READERS' COMMENT

Torsion Design of Prestressed Concrete*

By Paul Zia and W. Denis McGee

Comments by Michael P. Collins, Emory L. Kemp, G. S. Pandit, K. S. Rajagopalan, Charles H. Raths, and Authors

Michael P. Collins†

The paper by Professors Zia and McGee is of considerable practical importance as it shows the manner in which the present ACI torsion provisions could be extended to cover prestressed concrete. As the proposals set out in the paper will undoubtedly be considered for the next ACI Code the writer would like to comment on a number of sections of the paper and make a few specific suggestions.

Expressions for torsional shear stress

It can be assumed that a prestressed concrete member in torsion will crack when the principal tensile stress reaches the "tensile strength" of the concrete. To calculate the principal tension we need to know the magnitude of the shear stress caused by the torsion.

Following current ACI practice, the authors suggest empirical expressions to calculate the torsional shear stress, τ, for rectangular sections, flanged sections, and box sections. They show that in order to obtain consistent results for rectangular sections it is necessary to modify the current ACI expression.

The writer would like to suggest that torsional shear stresses can be calculated by procedures which are more general than those proposed and which are based on concepts familiar to most engineers.

The torsional shear stress in a thin-walled tube is given by the well-known expression[20]

$$\tau = \frac{T}{2A_o t} \tag{A}$$

where

A_o = area enclosed by centerline of tube
t = wall thickness of tube

For a thin-walled box section Eq. (A) could be used directly and it would lead to results quite similar to those obtained from the procedure outlined in the paper.

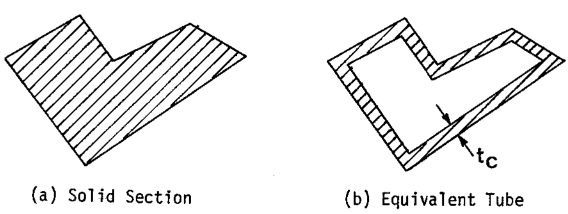

Fig. A. The equivalent thin-walled tube.

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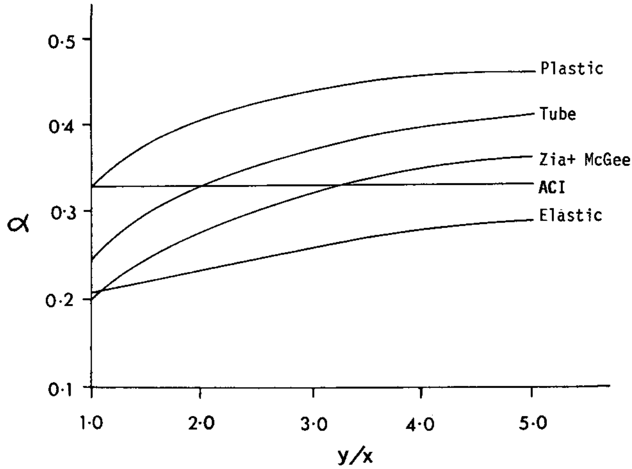

Fig. B. Torsion coefficients for rectangular sections.

To obtain the torsional shear stress for solid sections or thick-walled boxes it is suggested that an equivalent thin walled tube be considered. This tube would have the same external dimensions as the original section but would have a wall thickness of t_c (see Fig. A).

An appropriate value for the equivalent tube thickness, t_c, is

$$t_c = \frac{3A_c}{4p_c} \qquad \text{(B)}$$

where

A_c = area enclosed by perimeter of section

p_c = perimeter of section.

Of course, for hollow sections t_c should not be taken as greater than the actual wall thickness. Once the thickness of the tube is known, then the area enclosed by the centerline of the tube, A_o, can be calculated. However, for solid and thick-walled sections A_o can be taken as $2/3\ A_c$ without any great loss of accuracy. Eq. (A) then becomes

$$\tau = \frac{3T}{4A_c t_c} \qquad \text{(C)}$$

For rectangular sections the above equivalent tube concept could be used to determine the torsion coefficients given in Fig. 1 of the paper. These coefficients are compared in Fig. B of this discussion. It is of interest that the equivalent tube coefficients display the same trend as those suggested by the authors.

