonal compression field. A so-called concrete contribution (or better contribution of concrete tensile strength) exists only at the service stage, when the reinforcement is not yet yielding. From this viewpoint so-called concrete contribution terms are appropriate in serviceability checks. In strength checks they are relics of allowable stress design. The proposed design recommendations implement these concepts with exemplary consistency. They put the design for torsion and shear for both prestressed and non-prestressed beams on a common consistent basis. Moreover, they provide a consistent transition between the strength of rectangular beams subjected to torsion and the strength of plates subjected to twisting moments.

The designer is probably more concerned with practical advantages. The most important advantage may be safety. While provisions based on empirical relationships can only be memorized and apply only in the range covered by the underlying tests, recommendations based on first principles of mechanics can be taught and understood. This understanding allows the designer to properly interpret the intent of the code and to transfer the intent to situations not explicitly covered by the code (e.g., ledge design, Fig. 31). This should result in less human errors, the single most important cause of structural failures. While the design of non-prestressed beams for flexure and shear only, may appear slightly more complicated than according to the ACI Code,[1] this is certainly not the case for prestressed beams and when torsion is involved. The ACI Code does not even contain provisions regarding torsion in prestressed beams. Excluding shear-friction and all special provisions, the ACI Code contains 25 numbered equations, while the proposed design recommendations contain only half as many, namely 13. It is also important to realize that the proposed design recommendations can easily be written in the form of interaction relations for bending, axial force, shear and torsion and that these interaction relations generally agree well with test results. The ACI provisions, on the other hand, cannot readily be converted to interaction relations, because the need for longitudinal reinforcement due to shear is only implicitly recognized in the provisions on development of flexural reinforcement. Moreover, the use of a modified truss analogy with fixed 45-deg diagonals for both shear and torsion (equal volume of longitudinal and stirrup reinforcement!) does not recognize the stress redistributions possible between longitudinal and transverse reinforcement. However, in a time of rapidly increasing computer use in design offices, consistency between code provisions and realistic interaction relations is important.

While the writer welcomes the general concept of the proposal, both the proposed numerical values for the limits to τ_n and θ and the details of the recommendations of course require careful review. It is impossible to cover all aspects in this discussion. However, a few critical observations are briefly summarized in the following.

Figs. 17 and 25 show that according to the proposed recommendations the design of non-prestressed beams with Grade 60 steel is mainly controlled by the service stage. With regard to control of diagonal cracking the proposed recommendations are significantly more conservative than both the CEB and ACI Codes for $\tau_n/f_c' \geq 0.1$. It is particularly surprising to note, in Fig. 17, that providing more stirrups, i.e., increasing θ, is relatively ineffective to control diagonal cracking, because the serviceability limit is almost horizontal up to $\theta = 45$ deg.

As a brief check, the writer has evaluated a diagonal cracking limit as follows: The amount of stirrup reinforcement is determined as proposed from Eq. (1-8). The stirrup strains at service load, however, are calculated using the ACI shear equations. The assumption that the ACI equations conservatively predict the service load stirrup stresses for, say, $\tau_n/f_c' \leq 0.2$ appears as justifiable as the analogy used to introduce the transition function Eq. (30), which strongly controls the shape of the serviceability limit Eq. (1-7). In addition, $d_v = 0.9d$ is assumed.

The resulting serviceability limit cuts only negligibly into the feasible region from the diagonal crushing criterion, the maximum being 2.5 deg at $\tau_n/f_c' = 0.15$. In addition, this curve predicts that increasing θ, i.e., the amount of stirrups, is effective in controlling cracking. The writer recognizes that increasing θ also results in a decrease in longitudinal reinforcement which is not considered above. Nevertheless, in view of the significant differences, it would be helpful if the authors could support their recommendations with tests results.

Fig. 25 also shows that the proposed recommendations are significantly more conservative than the CEB Code with regard to diagonal crushing. However, it is well known that I-beams with closely spaced stirrups can resist nominal shear stresses twice as high as the maximum value according to the ACI Code.[4] Indeed, all web crushing failures reported in References 54 and 55 lie above the CEB curve. Disregarding serviceability limits, it is interesting to note that the proposed provisions would allow a τ_n as high as the CEB Code for I-beams with $A_v f_y/(b_w s f_c') \approx 0.3$, had the suggestion of Fig. 9, i.e., $b_v = b_w$ for I-beams, actually been implemented. For torsion, on the other hand, the $\tau_n - \theta$ relationship does not appear to be unnecessarily conservative.

In this respect it is interesting to note that the authors introduce a reduced diagonal compressive strength, Eq. (15), for shear but not for torsion. While the use of different values for the effective concrete strength for different situations is common in plastic analysis to empirically account for different strain conditions at ultimate, this is less understandable in an approach that explicitly considers these strain conditions. Within the framework of the presented theory and for given values of ϵ_l, ϵ_t, θ the strain conditions in the concrete in the plane of the reinforcement are the same for both cases. For torsion f_{du} would even decrease toward the interior of the beam because the controlling parameter, γ_m/ϵ_d, in Eq. (15) increases. In view of the above remarks and of the fact that Eq. (15) is based on only four shear panel tests,[22] it would be helpful if the authors could elaborate on their diagonal crushing criterion and support it with results from actual beam tests, particularly I-beams.

For flexure the ACI Code states only general principles of analysis, specifies the concrete strength to be used in the analysis and limits the depth of the neutral axis, the reinforcement ratio or the reinforcement index to ensure ductile behavior. It does not specify design charts and interaction diagrams for flexure and axial load. It might be advantageous to follow the same pattern for torsion and shear. In essence, this would mean to state the general principles of the underlying theory, to specify a diagonal concrete strength, f_{du}, to limit the compression block depth for torsion, a_o, and to specify a transition for the depth of the compression field between pure torsion and pure shear. Both, f_{du} and a_o, would be specified dependent on the strain condition and on θ as deemed necessary [e.g., Eqs. (15) and (B9)]. Design charts such as Fig. 12, Fig. 15 and Eq. (1.6) would be left to design handbooks.

This approach has the advantage of being more general. By directly specifying f_{du} the significantly high diagonal crushing limit of beams with inclined stirrups, which is not recognized in the proposed recommendations, would be considered. It would also allow to consider the effect of transverse bending moments in box girders, which reduce the diagonal crushing limit. It would allow to independently design each face of a box girder for the superposition of the shear flows due to shear and due to torsion, an advantage in bridge design. Thus, similar provisions for torsion and shear could be used for both buildings and bridges. Specifying directly f_{du} and a_o instead of τ_n would enhance transparency and intelligibility and reintroduce the concept of a diagonal compression field, which has somehow disappeared from the proposed recommendations and is not even mentioned in the general principles. It reminds the designer that he has to provide reinforcement that allows for a diagonal compression field satisfying the conditions of equilibrium and stability. The meaning and significance of this statement is illustrated in the following by some examples from the proposed design recommendations.

