READER COMMENTS

Serviceability Behavior of Post-Tensioned Beams*

by Edward G. Nawy and Jim Y. Chiang

Comments by Paul H. Kaar, Julius G. Potyondy, A. S. Prasada Rao, Himat Solanki, and Authors.

PAUL H. KAAR†

The authors' test series consisted of beams with effective depths from 7.25 to 9.41 in. (184 to 239 mm). These beams had a support span of 7½ ft (2.29 m).

In Appendix A, p. 94, the design example used a beam with an effective depth of 18 in. (457 mm). This depth is considerably larger than that employed in the test specimens. Applying the expressions derived entirely from the test results of small units to practical-sized members is misleading. Would it not be better to consider experimental data from beams of sizes larger than the small laboratory specimens before extrapolating to full-sized beams?

Researchers[21,22] have shown that crack widths of conventionally reinforced concrete beams are proportional to a fractional root of the scale factor, not directly proportional to the scale factor. It appears the authors are assuming direct similitude in their expressions for spacing and maximum crack widths.

It has also been shown through research[23] that non-prestressed steel in a prestressed member has a pronounced effect on crack formation. The non-prestressed steel tends to distribute cracks and restrict their progress. The authors' algebraic expressions only apply to specimens of the size and reinforcement arrangement tested.

References

21. Gergely, P., and Lutz, L. A., "Maximum Crack Width in Reinforced Concrete Flexural Members," *Causes, Mechanism, and Control of Cracking in Concrete,* (SP-20), American Concrete Institute, Detroit, 1968, pp. 1-17.

22. Kaar, Paul H., "High Strength Bars as Concrete Reinforcement—Part 8. Similitude in Flexural Cracking of T-Beam Flanges," Journal of the PCA Research and Development Laboratories, V. 8, No. 2, May 1966, pp. 2-12. See also PCA Development Department Bulletin D106.

23. Shaikh, A. F., and Branson, D. E., "Non-Tensioned Steel in Prestressed Concrete Beams," PCI JOURNAL, V. 15, No. 1, February 1970, pp. 14-42.

JULIUS G. POTYONDY‡

I wish to congratulate the authors on showing the advantages of using non-prestressed steel in post-tensioned concrete beams for controlling crack development. The

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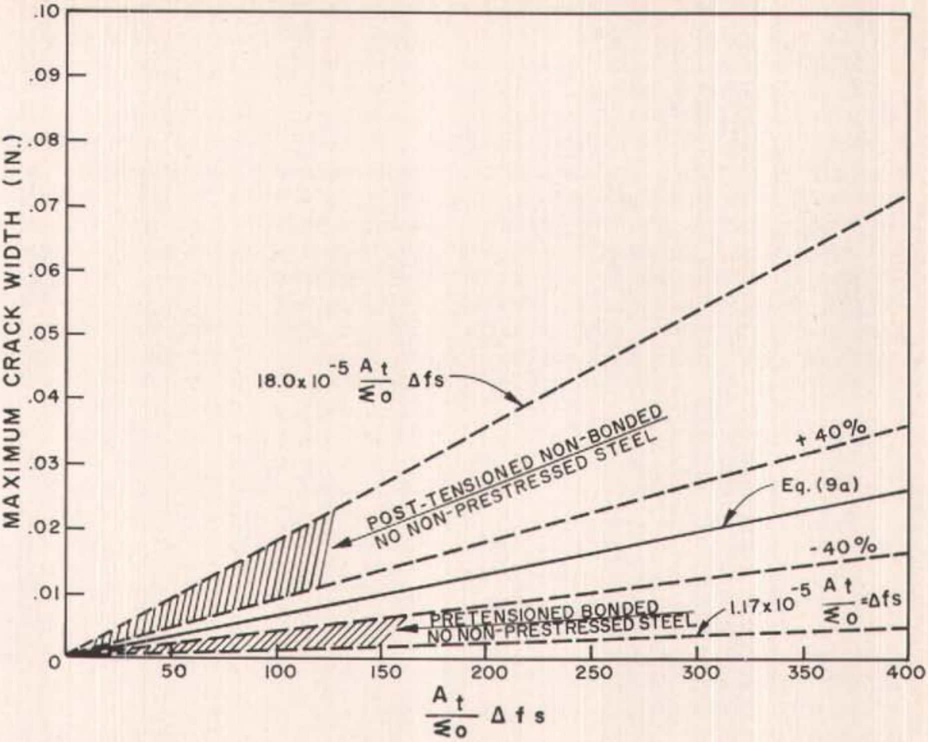

