

READER COMMENTS

Guide Specification for Precast Prestressed Concrete for Buildings*

by PCI Committee on Design Handbook

Comments by Anthony P. Chrest and
Anthony C. Martin, and Committee

**ANTHONY P. CHREST and
ANTHONY C. MARTIN†**

This Guide Specification will be very useful to the precast prestressed concrete industry. There are, however, several passages in the report that need clarification or further expansion.

1. Under Section 1.01 A1, the first sentence might read, "These specifications cover precast and precast prestressed structural concrete construction and associated elements, including . . ."

2. Section 1.01 B10, could read, "holes for other trades."

3. Under Section 1.01 C, the heading might read, "Material installed by precaster, but furnished by others."

4. Under Section 1.02 A, concerning the commentary note, any plant that has been in existence for 2 to 5 years can probably afford the PCI Certification Program.

5. Under Section 1.02 E, instead of "latest editions" the specific year should be mentioned.

6. Under the commentary notes for Section 1.02 E, the note regarding fire ratings should be expanded to point out that specific fire ratings for specific products should be listed if the designer is interested in fire rating information.

7. Under Section 1.03 A1, Erection Drawings, it has been our experience that erection drawings rarely show very much detail on items like bracing, guying, and sequencing. You usually have to keep after the precaster to get this sort of information. I suggest that something be added under this section to take care of the problem.

8. Under Section 1.03 A2, f and g, estimated cambers and method of transportation are normally not a function of production drawings.

9. Under Section 1.03 B1, Notes a through c should be noted on the erection drawing.

10. Under Section 1.03 B2, in some states,

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for instance Illinois, the wording should not be "registered engineer" but "registered structural engineer."

11. Under Section 1.03 D, Test Reports, should be submitted always, not by request.

12. Under Section 2.01 E1, Commentary, the welding of bars should be made a part of the specification whether welding is anticipated or not.

13. Under Section 2.01 J, add, "and/or deformed studs."

14. Under Section 2.02, the mix design should be verified by the owner's testing laboratory.

15. Under Section 3.01 A1, concerning a safety factor of 4: We've noticed that it seems to be a universal practice for manufacturers of lifting hardware to publish tables of capacities and in the fine print under the table say that these capacities must be divided by four. This practice is misleading and extremely dangerous, especially in the hands of an inexperienced designer. PCI should use its influence to have all of these tables revised and republished with the actual recommended allowables and not four times the recommended allowables.

16. Under Section 3.01 B3, the batten should be non-staining.

17. Under Section 3.02 B, before erection can proceed, the precaster or the erector must have his erection sequence approved by the engineer. Erection should not be allowed to commence unless an approved erection sequence, complete with all bracing, guying, etc., has been approved by the engineer.

COMMITTEE CLOSURE

The Committee on PCI Design Handbook appreciates the suggestions by Messrs. Chrest and Martin. All their points were reviewed by the Committee; those not commented on in the following were accepted:

1. The new words "and associated elements" are considered too vague; the section already refers to "other related items" and then gives examples of these items.

3. "Work" preferred over "material" to be consistent with Subsections A and B.

4. This observation is one with which the Committee heartily concurs.

7. Bracing, guying, and other erection procedures normally are not required on erection drawings. When he requests them, the engineer should be prepared to review and approve them.

8. Consensus that estimated cambers and method of transportation belong on production drawings.

9. Consensus that dead and live loads and any other applicable design loads should be shown on the erection drawings, but not the handling and erection stresses. This change will be made.

10. Consensus that present wording is proper.

11. Consensus that submittal of test reports should only be on request.

12. Bar welding requirement does not fit under materials; consensus that it remain as a note to the specifier. See also Section 3.03.

14. Consensus that industry should not recommend this as a standard practice because of added unnecessary cost; owner or his agent can always exercise this option in the specification.

15. Committee reported that some manufacturers of lifting hardware show working capacities and others list ultimate test values. The observation of misreading information is, of course, valid for many situations.

16. Rejected by Committee; falls in category of general yarding practice.

17. Consensus that most engineers do not and should not approve bracing, guying and other erection procedures. See remarks under Item 7.

The revisions have been incorporated into the Committee report and a revised version of these Guide Specifications is available from PCI Headquarters in separate form as Publication SPC 118-78.

Design of Partially Prestressed Concrete Flexural Members*

by Saad E. Moustafa

Comments by E. W. Bennett, M. Birkenmaier, T. Brøndum-Nielsen, A. S. G. Bruggeling, F. Levi, A. E. Naaman and A. Siriakson, A. H. Nilson, F. N. Pannell, K. Rajagopalan, and Author.

E. W. BENNETT†

The author has developed an interesting method for the design of partially prestressed beams, giving attention to the serviceability criteria of deflection and crack width which are often critical in this type of design. It does seem, however, that having obtained the strain distribution after cracking he could have gone further and used this not only to calculate the stress in the reinforcement, but also the curvature and hence the deflection.

In a cracked prestressed section the neutral axis position is obtained by taking account of the compressive action of the prestressing force and the section, as the author correctly points out, is different (i.e., stiffer) than an identical non-prestressed reinforced concrete section subject to pure bending. Since the position of the neutral axis changes with the applied moment, the stiffness of the section will also vary.

It must therefore be appreciated that the deflection should be based on the curvature of the beam at the appropriate load, and if, in accordance with convention, a value of I is derived, it must be regarded as an average value applicable to only one value of the applied moment.

The curvature method is essentially that recommended in the British Code of Practice (CP110:1972), although it has not been de-

veloped in detail for partially prestressed concrete. It may easily be applied to the author's design example, and should be more accurate than the use of the value $I_{cr} = 4000 \text{ in.}^4$ which he adopts. That value is much too low for the conditions represented by his Fig. 10. It is even less than the I_{cr} calculated for a cracked reinforced concrete section as suggested by Shaikh and Branson,⁷ the value of which is 4362 in.^4 , with the centroid axis 2.13 in. from the top.