To achieve similar behavior of top-, side- and bottom-loaded beams with respect to all failure modes, the reinforcement provided must also allow for the same compression field for all three cases. A side- or bottom-loaded beam is statically equivalent to a top-loaded beam subject to transverse tension between the top and the point of application of the load. Hence, additional transverse reinforcement is theoretically required only between the point of application of the load and the top; however, this reinforcement must be capable of transferring the total load to the top. The linear variation of the additional transverse reinforcement suggested in Section 1.8.17 for side-loaded beams violates this requirement. This reinforcement arrangement would require a diagonal compression field with a discontinuity at the point of application of the load. The derivation shown for Beam 2 in Fig. 23 amounts to an upper bound approach only in terms of plasticity theory, because the existence of a discontinuous compression field satisfying the conditions of equilibrium and stability is not demonstrated and is by no means automatically ensured.

In Design Example 1 the authors mention that different values of θ could be used in designing different regions along the length of the beam. In Reference 50 the writer has shown that changes in θ result in a fan-shaped transition compression field, in which the diagonal compressive stresses are significantly higher than the stresses calculated on the basis of a uniform compression field. Except at supports and locations of zero shear, the value of θ used for design should therefore not be changed along the length of the beam.

According to Section 1.8.14 the transverse reinforcement required within a segment $d_v/\tan\theta$ may be determined using the average values of A_v/s and A_t/s calculated from Eqs. (1.8), (1.9), and (1.10). While this procedure is safe for distributed loads, this is not necessarily so for concentrated loads. The increase in stirrup reinforcement due to a concentrated load must be provided in full amount immediately adjacent to the concentrated load. Providing the full amount only in the next segment $d_v/\tan\theta$ is equivalent to a change in θ and leads — in addition to the stress concentrations immediately below the load, which are less problematical due to a biaxial state of stress — to diagonal stress concentrations at the bottom face of the beam in the next segment.

Because the diagonal crushing criterion is based on the assumption of a uniform compression field, reinforcement should be provided that allows the avoidance of such stress concentrations as far as possible. For this reason it is considered unfortunate that the concept of a diagonal compression field has completely disappeared from the recommendations. It must be realized that the problem of stress concentrations due to concentrated loads or changes in θ becomes much more serious for the extremely low values of θ allowed by the recommendations than for more conventional values of θ.[50]

The "segment-wise" stirrup design procedure proposed in Section 1.8.13 also appears to be not fully coordinated with the minimum reinforcement requirements. Consider a simply supported, uniformly loaded (no moving loads) beam with span to depth ratio $l/d_v = 5$ and $f'_c =$ 5000 psi. (34.5 MPa). The shear at the supports is $\tau_n = 592$ psi (4.1 MPa). Using Section 1.3.4, minimum reinforcement requirements are satisfied by providing adequate longitudinal reinforcement. Assuming $f_v = 40$ ksi (276 MPa), diagonal cracking control requirements are satisfied (see Section 1.7.4.1). Section 1.6.3 allows the use of $\theta = 21.8$ deg. Because $d_v \tan \theta = 2.5\ d_v = l/2$, the lowest value of V_u within $d_v/\tan \theta$ is zero; hence Section 1.8.13 requires no transverse reinforcement. Section 1.8.15 contains only a "may" formulation, hence it can be disregarded. Thus, no provision seems to exist that would prevent the writer from designing this beam with a maximum nominal shear stress of $\tau_n = 8.4 \sqrt{f'_c}$ without any transverse reinforcement!

The writer also has some problems with regard to the design parameter ϵ_l. What is the meaning and significance of a longitudinal strain due to shear and/or torsion, when shear and torsion occur combined with bending (what they practically always do)? It appears that ϵ_l can be chosen rather arbitrarily and independently of the actual failure strain of the longitudinal reinforcement. Does ϵ_l have no physical significance for combined bending, torsion and shear? If it has, where is this strain located? If the code intention is to achieve yielding of transverse and longitudinal reinforcement before diagonal crushing, why can the limits to θ be evaluated on the basis of an elastic longitudinal strain? What is the rationale behind a shear and torsion design with $\beta_v > 1$ in the potential plastic hinge region as in Design Example 2?

Although there is still room for improvement, the writer feels that these proposed design recommendations deserve a serious effort toward implementation. As a first step the new concepts could be adopted as an alternative design method. This would allow the profession to gradually become familiar with it and adapt design aids. This procedure has proved successful elsewhere too.

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K. S. RAJAGOPALAN*

I wish to compliment the authors for their well-written paper and for distilling from their research findings a set of design recommendations. I studied the paper several times over. Only towards the end of my several readings of the paper, did I realize that my attempts to understand the paper would have been simpler, if I had started with the design examples in the Appendix, and worked my way through the proposed code recommendations and then to the body of the paper. My reason for expressing this realization is to encourage other design professionals to try this sequence in studying this paper.

I get the impression that the authors require the use of reinforcement in all members subject to shear and or torsion, to resist the shear forces. I recognize that the presence of reinforcement might be necessary most of the time; I also appreciate the fact that the presence of reinforcement would induce better ductility all the time. However, I am not convinced that the reinforcement for resisting shear and torsion is necessary all the time. I am aware of a sufficient number of flexural members without any web reinforcement that have functioned well over a number of years of design life. I would recommend to the authors that they should include some provision in their design recommendations that would tone down the effect of Section 1.3 requiring reinforcement in all cases of shear and or torsion.

It seems to me that the authors discourage (to put it mildly) the use of single-legged stirrups to resist shear (Section 1.6.1.1 of the proposed design recommendations require the use of 0.5 b_w for b_v). In the precast concrete industry, double-tees with thin webs represent a giant share of the market. To require the use of a double stirrup in these members would not be very practical. To penalize the design of such members would lead to serious economic consequences. Is there really a definite need to tighten the shear provisions in these members, especially in view of the lack of any serious shear problem in the structures that are already in service?

In the use of Eq. (1-6) of the proposed design recommendations, the authors seem to suggest that ϵ_l could be assumed to equal to ϵ_{ly}. In prestressed concrete members having high tensile strands for longitudinal reinforcement (some strands with high initial tension), the rationale of this assumption is not obvious to me. Is it always a safe assumption?

I concur with the authors that the presence of physical models is very helpful to the designer to visualize the action of the structure. I commend their lucid explanations of the various models of shear and torsion, as illustrated in Figs. 2, 6, 9, 18, 19, 21, 22, 23, and 24. However, I cannot share their sentiment when they castigate the present American practice as one "that lacks an understandable central philosophy."

I have recognized the following central philosophy in American practice that uses the interaction approach. The structural strength of a member in resisting shear or torsion consists of some concrete contribution that has been observed in tests. This concrete contribution can be augmented by the judicious use of web and longitudinal reinforcement. In the case of combined shear and torsion there is an observed interaction. Recognition of such interaction is not limited to shear and torsion; even the authors allude to such an interaction in Fig. 13. Indeed, such interactions are not limited to structural concrete.

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HIMAT T. SOLANKI*

The authors are to be congratulated for their outstanding presentation of a complete design proposal for shear and torsion for prestressed concrete beams. Several design methods for prestressed and reinforced concrete are also available.[57, 58]

Based on the authors' proposed design method, the writer has studied several beams including members with variable depth[59–62] and found the results to be in excellent agreement between the theoretical and experimental analysis.