Fig. A. Maximum crack width versus steel stress.

use of this hybrid type reinforcement in prestressed concrete beams has been increasing in recent years and will undoubtedly be used more widely in the future.

The authors provide a mathematical model to determine the expected crack width and crack spacing in post-tensioned beams. I refer specifically to p. 87 where Fig. 11 indicates the maximum crack width versus reinforcement stress.

The following comments are intended to amplify the major points put forth by the authors:

1. The expression for maximum crack width given by Eq. (9a) on p. 86 is applicable to the conditions described in the paper, namely, to non-bonded post-tensioned beams loaded at third points of the span. Previous research has shown that the variation of maximum crack width for non-bonded beams with no mild reinforcement is between:

$$8.9 - 18.0 \times 10^{-5} \frac{A_t}{\Sigma O} (\Delta f_s).$$

This variation is influenced by the shape of the tensile zone, the percentage of reinforcement, and the type of loading. The advantage in using mild steel reinforcement to control cracking far outweighs the cost.

2. The development of maximum crack width and spacing is also a function of the type of loading applied. My research indicates that a much wider range of variation exists as the function between loading types in post-tensioned non-bonded beams than in bonded ones.

In the case of non-bonded beams, the maximum crack width occurs under the load if the single concentrated load is applied at midspan. If two equal concentrated loads are applied at the quarter points of the span, the maximum crack width varies between 32 to 45 percent of the crack width of one single load case. This observation should be emphasized in these cases in order to provide the proper control of cracking.

Eq. (10), p. 86, is based on the work published in Reference 12 for pretensioned bonded beams reinforced with non-prestressed steel. I have previously found that the −40 percent spread is wider if a large number of data are analyzed with a wider range of variables.

The coefficient of Eq. (10) may vary between 1.17 and 5.85 as indicated by the authors. This change of coefficient results in an −80 percent spread for beams with no mild steel reinforcement. Since the spread is into the lower crack width, the proposed Eq. (10) for predicting crack width is on the safe side.

It would be interesting to study further the influence of mild steel reinforcement on the behavior of post-tensioned beams, particularly the effect of shrinkage, creep and stress distribution along the non-prestressed steel.

A. S. PRASADO RAO*

The authors' paper presents very much needed formulas for predicting the cracking and deflection behavior of non-bonded post-tensioned T-beams having non-prestressed reinforcement. Other than these formulas, most crack width equations available in the literature pertain to members having both pretensioned and bonded non-prestressed reinforcement.

Basically I agree with the authors' conclusions. However, I believe that the procedure for calculating Δf_s and M_{cr} should at least include the effect of time-dependent concrete strains such as creep and shrinkage in beams containing both unbonded post-tensioned and bonded non-prestressed reinforcement.

A method that considers the above effect, the only method I am aware of, is based on an uncracked section theory attributed to Arutyunyan.[24]

I have developed an alternative method that could be safely applied to partially prestressed concrete members having both prestressed and non-prestressed reinforcement, which generally crack at working load. This method also considers the tensile force developed due shrinkage and creep at the level of bonded non-prestressed reinforcement.

I believe that the crack width formula should be the same regardless of whether the beam has bonded pretensioned or post-tensioned reinforcement, or unbonded post-tensioned or bonded non-prestressed reinforcement, or any combination of the above reinforcements. Their effect can be taken care of by appropriately calculating the parameters A_t, ΣO, and Δf_s.

