Reader Comments

Design Proposals for Reinforced Concrete Corbels*

by Alan H. Mattock

Comments by Sepp Firnkas, Glyn Jones, Charles H. Raths, Paul F. Rice, Edward R. Sturm, Charles A. Matz, H. Carl Walker, and Author

SEPP FIRNKAS†

Professor Mattock's paper represents a clear and practical summation of his many years of research (as evidenced by his numerous previous publications) about behavior and strengths of corbels. Acceptance of the proposed "Model Code Clause" would eliminate the rather arbitrary ACI 318-71 limitations of the shear-friction concept to corbels of $a/d \leq 0.5$.

The possible reduction of reinforcement will not only be beneficial to the industry in terms of savings, but perhaps more so to designers and quality control personnel since they have to cope with the usual maze of criss-crossing bars and connection details in a congested corbel—especially the critical areas.

Amplifying Professor Mattock's reinforcing steel details and specifically the provision of adequate and proper anchorage development, I would like to add a few additional "non-column" corbel configurations and resulting differences in the corbel force model. In the past a number of these "non-column" corbels failed or developed distress due to lack of adequate anchorage of reinforcement or inadequacy of force transfer details.

Comments or corrections of my thoughts on the following items would be greatly appreciated.

It appears that the column free-body concept does not provide a complete force model. The vertical shear V_u must transferred into the body of the beam by "hanger" reinforcement since the vertical upwards reaction automatically present in the column is missing.

A typical column-corbel free body diagram and force model is shown for comparison in Fig. A.

(a) Beam corbel (Fig. B)

(b) Force applied at bottom of Corbel (Fig. C)

Has the author considered in his investigation and reinforcing details the

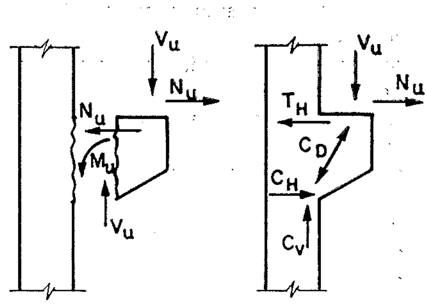
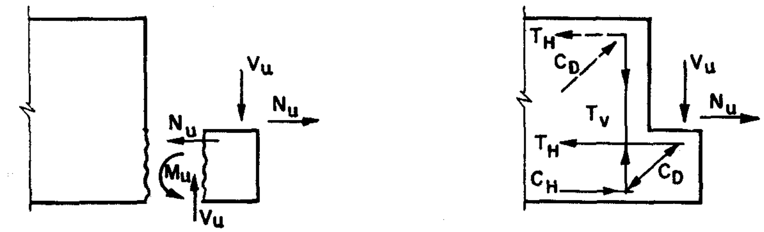


Fig. A. Typical column-corbel free body diagram and force model.

* PCI JOURNAL, V. 21, No. 3, May-June 1976, pp. 18-42.

† President, Sepp Firnkas Engineering, Inc., Boston, Massachusetts.



TYPICAL BEAM CORBEL AND FORCE MODEL

FREE BODY AND REACTIONS

Fig. B. Typical beam corbel showing force model and free body.

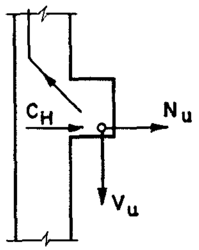


Fig. C. Force applied at bottom of corbel.

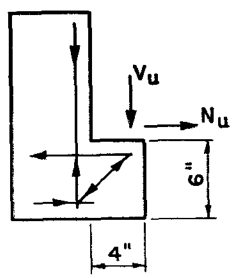


Fig. D. Beam shelf.

case of loads applied at the bottom of corbels. This is necessary for certain types of overhead trolley cranes. Again, a different force model may result (see Fig. C).

(c) Beam shelves (Fig. D)

Since the shelf width and height is usually kept to a minimum of 4 and 6 in., respectively, and similar to the "beam-corbel" condition the vertical upward reaction is not present, what precautions or changes have to be taken to apply the author's proposed design methods for this case?

The preceeding questions and notions may have (and I hope they do) quite simple answers but, nevertheless, should be clarified. I think Professor Mattock is most qualified to provide them to the engineering profession and the precast concrete industry.

GLYN JONES*

Dr. Mattock presents his proposals for the design of reinforced concrete corbels in a clear and practical manner. The design charts and worked examples are particularly useful.

In his previous article[1] Dr. Mattock and his coauthors describe a series of tests on corbels. He demonstrates that his design proposals will ensure adequate strength against collapse and that excessive crack widths do not develop under service loads.

One of the main criticisms of the corbel design method advocated in the British Code of Practice, CP 110:Part 1:1972, is that no guidelines are given to ensure that the design satisfies the serviceability limit state of local damage.

With this in mind, readers may be interested in the following quantitative comparison between the design method proposed by Dr. Mattock and the method given in CP 110.

The design method in CP 110 is based on the assumption that the corbel behaves as a strut and tie system. This involves a strain compatibility check based on the assumptions given in Section 3.3.5.1 of CP 110. Design charts have

* Chief Design Engineer, Fairclough Civil Engineering Ltd., Northern Division, Adlington, Lancashire, England.

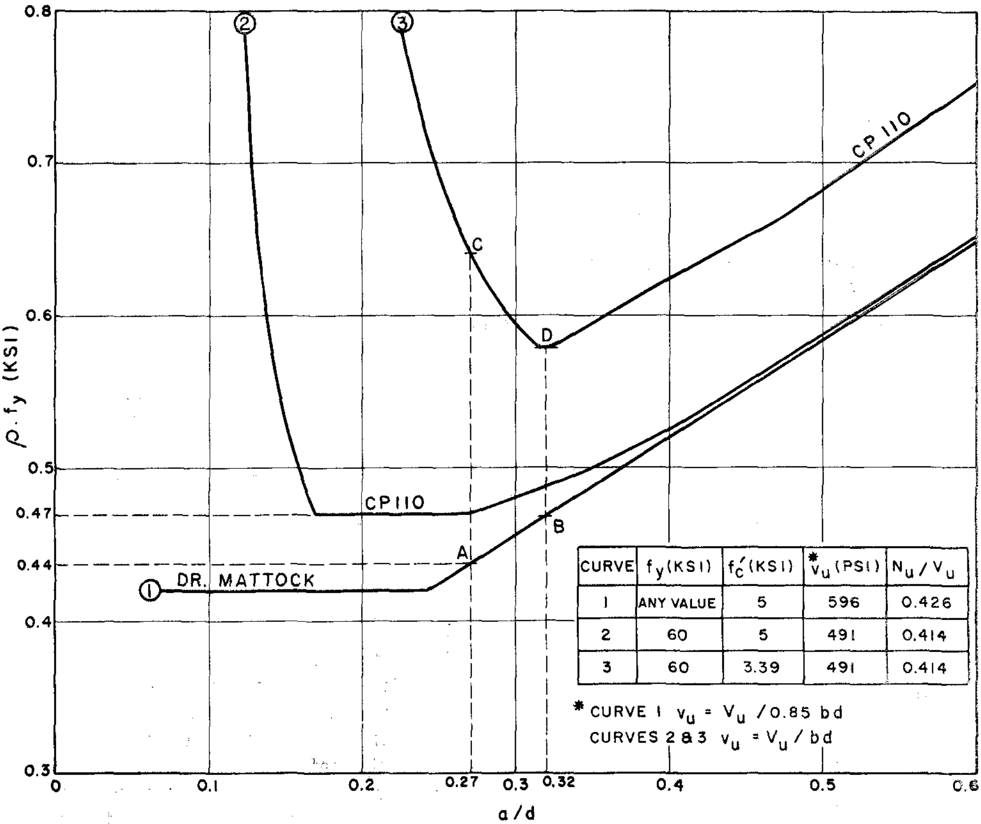


Fig. E. Comparison of Dr. Mattock's design method with provisions in British Code of Practice CP 110.

been produced[6] to reduce the tedium in carrying out this check.

The theory on which these charts are based can be manipulated into a form which will allow the plotting of design curves similar to those produced by Dr. Mattock, thereby providing a useful graphical comparison.

Design Example 1 given in Dr. Mattock's article is used as the basis for the *comparison and the relevant design* curves for the main tension steel are plotted in Fig. E (the units and notation used by Dr. Mattock have been maintained throughout to avoid confusion).

The characteristic strength f_{cu} of the concrete has been taken as $1.28f'_c$, and the values of V_u and N_u and hence (N_u/V_u) have been modified to conform with the partial factors of safety embodied in CP 110 as follows:

$$V_u = 1.4V_D + 1.6V_L$$
$$= 1.4 \times 32 + 1.6 \times 30$$
$$= 92.8 \text{ kips}$$
$$N_u = 1.6N = 1.6 \times 24 = 38.4 \text{ kips}$$
$$N_u/V_u = 0.414;$$
$$v_u = V_u/bd = 92.8 \times 10^3/(14 \times 13.5)$$
$$= 491 \text{ psi}$$

This value corresponds to the shear stress in ACI 318-71:

$$v_u = 95.8 \times 10^3/(0.85 \times 14 \times 13.5)$$
$$= 596 \text{ psi}$$
$$f_{cu} = 1.28 \times 5000 = 6400 \text{ psi}$$
$$f_y = 60 \text{ ksi}; \; a/d = 0.27.$$

Hence, from Design Curve 2 in Fig. E:

$$\rho f_y = 0.47 \text{ ksi}$$

Total area of main tension reinforcement:

$$0.47 \times 14 \times 13.5/60 = 1.48 \text{ sq in.}$$

The area of reinforcement required to resist N_u is:

$38.4/(0.87 \times 60) = 0.74$ sq in.

Hence:

$A_h = 0.5(1.48 - 0.74) = 0.37$ sq in.

It is evident that the design clauses in CP 110 are more stringent than those proposed by Dr. Mattock.

In Fig. E, Design Curve 3 is plotted using 30 grade concrete ($f_{cu} = 30$ N/sq $mm = 4350$ psi, $f'_c = 3390$ psi) and the same parameters that were used for Curve 2. It can be seen that the CP 110 method is very sensitive to changes in the characteristic strength of the concrete.

The corresponding but less sensitive requirements in the clauses proposed by Dr. Mattock are as follows:

$V_u/0.85bd \leqq 0.3f'_c$, $A_s/bd \geqq 0.04\ (f'_c/f_y)$

and

$A_f/bd \leqq 0.75\rho_b$ (being a function of f'_c)

A comparison of Curves 1 and 3 reveals yet another interesting difference between the two design methods when the quantity a/d is varied while the quantity v_u is kept constant. Supposing the distance a is increased from 3.67 to 4.32 in., then a/d increases from 0.27 to 0.32.

In this case Dr. Mattock's method would logically require an increase in the area of main tension steel corresponding to Points A and B on Curve 1. The CP 110 method, however, allows a decrease in the area of main tension steel corresponding to Points C and D on Curve 3. The reverse effect is seen when a/d is reduced in value.

In conclusion, points on Curve 3, at least to the left of Point D, should be disregarded since they correspond to low values of the lever arm factor which, in the case of Curve 3, is equal to 0.645 at Point D. This conclusion suggests a lower limit on the value of a/d for a given value of v_u when using the CP 110 method of design.

Reference

6. Jones, G., "Design Charts for Reinforced Concrete Corbels," *Concrete*, V. 10 No. 6 June 1976, pp. 28-30.

CHARLES H. RATHS*

Dr. Mattock's paper clearly and concisely summarizes his research studies of shear-friction relative to corbels. The contribution of this paper is his recommendations for using "modified shear-friction," and combining it with a free-body approach to connection design where shear span to depth ratios of up to 1 are allowed.

This discussion is directed toward Dr. Mattock's recommended design procedures using his alternate "modified shear-friction" method. The proposed procedures for corbel design, while certainly correct, require too many expressions regarding the upper limits of v_u and the relations between v_u and $\rho_v f_y$. Also, the method indicated for calculating flexural reinforcement could be simplified.

Based upon this writer's extensive background in connection testing, design and failure investigations, proposed connection design relations should employ one or two simple expressions which can be extrapolated to solve unusual design problems. It would appear that any proposed simple design procedure should maintain the approach developed by Mast[7] using an "apparent friction coefficient" combined with free-body concepts.

v_u maximum

The proposed design method for corbels requires that v_u maximum be based on f'_c, concrete unit weight, and the a/d ratio. It would appear that v_u maximum for monolithically cast concrete connection can be simplified to

$$v_u \leqq C^2_s 1000 \tag{9}$$

$$v_u \leqq C^2_s\ (0.25f'_c) \tag{10}$$

where the smaller value of Eqs. (9) and (10) controls. The coefficient C_s is defined as 1.0 for normal weight, 0.85 for sand-lightweight and 0.75 for all-lightweight.

These relations for v_u maximum result in somewhat lower limiting values than proposed by Dr. Mattock. Table A presents a comparison of Eqs. (9) and (10) to Dr. Mattock's v_u limits for non-normal weight concrete where the average dif-

* President, Raths, Raths and Johnson, Inc., Structural Engineers, Hinsdale, Illinois.

ference relative to Dr. Mattock's v_u maximum values is 14 percent.

Maximum v_u's in the range of 700 psi for sand-lightweight and in the range of 500 psi for all-lightweight as given by Table A seem practical for 5000-psi concrete. Moreover, for normal weight concretes, v_u maximum of 1000 psi appears also to be a reasonable upper limit based on Dr. Mattock's previous tests and those of Kriz-Raths.[8] Eqs. (9) and (10) make no provisions for direct shear interfaces that are not cast monolithic.

Eqs. (9) and (10) apparently are in keeping with what is assumed to be Dr. Mattock's intent regarding upper v_u limits. For a given a/d ratio and f'_c, the greatest allowable v_u would be for normal weight concrete, the next greatest for sand-lightweight and the least for all-lightweight concrete.