Referring to Fig. A, the effective value of I_{cr} for the cracked section corresponding to Fig. 10 is applicable to a total moment of $M_D + M_L - P_e$ and may be calculated as follows:

$$\frac{M_D + M_L - P_e}{E_c I_{cr}} = \frac{1}{r} = \frac{f_c}{E_c k d}$$

where

f_c = stress in concrete at top of section

r = radius of curvature

Hence:

$$\begin{aligned} I_{cr} &= \frac{(M_D + M_L - P_e) k d}{f_c} \\ &= \frac{(2257 + 3024 - 246 \times 13.65) \times 0.26 \times 20.5}{1.78} \\ &= 5758 \text{ in.}^4 \end{aligned}$$

Since the integration of curvature over the length of a beam is laborious, an effective

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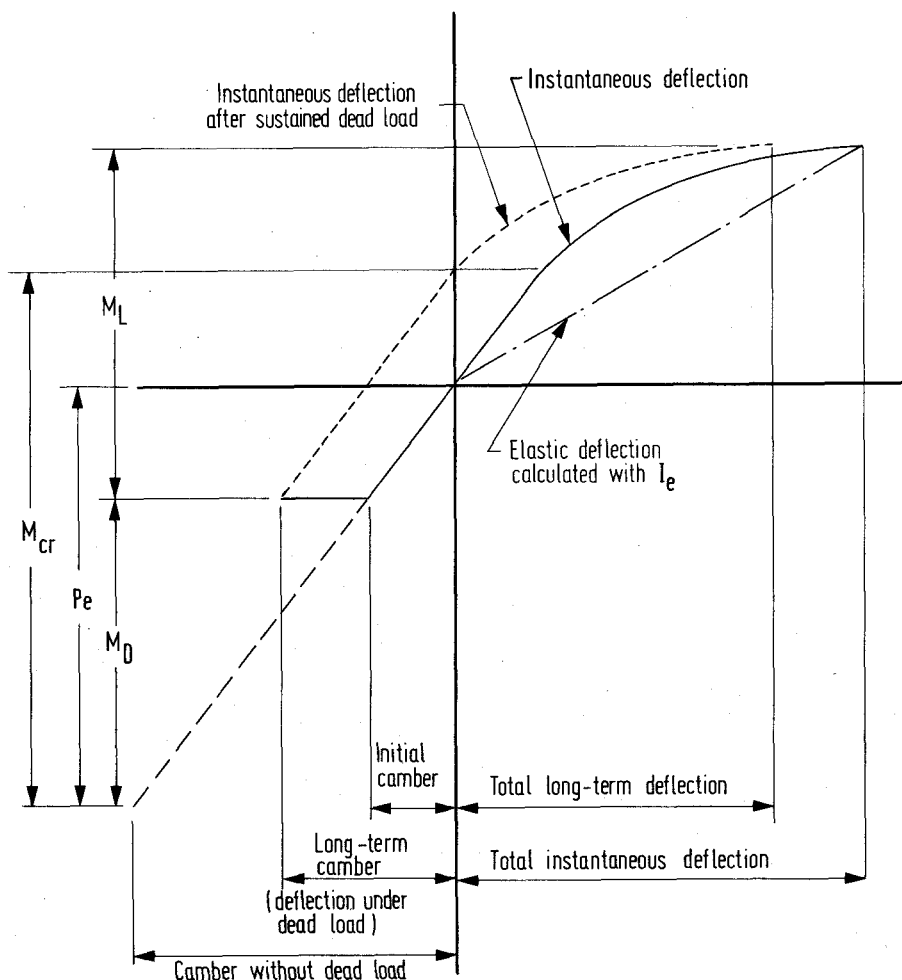


Fig. A. Instantaneous and long-term deflection.

moment of inertia for the whole beam may be estimated by the Branson-ACI formula. However, since I_{cr} has been calculated for the total moment and deflection (measured from the origin in Fig. A), M must be replaced by $M_D + M_L - P_e$ and M_{cr} by $M_{cr} - P_e$ as is clearly shown in the diagram.

Thus:

$$I_e = \left[\frac{M_{cr} - P_e}{M_D + M_L - P_e} \right]^3 I_g + \left\{ 1 - \left[\frac{M_{cr} - P_e}{M_D + M_L - P_e} \right]^3 \right\} I_{cr}$$

$$= \left[\frac{4201 - 246 \times 13.65}{5281 - 246 \times 13.65} \right]^3 20,985 + \left\{ 1 - \left[\frac{4201 - 246 \times 13.65}{5281 - 246 \times 13.65} \right]^3 \right\} 5758$$

$$= 7395 \text{ in.}^4$$

The total instantaneous deflection is:

$$\delta_{inst} = \frac{5(M_D + M_L - P_e)L^2}{48E_c I_e}$$

$$= \frac{5(5281 - 246 \times 13.65)(60 \times 12)^2}{48 \times 4286 \times 7395}$$

$$= 3.28 \text{ in.}$$

Assuming that the deflection under instantaneous live load is unaffected by long-term dead load:

$$\left[\begin{array}{l} \text{Total long-term} \\ \text{deflection (long-} \\ \text{term dead load plus} \\ \text{instantaneous live} \\ \text{load)} \end{array} \right] = \left[\begin{array}{l} \text{Instantaneous} \\ \text{deflection under} \\ \text{total load} \end{array} \right] + \left[\begin{array}{l} \text{Long-term} \\ \text{deflection under} \\ \text{dead load} \end{array} \right] - \left[\begin{array}{l} \text{Instantaneous} \\ \text{deflection under} \\ \text{dead load} \end{array} \right]$$

Taking account of the negative sign of the deflection under dead load:

$$\delta_{\text{long-term}} = 3.28 - 1.40 + 0.70 = 2.58 \text{ in.}$$

which is 15 percent less than the author's value.

If these theoretical calculations can be supported by experimental evidence, the deflection performance of partially prestressed beams may thus prove to be even better than the author's method would suggest.

Reference

7. Shaikh, A. F., and Branson, D. E., "Non-Tensioned Steel in Prestressed Concrete Beams," PCI JOURNAL, V. 15, No. 2, February 1970, pp. 14-36.

M. BIRKENMAIER*

Dr. Moustafa is to be commended for his interesting contribution to the design of partially prestressed concrete flexural members. My comments in this discussion are divided into four parts:

1. Non-prestressed steel in ultimate strength design

Dr. Moustafa's calculations in the design example show a value for the incremental steel strain of $\Delta\epsilon_s = 0.040$ in the ultimate condition. The total strain in the prestressing steel at this stage is

$$\epsilon_{ps} = \epsilon_{so} + \Delta\epsilon_s = 0.0451$$

which is a very high value. The corresponding

ultimate stress is $f_{psu} = 270$ ksi.

The increase in steel stress of

$$\Delta f_{ps} = f_{psu} - f_{ps0} = 270 - 139.4 = 130.6 \text{ ksi}$$

has to be achieved entirely through an increase in the bond stress between the pre-stressed steel and concrete. Tests on pre-stressed beams using 1/2-in. diameter strands have shown that such an increase can be accommodated in most cases.