In the case of prestressed beams that have a combined or unbonded prestressed and bonded non-prestressed reinforcement, Δf_s is the tensile stress in the non-prestressed reinforcement at any crack width load level. (The method for calculating Δf_s has been mentioned earlier.)

I believe that crack widths are controlled entirely by bonded reinforcement. Unbonded reinforcement does little toward controlling crack width. Hence, in calculating ΣO and A_t, it is appropriate to consider only bonded reinforcement. Accordingly, for the non-even case (see authors' Fig. 9, p. 85) A_t and ΣO work out to 9 in.^{2} and 2.3562 in., respectively.

As an example, let us consider Beams B3 and B4 (taken from Table 2, p. 79). The theoretical crack width at Δf_s of 30 ksi is given as 0.0064 in. (see Table 5, p. 83). Using my method, the following values result:

$$A_t = 3(1\tfrac{1}{2} + 1\tfrac{1}{2}) = 9 \text{ in.}^2$$

and

$$\Sigma O = 2(1 \times 1781) = 2.3562 \text{ in.}$$

Now, the crack formula becomes:

$$w_{max} = K \frac{A_t}{\Sigma O} \Delta f_s$$

Substituting the values of w_{max} A_t, ΣO, and Δf_s, K works out to be 5.58×10^{-5}. This is very close to the authors' value for pretensioned beams, which is 5.85×10^{-5}, given in Eq. (10), p. 86.

After finding the forces that act on the concrete section at the center of gravity levels of prestressed and non-prestressed reinforcement, the cracking moment M_{cr} can be easily worked from the following expression:

$$M_{cr} = F(e - ge_1) + \frac{F I_g}{A_g y_t}(1 - g) + \frac{f_r I_g}{Y_t}$$

where $g = F_1/F$ and

F_1 = tensile force at the center of gravity level of non-prestressed reinforcement, and

e_1 = eccentricity of F_1 with respect to center of gravity of concrete area.

It appears there is an error in the figure on p. 94; 12⁷⁄₁₅" ϕ should be written as 12⁷⁄₁₆" ϕ.

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Table A. Observed versus Theoretical Maximum Crack Width at Tensile Face of Beams.

Beam	Net Steel Stress Δf_s					
	40 ksi			60 ksi		
	w_{obs}	w_{theo}	$\frac{w_{obs}}{w_{theo}}$	w_{obs}	w_{theo}	$\frac{w_{obs}}{w_{theo}}$
B-3	0.0136	0.01337	1.017	0.0216	0.01996	1.082
B-4	0.0152	0.01337	1.137	0.0236	0.01996	1.182
B-5	0.0118	0.01191	0.991	0.0196	0.01778	1.102
B-6	0.0111	0.01191	0.932	0.0186	0.01778	1.046
O-II-1	0.0131	0.01331	0.984	0.0210	0.01996	1.052
O-II-2	0.0119	0.01331	0.894	0.0196	0.01996	0.982
O-III-1	0.0101	0.01146	0.881	0.0166	0.01771	0.970
O-III-2	0.0100	0.01146	0.872	0.0174	0.01771	1.017
O-IV-1	0.0073	0.00729	1.001	0.0114	0.01075	1.060
O-IV-2	0.0077	0.00729	1.056	0.0120	0.01075	1.116
U-II-1	0.0163	0.01788	0.912	0.0290	0.02700	1.074
U-II-2	0.0171	0.01788	0.956	0.0274	0.02700	1.015
U-IV-1	0.0073	0.00671	1.088	0.0118	0.01000	1.180
U-IV-2	0.0077	0.00671	1.147	0.0126	0.01000	1.260
			mean = 0.991 S.O. = 0.091			mean = 1.083 S.O. = 0.080

In Appendix B—Notation, p. 95 there also appears to be an error. "e" should be defined as the eccentricity of the prestressing steel with respect to center of gravity of concrete area, in., and not the eccentricity of center of gravity of steel with respect to the center of gravity of concrete area, in.

Reference

24. Arutyunyan, N. Kh., *Some Problems in the Theory of Creep in Concrete,* Chapter VII, Pergamon Press, London, 1966, pp. 217-251.