However, use of Dr. Mattock's limiting v_u relations indicates sand-lightweight can have a greater allowable v_u than normal weight in some cases. The term v_u maximum for sand-lightweight concrete having $f'_c = 6000$ psi and an a/d of 0.2 is 930 psi, according to Dr. Mattock's procedure, while normal weight v_u's are limited to 800 psi. This seems incorrect if past experience regarding v_u capacities and concrete unit weights is a guide.

Modified shear-friction

A concept similar to Dr. Mattock's "modified shear-friction" relation should be adopted and the present ACI 318-71 approach given by Sections 11.15.3 and 11.15.4 eliminated. Dr. Mattock's proposed relations between v_u and $p_v f_y$ more realistically reflect direct shear behavior and capacities than present requirements.

While this writer favors a "modified shear-friction" type concept, he would suggest that the proposed design relation reflect Mast's "apparent friction concept," where u is expressed as $\tan\theta$, or

$$V_u = \phi A_{vf} f_y \tan\theta$$

or

$$A_{vf} = \frac{V_u}{\phi f_y \tan\theta} \qquad (11)$$

The term $\tan\theta$ of Eq. (11) provides a means to develop a single direct shear expression which accounts for different unit weights and non-monolithic direct shear interfaces often encountered in design.

Review of Dr. Mattock's numerous research publications on which his "modified shear-friction" approach is based suggests that a parabolic relation can be used to obtain the same results. A parabolic function satisfying Dr. Mattock's data for direct shear is:

$$v_u = C_s \ (37.42) \sqrt{p_v f_y} \qquad (12)$$

or

$$v^2_u = C^2_s \, 1400 \ p_v f_y \qquad (13)$$

or

$$v_u = C^2_s \, 1400 \ (p_v f_y / v_u) \qquad (14)$$

Table A. Lightweight concrete v_u maximums for $f'_c = 5000$ psi.

v_u max psi	a/d = 0.3		a/d = 0.5		a/d = 1.0	
	110 pcf	95 pcf	110 pcf	95 pcf	110 pcf	95 pcf
$(0.2 - 0.07a/d)f'_c$	895	895	825	825	650	650
1000 - 350 a/d	895*	---	825*	---	650*	---
800 - 280 a/d	---	716*	---	660*	---	520*
$C^2_s\,(0.25f'_c)$	930	730	903	703	903	703
C^2_s 1000	722*	562*	722*	562*	722*	562*
Difference Controlling Values	173	154	103	98	72	42

*Controlling v_u maximum

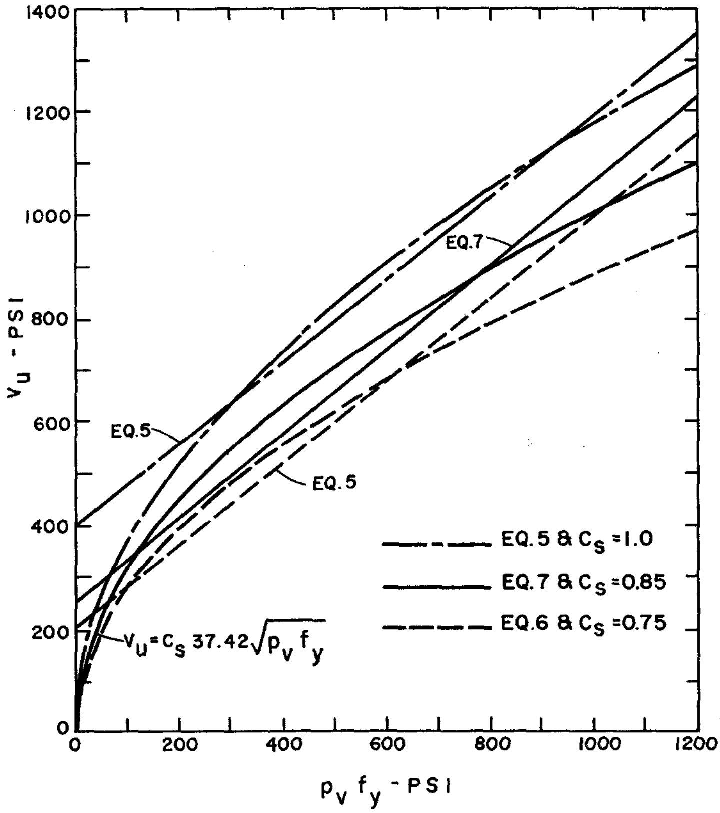


Fig. F. Comparison of Eq. (12) to Dr. Mattock's Eqs. (5), (6) and (7).

Expressing $v_u/(p_v f_y)$ equal to $\tan\theta$ and substituting $\tan\theta$ into Eq. (14) results in:

$$\tan\theta = C^2{}_s\, 1400/v_u \qquad (15)$$

Fig. F presents a comparison of Eq. (12) to those proposed by Dr. Mattock for direct shear capacity. It can be seen for monolithic cast concrete that little difference exists relative to design values.

If the constant "1400" is assumed to vary with the type of direct shear interface, then it is possible to develop one direct shear relation reflecting the influence of both the concrete unit weight and the type of direct shear interface assuming 1400 equals 1000*u* where *u* is 1.4 for a monolithically cast direct shear interface.

Review of test data by Gaston and Kriz[9] on clamped smooth concrete shear interfaces appears to indicate that *u* can be expressed as a variable relative to the type of direct shear interface. Fig. G presents a comparison to Gaston and Kriz' normal weight concrete test data where Eq. (15) has been modified to

$$\tan\theta = C^2{}_s\, 1000u/v_u \qquad (16)$$

or in terms of Eq. (12):

$$v_u = 24.49\sqrt{p_v f_y} \qquad (17)$$

when a $u = 0.6$ is selected.

The Fig. G comparison suggests, therefore, that the *u* values used with the Mast or ACI 318-71 shear-friction approach can realistically reflect the type of shear interface. Table B provides possible *u* values for design.

Table B. *u* values.

Shear Interface	u
Monolithic Cast	1.4
Cast to Roughened Concrete 1/2 in. Undulations	1.0
Cast to Smooth Concrete	0.6

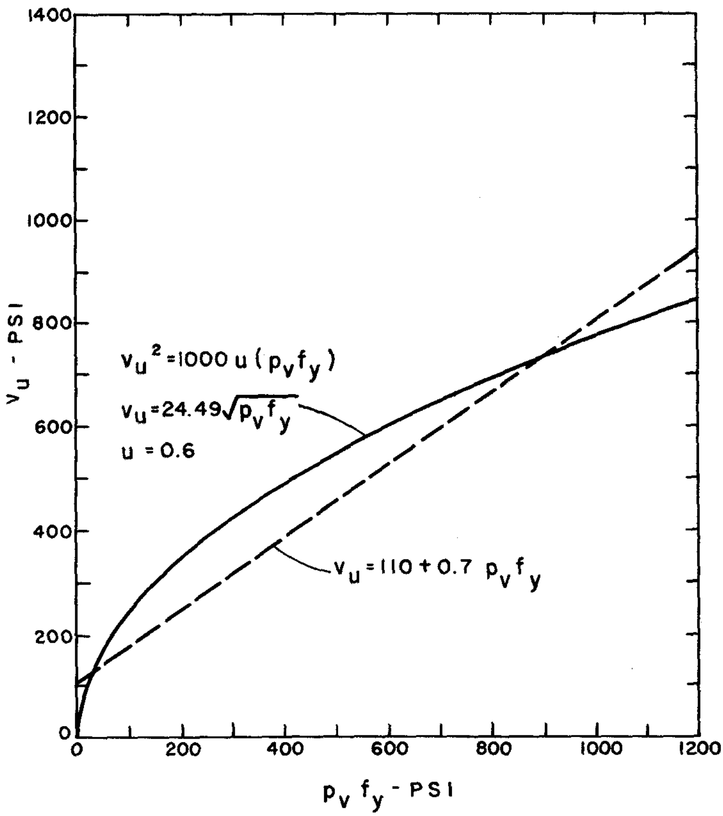


Fig. G. v_u capacity (Reference 9) smooth interface normal weight concrete—dowel action prevented.

Once different direct shear interfaces are considered, it seems appropriate to consider that the type of shear interface also influences the maximum v_u limit. Correspondingly, Eqs. (9) and (10) would require modification for u such that:

$$v_u \leqq C^2_s\, 1000\ (u/1.4) \tag{18}$$

$$v_u \leqq C^2_s\, (0.25\, f'_c)(u/1.4) \tag{19}$$

Applying the maximum v_u limitations as expressed by Eqs. (18) and (19), it is possible to develop a single direct shear relation for different unit concrete weights and different type shear interface conditions which can be expressed in terms of Mast's approach as:

$$A_{vf} = \frac{V_u}{\phi f_y \tan\theta} = \frac{V_u\,(v_u)}{\phi f_y C^2_s\, 1000u} \tag{20}$$

Connection moment reinforcement

Dr. Mattock's proposed corbel design procedures, and present ACI 318-71 requirements for shear-friction, use the methods of Section 10.2 of ACI 318-71 for the selection of flexural reinforcement. It seems expedient, in terms of connection design, to use an expression which directly determines flexural reinforcement rather than estimate the compression block depth a as discussed by Dr. Mattock.

From the rectangular stress block assumption it can be shown that the lever j_u is

$$j_u = 1 - \frac{p f_y}{1.7\, f'_c}$$

Knowing j_u, the required flexural reinforcement A_f can be calculated from

$$A_f = \frac{M_u}{\phi j_u d f_y} \approx \frac{M_u}{\phi (0.9) d f_y} \tag{21}$$

Eq. (21 indicates a j_u value of 0.9 which is appropriate upon considering the usual flexural parameters p, f_y and f'_c involved in connection design.

Alternate corbel relations

Using Dr. Mattock's free body approach (see Fig. 1), and his notations, his conclusion that 50 percent of the direct shear reinforcement is not considered part of the main tension reinforcement A_s, and Eqs. (20) and (21) as developed in this discussion, alternate corbel design relations for A_s and A_h can be advanced:

$$A_s = \frac{1}{\phi f_y}\left[\frac{V_u a + N_u (h\text{-}d)}{0.9d} + N_u\right] \qquad (22)$$

or

$$A_s = \frac{1}{\phi f_y}\left[\frac{V_u (v_u)}{2\, C^2{}_s\, 1000u} + N_u\right] \qquad (23)$$

where A_s is the greater of Eqs. (22) and (23). In Eqs. (22) and (23), V_u and N_u are in kips, v_u is in psi, f_y is in ksi, $\phi = 0.85$, u and C_s are as previously defined, and a, d and h are in inches.

$$A_h = \frac{V_u (v_u)}{\phi f_y\, 2C^2{}_s\, 1000u} \qquad (24)$$

where the same notation as above is used.

The above alternate corbel design relations do not reflect minimum reinforcement requirements. While the present ACI 318-71 minimum requirements are reflected in Dr. Mattock's proposed corbel design procedures, it appears that further study of all influencing design conditions and practices should be considered prior to making any new minimum recommendations since minimums can control in many design situations.

Design examples

Eqs. (22), (23), and (24 [with adjustments to include two-thirds of A_{vf} with A_s] can be used to calculate the reinforcement requirements in Dr. Mattock's design examples. Comparisons of total reinforcement ($A_s + A_h$) shows 4.8 percent more steel for Example 1 and 6.8 percent less steel for Example 2.

Experience has indicated that connection reinforcement congestion problems are more frequently a result of large amounts of A_s. And, experience has also shown that proper reinforcement placement is more critical than variations of 20 percent or more from required reinforcement areas. Furthermore, tests of actual field connections indicate it is immaterial whether two-thirds or one-half of A_{vf} is placed with A_s as long as the total amount of reinforcement satisfies shear, flexure, and horizontal tensile force N_u.

Reinforcing details

Dr. Mattock's paper presents a limited discussion on reinforcing steel details. Experience in precast concrete connection fabrication, design and connection problems indicates that connection details in general are equally as important, if not more so, than the connection design procedures.

Anchorage of reinforcement indicating weld sizes and electrode types, adequate room to place reinforcement within the connection (congestion), compatibility of connection reinforcement with other structural reinforcement (i.e., column reinforcement, beam reinforcement, and prestressing strands), connection reinforcement placement tolerances, reinforcement bends, employing mild steel structural bars rather than reinforcing bars, and so forth should be covered in any proposed design procedure or commentary. Most connection failures result from lack of considering reinforcement details rather than from improper design procedures.

Tolerances and other considerations

Since building tolerances play a major role in determining the centroid of load application, a recommended design procedure should make provisions regarding minimum shear spans a.

A major design factor is N_u minimum. Proposed design methods should make provisions for N_u. Again, this writer's experiences indicate that a N_u minimum of 1.4 (0.15 V_D), where V_D is working dead load, is more appropriate than present requirements, providing details are used minimizing N_u forces.

Connection design procedures should also provide for a minimum load factor greater than that for the members being connected. This results from connection failure modes not providing distress warnings such as abnormal beam deflections.

Summary

Obviously, the preceeding comments as well as alternate corbel design relations are based in large part on Dr. Mattock's outstanding research work on shear-friction and connection design. The comments presented here are not intended in any way to be critical of Dr. Mattock's proposed corbel design procedures. Rather, this discussion has been intended to reflect a designer's concerns for straight-forward design expressions for use in every day design practice taking advantage of Dr. Mattock's work; and further, to extend the same design principles to daps, confined bearing and composite connections.

References

7. Mast, F. R., "Auxiliary Reinforcement in Concrete Connections," *Journal of the Structural Division,* ASCE, V. 94, No. ST6, June 1968, pp. 1485-1504.
8. Kriz, L. B., and Raths, C. H., "Connections in Precast Concrete Structures—Strength of Corbels," PCI JOURNAL, V. 10, No. 1, February 1965, pp. 16-61.
9. Gaston, J. R., and Kriz, L. B., "Connections in Precast Concrete Structures—Scarf Joints," PCI JOURNAL, V. 9, No. 3, June 1964, pp. 37-59.