The writer believes that Eq. (7) could be simplified by using the balanced failure condition.[63]

The writer disagrees with the authors' combined loads interaction Eq. (26). The interaction equation for combined shear and moment could be expressed as:[32, 33]

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$$\frac{M_{cr}}{M_{ocr}} + \left(\frac{V_{cr}}{V_{ocr}}\right)^2 = 1 \quad (33)$$

Similarly, the interaction equation for combined bending and torsion could be expressed as:[64, 65]

$$\frac{M_{cr}}{M_{ocr}} + \left(\frac{T_{cr}}{T_{ocr}}\right)^2 r = 1 \quad (34)$$

where r is the ratio of the strength of the top to the strength of the bottom reinforcement.

Lastly, as a general expression, the interaction equation for combined bending, torsion, and shear could be expressed as:[66]

$$\frac{T_{cr}}{mM_{ocr}} + \left(\frac{T_{cr}}{T_{ocr}}\right)^2 r + \left(\frac{T_{cr}}{eV_{ocr}}\right)^2 = 1 \quad (35)$$

where $m = T_{cr}/M_{cr}$ and $e = T_{cr}/V_{cr}$.

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HRISTA STAMENKOVIC*

The authors are to be commended for their detailed study of shear and torsion in reinforced concrete beams. Nevertheless, some clarification is necessary to comprehend their design proposals and to avoid possible confusion.

In their discussion of "Control of Diagonal Cracking," the authors stated that, "The compression field theory, because it neglects concrete in tension, predicts that the transverse reinforcement will commence straining as soon as the load is applied. In reality, the transverse reinforcement will not begin to strain until cracking occurs." This statement gives the impression that some or all of the stirrups will be in tension as soon as some cracks have developed in a reinforced concrete beam. However, such a mechanism is incorrect because each stirrup in a reinforced concrete beam remains free of strain until it is crossed by cracks. Consequently, only stirrups crossed directly by cracks become loaded in tension while all other

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stirrups remain unstressed, even though the beam might be close to collapse. To support the above contention about the stressed condition of stirrups, let me quote from some recognized publications:

1. "When diagonal cracking occurs, web bars intersected by a crack immediately receive sudden increases in tensile stress in the vicinity of the crack, while web bars not intersected by diagonal cracks remain unaffected."[67]
2. "The web reinforcement being ineffective in the uncracked beam, the magnitude of shear force as stress which causes cracking to occur is the same as in a beam without reinforcement."[68]
3. "The largest strains measured occurred in *each stirrup* where it was crossed by the potential failure crack. It is seen that during the first five cycles of loading some 40 kips were resisted by mechanisms not involving stirrups."[69]
4. "Stirrups cannot be counted on to resist shear if they are not crossed by the inclined crack."[70]
5. "Although web reinforcement increases the ultimate shear strength of a member, it contributes very little to the shear resistance prior to the formation of the diagonal tension cracks."[71]

Because of the effect of Poisson's ratio, it is possible that negligible stresses develop in stirrups, but such stresses are much less than any diagonal stresses which must be present during diagonal cracking of concrete if diagonal tension indeed exists. Also, small stresses will be induced at stirrups by flexural compression and tension due to binding forces developed by the cement gel as bond between the aggregate particles and stirrups that are present.

Because of the above facts concerning the stressed condition of vertical stirrups and the authors' quoted statements, this writer is asking the authors for their interpretation and clarification of the following concepts in order to support their theory:

1. How can any concept of truss analogy be used, as the authors suggest, if a reinforced concrete beam can collapse only after the first crack has developed so that only one stirrup goes into tension while all other stirrups remain free of any tension?
2. How can a truss mechanism be triggered to work if only one stirrup is engaged in stress distribution (as a member of a truss) where other stirrups do not participate in load distribution?
3. Can only one, or maybe a couple of stirrups, engaged in stress distribution, achieve the essential concept of any truss known as "rigidity?"
4. How can the authors explain the phenomenon that any stirrup in tension will cause its own compressed concrete strut to co-exist with flexural tension in a flexural tensile zone of a bent reinforced concrete beam at the same time and at the same location?
5. How can flexural tensile stresses in a "compressive strut" be converted into compression if the neutral axis of a bent beam is present?
6. How can the neutral axis ever be eliminated in a bent beam (i.e., before the beam's collapse) in order to function as a truss?

It is hoped that the answer to the above questions will enable the authors to further support their shear and torsion design proposals for reinforced and prestressed concrete beams and help readers obtain a clearer understanding of the authors' ideas.

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CLOSURE by MICHAEL P. COLLINS* and DENIS MITCHELL†

The authors would like to take this opportunity to express their gratitude for the many useful comments and suggestions they have received concerning the PCI JOURNAL paper. In particular they would like to thank those engineers who have written formal discussions.

The primary objective of the paper was to demonstrate that it is possible to develop shear and torsion design recommendations from rational models. The design recommendations presented in the paper are comprehensive and because of this they appear rather complex. It must be appreciated, however, that the recommendations are attempting to cover a wide variety of design situations such as: prestressed and non-prestressed beams; shear and/or torsion; strength and serviceability considerations; inclined tendons and stirrups; deep beams; brackets and corbels; axial load (tension or compression); variety of cross-sectional shapes (including bridge girders); and detailing considerations.

In this first attempt the authors' prime concern was to arrive at a workable set of design recommendations that were both comprehensive and conservative. The authors feel sure that with the help of suggestions from other engineers it will prove possible to simplify the design recommendations and hopefully make the rational models on which they are based more evident.

Professors Cederwall and Elfgren ask for some discussion of the procedures used for determining the angle of inclination, θ, of the diagonal compression. The assumption made is that the angle of inclination of the diagonal compressive stress in the concrete (i.e., the principal compressive stress) coincides with the angle of inclination of the principal compressive strain. The direction of the principal compressive strain depends on the relative magnitudes of the longitudinal tensile strain, ϵ_l, the transverse tensile strain, ϵ_t, and the diagonal compressive strain, ϵ_d.

While the meaning of the term strain in an elastic homogeneous (i.e., uncracked) member is quite clear, the meaning of strain in a cracked reinforced concrete member perhaps deserves some discussion. The side view of a diagonally cracked member is shown in Fig. A. If the strains on this cracked surface were measured over very small base lengths (i.e., local strain measurements), severe discontinuities would be observed (e.g., very low values between cracks and very high values across the cracks).