HIMAT SOLANKI*

The authors are to be congratulated for presenting a complete method of predicting the maximum crack widths for partially prestressed concrete beams.

It should be noted that Eqs. (9a) and (9b), p. 86, are primarily derived from the experimental study of T-beams. Is it valid to apply any other shape of prestressed/reinforced concrete beams in these equations?

In Conclusion 4, p. 92, the authors mention that: "The presence of non-prestressed steel in prestressed members has a significant effect on crack control whereby cracks become more evenly distributed and their spacings and widths become smaller."

It is true that the crack widths and spacings have a direct effect on the tensile stress. Therefore, for calculating the effective moment of inertia, it can be assumed more appropriate to use the tensile reinforcement force ratio instead of M_{cr}/M_a as proposed by Branson in Eq. (15).[25] The authors' Eq. (12), could be modified to:

$$I_{eff} = \left(\frac{T_{cr}}{T_a}\right)^3 I_g + \left[1 - \frac{T_{cr}}{T_a}^3\right] I_{cr} \leq I_g \qquad (15)$$

where T_{cr} and T_a express the tensile reinforcement force in the cracked section at cracking and at an arbitrary load level, respectively.

Based on the new Eq. (15), lower values (up to 10 percent) of deflection were found as compared to the authors' Eq. (11).

Using the authors' approach, several beams[8,12,26,27] were analyzed and it was concluded that Eq. (9a) could be expressed as:

$$w_{max} = 6.21 \times 10^{-5} \frac{A_t}{\Sigma O}(\Delta f_s) \qquad (16)$$

Bennett and Veerasubramanian's[26] maximum crack width formula was modified as follows:

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$$w_{max} = \beta_1 + \beta_2 (\sigma_s - \sigma_{rt}) d_{cc} \quad (17)$$

where

- β_1 = constant representing residual crack width, the value suggested for deformed bars is 0.02 mm
- β_2 = constant depending on bond characteristics of the steel; the value of the deformed bars is 6.5
- σ_{rt} = residual tensile strain in concrete
- d_{cc} = cover over reinforcing bars to the tensile face

Based on Eq. (17), several beams[8,12,26,27] were analyzed and found to be satisfactory. (This is concluded from Table A.)

Finally, I believe that these equations would give a better correlation than the authors' formulas.

References

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26. Bennett, E. W., and Veerasubramanian, N., "Behavior of Nonrectangular Beams with Limited Prestressed after Flexural Cracking," *ACI Journal,* V. 69, No. 9, September 1972, pp. 533-543.
27. Tansi, P., Heaney, A. C., and Warner, R. F., "Serviceability Tests on Partially Prestressed Concrete Beams," UNICIV Report No. R-184 University of New South Wales, Kensington, Australia, May 1979, 33 pp.

CLOSURE by EDWARD G. NAWY* and JIM Y. CHIANG†

The authors wish to thank Messrs. Kaar, Potyondy, Prasado Rao, and Solanki for their interest in this paper.

In answer to Mr. Kaar's comments, the authors emphasize that the two parameters involved in the calculation of crack width are crack spacing and apparent average strain at the location where the crack width is measured. The stress in the steel, the depth of neutral axis, and the amount of clear cover control the average strain. Most crack width formulas, including the equations proposed by the authors, have this quantity, $R_i f_s$ as a variable.

The crack spacing depends on the relative bond strength and the tensile strength of the effective concrete area surrounding the steel bars. Using basic principles of mechanics, it can be concluded that crack spacing depends on: (1) bond strength of the concrete, (2) the available bonding area which depends on the sum of the perimeter, ΣO, (3) tensile strength or modulus of rupture, and (4) the effective tensile area.

Using statistics, Gergely and Lutz chose to use a fractional factor to determine the effective area. Some other researchers, however (including the CEB-FIP Standards[28]), have chosen to use both the effective area and the sum of the perimeters linearly. We have no reason to believe that one method is superior to the other. (Note that the statistics determine the key constant.)