In the case of the non-prestressed steel, however, Dr. Moustafa's proposal to use 1/2-in. diameter strands to take up a part of the ultimate tensile force ($T'_s = T'_u - T_{ps}' = 49.5$ kips) at an ultimate stress of 270 ksi and an ultimate strain of $\Delta\epsilon_s = 0.040$ would lead to problems. In this case, the increase in steel stress from 0 to 270 ksi must again be achieved entirely through an increase in the bond stress between the steel and concrete.

This is difficult to achieve with 1/2-in. diameter strands because even at much lower steel stress increases, slip due to bond failure occurs between the steel and concrete.

I therefore believe that the steel stress (f_s) in non-prestressed 1/2-in. diameter strand must be limited in the ultimate design phase to about 120 ksi because any further stress increase beyond this level is unlikely due to progressive bond failure. Where Grade 60 steel is used, an increase in steel stress of only 60 ksi must be accommodated and the ultimate strain of $\Delta\epsilon_s = 0.040$ can be achieved.

2. Steel stress at service load

Dr. Moustafa has developed an elegant procedure for calculating the increase in steel stress Δf_s in a cracked prestressed concrete section. The available steel cross-sectional area ($A_{ps} + A_s$) is replaced by an imaginary steel area A_s^* and the problem is reduced to that of an ordinary reinforced concrete section subject to a bending moment M .

The solution to the problem consists in choosing a suitable value of A_s^* or $n\rho^* = nA_s^*/bd$ so that both equations:

$$T/T_o = n\rho^*/(n\rho^* - n\rho_o) \quad (10)$$

$$T/T_o = m/j \quad (11)$$

are simultaneously satisfied. In effect, the author has converted Eqs. (10) and (11) in the form of interaction diagrams. This enables values for T/T_o and $n\rho^*$ to be read off the diagrams for given values of

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$$n\rho_e = n(A_{ps} + A_s)/bd \text{ and } m = M/T_o d$$

Unfortunately, the logarithmic scale to which the charts have been plotted does not lend itself to easy interpolation of the $n\rho_e$ and $n\rho^*$ values. For given values of $n\rho_e$ and m , a numerical iterative procedure for the design example (outlined below) could be used to determine the values of $n\rho^*$ and T/T_o .

1. Assume an estimated value of $j = 0.9$.

2. Calculate T/T_o using Eq. (11):

$$T/T_o = 1.21/0.9 = 1.34$$

3. Compute $n\rho^*$ from Eq. (10):

$$\begin{aligned} n\rho^* &= (n\rho_e)(T/T_o)/(T/T_o - 1) \\ &= (0.006)(1.34)/(0.34) \\ &= 0.024 \end{aligned}$$

4. With $n\rho^* = 0.024$, calculate k by solving the well-known quadratic equation and obtain $k = 0.227$ and hence $j = 0.924$.

5. Then calculate T/T_o using Eq. (10):

$$\begin{aligned} T/T_o &= (0.024)/(0.024 - 0.006) \\ &= 1.333 \end{aligned}$$

Finally, using Eq. (11):

$$T/T_o = 1.21/0.924 = 1.310$$

6. From the above it is seen that the two calculated values of T/T_o differ a little. Therefore, we can repeat the procedure using the value of $j = 0.924$ and $T/T_o = 1.310$, thus obtaining $n\rho^* = 0.025$, $k = 0.233$, $j = 0.922$, and $T/T_o = 1.312$. It is readily seen that the T/T_o values rapidly converge.

3. Accuracy of prestress force

The method outlined by Dr. Moustafa clearly shows the contribution of the prestressing force to the stress distribution in the cracked reinforced concrete section.

$$\begin{aligned} T &= T_o + \Delta T = f_{so} A_{ps} + \Delta f_s (A_{ps} + A_s) \\ T &= \Delta f_s A_s^* \end{aligned}$$

from which it follows that

$$A_s^* = A_{ps} (1 + f_{so}/\Delta f_s) + A_s \quad (13)$$

Eq. (13) shows that the prestressing leads to a considerable increase in the efficiency of the prestressed steel area A_{ps} . In the design example:

$$\begin{aligned} f_{so} &= 139.4 \text{ ksi, } \Delta f_s = 32.4 \text{ ksi, giving} \\ A_s^* &= 5.3 A_{ps} + A_s = 8.42 \text{ sq in.} \end{aligned}$$

The efficiency of the prestressed steel area A_{ps} is thus increased 5.3 times.

Eq. (13) shows that the value of A_s^* is very sensitive to changes in the value of f_{so} . If we assume in the design example that the prestress force at decompression, T_o , is 10 percent smaller, we obtain

$f_{so} = 0.9 \times 139.4 = 125.5$ ksi, $T_o = 192$ kips, $m = M/T_o d = 1.34$, and $n\rho_e = 0.006$, resulting in

$$n\rho^* = 0.020, T/T_o = 1.43$$

The incremental stress is now

$$\Delta f_s = (0.43)(192)/1.836 = 44.9 \text{ ksi}$$

which is 39 percent higher than the value $\Delta f_s = 32.4$ ksi. The imaginary steel area is now only

$$A_s^* = A_{ps}(3.79) + A_s = 6.10 \text{ sq in.}$$

These figures clearly show that especially in flexural members with low steel ratios $n\rho_e$, even small inaccuracies in estimating the available prestressing force lead to large variations in the calculated values of Δf_s .

In the case of partially prestressed concrete members it is therefore very important that the prestressing force be determined as accurately as possible. Similarly, losses due to shrinkage and creep in concrete or relaxation in steel, should be calculated precisely.

4. Crack control

In recent years many major bridges, buildings and other structures have been built in Switzerland using partial prestressing. The Swiss Code for reinforced and prestressed concrete (SIA-Norm 162, 1968) stipulates the following checks under service load conditions.

1. That the maximum tensile fiber stress in the homogeneous concrete section, under dead load alone, does not exceed 170 psi (12 kg/cm²).
2. That the increase in stress, Δf_s , in the tensile reinforcement in the cracked section under dead and live loads, does not exceed 21.4 ksi (1500 kg/cm²).

These requirements must be complied with in both pre- and post-tensioned structures. An additional Code requirement, especially for post-tensioned members, is that an adequate percentage of non-prestressed reinforcement must be provided in the tensile zone.

The limits placed on Δf_s in the Swiss Code have been based on a comprehensive series of tests carried out by Thürlimann/Caflisch⁸

on 14 beams of 20-ft span. These tests showed that at a value of $\Delta f_s = 21.4$ ksi, crack widths are likely to occur in the region between 0.0039 and 0.0063 in. (0.10 and 0.16 mm) and that on unloading, the cracks close up completely.

The inclusion of partial prestressing in the Swiss Code (under some fairly conservative restrictions as outlined above) resulted in a remarkable increase in the number of prestressed concrete structures in Switzerland. Nearly 10 years of experience in the design and construction of such structures have indicated mainly positive results.