PAUL F. RICE*

The proposed theory and simple rational method of design is needed and most welcome. I have no concern nor can I quibble with the shear-friction theory, but I do feel more attention is due to practical limitations in the examples given.

Two aspects of the paper are matters of concern. One is the complete indifference to calculation of development and anchorage length requirements per Chapter 12, ACI 318-71, in the examples.

Consider the example of the 14 x 14-in. column with all-lightweight aggregate concrete. The required area of steel is calculated (to two decimal points), shown as 1.99 sq in., and the next line reads "Use 2-#9 bars (2.00 sq in.)."

However, #9 top bars, Grade 60, in all-lightweight concrete $f'_c = 4000$ psi require an embedment, $E = 36.9$ in., with a hooked end; $E = 29$ in. for sand-lightweight; and $E = 21$ in. for normal weight concrete.

$$E = (l_d - l_e)\,1.4 + 0.5D + d_b$$

where $D =$ inside bend diameter of hook

$l_d = 38 \times 1.33$

$= 50.5$ in. for all-lightweight

Adding 30 percent for confinement:

$f_h = (540\sqrt{4000})\,(1.3) = 44{,}398$ psi

$l_e = (38)\,(f_h/f_y)$

$= 38(44{,}398)/60{,}000$

$= 28.1$ in.

$E = (50.5 - 28.1)\,(1.4) +$

$(0.5)\,(8 \times 1.128) + 1.128$

$= 31.22 + 5.64$

$= 36.86$ in.

The available embedment length is 13 in. The first two examples with normal weight concrete, #8 bars will pass if we consider the hooks as on "other bars," confined for 30 percent better anchorage, and apply the "top bar factor," 1.4, only to the lead-in lengths.

The other concern is the welding question. Ordinary (ASTM A615) reinforcing bars are not produced to meet weldability requirements. For ordinary A615 bars, specifying that welding procedures should follow AWS D12.1-75 for the chemical analysis of the particular bars used would be satisfactory.

Alternatively, it should be emphasized that special weldable bars conforming to ASTM A706 should be used and that the welding itself should conform to AWS D12.1-75. Design details like Fig. 2 or Fig. 3 should be accompanied by welding details.

The statement, p. 29, "The welds must be *sized* so as to transfer to the transverse reinforcing bar, a force equal to the yield strength of the main reinforcing bar," is by itself insufficient.

The 1975 *Reinforcing Steel Welding Code* does not indicate design for welds of crossing bars. Only butt welds, fillet

* Vice President, Engineering, Concrete Reinforcing Steel Institute, Chicago, Illinois.

welds, and groove welds are assigned design values. Full strength fillet welds around the periphery of a bar butted to a plate are shown and by extension such welds for a bar butted to the side of another larger bar might be acceptable.

The length and size of weld required for full strength is hardly available at the crossing of equal size bars. The strength of the flare V-groove weld to the #3 framing bar is limited by the length of the 90-deg hook tail (2.5 in.) and the small size of the framing bar.

EDWARD R. STURM*

Dr. Mattock is to be congratulated for this clear and concise report. It reads well, is simple to follow and I can find no fault with the theory. In fact, I checked his Example 1 using both the *PCI Design Handbook* and the 1965 paper "Connections in Precast Concrete Structures—Strength of Corbels" by L. B. Kriz and C. H. Raths.[6]

In all three methods, the same result (3-#7 bars) was obtained, although there is a reduction when you use Dr. Mattock's modified shear-friction equation. Therefore, my question is directed not to the author but to the industry as a whole. Why do we insist upon using deformed reinforcing bars in instances where it is impossible to obtain proper anchorage by bond?

Reinforcing steel (especially Grade 60) contains a high percentage of carbon. It thus is difficult to weld and requires special low hydrogen electrodes, and the bars should be preheated. Also, the curved surface of the bars and the deformations tend to increase the difficulty in obtaining a good weld.

With these facts in mind, I have, for the past several years, been designing corbel reinforcement with ordinary structural (A36) steel. This is a very weldable material and can be fabricated by a steel supplier to a much closer tolerance than the welded "rebar cage" shown in most publications.

For the sake of comparison, I have taken the author's first example using $V_u = 95.8$ kips, $N_u = 40.8$ kips, $f'_c = 5000$ psi NWT but reducing f_y to 36 ksi.

Instead of a bearing plate, I would suggest using a bearing angle. Adequate stiffness can be obtained using a thinner section and the vertical leg can replace the anchor bar. Also the width of the angle should be slightly less than the width of the corbel in order to allow tolerance for fitting it into the form.

Thus, I would try a 5 x 3 x 3/8-in. angle x 1 ft 1½ in., and check the bearing stress.

$$f_b = \frac{95{,}800}{0.7 \times 13.5 \times 4.625} = 2192 \text{ psi}$$

which is less than 2500 psi and thus satisfactory.

Assuming that the load is applied at the outer third point of the angle and allowing an approximate shear stress of 600 psi:

$$a = 1 + 0.67 \times 5 = 4.33 \text{ in.}$$

$$d \approx \frac{95.8}{0.85 \times 14 \times 600} \approx 13.4 \text{ in.}$$

Now, if we let the total depth of the corbel be 15 in. and assume that 3-in. plate will be used to develop the moment capacity, we can compute:

$$d = 15 - 0.375 - 0.5(3) = 13.13 \text{ in.}$$
$$[\approx 13.4 \text{ (ok)}]$$
$$\text{and } a/d = 0.33$$
$$[< 1.0 \text{ (ok)}]$$

Then using the author's modified shear-friction theory, the calculations become:

$$M_u = 95.8 \quad (4.33) + 40.8 \quad (15\text{-}13.13)$$
$$= 491.1 \text{ in.-kips}$$

Assume $x = 0.7$ in.

$$A_f = \frac{491.1}{0.9(36)\,(13.13\text{-}0.35)}$$
$$= 1.19 \text{ in.}^2$$

Check:

$$x = \frac{1.19\,(36)}{0.85\,(5)\,(14)} = 0.72 \text{ in.}$$
$$\approx 0.7 \text{ in.}$$

$$A_{vf} = \left[\frac{95.8}{0.8\,(0.85)} - 0.5\,(14)\,(13.13)\right]/36$$
$$= 1.36 \text{ in.}^2$$

but not less than 0.2 (14) (13.13)/36
$$= 1.02 \text{ in.}^2$$

---

* Vice President, Engineering, Thomas Concrete Products Company.

$$A_t = \frac{40.8}{0.85\,(36)} = 1.33 \text{ in.}^2$$

$$A_s = 1.19 + 1.33 = 2.52 \text{ in.}^2$$

Use two 3-⅜ × ⅜-in. plates ($A = 2.53$ in.^{2}).

$$A_h = 0.5\,(1.19) = 0.60 \text{ in.}^2$$

Use two 1⅛ × ⅜-in. plates ($A = 0.84$ in.^{2}

The steel requirements for A_s and A_h, though, can be combined so that we can use two- 4½ x ⅜-in. plates in order to resist the combination of bending, shear and horizontal tension with a single "U" shaped strap. This will result in the prefabricated steel assembly shown in Fig. H which has equal strength to the author's design shown in Fig. I.

There are, however, three big advantages in using the steel assembly shown in Fig. H. First of all, it can be completely prefabricated so that it can be delivered and placed into the form as a single unit. Secondly, it weighs less (36.2 lbs vs. 42.3 lbs). and finally, the biggest advantage of all: COST. I have received quotations from two steel fabricators for each of these assemblies, and the one shown in Fig. I is 50 percent more expensive than the one shown in Fig. H.

Now, let us consider a fourth advantage. Up to this point we have been discussing single corbels which are straight forward when columns are cast in piling forms. But what about the case of double, triple or even quadruple corbels which we sometimes have to produce?

There is a simple solution shown in Figure J. The portion of the "U" shaped strap within the column is replaced by an 11 x 11 x ½-in. steel tube 6 in. long (fabricated). The column can then be precast with a 1½ in. deep x 15 in. high recess on all sides requiring a corbel; so that after stripping, the remainder of the corbel assembly can be welded to the tube. Then

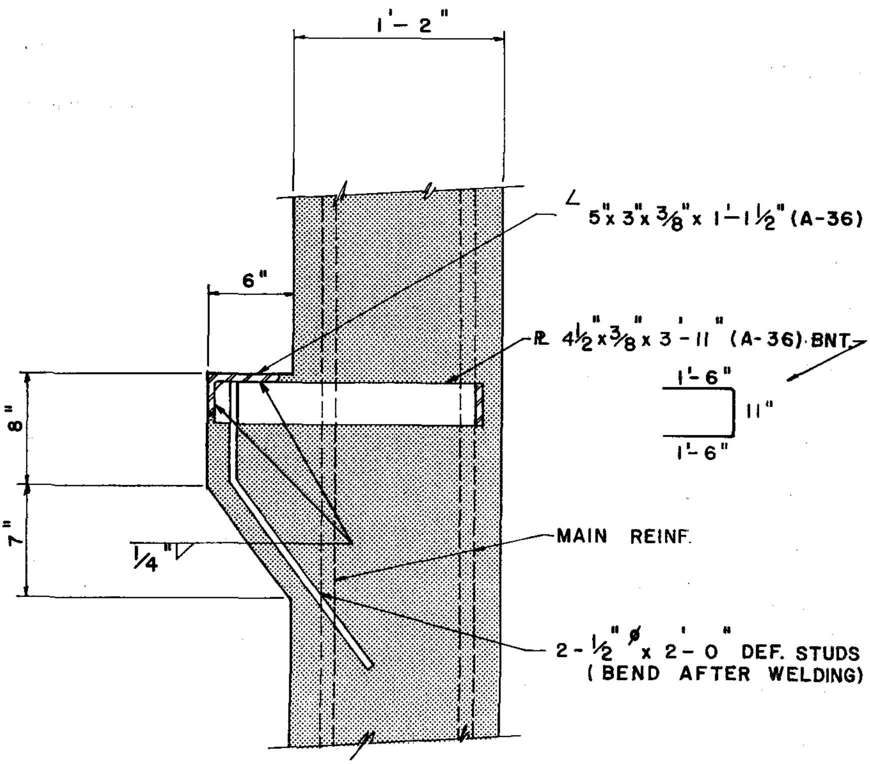


Fig. H. Modified detail using A36 steel.

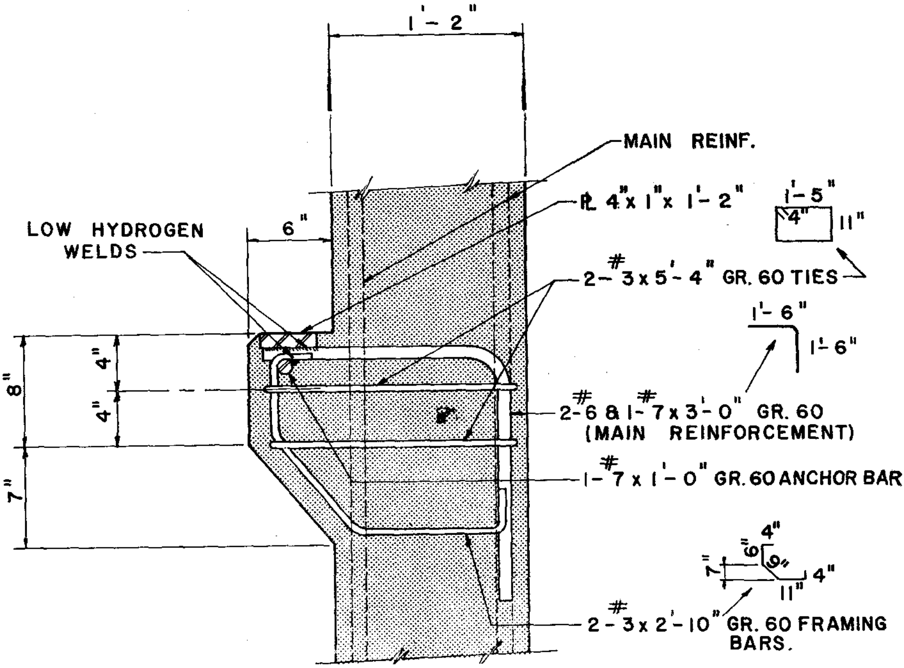


Fig. I. Author's detail using reinforcing steel cage.

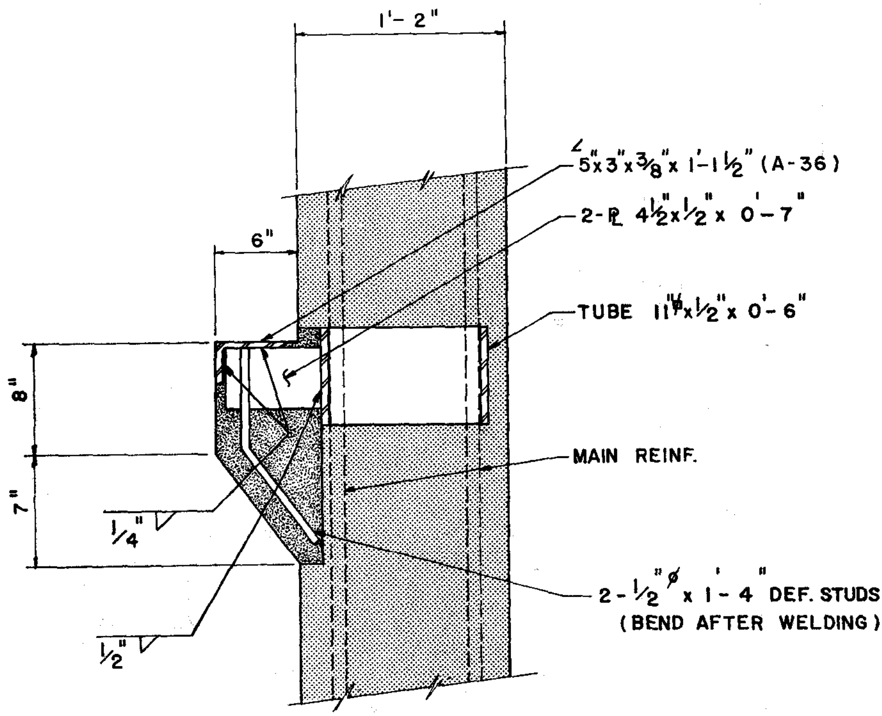


Fig. J. Modified detail using A36 steel with corbel attached after fabrication.

the concrete for the corbel itself can be cast.