On the other hand, if the strains were measured over base lengths that were several times the crack spacing (i.e., average strain measurements) consistent readings would be obtained [see Fig. A(a)]. These average strains in the various directions must be related to one another by the requirements of compatibility. These requirements can be represented by a Mohr's circle of strain [see Fig. A(b)]. From the Mohr's circle of strain the following equation for the angle of inclination of the diagonal compression can be derived:

$$\tan^2 \theta = \frac{\epsilon_l + \epsilon_d}{\epsilon_t + \epsilon_d}$$

It is important to realize that the angle of inclination of the diagonal compression does not remain constant throughout the life of a beam. Fig. B(a) compares the predicted and observed[46] angles of principal compressive strain for a non-prestressed concrete beam, PT5, loaded in torsion. Four rather distinct phases can be observed in the life of the member. Prior to cracking the angle remains constant at 45 deg, the value predicted by the traditional elastic theory. Because the beam contains more longitudinal than transverse steel, after cracking it is stiffer in the longitudinal direction (i.e., $\epsilon_l < \epsilon_t$) and hence θ becomes smaller than 45 deg. The third stage commences with yielding of the transverse steel which results in a continuous decrease of θ. Finally, after the longitudinal steel yields the angle θ remains constant until failure.

Fig. B(b) compares the predicted and observed[46] angles of principal compressive strain for a prestressed concrete member, P2, loaded in torsion. Due to prestressing there is a longitudinal compressive stress in the concrete (i.e., $\theta = 0$) prior to the application of torsion. As the torsional load is increased the angle of inclination of the principal compressive stress continuously increases until the torque is high enough to yield the transverse reinforcement. In view of the changes in θ it seems inappropriate to adopt one value of θ for all stages in the life of a beam.

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With respect to the influence of θ on the torsional stiffness, the determination of θ is an integral part of the determination of the torque-twist response. Fig. C compares the predicted and measured torque-twist responses of Beam PT5 (non-prestressed) and Beam P2 (prestressed). For the non-prestressed concrete member it can be seen that in the regions where θ was constant the torsional stiffness (i.e., the slope of the torque-twist curve) is also constant. For the prestressed concrete beam there is a continuous deterioration in torsional stiffness after cracking.

Professors Cederwall and Elfgren believe that it is very conservative to assume that the concrete outside of the hoops spalls off in a beam subjected to torsion. Fig. D compares the torque-twist curves of two non-prestressed beams, PT5 and PT6, which had identical reinforcing cages and similar concrete strengths. Beam PT5 had a concrete cover of 1/16 in. (1.6 mm) whereas Beam PT6 had a concrete cover of 1 9/16 in. (40 mm).

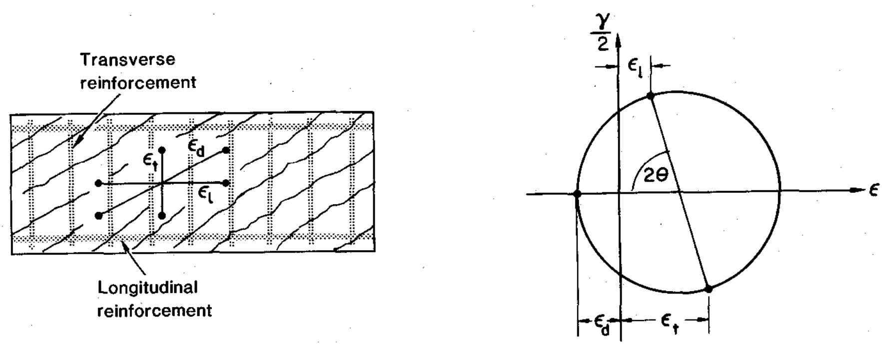
The test results for these two beams indicate that while the concrete outside of the hoops significantly affects the cracking torque of the member, it has no measurable influence on the failure torque. A photograph of Beam PT6 at failure is shown in Fig. 4 of the paper in which significant spalling of the concrete cover can be seen. It is difficult to understand how this spalling could be prevented by "detailing regulations."

The authors certainly agree with Professor Dilger's observation that the design proposals offer more freedom to the designer in determining the amount of transverse reinforcement needed to resist a given load. "Trade-offs" can be made between the amounts of longitudinal reinforcement and transverse reinforcement within certain limits. This flexibility can be of particular value when an engineer is faced with assessing the structural adequacy of an existing structure which may apparently be deficient in transverse steel but have more than enough longitudinal steel.

Professor Dilger states that the crack control requirement "needs some more study." He believes that the load at the formation of shear cracks has been overestimated and observes that for non-prestressed concrete beams $V_{ocr} = 4\sqrt{f'_c}b_w d$. The crack control expression [Eq. (31) or Eq. (1-7) of the recommendations] uses the term V_{cr}, not V_{ocr}, that is, the cracking shear under combined loading rather than the pure shear cracking strength. For non-prestressed concrete beams, V_{cr} will usually be taken as $2\sqrt{f'_c}b_w d$, as given in Section 1.4.6 of the recommendations.

It is possible that for the control of diagonal cracking in bridge members it may be appropriate to use a somewhat more conservative value of the limiting strain in the transverse reinforcement as suggested by Professor Dilger.

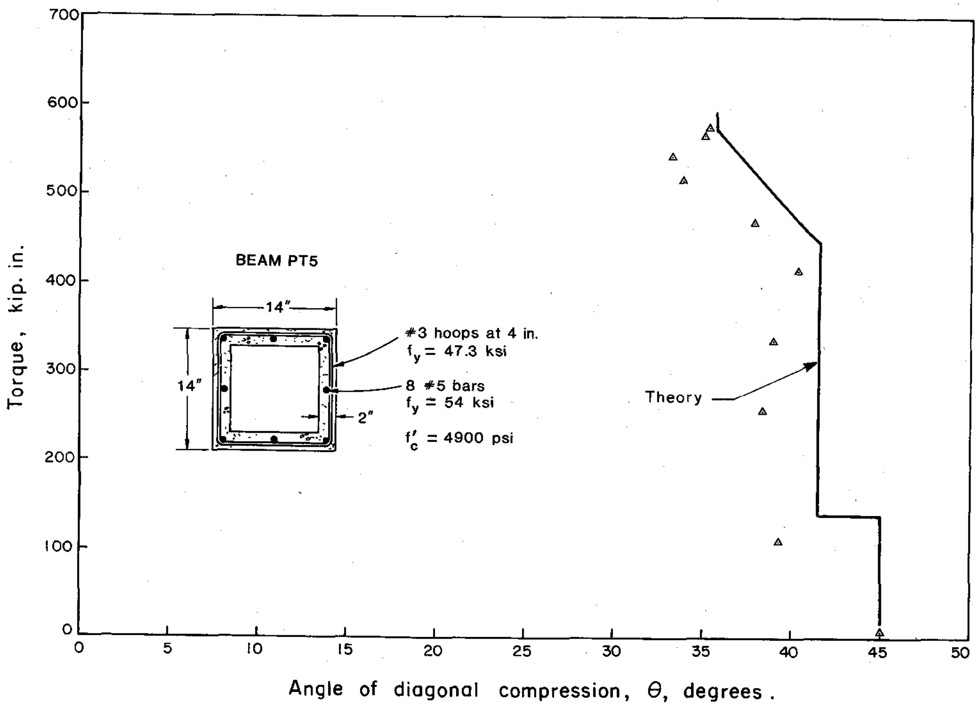
Professor Gergely expresses some concern about the complexity of the method. As stated above, the authors also hope that it will prove possible in the future to simplify the design recommendations. However, in comparing the complexity of the current design proposals with the shear and torsion provisions of the ACI Code allowance should be made for the fact that the familiarity of the code equations tends