Currently, the Swiss Code is being revised to include provisions which will extend the range of application of partial prestressing. One of the proposals being considered is to allow an increase in the permissible value of Δf_s , i.e., provided the resulting crack widths are calculated and found to be acceptable for the conditions of use.

The general consensus in the profession is that none of the well-known crack width formulas appear to be adequate when applied to partial prestressing. Thus, a considerable amount of research into this subject needs to be carried out so that more realistic design criteria can be formulated.

Reference

8. Thürlimann, B., and Caffisch, R., "Bending Tests on Partially Prestressed Beams (in German)," Report No. 6504-1, Institute of Structural Engineering, Swiss Federal Institute of Technology, Zurich, Switzerland, 1970.

T. BRONDUM-NIELSEN*

According to the author's notation on p. 28, b is defined as being equal to the width of the compression face of the member. Unfortunately, this parameter is not suitable for circular cross sections (e.g., piles and poles) or for other cross sections in which b is also equal to zero.

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For the cross section shown in Fig. 4, the author states that the depth of the compression zone can be computed directly from the solution of a cubic equation. This is generally not the case. However, it applies to symmetrical bending of cross sections with constant web thickness provided that the neutral axis intersects the web.

The interaction effect of concrete shrinkage and creep, steel relaxation and effects of load and prestress force may demand a more rigorous analysis than the approximation obtained by simply deducting prestress losses. In addition, there is the effect of combined sustained and instantaneous loads in all structures; with instantaneous load, because the dead load and possible prestress has the character of sustained loads.

The analysis and design of a structure obviously becomes more complicated when the above effects are taken into consideration. Nevertheless, their influence is substantial and must be considered in a design situation. In 1973 I proposed an approximate solution for such a problem⁹ and have recently developed a more rigorous analysis, which has not yet been published.

Reference

9. Brøndum-Nielsen, Troels, *Structural Concrete*, Technical University of Denmark, 1973, Polyteknisk Forlag, Lyngby, Denmark.

A. S. G. BRUGGELING†

Dr. Moustafa must be commended on his paper which presents a design method for partially prestressed concrete members. His statement that "the full potential of partially prestressed concrete has not been fully implemented by structural designers because readily applicable criteria have not been fully developed" also applies to the situation in the Netherlands.

To help remedy this situation we prepared a report¹⁰ with numerical examples to give structural engineers design guidance on the subject of partial prestressing. It is, therefore, interesting to compare the design method developed by the author with our methods and procedures.

1. General remarks

Partially prestressed concrete covers a rather wide range of possibilities between reinforced concrete and fully prestressed concrete. In some structures the partial prestressing force plays a dominant role and yet in other structures partial prestressing plays only a limited role. Partially prestressed structures can also be built using only prestressing steel. In this case cracking is allowed in the tensile zone. In other cases non-prestressed strands can be used as supplementary reinforcement.

In his interesting numerical example Dr. Moustafa uses two of the above mentioned possibilities in a beam by combining prestressing with a relatively small amount of non-prestressed reinforcement.

Design methods for partially prestressed concrete must, however, be applicable to all cases including those with a high percentage of non-prestressed reinforcement. In this case the influence of the reinforcement on the stress distribution after prestressing, and on the redistribution of stresses in the concrete with time, cannot be neglected. This will be explained further.

The analysis of a rectangular section is, in my opinion, of only academic interest because partially prestressed rectangular sections are not structurally efficient.

2. Direct strength design

Dr. Moustafa uses, in the expression for the ultimate bending moment, the effective reinforcement index:

$$\omega_e = \frac{A_{ps}f_{ps} + A_s f_s}{bdf'_c}$$

Using this index, the stresses in the prestressed reinforcement (f_{ps}) and in the non-prestressed reinforcement (f_s) at ultimate load can be determined. However, if no restrictions are given, it may be possible to produce a concrete structure susceptible to brittle failure. If the reinforcement in the tensile zone does not show sufficient ductility at ultimate load, the structure can rupture suddenly.

In the Dutch Codes of Practice, the height c of the compression zone in relation to the depth d of the beam is limited to a maximum ratio.¹¹

For reinforced concrete:

$$\frac{c}{d} \leq \frac{1}{1 + \frac{f_y}{71}}$$

where f_y is in ksi.

For prestressed concrete:

$$\frac{c}{d} \leq \frac{1}{1 + \frac{f_{p0.1}}{210}}$$

where $f_{p0.1}$ denotes 0.1 percent of stress elongation.

In the case of partially prestressed concrete, the lowest value of c/d for both cases must be taken into account in order to insure adequate structural behavior. In general, the c/d value is the smallest ratio for prestressed concrete. If we apply this criterion to the author's numerical example, we find that the allowable c/d value will be 0.47 assuming $f_p 0.1$ is 240 ksi.

In actuality, the real c/d value used in this design example is only 0.07, which is very much on the safe side. Note, however, that in the case of rectangular sections there will be a trend towards high c/d values. This is another reason why rectangular sections are not efficient for partially prestressed concrete.

In the case of T-beams, the ultimate bending moment (with low c/d values) can easily be calculated if we assume yielding of reinforcement, steel stresses in the prestressing steel strands at 95 percent of ultimate, and an estimated lever arm between 0.85 d and 0.95 d .

3. Optimization of the prestressing force

In the optimization procedure for partially prestressed concrete one cannot, in general, neglect the influence of the non-prestressed reinforcement. If the amount of reinforcing steel in the tensile zone is important, it will take over a part of the prestressing force at release, thus reducing the initial stresses in the concrete.

The value of the initial prestressing force P_i taken over by the bonded reinforcement (prestressing steel and non-prestressed reinforcement) can easily be calculated using:

$$\Delta P_i = P_i \frac{1}{1 + C}$$

$$C = \frac{1}{\frac{E_s}{E_c} \left(\frac{A_{ps} + A_s}{A} \right) \left(1 + \frac{A_e^2}{I} \right)}$$

$$= \frac{1}{n\omega f}$$

where $f = 1 + A_e^2/I$.

Applying the above formula to the numerical example (assuming that $E_s = fE_c$ at release), we obtain:

$$\Delta P_j = 0.158 P_j (10 + 2 \text{ strands})$$

$$\Delta P_j = 0.194 P_j (10 + 2 \text{ \#6 bars})$$

This example shows that already 20 percent of the original prestressing force is taken over by the total reinforcement (of course also reducing the tensile stress in the prestressing steel).

With higher values of $(A_{ps} + A_s)$, this reduction of P_j increases considerably and cannot be neglected.