In order to allow for the fact that the concrete is not cast monolithicly, the steel requirements should be increased by a ratio of 1.4/1.0.

$$A_s + A_h = 1.4\ (2.52 + 0.60)$$
$$= 4.37\ \text{in.}^2$$

Use two 4½ × ½-in. plates.

Thus, the thickness of the 4½-in. straps and of the tube should be increased from ⅜ to ½ in. Also, note the height of the tube has been increased from 4½ to 6 in. in order to allow for fabrication tolerances.

CHARLES A. MATZ*

The author has provided a valuable contribution towards the analysis and design of reinforced concrete corbels. Hopefully the "Model Code Clause" can be acknowledged and adopted as a replacement for Section 11.14 of ACI 318-71.

In Appendix B, Item 3a, the equation should be amended to include the shear-friction coefficient (μ) in the denominator in accordance with ACI 318-71, Section 11.15.4, Eq. (11-30).

Many thanks to Professor Mattock for also including an HP-65 calculator pro-

* Design Engineer, Mullen & Powell, Inc., Consulting Engineers, Dallas, Texas.

Table C. Corbel design procedure using SR-52.

STEP	PROCEDURE	ENTER	PRESS		DISPLAY
1.	Insert Corbel #1 Read Sides A & B				
2.	Enter Data		E		1.
		V_u (kips)		Run	2.
		N_u (kips)		Run	3.
		a (in.)		Run	4.
		h (in.)		Run	5.
		h-d (in.)		Run	6.
		b (in.)		Run	7.
		f_y (ksi)		Run	8.
		f'_c (psi)		Run	0.
3.	Calculate $\beta_1 f'_c$		A		$\beta_1 f'_c$ (psi)
4.	Insert Corbel #2 Side A & B				
If ACI 318-71 Shear Friction is to be used					
5a.	Calculate A_f	1	A		A_f (Sq.in.)
6a.	Calculate 2/3 A_{vf}		B		1.
7a.	Enter μ = 1.4 Normal wgt. conc. 1.05 All lgt. wgt. conc. 1.15 Sand lgt wgt. conc.	μ		Run	2/3 A_{vf} (in^2)
8a	Calculate A_t		C		A_t (sq.in.)
9a.	Calculate A_s		D		A_s (sq.in.)
10a.	Calculate A_h		E		A_h (sq. in.)
If Modified Shear Friction is to be used					
5b.	Calculate A_f	2	A		A_f (sq.in.)
6b.	Calculate 2/3 A_{vf}		B		2.
7b.	Enter K = 0.5 Normal wgt. conc. 0.25 All lgt. wgt conc. 0.31 Sand lgt. wgt. conc.	K		Run	2/3 A_{vf}(in.^{2})
8b.	Calculate A_t		C		A_t (sq.in.)
9b.	Calculate A_s		D		A_s (sq.in.)
10b.	Calculate A_h		E		A_h (sq.in.)

At step 5 either 5a or 5b is selected in accordance with the entry (1 or 2). At any point after step 4, 5a or 5b may be restarted by entering the appropriate value (1 or 2) and Press A. Hence after 5a. thru 10a., 5b. thru 10b. may be run by keying.2 Press A.

Table D. Program for Corbel #1 using SR-52.

LOC	KEY	LOC	KEY
000	*LBL E *CMS	122	- RCL 05
003	1 STO 19	126	)) =
007	RCL 19 HLT	129	STO 10
011	*IND STO 19	132	+/- SUM 09
013	1 SUM 19	136	RCL 09
019	RCL 19	139	*ifpos 144
022	- 9 =	143	+/-
025	*if zro 033	144	-
029	GTO 007	145	.005
033	HLT	149	=
034	*LBL A	150	*ifpos 042
036	.95	154	RCL 08 ÷
039	STO 10	158	1000
042	RCL 10	162	- 4 =
045	STO 09	165	*ifpos 170
048	RCL 01	169	0
051	x RCL 03	170	x .05
055	+ RCL 02	174	+/- = +
059	x RCL 05	177	.85 =
063	= ÷ .9	181	x RCL 08
067	÷ RCL 07	185	= STO 12
071	÷ (	189	÷ 18 ÷
073	RCL 04 -	193	RCL 07 ÷
077	RCL 05)	197	(87 +
081	÷ RCL 09	201	RCL 07)
085	= STO 11	205	x
089	1 - (	206	(RCL 04
092	RCL 11 x	210	- RCL 05
096	RCL 07 ÷	214	) =
100	(RCL 08	216	STO 13
104	+ RCL 09	219	RCL 12
108	) x	222	HLT
110	588 ÷		
114	RCL 06 ÷		
118	(RCL 04		

Table E. Program for Corbel #2 using SR-52.

LOC	KEY	LOC	KEY
000	RCL 00	117	= STO 19
003	- 2 =	121	- RCL 18
006	*ifpos 012	125	=
010	*stflg 1	126	*ifpos 134
012	RCL 04 -	130	RCL 18 *rtn
016	RCL 05 =	134	RCL 19 *rtn
020	x RCL 06	138	1 HLT
024	= STO 09	140	*1/x x
028	RCL 06 x	142	RCL 01 ÷
032	RCL 13 =	146	.85 ÷
036	STO 12 -	150	RCL 07
040	RCL 11 =	153	GTO 103
044	*ifpos 057	157	*LBL C
048	0 *1/x HLT	159	RCL 11
051	*LBL A	162	*E'
053	STO 00 *rset	163	STO 16
057	RCL 11 HLT	166	RCL 02 ÷
061	*LBL B	170	.85 ÷
063	*ifflg 1	174	RCL 07
065	138	177	= STO 17
068	2 HLT	181	HLT
070	x RCL 09	182	*LBL D
074	+/- +	184	+ RCL 16
076	RCL 01 ÷	188	= STO 18
080	.68 =	192	RCL 09 x
084	÷ RCL 07	196	RCL 08 x
088	= STO 18	200	4 EE +/- 5
092	RCL 09 x	204	INV EE ÷
096	.2 ÷	207	RCL 07 *E'
099	RCL 07 *E'	211	HLT
103	x 2 ÷ 3	212	*LBL E
107	= STO 14	214	- RCL 17
111	STO 18	218	= ÷2 =
114	HLT	222	HLT
115	*LBL *E'	223	HLT

Table F. Example run of Corbel design (data from Dr. Mattock's Example I).

ENTER	PRESS	DISPLAY	COMMENT
Enter Card 1	Side A & B		
	E	1.	
95.8	Run	2.	V_u (kips)
40.8	Run	3.	N_u (kips)
3.67	Run	4.	a (in.)
15.	Run	5.	h (in.)
1.5	Run	6.	h-d (in.)
14.	Run	7.	b (in.)
60.	Run	8.	f_y (ksi)
5000.	Run	0.	f'_c (psi)
	A	4000.	$\beta_1 f'_c$ (psi)
Enter Card 2	Side A & B		
For Shear Friction			
1	A	.57912...	A_f(sq.in.)
	B	1.	!!ENTER μ VALUE!!
1.4 (=μ)	Run	.89449...	2/3 Avf (sq.in)
	C	.8	At (sq.in.)
	D	1.69449	A_s (sq.in.)
	E	.44724...	A_h (sq.in.)
For Modified Shear Driction			
2	A	.57912...	A_f (sq.in.)
	B	2.	!!ENTER K VALUE!!
0.5 (=K)	Run	.51535...	2/3 A_{vf} (sq.in.)
	C	.8	At (sq.in.)
	D	1.37912...	A_s (sq.in.)
	E	.28956	A_h (sq.in.)

gram to alleviate the routine calculation requirements. I feel certain that in the near future we will see more professional programs available in all areas of design.

I have included a similar program written for the Texas Instruments SR-52 (see Tables C, D, E, F). It follows a design procedure identical to that shown for the HP-65 in Appendix B.

Hopefully, owners of the SR-52 will now also be able to take advantage of the streamlined design tool Professor Mattock has provided.

H. CARL WALKER*

The author is to be commended for developing a neat blend of structural research, code recommendations, and step-by-step design examples.

In the proposed model code clause, Section 11.14.2.5 states: "The main tension reinforcement shall be anchored as close to the outer face of the corbel as cover requirements permit, by welding a bar of equal diameter across the ends of the main reinforcing bars, or by some other means of positive anchorage."

The question then arises: What is the required capacity of this weld or anchorage? Is it intent to develop the full strength of the main tension reinforcement? If so, this will require full penetration weld, which may have questionable capacity when considering the variable carbon contents in reinforcing steel.

Fig. 3, Design Example 1, shows a steel bearing plate welded to the main reinforcement. Many concrete haunches are designed and fabricated without such a steel plate.

The question then becomes: What should be the outermost dimensional limit of the applied load? The limit could be as far out as the edge of the chamfer; could be limited to the extreme end of the main reinforcing; or could be held back on some angle, say a 45 deg slope from the end of the main reinforcing; or held back on a 45 deg slope from the anchor bar. Since bearing surfaces are not fabricated and erected coplanar, the author's comments on edge bearing limits would be appreciated.

Professor Mattock uses a 1-in. bearing plate welded to the main reinforcement in his design example which can serve many purposes. It would appear that a fillet weld on both sides of the main reinforcement to the bearing plate would easily develop the full strength of the main reinforcement, thus eliminating the need for the transverse anchor bar. The exterior edge of the bearing plate also limits the extreme of the load bearing area of the haunch. Also, the framing bars could be tack welded directly to the plate.

I trust that the 1-in. plate was for illustration purposes and not intended to indicate a minimum plate thickness. Professor Mattock's comments on elimination of the anchor bar in this case would be appreciated.

Haunches are very sensitive to the dimensional tolerances. Haunch reinforcing cages that are attached only to the main column reinforcing cage are subject to adjustments of the main column reinforcing cage and often are not held within tolerance location limits prescribed for the haunch reinforcing.

Therefore, a detail such as shown in Design Example 1, where the bearing plate can be attached directly to a form surface and the haunch reinforcing cage is then fixed in position relative to that form surface, will provide excellent tolerance control.

Again, the author is to be commended on his fine advancement to the state of the art of precast concrete connection design and his clear presentation of the transition from research to code recommendation and design use.

* President, Carl Walker & Associates, Inc., Kalamazoo, Michigan.

AUTHOR'S CLOSURE

The author thanks all the contributors for their thoughtful discussions. It is through such an exchange of ideas that progress is made.

Mr. **Sturm's** use of adequately anchored structural steel plates for the main tension reinforcement of corbels, instead of reinforcing bars, is quite proper. However, I do not think it is appropriate to combine both steel areas A_s and A_h into a single U-shaped strap. The horizontal stirrups having a total area A_h are provided in order to prevent a premature diagonal tension failure occurring within the corbel, as well as to contribute to shear resistance at the corbel-column interface.

To control diagonal tension cracking, the reinforcement of area A_h needs to be distributed over the upper two-thirds of the effective depth of the corbel. I fear that with the reinforcement detail shown by Mr. Sturm, a premature diagonal tension failure could occur. I think that cost savings would still result if the U-shaped steel strap were used to supply A_s, and stirrups were provided for A_h. The assembly could still be prefabricated and placed in the form as a single unit.

Mr. Sturm's solution to the problem of reinforcing a multiplicity of corbels at one level is ingenious. However, horizontal stirrup reinforcement should be provided in the corbels in this case also. Perhaps U-shaped pieces of rebar or steel strip could be welded to the steel tube, to provide A_h.

Both Mr. **Firnkas** and Mr. **Rice** are properly concerned about what happens on the other side of the interface between the corbel and whatever member is supporting it. This paper was primarily concerned with providing a design method for the reinforcement within the corbel, and the proposals would be applicable to corbels projecting from beams as well as from columns.

A corresponding set of reactive forces to those shown acting on the corbel "free body" in Fig. 1b, are of course acting on whatever member is supporting the corbel. Mr. Firnkas correctly points out that the member itself must be appropriately reinforced to carry these forces. In the case of a column, the vertical shear coming from a corbel can be resisted directly by compression in the column below the level of the corbel.

However, in the case of the beam supporting a corbel it is absolutely essential to provide "hanger" reinforcement as indicated by Mr. Firnkas, in order to transfer the vertical load to the flexural compression zone of the beam. If this is not done, failure can occur by the formation of diagonal tension cracks in the beam on both sides of the corbel.

Hanger reinforcement connecting the flexural tension and flexural compression regions should always be provided whenever vertical loads are applied to a beam in the flexural tension region. The hanger reinforcement should have a yield strength equal to the total vertical load being supported by the tension zone, V_u, from the corbel in this case, and should encircle the flexural tension reinforcement.

The studies at the University of Washington have not included either the case of the corbel loaded at its bottom face or the "beam shelf." As indicated by Mr. Firnkas, a completely different force system exists when the load is applied at the bottom of the corbel, and the reinforcement would have to be arranged accordingly.

As regards the "beam shelf," tests made at the University of Texas[10,11] appear to indicate that the present corbel design proposals could appropriately be used to design such "beam shelves" to carry a series of concentrated loads along the length of the shelf. The concentrated load is distributed along the shelf to some extent, and it is appropriate to consider that the concentrated load is carried by a length of shelf equal to $(4a + w)$, where a is the distance between the line of action of V_u and the face of the beam and w is the length of the bearing surface on the shelf, measured parallel to the beam axis.

The reinforcement A_s and A_h should be provided within this length $(4a + w)$. The reinforcement A_s must be anchored positively in the same way as in a corbel and U-shaped framing bars should be provided following the side and bottom faces of the shelf. The steel area A_h may be concentrated at one level, between one third and half the effective depth below the main reinforcement. It may be in the form of closed stirrups or may be similar in form to the reinforcement A_s.