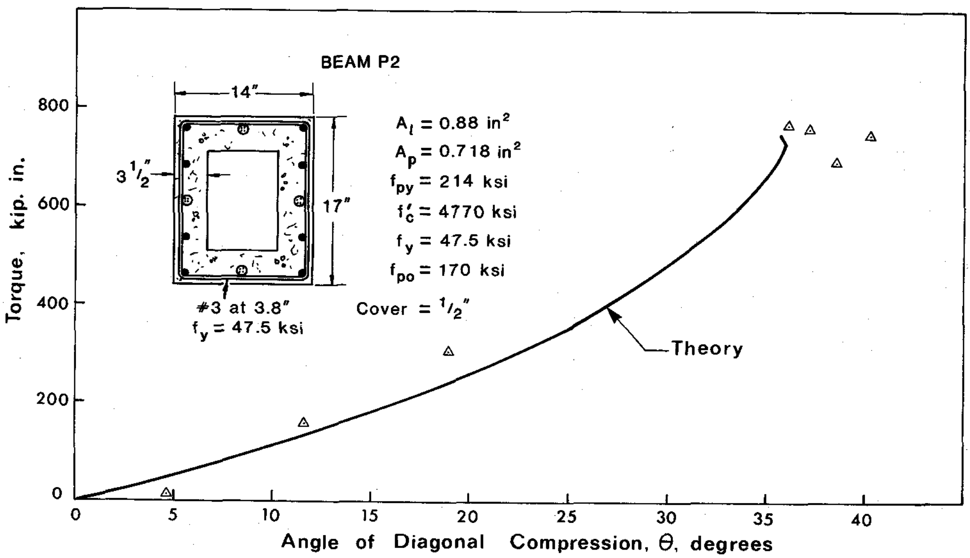
(a) Diagonaly Cracked Reinforced Concrete

(b) Mohr's Circle of Strain

Fig. A. Compatibility conditions for diagonally cracked reinforced concrete.

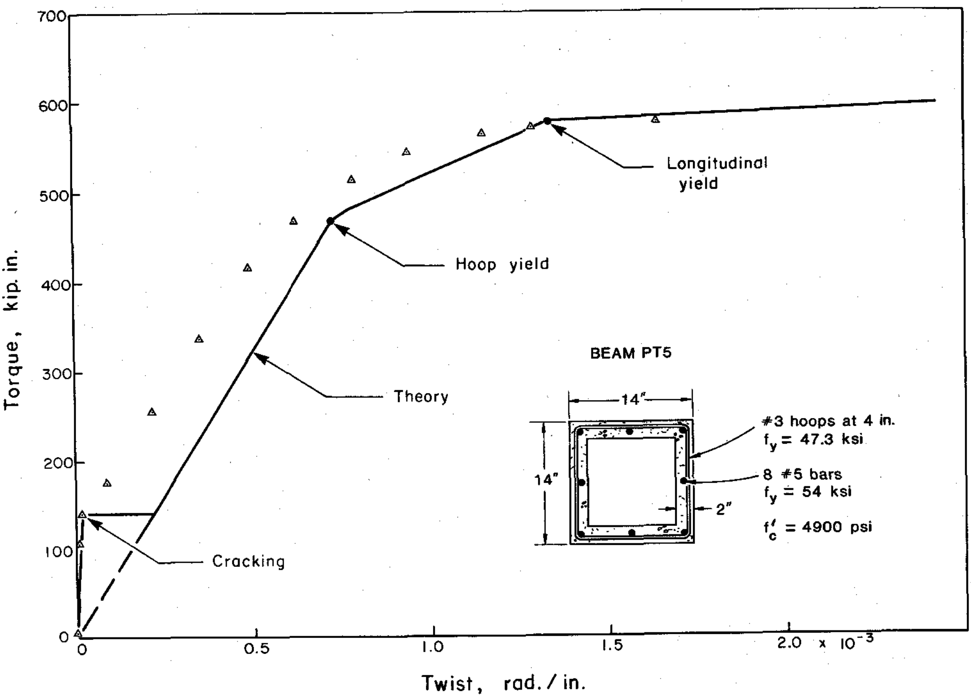


(a) Non-Prestressed Beam

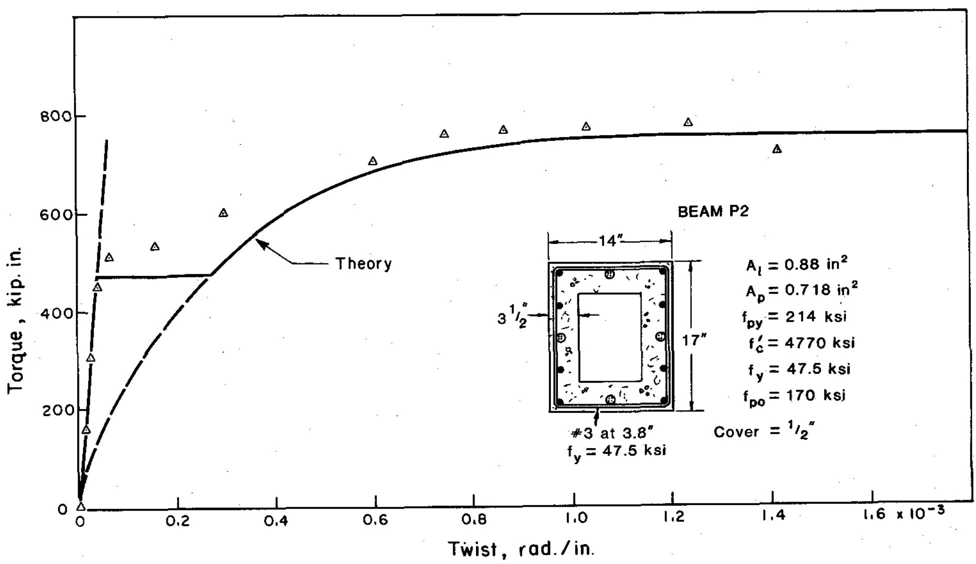


(b) Prestressed Concrete Beam

Fig. B. Comparison of predicted and observed angle of principal compressive strain.



(a) Non-Prestressed Beam



(b) Prestressed Concrete Beam

Fig. C. Comparison of predicted and observed torque-twist response.

to disguise their complexity. In this regard it is of interest that when Professor Dilger applied the proposed method to the design of a bridge he found that it was "less cumbersome than the existing approach."

The authors agree with Professor Gergely that comparisons of the theory with "carefully performed and well documented tests" are "highly useful." In the paper it was impossible to present all of the experimental data used in the development of the models. Many more comparisons of the analytical predictions with experimental results can be found in References 16, 17, 18, 21, 23, 24, 25, 46 and 72. In this regard it is pleasing to learn that additional comparisons made by Mr. Solanki have found "an excellent agreement between the theoretical and experimental analysis."