4. Time-dependent effects

The bonded reinforcement not only influences the stress distribution over the concrete sections shortly after release of the prestressing force but also with the passage of time.

The magnitude of the redistribution of stresses depends on the stress distribution under load conditions, creep and shrinkage behavior of the concrete, and the area of steel reinforcement.

We calculated this quantity for the numerical example assuming a shrinkage shortening of 30×10^{-5} , a creep factor of 2.5, and a 6 percent relaxation loss in the prestressing steel.

Our results showed that there was a compressive force in the center of the reinforcement, acting on the concrete section at midspan, at a decompression load of:

$$273 - 43.4 = 229.6 \text{ kips (10 + 2 strands)}$$

$$273 - 48.2 = 224.8 \text{ kips (10 strands + 2 \#6 bars)}$$

The bending moment due to sustained load is assumed to be 3900 in.-kips. This example clearly shows that already in the case with a low percentage of reinforcement, the influence of time-dependent behavior on the loss of compression force is 10 percent.

This 10 percent differential between both cases has an influence on the cracking moment and decompression load. With higher

percentages of reinforcement, which are often used in practice, these differences become more significant. The redistribution usually reduces the concrete stresses in the bottom of the beam section thereby decreasing the cracking moment.

After cracking a more important increase in steel stresses in the tensile zone takes place for the same maximum service load. Therefore, a reduction of the cracking moment leads to a larger increase of steel stresses at the same maximum service load.

The crack width will therefore increase and the durability of the partially prestressed concrete structure can be reduced.

Therefore, one must be very careful with the time-dependent effects. We developed a relatively simple design method to calculate the time-dependent influence which cannot be given here because of space limitations. This method has, however, some relationship with another very interesting paper published in the PCI JOURNAL.¹²

5. The numerical example

The numerical example illustrates clearly the design method developed by the author. It is well-conceived because the influence of the bonded reinforcement is small. I believe, however, that the maximum concrete release strength of 2.5 ksi used in the example is too low for pretensioned prestressed concrete. With such low values of concrete strength the transmission length of the bonded strands will be too large, thus reducing the shear strength of the beams.

In the Netherlands a minimum concrete release strength of 4 ksi must be guaranteed by the producer. I appreciate the fact that the author has used this low value of release strength only to illustrate his design procedure, but we have to realize that in practice a higher strength is necessary. In this particular case no additional non-prestressed reinforcement is actually necessary.

I previously mentioned that stress redistribution was neglected in the author's design method. The numerical example assumes a value of prestress losses of 39 ksi, which may be higher as shown before. With this value an incremental stress in the prestress strands after cracking is calculated as $\Delta f_s = 32.4$ ksi and 35.6 in the non-prestressed strand.

Dr. Moustafa states that, based on avail-

able test data, the above stresses are acceptable for crack control.

Results of tests carried out in Switzerland (Empa), The Netherlands (Stevinlaboratory T. U.-Delft) have shown that with $\Delta f_s = 20$ ksi the maximum crack width will be safely limited to 0.008 in., a value that in my opinion can be accepted only under normal conditions, i.e., in a non-aggressive atmosphere and in structures where the cracks are open during only very short periods. I believe that, without giving adequate restrictions to the conditions of the structure under service, a value of Δf_s varying from 32 to 35 ksi might not be generally acceptable.

Lastly, I wish to thank Dr. Moustafa for the opportunity to discuss some of the design considerations involved in this important subject of partial prestressing which I believe holds great promise for the future.

References

10. CUR-Rapport 87: Gedeeltelijk voorgespannen beton.
11. Voorschriften Beton, deel E en F, Art. 503.2.
12. Tadros, Maher K., Ghali, Amin, and Dilger, Walter H., "Effect of Non-Prestressed Steel on Prestress Loss and Deflection," PCI JOURNAL, V. 22, No. 2, March-April 1977, pp. 50-63.

F. LEVI*

Dr. Moustafa has written a very interesting paper which should be useful to practicing design engineers. I have only one important remark to make. In partially prestressed flexural members the effects of creep and shrinkage are significantly increased by the presence of a rather high percentage of non-prestressed steel (i.e., there will be local interaction between the concrete and steel).

This interaction can reduce the effective prestressing to a level as low as 60 percent of

the initial value. It seems to me that this aspect of the problem has not been sufficiently examined in the paper.

A. E. NAAMAN and A. SIRIAKSORN†

Dr. Moustafa is to be commended for presenting a useful and practical method for the approximate design of partially prestressed beams.

The writers were particularly interested in the paper since they are doing research on partial prestressing (sponsored in part by a Fellowship Award from the Prestressed Concrete Institute). In this study a computer program is being developed with the purpose of analyzing accurately or designing partially prestressed beams to satisfy serviceability criteria.

The example problem described in the paper was fed into the computer and a comparison of results undertaken; the analysis of some of the observed discrepancies lead to a number of remarks, some of which are conceptual and some purely numerical; they are as follows:

1. The author approximates the stress in the prestressing steel at decompression f_{so} by the effective prestress f_{se} , i.e., $f_{so} \approx f_{se}$; this approximation is not acceptable in many cases. The effective prestress f_{se} must be defined as the stress in the prestressing steel under the combined effect of the final prestressing force (after all losses) and the dead load moment, i.e., $(F + M_D)$; the presence of the dead load moment is essential so as to allow the determination of prestress losses due to creep; if f_{so} were taken equal to f_{se} then creep losses would be equal to zero and the total prestress losses would be substantially reduced.

2. In a second approximation the author assumes the state of decompression as a reference state (Figs. 1 and 4) to compute the changes in stress and strain in the steel due to the application of live load moment, while the reference state should be that under $(F + M_D)$. The exact knowledge of stress changes in the steel in the elastic range of behavior is very important in assessing fatigue damage.

In the example problem treated by the au-

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thor the stress in the concrete at the level of the steel due to $(F + M_D)$ is equal to 957 psi. At decompression the increase in stress in the prestressing steel is approximated by $n_p(957) \approx 6.12$ ksi. This stress increment represents 23 percent of the ACI allowable stress range to design against fatigue ($f_{pr} = 0.10 f_{pu} = 27$ ksi).

Thus, using the state of decompression as a reference will underestimate the change in stress due to the application of live load and may lead to an unsafe design. Furthermore, if an analysis in the nonlinear range of behavior is performed, the above neglected stress increment may substantially affect the determination of strains in the steel and corresponding curvature of the section. Note, however, that using the state of decompression as a reference to determine stress changes in the steel for the purpose of evaluating crack width is correct.