Tests made by the Cement and Concrete Association,[12] C&CA, in which beam shelves were subjected to a continuous strip load rather than a series of concentrated loads, also indicate that the proposals for corbel design may appropriately be used in this case also.

In general, the shear stress in the shelf will be quite low for this continuous strip loading. If it is less than $2\sqrt{f'_c}$, the reinforcement in the shelf can be concentrated in one layer near the top face. In this case, its area should be the greater of $(A_f + A_t)$ or $(A_{vf} + A_t)$. The C&CA tests indicated the vulnerability of the outer edge of the shelf, and the recommendation was made that the outer edge of the bearing strip should be a distance inside the anchor bar equal to the cover to the main shelf reinforcement A_s. A chamfered corner was also recommended.

Mr. **Rice** is concerned about the anchorage of the main reinforcement in the column. The examples included in the paper were simply to illustrate the calculation of the amounts of reinforcement required for the corbels. In practice the anchorage requirements of the reinforcement would of course have to be investigated as part of the total design process.

In the case of corbels on columns precast horizontally, the main tension reinforcement of the corbels would not be "top reinforcement," and therefore, the 1.4 factor on l_d would not apply. For the examples using normal weight concrete adequate anchorage would be provided for all the bar sizes used, by a standard 90-deg hook as indicated in Fig. 3.

In the example in which lightweight concrete was specified, it would be necessary to used a bend radius larger than the four bar diameters specified for the standard hook. The required 38-in. embedment length could then be provided from the corbel-column interface to the end of the bar, (as shown in Fig. 12.6 of the "Commentary on Building Code Requirements for Reinforced Concrete (ACI) 318-71").

Both Mr. **Rice** and Mr. **Walker** express concerns regarding the welding of the transverse bar to the outer ends of the main tension reinforcement to provide a positive anchorage. The problems associated with the welding of reinforcing bars are appreciated, and Mr. Rice's comments on the chemical analysis of the steel and on welding procedures are appropriate.

It is true that AWS D12.1-75 does not indicate design for welds of crossing bars. However, such a detail is shown in Fig. 6.1.21 of the *PCI Design Handbook.* This detail was used successfully in the corbels tested at the University of Washington.[1]

When a horizontal force N_u is applied to the corbel through a steel bearing plate, the welds between the bearing plate and the main reinforcement must be sized to carry the force N_u. The welds between the transverse anchor bar and the main reinforcement need then only be sized to carry a force $(A_s f_y - N_u)$.

Although, as Mr. Walker remarks, the welds between the main reinforcement and the bearing plate could be made to have a strength equal to the yield strength of the reinforcement, I do not believe that the bearing plates alone could be relied upon to anchor the reinforcement. The concrete above the rebars and adjacent to the bearing plate would probably spall under the lateral pressure from the edge of the bearing plate, and the anchorage would be lost.

Mr. Walker suggests that the corbel reinforcement could be positioned accurately in the form by attaching the bearing plate to the form surface. This was in fact the method used to locate the reinforcement in the corbels that were tested at the University of Washington.[1] It was found to be very convenient and resulted in accurate location of both the bearing plates and the corbel reinforcement.

The 1-in. thick bearing plate used in the examples was for the purpose of illustration and was not intended to indicate a minimum plate thickness. A steel bearing plate should be provided if a significant horizontal force must be carried by the corbel. This force can then be transferred directly to the main reinforcement without risk of pulling the outer corner concrete off the corbel.

If a significant horizontal force is not expected and an inset steel bearing plate is not provided, then the outer edge of the loaded area should preferably be not less than 2 in. from the outer face of the corbel. Precautions should be taken to ensure that if rotation of the end of the beam occurs as a result of creep, the bottom of the beam will not come into contact with the outer corner of the corbel and cause damage to the concrete.

The saying goes that, "There are many ways to skin a cat." Mr. **Raths** has chosen to skin this cat in a somewhat different way from that proposed by the author, although starting from the same premise that the design of corbels can be based upon satisfaction of the laws of statics, when the corbel is considered as a "free body" (see Fig. 1b).

The heart of the paper was the "Proposed Model Code Clause." The form of this Model Code Clause is consistent with the current trend in codes; that is, wherever possible to specify basic principles and limiting conditions to be observed, rather than to specify a unique set of design equations. How the requirements of the Code Clause are met in design is the responsibility of the designer.

In drawing up the design examples included in the paper, the author's purpose was to illustrate the application of the provisions in the clearest possible fashion; therefore, no "short cuts" or approximate simplifications were included. It is obviously possible to abbreviate the calculation of A_s by assuming that $j_d = 0.9$, as suggested by Mr. Raths in Eq. (21), (provided that the designer remembers this becomes a non-conservative approximation, if ρf_y exceeds 0.17 f'_c).

The use of equations such as (22) and (23) as design aids is quite proper, providing the designer is familiar with the principles upon which the design procedure is based and with any simplifications and assumptions made in the derivation of the equations. [Eq. (22) is simply an algebraic expression of those requirements of the proposed Sec. 11.4.2, which relate to moment and horizontal force. An equation similar to Eq. (23) could be written, but basing the first term within the bracket on the ACI 318-71 shear-friction provisions or on the author's modified shear friction proposals.]

Mr. Raths infers that Eqs. (22) and (23) "can be extrapolated to solve unusual design problems." The author believes that it is preferable to consider each such problem individually, taking into account the likely location of cracks, and the forces and moments to be carried across critical planes. The reinforcement should then be designed making use of basic principles. Design aids can be developed for these more unusual design problems, if their frequency of occurrence is sufficient to justify this.

Mr. Raths proposes the use of a parabolic relation relating v_u and $\rho_v f_y$, Eq. (12), for shear transfer calculations. This equation is of the same form as that proposed in 1968 by Birkeland, and discussed in References 2 and 13.

$$v_u = 33.5\sqrt{\rho_v f_y}$$

(for sand and gravel concrete)

The coefficient 33.5 was chosen in this case so that when $\rho_v f_y = 571$ psi, v_u would equal 800 psi, (as is the case when applying the simple shear friction provisions of ACI 318-71). This equation is a lower bound to the available shear transfer test data.

Mr. Rath's Eq. (12) is 12 per cent less conservative than Mr. Birkeland's equation. As seen in Mr. Raths' Fig. F, it is also unconservative relative to the author's modified shear friction Eqs. (13), (14), and (15), over the range of values of $\rho_v f_y$ commonly used in corbel design. In this connection, it should be remembered that the modified shear friction equations yield mean values for shear transfer strength, *not* lower bound values.

If Mr. Raths wishes to use a parabolic form of equation, it would seem preferable to choose a coefficient so that the predicted values of v_u do not exceed the mean values of the test data, say $\sqrt{900\mu}$ instead of $\sqrt{1000\mu}$. This would also yield simpler coefficients of 35.50 for $\mu = 1.4$ and 30.00 for $\mu = 1$.

Mr. Raths proposes to use a value of $\mu = 0.60$ for the case of a smooth interface between concretes cast at different times. This leads to the equation:

$$v_u = 24.49\sqrt{\rho_v f_y}$$

shown in Mr. Raths' Fig. G. Mr. Raths justifies this equation with reference to friction tests by Kriz involving smooth concrete interfaces which were clamped together by externally prestressed steel bars.

More recent tests* of shear transfer across a smooth interface reinforced with ordinary rebars, indicate a shear transfer strength corresponding almost exactly to the equation:

$$v_u = 0.6\, \rho_v f_y$$

This is much below Kriz' data and even farther below the line representing Mr. Raths' equation. Very little separation across the interface occurred in these tests, so that the shear transfer reinforcement was not stressed in tension to any significant extent.

The shear transfer strengths developed correspond to the shear yield strength of the shear transfer reinforcement. The value of $\mu = 0.6$ should not, therefore, be used in Mr. Rath's equation for the case of a smooth interface.

Mr. Raths takes issue with the multiplicity of upper limiting values for v_u included in the proposal. These various values allow maximum advantage to be taken of each different type of concrete and also reflect the not insignificant influence of a/d on the maximum shear stress which can be developed in lightweight concrete corbels.[1]

There is nothing to prevent a designer adopting a single but more conservative limiting values for v_u for all concretes, if he so wishes. For instance, $(0.2\, f'_c - 0.07\, a/d)$;$f'_c$ in this expression to be not more than 4000 psi. This upper limit would be correct for all-lightweight concrete, but conservative to varying degrees for sanded-lightweight and normal weight concrete.

The upper limits that Mr. Raths proposes ignore the effect of a/d and yield values varying between 33 percent conservative to 11 percent unconservative, as compared to the upper limits proposed in the paper, (which correspond to actual behavior). The average difference of 14 percent quoted by Mr. Raths in his discussion is misleading, since the differences in his Table E vary from 22 percent conservative to 11 percent unconservative.

The upper limit of $0.2\, f'_c$ or 800 psi specified for normal weight concrete when using the shear-friction provisions of ACI 318-71 does not reflect the actual maximum attainable shear transfer strength for a given value of f'_c, (which is $0.3\, f'_c$). It rather stems from the fact that using $\mu = 1.4$, the shear friction equation becomes unconservative for values of $\rho_v f_y$ greater than about 570 psi, which corresponds to $v_u = 800$ psi.

This situation does not arise in the case of all-lightweight and sanded-lightweight concretes if the values of μ are multiplied by 0.75 and 0.85 as proposed. In these cases the upper limit specified is the actual maximum shear transfer strength attainable.

This gives rise to the situation pointed out by Mr. Raths, in which for $f'_c = 6000$ psi and $a/d = 0.2$, the maximum value of v_u when using shear friction, is 800 psi for normal weight concrete and 930 psi for sanded-lightweight concrete. However, if the author's modified shear-friction equations are used, then the upper limit for v_u in normal weight concrete becomes $0.3\, f'_c$, or 1800 psi in this case. (Such shear transfer strengths have been obtained.)

Mr. Raths proposes to include only half the required A_{vf} in A_s and to make A_h equal $A_{vf}/2$. This would be acceptable as long as A_h is not less than $\frac{1}{2}(A_s - \frac{N_u}{\phi f_y})$.

The term $A_{vf}/^2$ could possibly become less than this quantity when a/d is large. The presence of a sufficient quantity of horizontal stirrups in the corbel is particularly important when a/d is large, if behavior is to be satisfactory.

Mr. Raths' comments on minimum reinforcement and an appropriate value for N_u minimum are well taken. The author agrees with Mr. Raths completely regarding the need for accurate placement of the reinforcement and for careful attention by the designer regarding all aspects of detailing the reinforcement.

Mr. **Matz'** contribution of the corbel design program for the Texas Instruments

* Mattock, A. H., Discussion of "Considerations for the Design of Precast Concrete Bearing Wall Buildings to Withstand Abnormal Loads," by PCI Committee on Precast Concrete Bearing Wall Buildings (accepted for publication in the PCI JOURNAL).

SR-52 calculator should be very welcome for owners of that calculator. The μ missing from the denominator of the equation in Item 3(a) of Appendix B was in the original manuscript, but somehow got lost in the printing process!

Mr. **Jones** makes an interesting comparison of corbel design based on the *British Code of Practice* CP110 and on the proposals made by the author. The truss model used in CP110 is an appealing one on first sight, but does apparently result in some illogical reinforcement requirements at low values of a/d. Its sensitivity to the concrete strength is also interesting.

The author would like once more to thank all the contributors for discussing this paper and for the time and energy they have put into the task.

References

10. Furlong, R. W., Ferguson, P. M., and Ma, J. S., "Shear and Anchorage Study of Reinforcement in Inverted T-Beam Bent Cap Girders," Research Report 113-4, Center for Highway Research, University of Texas at Austin, July 1971.
11. Furlong, R. W., and Mirza, S. A., "Strength and Serviceability of Inverted T-Beam Bent Caps Subject to Combined Flexure, Shear and Torsion," Research Report 153-1F, Center for Highway Research, University of Texas at Austin, August 1974.
12. Clarke, J. L., "Behavior and Design of Small Continuous Corbels," Technical Report 42.513, Cement and Concrete Association, Wexham Springs, Slough, England, March 1976.
13. Birkeland, H. W., Discussion of "Shear Transfer in Reinforced Concrete with Moment or Tension Acting Across the Shear Plane," by Mattock, A. H., Johal, L., and Chow, H. C., PCI JOURNAL, V. 21, No. 3, May-June 1976, pp. 112-113.

Prestressing the CN Tower*

by Franz Knoll, M. John Prosser, and John Otter

Comments by Morris Schupack and Authors

MORRIS SCHUPACK†

The authors are to be complimented on a fine paper, bringing forth many of the details of post-tensioning which are generally not publicized. The problems of grouting vertical tendons have been seriously investigated in order to resolve problems that have generally been ignored. The testing which was done to work out grouting procedures was certainly most thorough and advanced the state of the art of grouting vertical strand tendons.

The farsightedness of the owner, architect, engineer and post-tensioner supplier in doing these tests is to be highly complimented. General experience with grouting vertical strand tendons has indicated bleed phenomena or water separation of about 5 percent or more of the vertical height. This was apparently also experi-

*PCI JOURNAL, V. 21, No. 3, May-June 1976, pp. 84-11.

†President, Schupack Associates and Company, Consulting Structural Engineers, South Norwalk, Connecticut.

enced in general at the CN Tower.

In this writer's own experience, as described in References 1 through 4, water separation with unstressed tendons has been in the order of 5 to 20 percent for strand tendons about 20 ft high. In order to see if this phenomenon would occur under stressed tendons, tests were performed with unstressed Dyform strands. Dyform strand is a strand which is drawn through a die after it is formed into the strand pattern. This makes very intimate contact between the seven wires of the strand.

These tests indicated that no appreciable decrease in water transport mechanism occurred. For stressed vertical strand tendons, the experience of others, both in the Americas and overseas, has generally indicated a water separation of about 5 percent. Actually, the authors' experience falls into the general experience of this writer.