To give an indication of the type of comparisons that can be made between experimental results and analytical predictions, Table A was prepared. The eight large reinforced concrete specimens listed in this table were tested as parts of continuous frames. In each case the loading was arranged so that the moment was zero at the midpoint of a 5-ft (1.52 m) long test region. The shear strength predictions listed in Table A were obtained by using Eqs. (1), (3), (23) and (24) of the paper along with the assumption that for these symmetrically reinforced beams subjected to low moment:

$$\Delta N = (A_l + A_p) E_s \epsilon_l + A_p f_{po}$$

It can be seen from Table A that the failure shear of these beams is predicted accurately.

Professor Kordina and Dr. Teutsch, together with Professor Mueller, are to be complimented on their thoughtful and very detailed discussions of the paper. These three discussers raise the point that in the model for torsion the full cylinder strength of the concrete is used whereas in the model for shear a reduced compressive strength is employed to allow for the detrimental effect of having severely cracked and severely deformed concrete. The authors agree with the discussers that this is inconsistent and in a fully rational model the same concrete stress-strain characteristics would be used for both torsion and shear. The torsion model and the shear model were developed at different times and it so happens that the torsion model, which was developed first, appears to be less sensitive to the actual stress-strain characteristics of the concrete. Perhaps this is due to the large strain gradient through the thickness of the walls for beams in torsion (see Fig. 5 of the original paper).

Recent research[73] on the stress-strain characteristics of diagonally cracked concrete indicates that the presence of high shear strains does not degrade the compressive stiffness of lightly stressed concrete. Hence, for a beam in torsion only the highly stressed outer regions would be affected by this degradation. On the other hand, for a beam in shear the entire web width is uniformly stressed and therefore at failure all of the concrete would be affected.

Professor Kordina and Dr. Teutsch regard spalling as a post-failure "secondary phenomenon." The test results of Beams PT5 and PT6 shown in Fig. D convince the authors that spalling can occur before the maximum torque is reached and hence this possibility should be taken into account in predicting the failure load. It is possible that in beams with small covers the tensile stresses in the concrete will enable the cover to stay on up to the failure load. The authors felt that it was not prudent to rely on these concrete tensile stresses and hence neglected the cover.

Professor Kordina and Dr. Teutsch are of the opinion that the authors' model significantly underestimates the depth of compression in torsion (t_d in Fig. 5). This depth of compression can be estimated experimentally if strain measurements are taken at different depths below the surface of the concrete. For Beam PT5, readings such as these indicated a value of t_d of approximately 1.5 in. (38 mm) at failure while the value of t_d predicted by the theory was 1.2 in. (30 mm). This difference is probably due to neglecting the concrete strength degradation. For example, if α_1 in Eq. (10) was taken as 0.7 instead of 0.85, then the predicted value of t_d would be 1.5 in. (38 mm). It is worth noting that the prediction of torsional strength is relatively insensitive to this possible error so that for Beam PT5 changing the value of t_d by 25 percent only changes the predicted torsional strength by 3 percent.

The discussers also question the appropriateness of neglecting the concrete cover for beams subjected to shear. To investigate the influence of cover on shear capacity, two similar non-prestressed box beams, SA3 and SA4, were tested.[74] These two beams (see Fig. E and Table A) had identical concrete dimensions, reinforcement areas and material properties. The only difference between the two beams was the location of the stirrups within the wall thickness. The stirrups in Beam SA3

Table A. Comparisons of observed and predicted shear strengths.

Beam	Dimensions			Concrete	Stirrups			Longitudinal steel		Prestress		Shear strength	
	b_w (in.)	b_v (in.)	d_v (in.)	f'_c (psi)	A_v (in.^{2})	s (in.)	f_y (ksi)	A_l (in.^{2})	$A_l f_p$ (kips)	$A_p f_{po}$ (kips)	f_{pc}/f'_c	Observed (kips)	Predicted (kips)
SA1*	12.0	11.5	20.9	6000	0.22	8.57	54.6	4.80	312	0	0	84	94
SA2*	6.0	5.5	20.9	6000	0.22	8.57	54.6	4.80	312	0	0	73	76
SA3*	6.0	5.5	20.9	5835	0.22	2.85	54.6	14.40	756	0	0	163	151
SA4*	6.0	3.0	20.9	5835	0.22	2.85	54.6	14.40	756	0	0	120	115
SK1†	12.0	10.13	20.75	3900	0.22	4.0	58.0	6.32	404	332	0.277	155	145
SK2†	7.25	5.38	20.75	3900	0.22	4.0	58.0	6.32	404	332	0.368	119	113
SK3†	12.0	11.25	20.75	4090	0.22	4.0	58.0	12.64	809	0	0	167	147
SK4†	7.25	6.50	20.75	4090	0.22	4.0	58.0	12.64	809	0	0	135	118

*From Reference 74.
†From Reference 75.
Note: 1 in. = 25.4 mm; 1 in.^{2} = 645.16 mm^2;
1 kip = 4.448 kN; 1 psi = 0.006895 MPa.

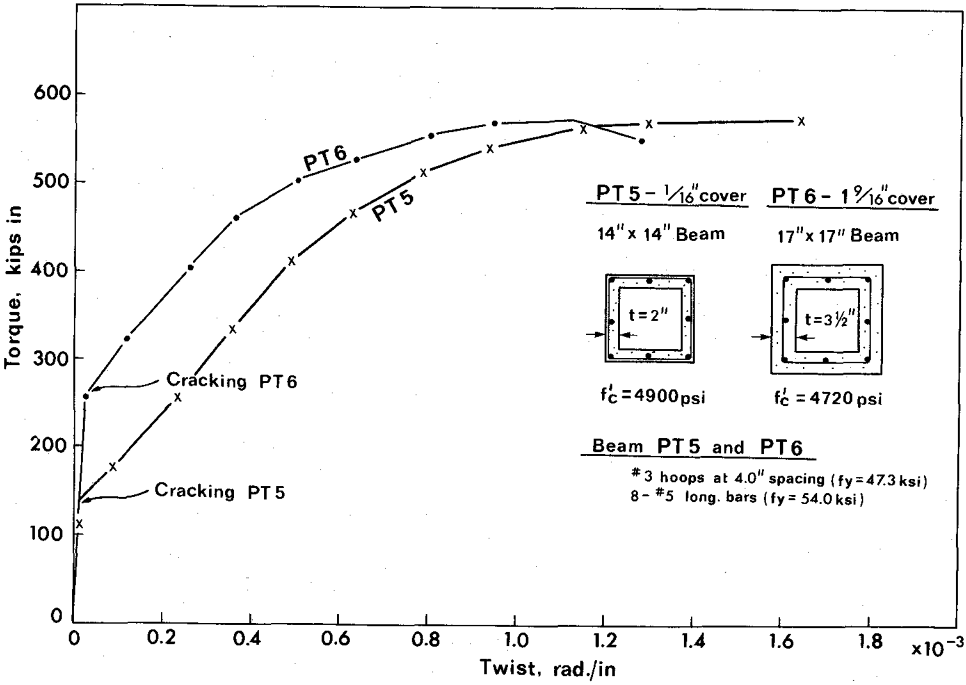


Fig. D. Influence of concrete cover on torque-twist response.