3. In view of comment 2 above, the observed stress changes of 32.4 ksi and 35.6 ksi in the prestressed and non-prestressed strands would be unacceptable strictly according to the ACI recommendations. It would have been even more so if reinforcing bars were used as, for them, the allowable stress change is only 20 ksi. However, engineering experience may indicate, that for the type of structure considered, the actual magnitude of the most frequent live load amounts to about 50 percent of the specified live load. This will reduce the effect of fatigue.

4. Some numerical errors were noted in the example problem:

- (a) In computing the additional long-term deflection, the value of the final prestressing force must be used, not the initial value P_i .
- (b) The cracked moment of inertia should be $I_{cr} \approx 6520$ in.⁴ instead of 4000 in.⁴ Fortunately, in this particular case the value of the effective moment of inertia I_e is not greatly affected, it is $I_e = 13,830$ in.⁴ instead of 12,590 in.⁴
- (c) In checking actual deflections against allowable, according to the ACI Code, one must evaluate the sum of the long-time deflection due

to the dead load and the live load deflection. For the example problem, using values derived by the author leads to:

$$-1.4 + 3.03 = 1.63 \text{ in.}$$

$$\text{i.e., } < L/240 = 3 \text{ in. (ok)}$$

The live load deflection $\delta_L = 3.03$ should be checked against $L/180 = 4$ in. and is satisfactory.

The writers agree with the author that additional research is needed in many areas related to the analysis and design of partially prestressed members.

A. H. NILSON*

The most important single property of a prestressed beam is its strength, after which come the lesser, but still important, serviceability considerations of deflection and cracking. Concrete stress limits, although emphasized in most current design specifications, may properly be considered of minor importance and, at best, are an indirect and unsatisfactory gauge of strength and serviceability.

Accordingly, logic favors a design method by which concrete and steel dimensions are selected on the basis of required strength at a hypothetical overload stage, after which the amount of prestress force may be selected to impart the desired deflection at some specified load, or to control cracking to the desired degree.

The design approach proposed by the author, which initiates with provision of adequate strength, is appealing for this reason, and is fully consistent with current reinforced concrete design procedures, which are based on a strength approach (Ref. 4). It is encouraging to find the author, a designer of considerable experience, advocating such procedures.

For partially prestressed beams, the author establishes an effective reinforcement index:

$$\omega_e = \frac{A_{ps} f_{ps} + A_s f_s}{b d f'_c}$$

in place of the separate reinforcement indices:

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$$\omega = \frac{\rho f_y}{f'_c} = \frac{A_s f_y}{b d f'_c}$$

and

$$\omega_p = \frac{\rho_p f_{ps}}{f'_c} = \frac{A_{ps} f_{ps}}{b d f'_c}$$

of the ACI Code (Ref. 4).

The ultimate flexural strength M_u/bd^2 may then be expressed in terms of ω_e and the concrete strength f'_c [Eq. (4a)]. For design, given M_u/bd^2 , and f'_c , the required ω_e may be found [Eq. (5b)]. After this, the incremental steel strain $\Delta\epsilon_s$ may be determined [Eq. (6)] and the relative depth to the neutral axis, c/d , found [Eq. (7)].

The ultimate strength design chart of Fig. 2 presents $K = bd^2/M_u$ as a function of the effective reinforcement index ω_e . It is exactly analogous to design charts for reinforced concrete members, such as are found on pp. 593-594 of Ref. 13, which relate the flexural strength parameter M_u/bd^2 to the tensile steel ratio $\rho = A_s/bd$. Such charts could as easily be presented in terms of $\omega = \rho f_y/f'_c$.

But an essential difference is that, while for reinforced concrete members the steel stress at failure is known to equal f_y (because under-reinforced members are mandated by the Code), for prestressed members the steel stresses f_{ps} and f_s are not known at this stage. Consequently, while the required ω_e may be read from Graph 2, the steel areas A_{ps} and A_s may not be determined, and "direct ultimate strength design" is not possible.

This point is avoided in the author's example by assuming the stress in the tensioned strands at decompression to be 39 ksi less than the jacking stress of 178.4 ksi. On this basis, the strains ϵ_{so} and ϵ_{ps} are found, and the stress f_{ps} read from Fig. 8 to be 270 ksi. After this, the stress in the mild steel reinforcement is assumed to equal $f_y = 60$ ksi at failure in calculating the area A_s . Alternately, the stress in the non-prestressed strands is assumed to be $f_s = 270$ ksi at failure in calculating the supplemental strand area A_s .

These assumptions can not be justified in general. Their introduction is contradictory to Conclusion 4, which states: "Ultimate strength criteria are based on satisfaction of compatibility of strains, and therefore, actual tensile stresses in the reinforcement are used in the computations."

While the writer fully supports a transition to strength-based design of prestressed

members, he sees no way of avoiding a certain amount of iteration in the design process, resulting from the interdependence of the steel stresses at failure and other design parameters. However, a strength-based design approach which converges rapidly, and which does not require the construction of design charts or interaction diagrams, will be found in Ref. 14.

The writer wishes to add one further comment. While the author has assumed the validity of the customary rectangular equivalent stress distribution, and has made the usual assumption that $\epsilon_{cu} = 0.003$, in accordance with the ACI Code, recent research indicates that such assumptions may not be valid for high strength concretes, such as are used commonly for prestressed members. Such concretes are characterized by a limiting compressive strain in flexure less than 0.003, and a stress-strain relation which is nearly linear to failure.

References

13. Winter, G., and Nilson, A. H., *Design of Concrete Structures*, 8th Edition, McGraw-Hill Book Co., New York, N.Y., 1972.
14. Nilson, A. H., *Design of Prestressed Concrete*, John Wiley and Sons, New York, N.Y., 1978.

F. N. PANNELL*

This is a very interesting paper, giving a straight-forward method of design for a particular class of beam.

The article might perhaps have been entitled "Design of fully-cracked partially prestressed flexural members" since it makes no attempt to satisfy conditions of no cracking, and permits the use of sections outside the range of conventional prestressed concrete design; this may be seen from the author's example in which the values adopted for Z_t and Z_b (3064 and 1224) are almost the inverse of those required to satisfy the non-cracked condition (1100 and 2600).

The author's method will undoubtedly lead

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to economies of design over existing methods, but it may be desirable to proceed with considerable caution. There is possibly a case for limiting the use of cracked sections to unbonded prestressed concrete beams with sheathed tendons. Because of the higher stress ratios, the capacity of prestressed tendons to accept corrosion without fracture is much less than that of conventional reinforcement, and failure may be without warning.