Previous research by the authors[13,29,30] on beams and slabs has shown that the scale effect is insignificant in determining the crack width. This conclusion also agrees with CEB analyses of prestressed beams[31] in which no significant difference could be detected between 400 and 600 mm (approx. 16 and 24 in.) deep beams.

Mr. Kaar may not have realized that in partially prestressed concrete, standard deviations of crack width are higher than those in reinforced concrete.[32] This happens because final randomness is a combination of cracking phenomena randomness with randomness in reaching the decompression state. Therefore, data from a few larger beams cannot be expected to fit minutely the equations developed by the authors.

The authors' expressions developed were also corroborated by analyses and comparison[18] with an extensive population of data study from other investigations (including the CEB-FIP) and found to be closely fitting.

The authors agree with Mr. Kaar's comment that, "the presence of non-prestressed steel substantially affects the crack formation." That is why the authors undertook this investigation.

Mr. Kaar's last sentence implies that for every structural unit there should be pro-

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totype testing. If this were to occur, design procedures as they exist today would never have become available for the engineering profession.

We would like to thank Dr. Potyondy for his contribution to the discussion and for his valuable remarks. His research confirms the conclusion that when no mild steel reinforcement is used, the crack width can be several times larger. He is also correct in stating that the benefits in using mild steel reinforcement exceed the initial cost.

The authors also agree with Dr. Potyondy's remarks in Paragraphs 4, 5, and 6 and also concur with him that additional research is needed on the effect on crack width in the areas of long-term shrinkage and creep behavior.

Dr. Prasado Rao's remarks are welcomed. The expressions developed in our research were not for long-term loading. Only limited work exists on long-term effects on crack width, for reinforced or prestressed concrete beams. Modification factors have been proposed by CEB[28] to take such effects into account in adjusting the value of Δf_s for long-term effects.

We disagree with Dr. Prasado Rao in his hypothesis that the same crack width formula can be used for both pretensioned and post-tensioned beams. The research results of the present investigation show the lesser number, hence wider cracks developed in the non-bonded post-tensioned beams. This is due to the lesser bonded area of reinforcement whose effect cannot directly be taken by simply evaluating the perimeters ΣO, which Dr. Rao suggests. There is the frictional effect of the interface of the interface of the non-bonded tendons with the adjacent concrete which requires that the areas and circumferences of the non-bonded tendons be included in the equation for crack width evaluation.

The example presented by Dr. Prasado Rao, in which he neglected taking into account the existence of the prestressing tendons in the calculation of the perimeter sum ΣO and the concrete area in tension A_T, by coincidence gave K value close to that for pretensioned beams. This is because the beams he chose to use for illustration (Beams B3 and B4) contain the lowest steel percentage of the 14 beams in Table 5, p. 83.

Dr. Prasado Rao is correct about the typographical error in the figure on p. 94, while he is incorrect with respect to the definition of the eccentricity, e (see Appendix B, p. 95) based on the above discussion.

Finally, the authors are grateful for the amplifications presented by Dr. Solanki's discussion of the paper and his complimentary remarks.

Dr. Solanki asks the question: Is it valid to apply any other shape of prestressed/reinforced concrete beams in these equations? These expressions apply to other shapes of prestressed beams since the parameters of the equation developed by the authors deal with stress levels, perimetric values, and the concrete area in tension. The formulas, of course, do not apply to reinforced concrete beams.

While there might be some merit to using force ratios for computation of deflection, the discussion on deflection was included to test the behavior of the beams against the codes which are based on the generally accepted Branson equation.

On his question of using a factor of 6.21 [see Eq. (16)] in the equation the authors proposed, it is not clear why Dr. Solanki chose only part of the population of beams for coming up with the 6.21 factor, and whether his suggestion in expressing Eqs. (16) and (17) resulted from excluding the lower stress level of $\Delta f = 30$ ksi. If this is the case, the authors are convinced that in prestressed beams, the lower stress levels of $\Delta f_s = 30$ ksi and less should be at least as important as the higher stress levels in the regression analysis for developing the crack width expressions.

References

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