In References 1 through 4 it was not meant to imply that all tendons would be subject to the upper bound of test experience. It would be the writer's expectation that if a vertical tendon is grouted in stages, and successive stages are grouted after at least initial set, the next stage would not be affected by the previous stages. Each vertical stage would bleed as if it were independent. It is not clear what the magnitude of bleed for each stage was approximately. This would be masked by a substantial standpipe, which is a reasonable approach to alleviate, but not necessarily cure, the problem.

In the writer's experience in grouting 180-ft high tendons, using a special water retentive grout aid, Sika Grout-Aid PT, it was found that less than 1 percent of water separation occurred. The bleed phenomenon was comparatively well controlled. This probably could have been controlled with a lower water-cement ratio and by use of a standpipe. Without using the water retentive admixture, based on previous tests done and others' experience, water separation related to the height of the tendon could have been 5 percent or more.

For 180-ft tendons, this would have been 9 ft plus. It was also found that with the water transport mechanism, non-cementatious fractions of the cement were brought to the top surface leaving a quantity, possibly of 2 or 3 percent of the vertical height, of a white, non-binding material which was alkaline but had few cementatious properties. It was felt that this was unsatisfactory and had to be avoided.

The authors relate that the use of aluminum powder as an expansion agent caused continued evolvement of hydrogen gas, which caused a bubbly or foam-like grout at the top-most surface. We have had some interesting experience in this regard, which was both confusing and obscure, until a separate test was made to determine the evolvement of gas based on the zinc alkali reaction with a galvanized conduit.

Fig. A indicates the evolvement of gas as related to time when a strip of a galvanized conduit was placed in a cement grout. This test was performed when it was found, several days after the grout had apparently set, that excessive hydrogen gas was being evolved in a vertical tendon contained in a galvanized conduit. It was assumed that the gas was being transmitted from the tendon system through the interstices of the strand. Tests have indicated aluminum alkali reaction in a grout occurs for about an hour.

Fig. B indicates the typical reaction of a Type II cement and aluminum powder contained in a grout additive. Information for both figures was obtained from the Sika Chemical Company. Note that the expansion process can go on for almost 3 hours, but the major portion of the expansion occurs within the first hour.

It should be realized that when the maximum gas is being evolved from the alkali-aluminum reaction, it is generally occurring during the mixing and grout holding, prior to pumping and during the pumping procedure itself. Therefore, measuring expansion of a grout with aluminum powder immediately after it is mixed is not representative of what occurs in the tendon. Excessive expansion would generally be found.

Based on the writer's experience, from the time the grout is mixed to the time it is pumped into the structure and comes to rest (particularly for a larger tendon, requiring approximately a cubic yard of

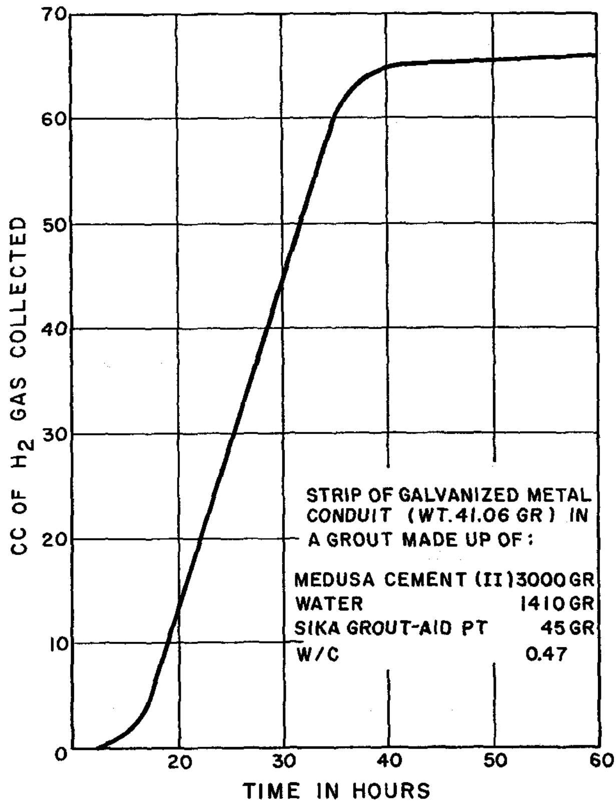


Fig. A. Gas evolved from the reaction of galvanized steel and a cement grout.

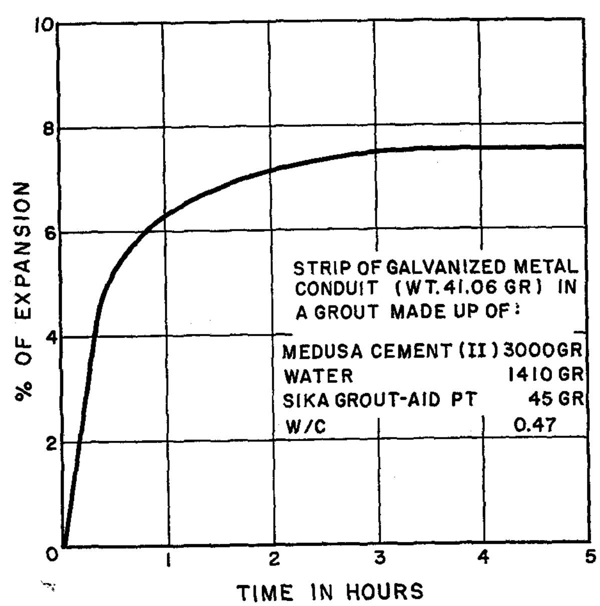


Fig. B. Expansion of cement grout containing aluminum powder (data obtained from same test as Fig. A).

grout), the first placed grout would be a ½ to ¾ hour old prior to coming to rest. The last placed grout would be 10 to 20 minutes old prior to coming to rest.

Our experience to date would indicate that to eliminate the excessive formation of gas for a long vertical tendon, galvanized conduits have to be protected from the alkali action or not used. This is always a difficult choice between obtaining the interim corrosion protection for the conduit and a second line of corrosion protection versus the possible gas formation.

In the experience of grouting a draped tendon with a 30-ft vertical rise, it was found that with a galvanized conduit no long-term gas was generated. The difference in performance between a 180-ft vertical tendon and a tendon with a 30-ft vertical rise is not understood at this time. It is pointed out that if conduits other than galvanized can be used, they would be preferred. This preference is only to avoid the late molecular hydrogen evolvement and not because of corrosion concerns. No corrosion problems associated with galvanized conduit and stress relieved tendons have been reported in the United States, to the writer's knowledge.

In grouting in successive lifts, such as in the CN Tower, the need for expansion is not required since expansion vertically is not important at the top of a lift. The only place where this is important is in the vicinity of the anchorage where there may be settlement under the anchorage plate and in the interstices of the anchorage hardware.

In the tests in which the writer has been involved, it was found that without some expansion occurring, voids can be expected to occur under the bearing plate and in the anchorage hardware itself. Also, if some bleed does occur, having a moderate amount of expansion (say, ½ to 1 percent), would tend to push the undesirable material away from the anchorage, giving the critical point of the tendon the best protection. This also can be aided by using a standpipe which will not necessarily eliminate a void under the bearing or anchorage plate if a grout which bleeds is used.

It was interesting to note that in the testing that was done for the CN Tower, if a water-cement ratio of 0.43 was used, the grout did not adequately fill the duct. The type of voids found in the ducts is disconcerting, because this reflects what must be occurring with many post-tensioning tendons using a water-cement ratio of less than 0.5. In the numerous tests in which the writer has been involved, both for horizontally curved tendons and vertical tendons, using a water-cement ratio of 0.47 with a highly thixotropic grout and Sika Grout-Aid PT, sampling indicated excellent filling of the conduit.

It is felt that with a grout which has a flow cone of 12 seconds, a viscosity that is too low is being used. For thixotropic grout with a water retentive admixture such as Grout-Aid PT, flow cone time using the Corps of Engineers' test standard CRD-C-79-58, "Method of Test for Flow of Grout Mixtures," is not meaningful for larger quantities of Grout-Aid. For a thixotropic mix, this could be anything from 30 to 90 seconds.

This method of testing viscosity is inadequate and inappropriate for thixotropic grout, in the writer's opinion. A more desirable procedure for measuring flow is to fill the flow cone to the top with grout and then measure the time it takes to fill a one liter bucket with grout. We have found this to be more reproducible and a more meaningful test. It is also felt that this is a more meaningful test for a non-thixotropic grout.

At the risk of being repetitious, it is the writer's opinion that, in order to get the best grouting of a post-tensioned tendon, the following points be followed:

1. Use a grout which has the required water retentivity for the particular use. That is, for a horizontal tendon, the water retentivity could be less than that for a tendon of a 100-ft height. If the water separation is not controlled, then voids at the top of the duct can be expected. Also, if tendons are in a vertical drape, voids can be expected at the high point of the tendon.

2. Do not use galvanized sheath if other type sheaths are available which will give similar type corrosion protection during the construction intervals and during the life of the structure. The use of plastic sheath or plastic coated steel is a

possibility. The use of a chromate treatment of the galvanized surface has been reported to control the formation of hydrogen gas.

3. Use the minimum water-cement ratio which can be mixed and pumped.

4. Use a positive displacement pump, such as a Moyno screw pump.

5. For mixing the grout, use a high energy mixer, such as a shear mixer.

6. For the grouting of one tendon at a time, use an expansion agent, but do not lock off the high points of the tendon. Grout should be free to expand. When grout is set, the conduit should be sealed.

References

1. Schupack, Morris, "Grouting Tests on Large Post-Tensioning Tendons for Secondary Nuclear Containment Structures," PCI JOURNAL, V. 16, No. 2, March-April 1971, pp. 85-97.
2. Wern, Allen H., Schupack, Morris, and Larson, Walter, "Prestressing System for H. B. Robinson Nuclear Plant," *Journal of the Power Division,* American Society of Civil Engineers, March 1971.
3. United States Patent No. 3,762,937 October 2, 1973, "Tendon Grouting Means."
4. Schupack, Morris, "Admixture for Controlling Bleed in Cement Grout Used in Post-Tensioning," PCI JOURNAL, V. 19, No. 6, November-December 1974, pp. 28-39.

AUTHORS' CLOSURE

The authors wish to thank Mr. Schupack for his very thoughtful and constructive comments.

It appears that clarification of certain aspects of our experience in grouting the CN Tower is required.

1. Water separation—The maximum water separation observed was about 14 percent. The shortest cables averaged about 8 percent. All other cables showed amounts of bleedwater consistently ranging from negligible to 5 percent. We estimate the average bleed percentage to have been about 3 percent.

Our findings therefore appear to be inconsistent with other experience and with our own preliminary tests. The questions arise; why the inconsistency; and, would it have mattered if we had had a higher separation percentage? This will be considered at another point in this reply.

2. Materials—Though it is not clear in our paper, the cement used in grouting the Tower, and in the *successful* preliminary testing, was a Type III high early strength cement. Type I was used in some of the testing but proved unsatisfactory. We believe that most other experience with vertical grouting has been with a Type I cement. This difference may be very significant.

The duct used in the Tower and in the tests was electrogalvanized.

The only admixture used in the actual grouting was a water reducer. It was in a liquid form and contained no expansive agent whatsoever.

3. Voids—We most definitely found that the stiff grout (using Type I) tested produced an unacceptably high degree of voids in the duct. It did so even when used without an expansive agent. The less viscous grout tested (using Type III) did not have this poor characteristic.

Our conclusions, which are particular to the situation, were as follows.

The reconciliation of our conclusions and Mr. Schupack's will be discussed afterwards.

1. Grout viscosity

A stiff grout with Type I cement is not satisfactory, in that voids are produced. A less stiff grout (10 seconds flow cone) with Type III cement is satisfactory from the viewpoint of filling the duct. The pumping characteristics of the less viscous grout are more satisfactory as well.

A less viscous grout with Type I cements was not tested.

2. Bleedwater

A Type III grout when used with galvanized duct, unstressed strands and without predrying of the duct produces bleedwater of the order of 7 percent, i.e., similar to that observed by others. Where the strand is stressed and where the duct is very dry, bleedwater is very small and may be neglected except at cable terminations (i.e., at the top anchorage). Bleedwater is not cumulative from lift to lift and would therefore not be of concern, except at anchorages, even if the percentages of bleeding reported by others did occur.

3. Expansion agent

The expansion agent used (aluminum powder—2 gr/bag of cement) was potentially detrimental in that it caused foaming. The foaming was not due to a grout-galvanizing reaction since it did not occur when the expansion agent was not used. However, the resulting foam was broken down by the weight of the subsequent grout lift and no area of gross weakness was found at the lift interface in the tests.

Conclusion

Our observations and Mr. Schupack's can perhaps be reconciled in the following hypotheses and observations:

1. The only cables to show substantial bleeding were those that were grouted before the winter, i.e., those to which the dry air was not applied. The one high value (14 percent) of bleedwater was explained at the time as being due to the delivery hoses still containing some wash-out water. Invariably, therefore, when the duct was dry, the segregation was very small.

2. We hypothesize that a thin layer of grout would set very quickly upon contact with the dry strand and that this layer would prevent infiltration of water into the strand interstices.

3. Excessive gassing due to a reaction between the grout and the galvanized sheathing was not noted on the CN Tower project. It is, of course, possible that it did occur and was not noticed. What is clear is that, in the testing, the chosen grout filled the duct most satisfactorily. We conclude, therefore, that if gassing did occur, the chosen grout dissipated this gassing without difficulty.

On the other hand, it is plausible that the more viscous grout that was also tested would not allow this gas and/or entrapped air to escape, that "lenses" of gas or air were thus formed and that these "lenses" were the voids observed in the tests. It is, therefore, our tentative conclusion that, where galvanized sheathing is desirable, a low viscosity grout is to be preferred, since it is known to work.