The advantage of using the equivalent tube concept for calculating St. Venant torsional shear stresses is that any cross-sectional shape can be dealt with. It would be of interest to know how the authors would calculate the torsional shear stresses for members with circular, triangular or trapezoidal sections.

Basic equation for torsional strength

The authors express the torsional strength of a prestressed beam as the sum of a concrete contribution and a web reinforcement contribution. Further they assume that the concrete contribution is some portion of the cracking torque of the member.

This empirical procedure is an extension of the ACI shear design approach in which it is necessary to invoke a concrete contribution in order to compensate for the conservative nature of the 45-deg truss equation for the "steel" contribution in shear. Several research workers[21-24] have shown that torsion results can be explained by truss equations without having to invoke a concrete contribution. In particular, a recent rational model[15,26] which has no empirical fitting factors and no "concrete contribution," is capable of explaining fully the post-cracking response of symmetrically prestressed beams in pure torsion.

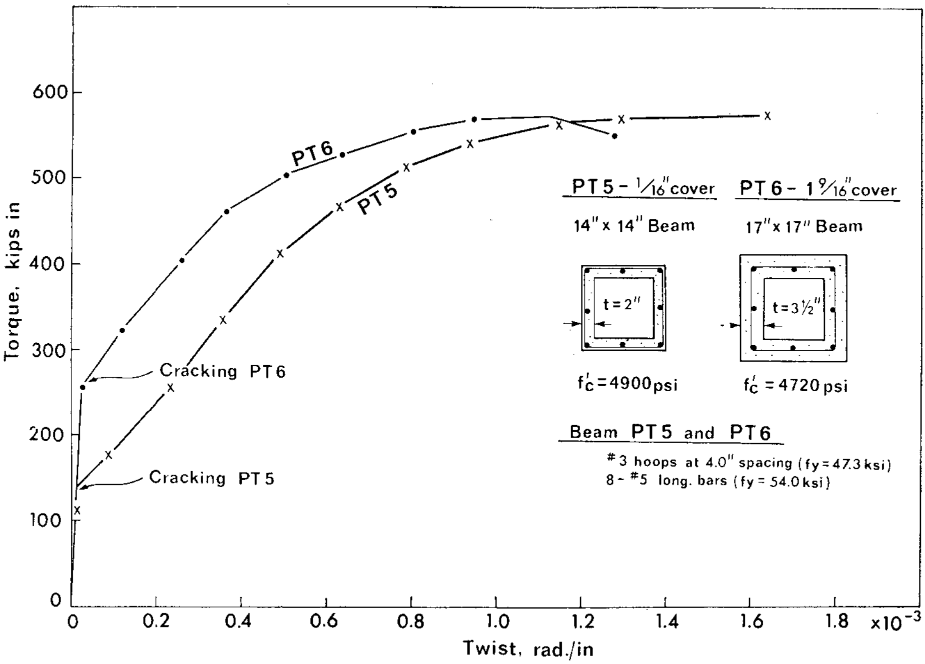

Fig. C. Effect of concrete cover on torsional response.

Fig. C illustrates one of the reasons why the concept of a "concrete contribution" to torsional strength is open to question. This figure gives the experimental results of two reinforced concrete beams tested in pure torsion at the University of Toronto. These two beams had identical reinforcing cages but Beam PT6 had a significantly greater concrete cover and hence larger external dimensions. It can be seen that the additional concrete outside the hoops significantly increased the cracking torque of the member and hence should have significantly increased the concrete contribution. What happened, however, is that at higher torques the concrete outside of the hoops spalled off and hence had no measurable influence on the ultimate torque. These tests demonstrate that it is the dimensions of the reinforcing cage and not the external dimensions of the concrete which govern the torsional strength.

The effect of prestress on torsional behavior is more complex than the simple increase in strength shown in Fig. 2 of the paper. In fact if two beams, one of which is prestressed, have the same dimensions, the same web steel, the same yield force of longitudinal steel, and are both under-reinforced they will fail at the same torque. After the longitudinal steel has yielded it does not "remember" that it was prestressed. The increase in strength shown in Fig. 2 of the paper is due to the fact that the prestressed beams contained more longitudinal steel.

Design for combined torsion and shear

For combined torsion and shear the ACI design procedure, used by the authors, assigns a portion of the concrete contribution to shear and a portion to torsion. This division of the concrete contribution is achieved by means of rather complex interaction equations.