If it could be established for any class of bonded prestressed member that cracking within specified limits would not give rise to corrosion, then the author's method (which is well-devised and well-presented) could be brought into use. The relevant crack widths would, of course, need to be those obtained in the stable long-term condition of the member.

It may not be adequate to adopt incremental stress as the sole criterion of crack control; since strand sometimes has less satisfactory stress-strain characteristics than the strand adopted by the author, it might be advisable to consider also the instantaneous modulus of elasticity of the strand at working stress.

K. RAJAGOPALAN*

Dr. Moustafa deserves praise for his worthwhile contribution to the design of partially prestressed flexural members. The author has presented an elegant method for the computation of stresses in service in a cracked prestressed concrete section using a fictitious non-prestressed steel area concept.

Computationally, Dr. Moustafa's method would involve an iterative solution for the stresses but, in each iteration, only a quadratic equation is solved. In contrast the method based on phantom forces propounded by Nilsson⁵ is non-iterative but involves the solution of a cubic equation.

The writer believes, however, that the scope of the interaction diagrams presented by the author is limited to only preliminary designs due to the following two reasons. The first one is concerned with accuracy as ac-

knowledge by the author himself.

Secondly, the fact remains that the interaction diagrams are not section-independent. Though the author has introduced the concept of an overlay, several interaction diagrams must still be made available to the designer, one for each stemmed section.

The writer believes that Dr. Moustafa's method is useful as a computational tool either for longhand or computer applications. For example, an iterative solution for the analysis of flexural stresses in service using the author's method can be set out as follows:

1. Assume j .
2. Assume d .
3. Compute $n\rho_e$ from Eq. (10b).
4. Determine the ratio (T/T_o) from Eq. (11).
5. Compute $n\rho^*$ from Eq. (10) and Δf_s from Eq. (8).
6. Compute k by solving the quadratic equation:

$$a_1 k^2 + a_2 k + a_3 = 0$$

where

$$a_1 = b \cdot d^2 \Delta f_s$$

$$a_2 = 2d [(b - b_1) t_1 \Delta f_s + nT]$$

and

$$a_3 = - [b - b_1] t_1^2 \Delta f_s + 2ndT]$$

in which b_1 is the sum of the average widths of the stems and t_1 is the thickness of the flange.

7. Compute the location g_1 and g_2 of the compressive forces C_1 and C_2 in the flange and stems, respectively.

$$g_1 = \left(\frac{3kd - 2t_1}{2kd - t_1} \right) \frac{t_1}{3}$$

and

$$g_2 = \frac{kd + 2t_1}{3}$$

8. Compute the forces C_1 and C_2 :

$$C_1 = \frac{\Delta f_s b t_1 (2kd - t_1)}{2nd (1 - k)}$$

and

$$C_2 = \frac{\Delta f_s b_1 (kd - t_1)^2}{2nd (1 - k)}$$

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9. Compute the location g of the resultant compressive force:

$$g = \frac{C_1 g_1 + C_2 g_2}{C_1 + C_2}$$

10. Compute j :

$$j = 1 - \frac{g}{d}$$

11. If the value of j computed in Step 10 agrees with the value assumed at the start of the iteration, Step 12 is followed. Otherwise, the iteration is restarted from Step 4 using the value of j computed in Step 10.

12. Compute the value of d , the location of the center of tension, and if necessary iterate again starting from Step 3.

A computer program written by the writer, which implements the above procedure, gives the following results when applied to the author's example:

$d = 20.56$ in.; $j = 0.9494$; $k = 0.2517$; $f_c = 1.65$ ksi; $\Delta f_s = 31.36$ ksi; $C_1 = 255.3$ C₂ = 15.3 kips; $T = 270.6$ kips and $M = 5281$ in.-kips.

The last mentioned values display the equilibrium check. It must be remembered that $\Delta f_s/n$ would give the stress in concrete at depth d from which the stress in the non-prestressed steel and the incremental stress in the prestressed reinforcement can be calculated. These values, for the author's example are 34.3 ksi and 31.2 ksi, respectively.

A comment should be made on the question of optimization. Keeping the dimensions of the section fixed, the author has merely selected the value of the prestressing force to suit certain conditions. Optimization in a real sense must, however, be concerned with at least the determination of the depth of the section and the prestressing force so as to minimize a criterion function.

The writer¹⁵ has recently formulated the problem as a minimum cost optimization and has also described an "empathic" solution of the problem. One interesting aspect of the study is that these solutions also correspond to minimum energy requirements since they result in minimum high tensile steel. It is well known that the energy input needed in steel production is many times that needed to produce concrete. Designs which result in

minimum steel can therefore help in energy conservation.

The concept of partial prestressing itself brings in energy savings as it involves the use of less prestressing steel. The author's interesting paper on the subject is therefore very relevant in the context of the present energy climate.

Reference

15. Rajagopalan, K., Discussion on "Pre-stressed Concrete for Buildings," PCI JOURNAL, V.22, No.3, May-June, 1977, pp. 116-119.

AUTHOR'S CLOSURE

The author deeply appreciates the comments and discussions offered by such an outstanding group of leading experts from seven countries. Their comments are extremely valuable and add significantly to the paper.

Dr. Bennett presents an interesting approach to the computation of deflection of partially prestressed beams. The first concept of computing the cracked moment of inertia using curvature is rational. However, in accordance with the ACI Code, Section 18.4.2, a bilinear moment-deflection relationship should be used. Assuming that the gross moment of inertia prevails for all moments equal to or less than:

$$P \left[\frac{Z_b}{A} + e \right] = M_o$$

and that an effective cracked section moment of inertia is to be used for additional moment, a bilinear moment-deflection relationship is obtained.

When this approach is applied to the design example the following results are obtained:

$$M_s = 5281 \text{ in.-kips}$$

$$\begin{aligned} M_o &= P \left[\frac{Z_b}{A} + e \right] \\ &= 213 \left[\frac{1224}{401} + 13.65 \right] \\ &= 3557 \text{ in.-kips} \end{aligned}$$

$$M_{cr} = P \left[\frac{Z_b}{A} + e \right] + Z_b (7.5 \sqrt{f'_c})$$

$$= 4206 \text{ in.-kips}$$

To use a bilinear curve:

$$M'_t = M_s - M_o$$

$$= 5281 - 3557$$

$$= 1724 \text{ in.-kips}$$

$$M'_{cr} = M_{cr} - M_o$$

$$= 4206 - 3557$$

$$= 649 \text{ in.-kips}$$

To compute I_{cr} :

$$\frac{M_o}{E_c I_g} + \frac{M_s - M_o}{E_c I_{cr}} = K_1 + K_2$$

where K denotes the curvature with K_1 and K_2 given by the first and second terms, respectively.