On the subject of gassing due to a reaction between grout and galvanizing, we note that the tests by Mr. Schupack were conducted using a Type II cement. Perhaps there is something peculiar to Type II cement that causes the reaction we noted. If this were the case, the gassing would not necessarily be found at all with the Type I and Type II cements normally used in grouting.

4. It appears that Mr. Schupack's preference for the thicker grout is based in a desire to reduce bleedwater as much as possible, and to thereby avoid any zones of comparative weakness where bleedwater may weaken the grout. Since the strength of the grout was not of first order importance in the CN Tower, this consideration is less persuasive.

We feel that, wherever the effects of bleeding can be reasonably tolerated, controlled or overcome, this consideration has less importance than the possibility of incomplete filling. Proper procedures at top anchorages can eliminate voids due to bleeding.

5. Tentatively, it seems reasonable to conclude that, in vertical grouting, the choice lies between a thixotropic grout that does not bleed but may not be compatible with galvanized sheating, and a thinner, Type III grout that will bleed to an extent that is usually manageable.

6. We do not agree with Mr. Schupack's statement that, in typical grouting situations, vents should be left open until the grout has set. It appears to us that this may defeat the purpose of an expansion agent, in that, if the grout is free to escape via the vent, little pressure is generated in the grout itself. Without internal pressure the grout cannot compress and, hence, fill air inclusions in the ducts. The result is unfilled voids.

Locating the Critical Flexural Stress Points in Single and Double Draped Prestressed Members*

by James J. Mallett

Comments by Leslie D. Martin and Author

LESLIE D. MARTIN†

Mr. Mallett has presented equations and programming data for determining critical stress points which should be useful to those who are writing computer programs for the analysis of pretensioned members, and who are interested in the degree of precision indicated.

Table 1 of his paper indicates a maximum error of 21 psi in the calculated bottom tension if the stress is calculated at midspan rather than at the theoretical critical point (for the 8DT24 section).

With a limiting tension of 424 psi ($6\sqrt{f'_c}$), his computer output indicates a superimposed capacity of 96 psf, which results in a midspan tension of 403 psi.

This midspan stress caused by the superimposed load can be calculated thus:

$$M = \frac{wL^2}{8} = \frac{(8 \times 96)\,(40)^2}{8}$$

$$= 153{,}600 \text{ ft-lb}$$

$$f = \frac{M}{Z_b} = \frac{153{,}600 \times 12}{1223.62}$$

$$= 1506 \text{ psi}$$

If the tension at midspan is allowed to reach 424 psi, then the calculated superimposed capacity can be calculated thus:

$$M = fZ_b = \frac{(1506 + 21)\,(1223.62)}{12}$$

$$= 155{,}741 \text{ ft-lb}$$

$$w = \frac{8M}{L^2} = \frac{8(155{,}741)}{(40)^2}$$

$$= 779 \text{ lb per ft (or 97 psf)}$$

or an error of about 1 percent.

AUTHOR'S CLOSURE

For Example 1, Mr. Martin demonstrates that the error resulting from computing the flexural capacity at 0.5*L* rather than the exact location is 1 percent. The magnitude of error depends upon the slope of the tendons as well as the allowable tensile stress and may or may not be significant for a particular design.

Applying Mr. Martin's procedure to Example 2 and checking at the midspan only, results in a tensile error of 215 psi and an error in capacity of 9.62 percent. as shown in the following calculations (see Table I):

Bottom stress at critical location .... 527 psi
Bottom stress at midspan...... 312 psi
Maximum error in stresses..... 215 psi

$$M = \frac{wL^2}{8} = \frac{8 \times 63.3 \times 60^2}{8}$$

$$= 227{,}880 \text{ ft-lb}$$

*PCI JOURNAL, V. 21, No. 4, July-August 1976, pp. 82-95.

†Consulting Engineer, The Consulting Engineers Group, Inc., Glenview, Illinois.

$$f = \frac{M}{Z_b} = \frac{227{,}880 \times 12}{1223.62}$$

$$= 2234.81 \text{ psi}$$

$$M = fZ_b = \frac{(2234.81 + 215)\,1223.62}{12}$$

$$= 249{,}803 \text{ ft-lb}$$

$$w = \frac{8M}{L^2} = \frac{8 \times 249{,}803}{60^2}$$

$$= 555.16 \text{ per ft} = 69.39 \text{ psf}$$

$$\text{Error} = \frac{(69.39 - 63.3)}{63.3} \times 100$$

$$= 9.62 \text{ percent}$$

For manual designs it is prudent to check both $0.4L$ and $0.5L$. Considering that the cost of an exact solution as shown by Example 1 is about 1/3 to 1/5 the hourly cost of an experienced designer, the computer offers a better design at less cost.

Flexural Stresses After Cracking in Partially Prestressed Beams*

by Arthur H. Nilson

Comments by Saad E. Moustafa, Hugh M. O'Neil, and Author

SAAD E. MOUSTAFA†

The author is to be commended for presenting an interesting and useful method for calculating elastic flexural stresses in partially prestressed beam after cracking. A careful review of the numerical example given in the paper revealed the following inconsistency:

1. The value of the distance y (distance from extreme compression fibers to neutral axis of cracked section) obtained from the solution of a cubic equation is 13.4 in.

2. The value of the same distance y, when computed from the resulting stress distribution across the cracked transformed section (as shown in Fig. A), is 14.1 in.

Although the difference between the two values of y is rather small, it does not appear that it is the result of a routine calculation round-off type of error. Therefore, it would be of interest to the user if the author would clarify the possible causes of such inconsistency. It is also suggested that the user should check his results by plotting the stress distribution across the cracked section.

*PCI JOURNAL, V. 21, No. 4, July-August 1976, pp. 72-81.

†Associate Professor of Engineering, School of Engineering, University of New Orleans, New Orleans, Louisiana.

HUGH M. O'NEIL*

The approach by Professor Nilson has merit in that it can help the designer investigate potential cracking in partially prestressed beams with bonded tendons.

When bonded bar reinforcement is reasonably near the surface of the beam and the tensile strain in that reinforcement due to the applied dead plus live load does not exceed values commonly deemed to be acceptable in conventional

*President, Hugh M. O'Neil Company, Consulting Civil and Structural Engineers, Oakland, California.

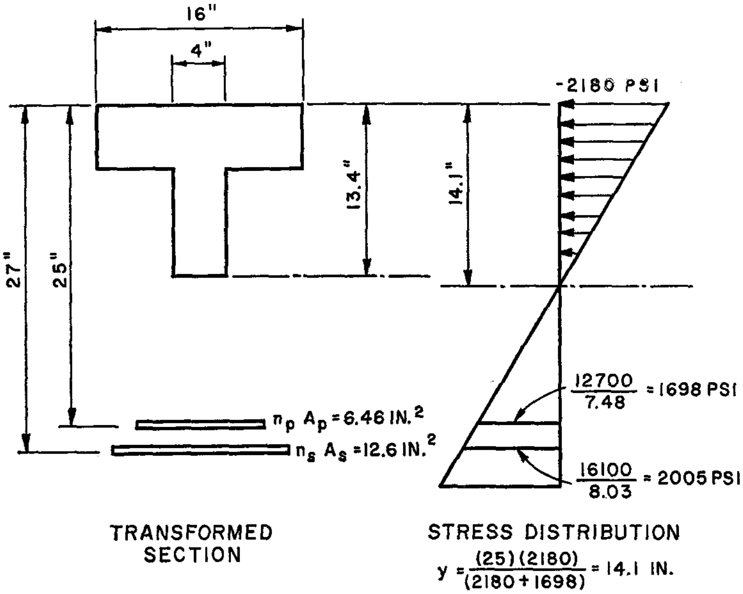


Fig. A. Stress distribution across cracked section.

reinforced concrete construction, one can conclude that the cracking should not be more severe than that commonly experienced for conventional reinforced concrete.

Also, the tensile strain in the rebar due to applied loads can be used as a rough indication of the tensile strain in the adjacent concrete and therefore can serve in estimating the severity of the cracking.

Professor Nilson confined his analysis to bonded tendons and apparently ignored the added compressive stress in the tensile rebar due to any initial shortening strain from shrinkage, creep, and elastic shortening.

Several years ago POSTEN PROGRAMS (a subsidiary of Hugh M. O'Neil Company) incorporated a design option using concepts very similar to those of Professor Nilson in computer program POSTEN,† for the design of continuous post-tensioned beams, flat plates, etc. A number of consulting firms using POSTEN have found the feature to be valuable in appraising the likelihood of severe cracking in special situations.

The procedures used by program POSTEN differ from those proposed by Professor Nilson in that the tendons are assumed to be unbonded, the compressive stress in the bonded rebar is not ignored, the tensile strength of the concrete is ignored for all cases, and the calculations are oriented toward design rather than analysis.

If the user specifies a maximum desired tensile stress, called the "limiting steel stress," the program will determine an area of bonded rebar sufficient to make the tensile stress at service loads in that same bonded rebar equal to the "limiting steel stress" specified. The initial stress in the rebar due to shrinkage, creep, etc., is computed to be equal to the modulus of elasticity of the steel times the average shortening strain.

The user has the option, however, of arbitrarily specifying the value which is called the "initial compressive stress." The solution is made by the program by means of a series of successive approximations.

A computer run was made (see Figs. B and C) using the same cross section,

†"Computerized Design of Post-Tensioned Continuous Beams and Flat Plates," PCI JOURNAL, V. 18, No. 3, May-June 1973, pp. 42-50.

Fig. B. Sample computer input for author's design example.

```
CONTINUOUS POSTENSIONED GIRDER OR SLAB PROGRAM
INPUT DATA
ARTHUR H. NILSON  -  EXAMPLE COMPARE  PCI JOURNAL

PERMANENT        A   BOND      B    C        D       E          F     G          H           I  J    K1  K2      L
OPTIONS          2    0    16100.  0       .00      0.        .00   .00       .00000       0  0    0   0     .0  .0

NO. OF BEAM     DEGREE OF     SPLITTING    28 DAY CONCR      ALLOW TENS.     ALLOW TENS.     ALLOW TENS        WEIGHT OF
   BENT         SKIP LOADG    STRTH FSP    STRENGTH          CONCR. TOP      CONCR. BOTT.    INIT CONDITIONS   CONCR.
   1               .00           .00          4000.              .0              .0               .0              144.

ULTIMATE STR.  YIELD STR.    YIELD STR.   YEILD STR.        ALPHA DIST     SAND LIGHT-
TENDONS        TENDONS       DEF BARS     STIRRUPS          DECIMAL FRAC   WT. CONCR.
    250000.    210000.        60000.       40000.             .000            0

                                         M        N                 O                  P               Q          R     BALANCING LOAD
TEMP.  AUTOMATIC  DEPTH     AXIAL    BAY       MIN. AVG. PRESTR.    OMIT SPANDREL      MULT JOBS   ABBREV. SPAN     BAL
OPTIONS  L    R    FTHIN     FORCE    WIDTH            FMIN          LEFT    RIGHT       1 RUN      INPUT   NO.     LOAD
         0    0      .0       .00      .00              .0            0        0           0          1       0        0.

NO      NO.LEFT   NO.RIGHT  EXT PREST ALLWD.   CONCR COVER-TENDONS   CONCR COVER,BARS   SLAB OR JOIST   2 WAY   SPAN EFFECTIVE
SPANS   CANTVR    CANTVR      LEFT   RIGHT      TOP       BOTTOM      TOP    BOTTOM     CONSTRUCTION    SLAB    NO. PRESTRESS
  1       0         0          0       0        5.00       5.00       3.00    3.00           0            0       0        .0

SPAN NUMBERS                            1
SPAN NAMES                             B1
SPAN                                  40.00
TOTAL DEPTH                           30.00
BOTT WIDTH B                           8.00
FLANGE WIDTH, BT                      16.00
FLANGE THICKNESS, T                    5.00
SHOULDER, G                           -2.00
BOT NOTCH HT., HB                      8.00
SLOPE WIDTH, SW                         .00
UNIFORM LOAD                          1560.
RATIO LL TO TOT. LOAD                  .612
INITIAL LOAD/TOTAL LOAD                .266
FRACTION OF LL (FOR DEFLECTION)       1.000
EST. FRICTION LOSS                     .000
NUMBER OF CONC LOADS                    0
NUMBER OF PARTIAL UNIF LOADS            0
NUMBER OF PARTIAL TRIANG LOADS          0

 REDESIGN DATA
DRAPE                                 11.73
PRESTRESS    (NOT INCL. FRIC.)       123000.

COLUMN NUMBERS             1          2
NET HEIGHT OF COLUMN90000.00    90000.00
DIM PARLLEL TO SPAN       .01        .01
DIM. PEPEN. TO SPAN     10.00      10.00
KEY TO OPTIONS:
 OPTION A.                    F. U - TOP CRACKING
  1 SUPRESS 18.5 & 18.6       G. U - BOT CRACKING
  2 LONG OUTPUT               H  SHRINK. FACTOR
  3 SUPRESS 18.5 & 18.6       I. ARBIT. STRESS LOSS
    +LONG OUTPUT              J. ELIM BOTT. REBAR
  4 VARIABLE I                K1.MOM REDIS,AREAS BEFORE & AFTER
  5 VARIABLE I                K2.MOM REDIS,AREAS AFTER ONLY
    +SUPRESS 18.5 & 18.6      L. LOAD FACTORS
 B. LIM STL. STRESS           OPTION TEN.-EXTRA PRESTR TO
 BOND.-BONDED TENDONS            SUIT TENSILE STRESS
 C. ADD COL. MOMS                INSTEAD OF LOAD BAL.
 D. BOT. COVER END SPANS
 E. INIT COMPR STRESS
TENDONS ARE UNBONDED
OPTION - SPECIFIED COMPRESSION STRESS IN BONDED
STEEL DUE TO CREEP +SHRINK.+ELASTIC SHORTENING=     0.
***NOTE*** MAX COMPR. STRESS EXCEEDS 0.45 X COMPRESSIVE STRENGTH.
```

SPAN B1   SPAN C. TO C. COLS.= 40.00 FT.   CLEAR SPAN= 40.00 FT.   MEMBER SIZE 8.00 IN. X 30.00 IN.