If we assume that there is no "concrete contribution" in torsion, then a considerably more simple design approach becomes possible. We can now give all of the "concrete contribution" to the shear and carry all of the torsion with steel. This has the great advantage that the shear

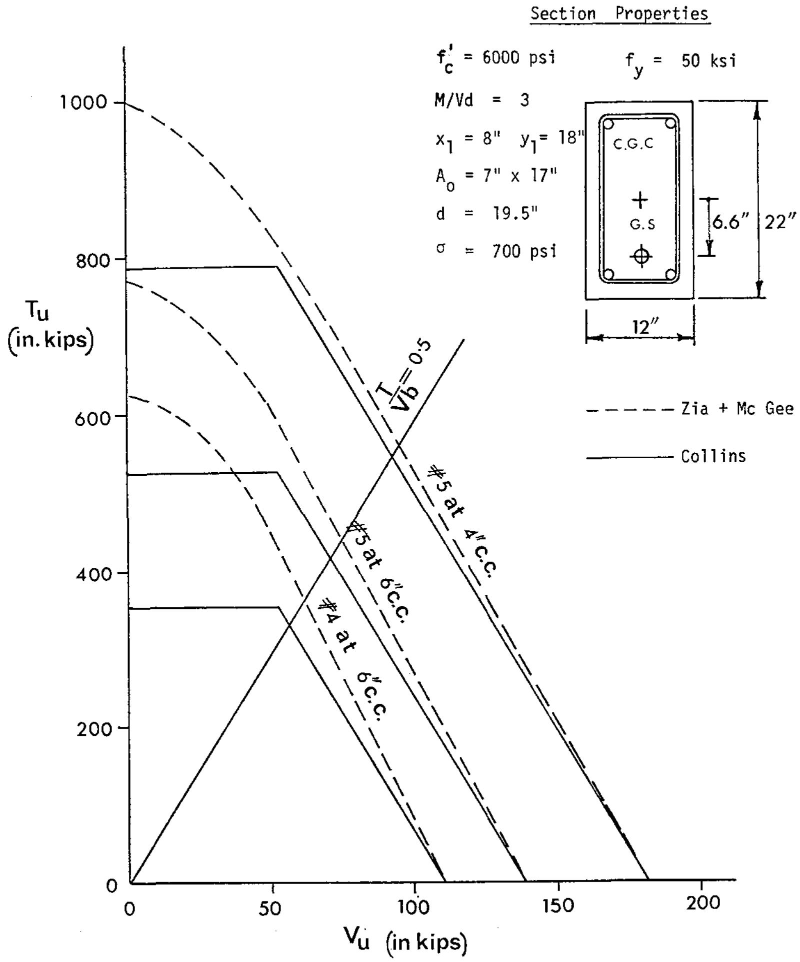

Fig. D. Effect of amount of web steel on shear-torsion capacity.

design is not at all influenced by the presence of torsion. Thus we can design web steel for the shear using available design aids, design web steel for the torsion separately, and then add the two steels.

Alternative design procedure

While it is possible to develop more accurate design procedures for prestressed concrete in combined torsion and shear,[27] the following method, which is based on 45-deg truss equations and neglects the beneficial effects of prestress in pure torsion, is simple and conservative.

1. Calculate the reinforcement required to resist the shear, flexure, and axial load acting in combination with torsion, by the usual procedures (i.e., those that are used when torsion is not present).

2. Calculate the required additional area of web steel (which must be provided as closed stirrups) to resist the torsion by the expression