$$K_{total} = \frac{f_c}{E_c k d}$$

$$= \frac{1.78}{E_c (0.26) (20.5)}$$

$$= \frac{0.3340}{E_c}$$

$$K_1 = \frac{M_o}{E_c I_g}$$

$$= \frac{3557}{E_c (20,985)}$$

$$= \frac{0.1695}{E_c}$$

$$E_c (K_{total} - K_1) = E_c K_2$$

$$= 0.3340 - 0.1695$$

$$= 0.1645$$

$$E_c K_2 = \frac{M_s - M_o}{I_{cr}}$$

$$\text{or } I_{cr} = \frac{M_s - M_o}{E_c K_2}$$

$$= \frac{1724}{0.1645}$$

$$= 10,480 \text{ in.}^4$$

$$I_e = \left(\frac{M'_{cr}}{M'_t} \right)^3 I_g + \left[1 - \left(\frac{M'_{cr}}{M'_t} \right)^3 \right] I_{cr}$$

$$= \left(\frac{649}{1724} \right)^3 (20,985) +$$

$$\left[1 - \left(\frac{649}{1724} \right)^3 \right] 10,480$$

$$= 11,040 \text{ in.}^4$$

Therefore, the live load deflection is:

$$\delta_L = \frac{5L^2}{48E_c} \left[\frac{M_o - M_D}{I_g} + \frac{M'_t}{I_e} \right]$$

$$= \frac{(5) (60 \times 12)^2}{(48) (4286)} \left[\frac{3557 - 2257}{20,985} + \frac{1724}{11,040} \right]$$

$$= 2.75 \text{ in.}$$

This value is very close to the deflection of 3.03 in. given in the paper on p. 27.

The author agrees with Dr. Bennett concerning the need for experimental verification of these theoretical deflection predictions.

Dr. Birkenmaier questions the reliability of achieving an ultimate stress of 270 ksi in a non-prestressed strand especially when the entire value of 270 ksi has to be supplied by bond. Based on personal experience and a limited number of tests performed at Concrete Technology Corp., Tacoma, Washington, non-prestressed strands had achieved their ultimate strength and actually fractured without any bond failure. It should be noted, however, that the beam length should be taken into consideration to insure enough development length. Again this is an area where experimental data are badly needed.

Dr. Birkenmaier is correct in emphasizing the importance of accurately estimating the prestress losses to avoid gross errors in estimating the incremental steel stress at service load.

The other two comments by Dr. Birkenmaier are very valuable and the author wishes to thank him for his contribution.

The author agrees with Dr. Brøndum-Nielsen concerning the unsuitability of the parameter b for circular members. However, it should be noted that these members do not lend themselves to partial prestressing economically. The author further agrees with Dr. Brøndum-Nielsen concerning the fact that the cubic equation is valid for cross sections

of constant web thickness and one axis of symmetry.

Prof. Bruggeling's comments are very interesting and valuable. The author concurs with him concerning the importance of including the effect of non-prestressed reinforcement on prestress losses and deflections especially when a high percentage of this reinforcement is used. The ACI Code, Section 18.8.1, sets a maximum value of 0.3 on the effective reinforcement index ω_e . This value corresponds to a c/d value of 0.45 and 0.54 for concrete strengths of 5000 and 8000 psi, respectively.

Prof. Bruggeling's third comment concerning the effect of non-prestressed reinforcement on the stresses in the concrete could be significant when a high percentage of this reinforcement is used, is correct. However, the author disagrees with him concerning the magnitudes of ΔP_j . A simple method of obtaining this value is to compute the stress in the concrete at the centroid of the steel, and use this value to compute ΔP_j . Applying this to the design example one obtains:

$$\begin{aligned} f_{ci} &= P_j \left(\frac{1}{A} + \frac{e^2}{I_g} \right) - \frac{M_D e}{I_g} \\ &= 246 \left(\frac{1}{401} + \frac{(13.65)^2}{20,985} \right) \\ &\quad + \frac{(2257)(13.65)}{20,985} \\ &= 1.33 \text{ ksi} \\ f_{sj} &= n f_{ci} \\ &= (9)(1.33) \\ &= 11.97 \text{ ksi} \end{aligned}$$

$$\begin{aligned} \Delta P_j &= (11.97)(2)(0.153) \\ &= 3.66 \text{ kips} \end{aligned}$$

$$\begin{aligned} \text{or } (3.66/246) 100 &= 1.49 \text{ percent} \\ &\quad (10 + 2) \text{ strands} \end{aligned}$$

Similarly, the percentage becomes 4.3 percent when using 10 strands plus 2 #6 bars.

In the case of two strands the effect on ΔP_j is negligible.

A concrete strength at prestress release of 2500 psi is more than adequate for providing a 24-in. transmission length based on tests reported in Reference 16.

The author concurs with Professor Levi on the important effect creep and shrinkage has

on the prestressing force. The subject, however, was considered beyond the scope of the paper since it would have obscured the main thrust of the paper.

With reference to the first three comments by Drs. Naaman and Siriakson, the author wishes to note that the approximations mentioned in their comments were not made explicitly or implicitly. The reason for assuming a certain prestress force at the decompression stage is for the sake of simplicity, and should not be confused with the effective prestress.

The state of decompression is used as a reference state to compute changes in stress and strain in the steel for the purpose of evaluating crack width. Designing against fatigue was not considered important for a roof member. However, when fatigue is considered, the author agrees with the writers about the need for consideration of stress range. Deflection was treated above.

In reference to Prof. Nilson's remark concerning the direct determination of the steel stresses f_{ps} and f_s at ultimate load, it should be noted that the assumed prestress loss at decompression has a very small effect (if any) on the value of f_{ps} and has no effect on the value of f_s , since f_s is a function of $\Delta \epsilon_s$. Thus, in the range of practical values of $\omega_e \leq 0.3$ as mandated by the ACI Code, small errors in estimating prestress losses will have no effect on the accuracy of the direct determination of steel stresses at ultimate load as presented in the paper.

The author agrees with Prof. Nilson regarding the need for a more accurate estimate of ϵ_{cu} and concrete stress distribution for high strength concretes.

The author agrees with Dr. Pannell concerning the limitation of using unbonded tendons in partially prestressed beams, and for crack width criteria for prestressed concrete.

The iterative procedure for the analysis of flexural stresses in service presented by Dr. Rajagopalan is a valuable contribution which extends the practical application of the author's method.

Reference

16. Kaar, P. H., LaFraugh, R. W., and Mass, M. A., "Influence of Concrete Strength on Strand Transfer Length," *PCI JOURNAL*, V. 8, No. 5, October 1963, pp. 15-23.