EFF. PRESTR 123.0 K = 580.PSI   DRAPE 11.73 IN L.&R. TEND ORDS 16.88 & 16.88 TEND LOSS= 33.8 KSI UNIF.LOAD 1560.PLF

10TH POINTS OF CLEAR SPAN	0	1	2	3	4	5	6	7	8	9	10
DIST FROM CTR LEFT COL, FT	.00	4.00	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00
DIST FROM SUPPORT FACE, FT	-.00	4.00	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00
TENDON ORDINATES FROM BOTTOM, IN.	16.88	12.65	9.37	7.02	5.62	5.15	5.62	7.02	9.37	12.65	16.88
MAX TEN. STRESS TOP , PSI	-580.	-1079.	-1467.	-1744.	-1910.	-1966.	-1910.	-1744.	-1467.	-1079.	-580.
MAX TEN. STRESS BOTT, PSI	-580.	61.	560.	917.	1130.	1202.	1130.	917.	560.	61.	-580.
MAX COMPR STRESS TOP , PSI	-580.	-1079.	-1467.	-1744.	-1910.	-1966.	-1910.	-1744.	-1467.	-1079.	-580.
MAX COMPR STRESS BOTT, PSI	-580.	61.	560.	917.	1130.	1202.	1130.	917.	560.	61.	-580.
BENDING MOMENTS (FT-K)	.0	112.3	199.7	262.1	299.5	312.0	299.5	262.1	199.7	112.3	.0
MOMENTS BALANCING LDS.(FT-K)	-.0	-43.3	-77.0	-101.0	-115.4	-120.2	-115.4	-101.0	-77.0	-43.3	-.0
COMBINED MOMENTS, (FT-K)	.0	69.0	122.7	161.1	184.1	191.8	184.1	161.1	122.7	69.0	.0
2NDARY MOMENTS (FT-K)	-.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
MAX SHEARS (K)	31.20	24.96	18.72	12.48	6.24	.00	-6.24	-12.48	-18.72	-24.96	-31.20
SHEARS, BALANCING LOADS (K)	-12.02	-9.62	-7.21	-4.81	-2.40	.00	2.40	4.81	7.21	9.62	12.02
COMBINED SHEARS (K)	19.18	15.34	11.51	7.67	3.84	.00	-3.84	-7.67	-11.51	-15.34	-19.18
VCI X PHI/U (K)	********	19.44	11.02	7.18	5.63	5.74	5.63	7.18	11.02	19.44	********
VCW X PHI/U (K)	26.83	25.54	24.25	22.96	21.99	21.10	21.99	22.96	24.25	25.54	26.83
WORST REVERSE MOMENTS (FT-K)	.0	112.3	199.7	262.1	299.5	312.0	299.5	262.1	199.7	112.3	.0
STRESSES AND MOMENTS - INITIAL CONDITIONS											
COMBINED STRESS TOP PSI	-679.	-529.	-412.	-329.	-279.	-262.	-279.	-329.	-412.	-529.	-679.
COMBINED STRESS BOT PSI	-679.	-872.	-1022.	-1129.	-1193.	-1215.	-1193.	-1129.	-1022.	-872.	-679.
MOMENTS DEAD LOAD ,(FT-K)	.0	29.9	53.1	69.7	79.7	83.0	79.7	69.7	53.1	29.9	.0
MOMENTS BALANCING LDS.(FT-K)	-.0	-50.6	-90.0	-118.2	-135.0	-140.7	-135.0	-118.2	-90.0	-50.6	-.0
RE-BAR FOR ULTIMATE STRENGTH (LOAD FACTOR FOR THIS SPAN= 1.584)											
AREA RQD AT TOP, SQ. IN.	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
AREA RQD AT BOTT.,SQ. IN.	.00	.06	.99	1.69	2.11	2.26	2.11	1.69	.99	.06	.00
RE-BAR , LIMITED STEEL STRESS											
AREA RQD AT TOP, SQ. IN.	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
AREA RQD AT BOTT.,SQ. IN.	.00	.00	.58	1.82	2.56	2.81	2.56	1.82	.58	.00	.00
RE-BAR , FORMULA 18-5 ACI CODE											
AREA RQD AT TOP, SQ. IN.	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
AREA RQD AT BOTT.,SQ. IN.	.00	.10	1.00	1.77	2.24	2.40	2.24	1.77	1.00	.10	.00
RE-BAR ,INITIAL CONDNS.											
AREA RQD AT TOP, SQ. IN.	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
AREA RQD AT BOTT.,SQ. IN.	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00

BONDED STEEL AREA FORMULA 18-6, ACI CODE 318-71   TOP .450 SQ IN.   BOTTOM .398 SQ IN.

MOMENT OF INERTIA,IN 4TH 21795.   LEFT INFLEC. PT. ORD. 16.88 IN.   RIGHT INFLEC. PT. ORD. 16.88 IN.

SECT. MOD. TOP, IN. CUBED 1661.   DIST. TO INFL.PT OF TENDON FROM   DIST. TO INFL. PT OF TENDON FROM

SEC MOD BOTT,IN. CUBED 1291.   CTR OF COL. AT LEFT .00 FT.   CTR OF COL AT RIGHT .00 FT.

AREA OF SECTION, SQ IN 212.0   DIST. TO C.G. OF MEMBER FROM BOTTOM 16.877 IN.

TENDON LOADS (BALANCING LOADS)   LEFT=300581.12 K/FT.   CENTER= -.601 K/FT.   RIGHT=300581.12 K/FT.

STIRRUP DIAM. = .375 IN.   SPACING LEFT= 22. 22. 22. 15. 14. 14. 15. 17. 20. 22. 22. 22.

SPACING RIGHT= 22. 22. 22. 15. 14. 14. 15. 17. 20. 22. 22. 22.

DEFLECTIONS- IMMEDIATE (DL + LL) 1.37 IN.   CREEP+SHRINKAGE(DEAD LOAD) -.13 IN.   TOTAL LONG TIME DEFL 1.239 IN.

MOM. IN COL LEFT END THIS SPAN(FT-K)= .00   .00   2NDARY COL MOM= .00   1/360TH CLEAR SPAN= 1.333 IN

MOM. IN COL RIGHT END THIS SPAN(FT-K)= .00   .00   2NDARY COL MOM= .00   CREEP+SHR+ELAST STRAIN=.00083

Fig. C. Sample computer output for author's design example.

bending moment, effective prestress, etc., as shown by Professor Nilson's design example in Fig. 3, p. 78 of his paper. A concrete strength of 4000 psi and a weight of 144 psi was used to give approximately the same modulus of elasticity of concrete. A "limiting steel stress" of 16,100 psi was used, which matches the value calculated by the design example. The initial compressive stress in the rebar was ignored by setting it equal to a very small value.

If program POSTEN and the author's design example were identical, the program would, ideally, determine an area of rebar equal to that assumed in the original data of the design example, which would be the area of two No. 8 bars or 1.56 sq in. The area of rebar determined by the program was 2.81 sq in., however.

Because the two cases are identical in all other respects, the increase is apparently due to the fact that the program assumed the tendons were unbonded instead of bonded as in the example. The value of Y and the compression stress in the concrete were computed internally but not given in the printout. The values were 14.0 in. and 2172 psi, which compare reasonably well with the values of 13.4 in. and 2180 psi in the example.

It is interesting to note that according to the computer output, Fig. C, the rebar requirements for this beam are slightly larger for "limited steel stress" than for ultimate strength or Eq. (18.5) of ACI 318-71.

Another computer run was made (output not included here) with identical input except that the initial compressive stress in the bonded rebar was not ignored, but arbitrarily set to a value of 10,000 psi and the final "limiting tensile stress" set at 16,100 − 10,000 or 6100 psi to give the same tensile strain in the rebar due to dead plus live load application as before.

Including the 10,000 psi of initial compressive stress in the rebar increased the required rebar area for "limited steel stress" from 2.81 to 7.40 sq. in., a rather significant increase. It would appear that a significant increase could also be expected for bonded tendons.

The above comparisons demonstrate that the amount of rebar necessary to limit tensile strain, and therefore cracking, is considerably greater for unbonded than for bonded tendons. Also, taking into account the initial compressive stress in the rebar increases the computed tensile strain substantially, suggesting that it should be considered in design calculations.

AUTHOR'S CLOSURE

The author greatly appreciates the interest shown by Mr. O'Neil and Dr. Moustafa in their written comments.

Mr. O'Neil notes the availability of his program POSTEN for the design of continuous beams and slabs, and comments that a treatment similar to that described by the author is incorporated for designs using unbonded tendons.

The author's presentation was restricted to pretensioned or bonded post-tensioned construction. It is agreed that the width of cracks may represent a subject of special concern for types of construction where grouting of post-tensioning tendons is not possible, as for slabs using wrapped tendons. An adaptation of the analysis to the unbonded case must account for the fact that the increase in strain in the tendon, as the member is loaded, will be much less than the change in concrete strain at the level of the tendon, in general, and consequently Eq. (4) must be modified.

The compressive stress in the reinforcing bar due to concrete shrinkage and creep was not accounted for in the analysis, as was stated just prior to Eq. (6). However, elastic shortening is included explicitly by Eq. (3). It would not be difficult to account for creep and shrinkage, determining these respective strains and modifying Eq. (6) for steel stress accordingly, although this was not done for the sake of simplicity. Whether or not creep would be significant would depend on the magnitude of the concrete compression at the level of the bars under combined action of all sustained loads, including transverse loads as well as prestress.

It was stated by the author that ". . . crack widths at service load are related to the increase in steel stress past the stage of concrete decompression . . ." It should be emphasized that steel stress is only one of several important parameters, which also include the size and distribution of the bars in the concrete tension zone, and the amount of concrete cover provided for the bars.

The author questions Mr. O'Neil's implication that cracking should be reduced solely by increasing the area of the nonprestressed steel. It would seem more efficient to consider use of a larger number of smaller bars. ACI 318-71 Eq. (10-2) may serve as a guide for partially prestressed beams, as well as for ordinary reinforced beams in this respect.

Dr. Moustafa properly notes the importance of checking the results of the calculations by plotting the stress distribution. By this means a small error was disclosed in the solution for y, the distance from the compression face to the neutral axis of the cracked section. The corrected values of y and the cracked centroid distance c_1^* are 14.1 in. and 7.75 in., respectively, and $A_{ct} = 135$ sq. in. No significant change results in concrete or steel stresses.

The Baton Rouge Hilton Tower—An All Precast Prestressed Systems Building*

by Sepp Firnkas

Comments by Colin H. Campbell and Author

COLIN H. CAMPBELL†

Mr. Firnkas' description of this building was clear and informative. In the Calgary area we have designed several buildings of this type although without the vertical post-tensioning. I would appreciate the author's comments on two points:

1. The slabs bear upon plastic strips on top of the wall. Are these strips compressible and if so, is the vertical load transferred through the grout only?

2. Slab to wall connections consist only of the friction aided by the vertical post-tensioning. I don't see a big problem for the interior walls, but what about the external walls? I'm thinking of the Ronan Point disaster in England a few years ago.

*PCI JOURNAL, V. 21, No. 4, July-August 1976, pp. 96-110.

†Consulting Engineer, The Grimble Consulting Group, Calgary, Alberta, Canada.

AUTHOR'S CLOSURE

Mr. Campbell's questions point precisely to the main features of the system and the two items the author considers among the most important ones, namely, Connections. Many systems have been invented, developed, imported and adapted to United States Codes and engineering standards since the mid 60's.

The discontinuity of most of them was predictable since the two points raised by Mr. Campbell were not resolved in compatability with United States production and labor habits and within the frame of United States economy.

In regard to Point No. 1:

The performance of various support details of long span extruded, prestressed hollow core planks between bearing walls or in short: the floor-wall connection has been researched and surveyed on numerous projects. We found need for rotation of planks, need and control of limited force transfer through the planks, avoidance of point pressure and stability of materials used for bearing pads as important criteria.

None of the available materials quite fitted our specifications and therefore the plastic "Korolath" bearing strips and shim packs were developed. The engineering characteristics of these bearing pads together with a design example of a typical floor-wall joint have been included in the PCI Committee report on: "Considerations of the Design of Precast Concrete Bearing Wall Buildings to Withstand Abnormal Loads," Appendix 2: "Load Capacity of Horizontal Joints" prepared by this author and published in the PCI JOURNAL of March-April, 1976 (reprints available from PCI). The percentage of load transfer through planks, hollow-core slabs and grout, respectively, is demonstrated in the joint design example.

In regard to Point 2:

The slab to wall connection is based on the shear-friction concept with post-tensioning functioning as a structural element and as a connection detail. Post-tensioning introduces predetermined compression forces and continuity of reinforcing steel at the joint. The connection detail at each floor level is hidden, makes a dry erection possible and can accommodate large fabrication and erection tolerances.

End walls are similarly connected. However, as an integral part of the system we arrange the building layout to have a shear wall from a stair-elevator or tower, a returning concrete curtain wall or a spandrel beam available to brace the end walls. The design of the brace-shear wall is based on all lateral design loads plus loads resulting from eccentricities of manufacturing and erection of the end walls. See plan view and various elevations in paper.

This design precaution has been proven in at least one dramatic incident. By accident, excessive loads together with debris loads developed in an end bay of an eight-story building. Shear failures of the majority of the planks developed and left the end wall completely unbraced in the lower three and the upper two floors.

The ensuing thorough and detailed analysis of the accident indicated undoubtably that the vertical post-tensioning of end and bracing walls saved the building from a type "Ronan Point Failure."

The effectiveness of post-tensioning as clamping force was additionally demonstrated. The floor planks failed in shear, not by pulling out, with the remaining stubs firmly clamped in the joint.

The Editors welcome discussions of papers published in the PCI JOURNAL. The comments must be confined to the scope of the paper under discussion. Please note that discussion of papers appearing in this current issue must be received at PCI Headquarters by Sept. 1, 1977.