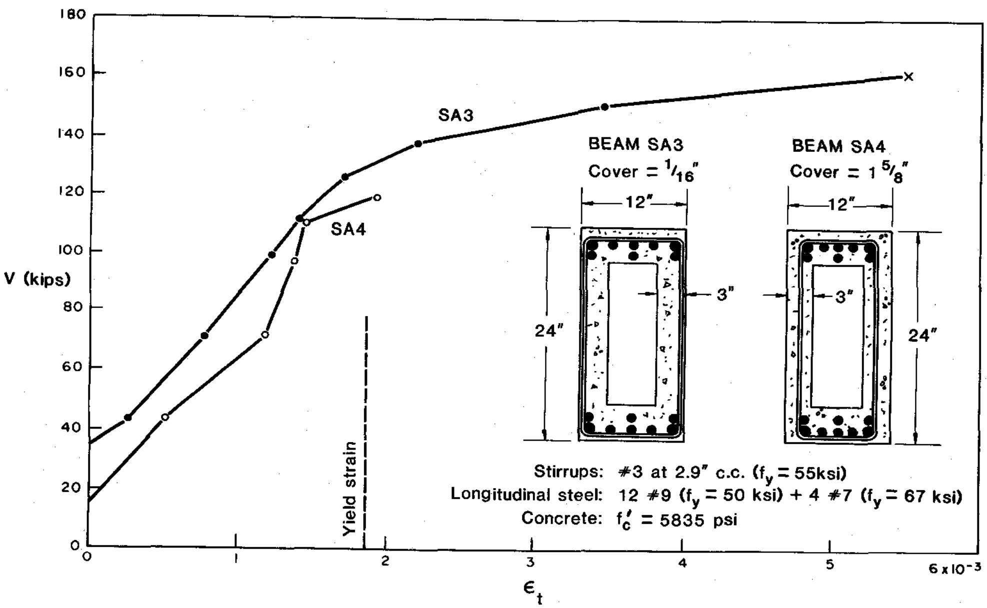


Fig. E. Influence of concrete cover on shear response.

had a cover of 1/16 in. (1.6 mm) while in Beam SA4 the stirrups had a cover of 1⅝ in. (41 mm).

Fig. E shows the measured relationships between the applied shear and the stirrup strains for these beams. It can be seen that the member with the larger concrete cover had a significantly lower shear strength. If the concrete cover had not spalled these two beams would have had the same shear strength. The compression field theory predicts that beam SA3 would fail at a shear of 151 kips (672 kN) with the strain in the stirrups at failure being 5.2 $\times 10^{-3}$. If it is assumed that all of the concrete cover has spalled off, then the compression field theory predicts that Beam SA4 fails at a shear of 103 kips (458 kN) with a stirrup strain of 2.0×10^{-3}.

By comparing these two predictions with the results in Fig. E, it can be seen that they are reasonably accurate if somewhat conservative. For Beam SA4 it would be very unconservative to assume that the concrete in the cover was fully effective. The appearance of Beam SA4 after failure is shown in Fig. 8 of the paper.

There seems to have been some misunderstanding of the purpose of Eq. (19) which defines a nominal shear stress. The term $T_n p_h / A_{oh}^2$ appears in the expressions derived from the theoretical model. The authors decided to label this term, which has units of stress, the nominal shear stress, τ_n. It is a convenient indicator of the intensity of torsional loading since it includes both the magnitude of the torsion and the important cross-sectional dimensions. Professor Kordina and Dr. Teutsch have attempted to transform this nominal shear stress definition into an expression for the depth of compression a_o by using the stress equations for a thin-walled hollow box cross section. Eq. (19) is merely a convenient definition. It is Eq. (20) which gives the theoretical expression for the depth of compression, a_o.

The authors agree with Professor Kordina and Dr. Teutsch that it is necessary to insure that the actual wall thickness of a thin-walled hollow box section is not too small. This limitation is given in the design recommendations as Section 1.8.12.

In the design recommendations members subjected to combined loading were treated by a simple superposition procedure. As stated in the paper, this procedure was developed for "under-reinforced beams" and Section 1.9.2 of the recommendations was intended to exclude beams over-reinforced for flexure. The authors agree with Professor Kordina and Dr. Teutsch that it would be better to directly check the possibility of a bending compression failure for members under combined loading. To this end the authors suggest the following modification to Section 1.9.2:

1.9.2 — Cross-sectional properties shall be chosen such that premature flexural crushing failures in the concrete are avoided.

1.9.2.1 — For members not subjected to axial compression the requirements of Section 1.9.2 will be satisfied if the ratios of longitudinal reinforcement satisfy the requirements of Sections 10.3.3 and 18.8.1 of ACI 318-77.

1.9.2.2 — For members subjected to axial compression the requirements of Section 1.9.2 will be satisfied if the cross section is designed to resist the factored axial moment, M_u, the factored axial compression, N_u, together with an additional factored axial compression, ΔN_u, acting at mid-depth of the stirrups where ΔN_u is given by Eq. (1-12).

The authors were impressed by Professor Mueller's knowledgeable, thoughtful and critical review of the paper. They are in full agreement with his assessment of what are the salient features of the new proposals.

Professor Mueller has some questions about the diagonal crack control requirements. Based on Fig. 17, he has concluded that the recommendations imply that "providing more stirrups . . . is relatively ineffective to control diagonal cracking." The effect of increasing the amount of stirrups can be seen more clearly in Fig. 25. The compression field theory prediction for the non-prestressed beams containing 60 ksi steel is governed by the crack control requirements for values of $A_v f_y/(b_w s f'_c)$ between 0.03 and 0.25. Within this range it can be seen that increasing the amount of transverse steel significantly increases the allowable shear.

Professor Mueller asks for some experimental verification of the diagonal crack control expressions. By rearranging Eqs. (30), (C2), (C5) and (1-8) the following expression for the strain in the transverse steel at service load can be obtained:

$$\epsilon_{te} = \left[1 - \left(\frac{V_{cr}}{V_{se}} \right)^3 \right] \frac{V_{se}}{E_s} \; \frac{s}{A_v d_v} \times \sqrt{\frac{A_v f_y d_v}{s V_n} \left(1 - \frac{f_y}{30} \; \frac{f_{pc}}{f'_c} \right)}$$

Table B. Comparisons of observed and predicted stirrup strains at service load.

Beam	Observed V_{cr} (kips)	Observed V_n (kips)	$V_{se} = 0.55\, V_n$ (kips)	$\epsilon_{te} \times 10^3$ Observed (avg. values)	Predicted
SA 1	44	84	46	0.27	0.25
SA 2	16	73	40	0.70	1.52
SA 3	36	163	90	1.11	1.32
SA 4	17	120	66	0.98	1.19
SK 1	80	155	85	0.08	0.20
SK 2	61	119	65	0.17	0.15
SK 3	44	167	92	1.16	1.56
SK 4	44	135	74	0.90	1.25

This expression was used to predict the stirrup strains for the two series[74-75] of large, well instrumented beams described in Table A. The comparisons between the predicted and measured values of transverse strain were made at a load corresponding to 55 percent of the failure load. It can be seen in Table B that the theoretical predictions agree reasonably well with the experimental observations.

Professor Mueller also asks for further elaboration on the diagonal crushing criterion and is particularly interested in seeing this criterion supported "with results from actual beam tests particularly I-beams." It is the authors' opinion that the failure criterion for diagonally cracked concrete cannot be adequately investigated with beam tests. Too many assumptions need to be made in interpreting the results of such tests. In an effort to investigate this aspect more thoroughly a large experimental program is currently underway at the University of Toronto. The preliminary results of this program are available in Reference 73. While the failure criterion will undoubtedly be modified as a result of these new tests we do not believe that this will have a significant effect on Eq. (1-6) of the design recommendations.

The authors thoroughly agree with Professor Mueller that ideally the shear and torsion recommendations should be formulated in a manner similar to those for flexure and axial load. However, the authors felt that initially it was necessary to present the method in the form of immediately usable design expressions even if this meant that the underlying principles became somewhat disguised.

The authors agree with Professor Mueller that to achieve similar behavior of top and side loaded beams the side loaded beams should contain additional transverse reinforcement between the point of application of the load and the top of the beam. It was, however, felt that an additional set of partial depth stirrups supplementing the usual full depth stirrups would be awkward in practice. The approach taken in the design proposals of providing additional full depth stirrups was taken from Reference 76.

The authors are not as concerned as Professor Mueller about changes in θ along the length of the beam though they agree that it would be unwise to change the value too abruptly.

Professor Mueller has given an interesting example of applying the design proposals to a simply supported uniformly loaded deep beam. It is surely one of the advantages of the design proposals that for such a beam they will focus the engineer's attention on the critical aspects of the design, that is, the distribution and anchorage of the longitudinal reinforcement and the bearing details. Such a beam will act essentially as a tied arch and thus will not require transverse reinforcement for strength. The design provisions for this case would, however, require transverse reinforcement spaced at 12 in. (300 mm) for control of diagonal cracking (see Section 1.7.4b).

The design proposals treat beams subjected to combined torsion, shear and bending by a superposition approach. Part of this approach involves treating the beam as though it were subjected to pure torsion or pure shear. It is at this stage of the process that the design parameter, ϵ_l, appears. It is not the intention of the recommendations to require that the longitudinal reinforcement yields prior to diagonal crushing of the concrete since the yielding of the transverse reinforcement will provide adequate ductility (see Section 1.6.2). In certain cases (e.g., support regions of prestressed concrete beams) it may be advantageous to use a value of ϵ_l considerably less than that corresponding to the yield strain. This would enable a lower value of θ to be used and hence less transverse steel would be required. While it appears that ϵ_l "can be chosen rather arbitrarily," it should be recognized that there is a penalty for choosing low values of ϵ_l. Such a choice could result in very large amounts of longitudinal reinforcement being required in members.

In reply to Dr. Rajagopalan's query regarding "flexural members without any web reinforcement," the authors would like to emphasize that the purpose of the paper was to present design recommendations for members containing web reinforcement. They agree that in certain circumstances it may be appropriate to use members without web reinforcement. However, the authors have no special recommendations about the design of such members.

The authors have two main concerns about the use of single-legged stirrups in members such as single and double-tees. To be fully effective the transverse steel must be properly anchored around the longitudinal steel and this is difficult to achieve with a single-legged stirrup. In addition experimental evidence (see Figs. 4, 8, D and E) strongly suggests that the unrestrained concrete cover is not fully effective. These reasons lead to the more stringent requirements incorporated into the design proposals.

In response to another of Dr. Rajagopalan's questions, it will always be safe to use $\epsilon_l = \epsilon_{ty}$ in Eq. (1-6).

The authors are grateful to Mr. Solanki for using the design recommendations to predict the behavior of several test beams. However, the authors do not agree with Mr. Solanki's suggestions for changes to interaction Eq. (26). It must be appreciated that this equation is concerned with predicting cracking loads. Mr. Solanki seems to be suggesting that equations derived for predicting the ultimate strength of under-reinforced beams be used to predict cracking.

In reply to Mr. Stamenkovic's questions, the authors would like to make the following observations:

Prior to cracking a reinforced concrete beam resists shear by a combination of tensile and compressive stresses in the concrete. In this uncracked stage the beam's behavior can be predicted by traditional elastic theory (see the line labelled "uncracked prediction" in Fig. 10 of the paper). During this stage the transverse reinforcement in the beam will be unstressed. After the diagonal cracking has fully developed (see Fig. 8 of the paper) the beam resists shear by compressive stresses in the concrete and tensile stresses in the steel.

It is in this fully cracked stage that the beam's behavior can be predicted by the compression field theory (see the line labelled "cracked prediction" in Fig. 10 of the paper). There will be a transition stage as the beam goes from the uncracked state to the fully cracked stage. During this transition stage many of the stirrups will not be crossed by diagonal cracks and consequently will be unstressed.

If a beam collapses immediately "after the first crack has developed" it either contains no web reinforcement or the web reinforcement present violates either the detailing requirements or the minimum reinforcement requirements. The "truss analogy" was never intended to apply to such members.

It is possible for diagonally compressed concrete to co-exist with stretched longitudinal steel because these strains are in different directions (see Fig. A).

Flexure will compress the top of the beam and stretch the bottom of the beam. Since ϵ_l will no longer be constant over the depth of the beam, θ will also change over the depth being smaller near the top of the beam and greater near the bottom [see Eq. (7) and Fig. 14 of the paper]. In the simple superposition design procedure proposed, this variation is neglected and an average value of θ is used (see Fig. 14).

In ending this closure the authors would like to publicly express their deep gratitude for the magnificent editorial job performed by the staff of the PCI JOURNAL.

(See next page for list of References and Errata.)

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Errata

The following corrections should be made in the paper being discussed (September-October 1980, PCI JOURNAL, pp. 32-100).

Fig. 23—In the equations for V_{u1}, V_{u2} and V_{u3}, the 2 in the denominators should be eliminated.

Fig. 27—In the upper figure the arrow labelled "support face" is pointing to line representing the center of the support.

Fig. 27—In the lower Figure the term labelled $0.5\, d_v \frac{\Delta N_u}{0.35}$ should be labelled $0.5\, d_v \frac{\Delta N_u}{0.85}$.

Design Example 2, Step 8—The calculation for d_{ve} should read:

$$d_{ve} = 1.5 + 6 \tan 48.4 = 8.3 \text{ in.}$$

Reference 28 should refer to V.99, June 1973, pp. 1091-1187.

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DISCUSSION NOTE

The Editors welcome discussion of papers published in the PCI JOURNAL. The comments must be confined to the scope of the article being discussed. Please note that discussion of papers appearing in this current issue must be received at PCI Headquarters by July 1, 1982.