$$\frac{A_t}{s} = \frac{T_u}{\phi 2 A_o f_y} \tag{D}$$

where

A_o = area enclosed by a line connecting

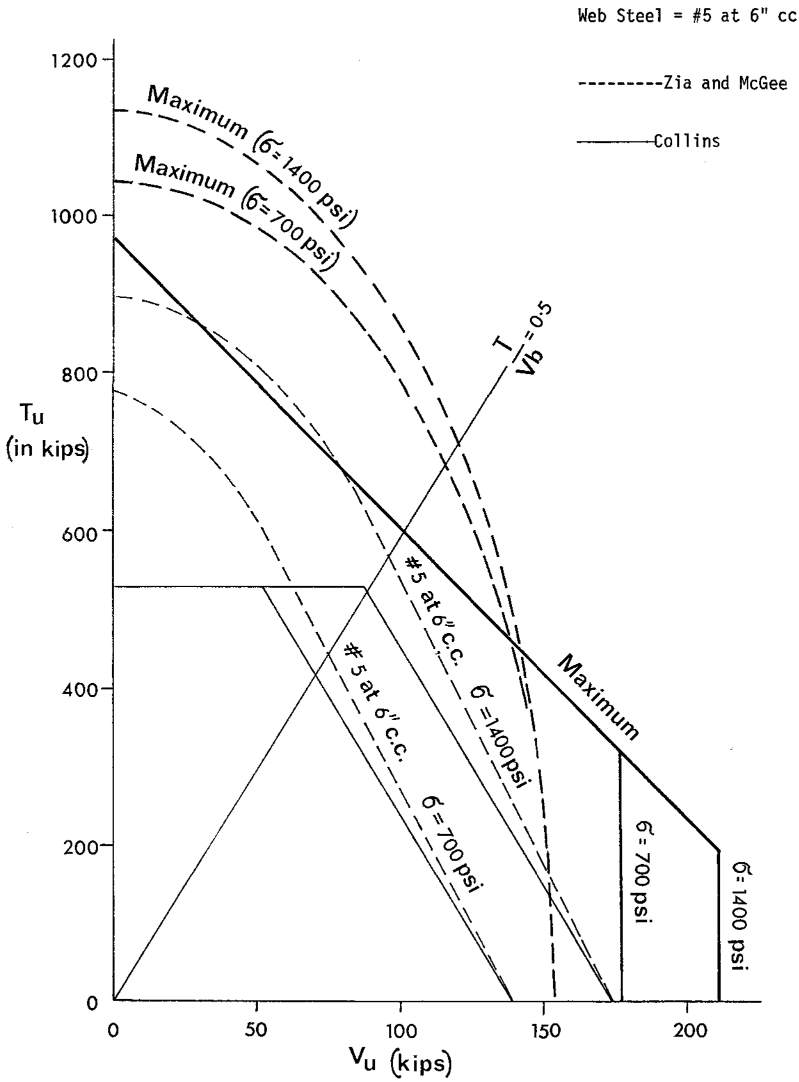

Fig. E. Effect of prestress on the shear-torsion capacity (section same as Fig. D).

the centers of the longitudinal bars in the corners of the closed stirrups

3. Calculate the required additional area of longitudinal steel (which must be distributed symmetrically around the section) by the expression

$$A_l = \frac{T_u p_o}{\phi 2 A_o f_y} \qquad \text{(E)}$$

where p_o = perimeter of A_o.

4. Check that the section is not over-reinforced. Calculate the nominal torsional shear stress, τ_u, by:

$$\tau_u = \frac{T_u}{\phi 2 A_o t_o} \qquad \text{(F)}$$

where

$t_o = (3/4)\ (A_o/p_o)$ but not greater than the actual wall thickness.

(Note that we use the cage dimensions to calculate, τ_u. At ultimate the beam is spalled.)

Then check that the sum of the nominal torsional shear stress, τ_u, and the nominal shear stress, v_u does not exceed $0.22f_c'$.

Comparison of the two design procedures

In order to compare the results obtained by the two design procedures a number of design charts for specific sections have been prepared. Typical examples of these charts are given in Figs. D and E.

The allowable torsions and shears with changing amounts of web steel for a specific rectangular section are shown in Fig. D. While the two procedures lead to significantly different results for the case of pure torsion, for practical loading ratios (say $T/(Vb) < 0.5$) the results are very similar.

Fig. E illustrates the manner in which the two procedures account for the effect of prestressing. The figure gives the capacities of a series of beams all containing the same amount of web steel but having different levels of prestress. Once again, for practical loading ratios, the differences between the results of the two methods are minor.

Also shown in Fig. E are the maximum loads (irrespective of the amount of web steel) allowed by the two methods. It can be seen that the authors' method allows higher maximum torsions as the prestress is increased but keeps the maximum shear constant. On the other hand, the method proposed by the writer, allows the maximum shear to increase (and hence agrees with the ACI Code shear provisions) but keeps the maximum torsion constant.

As a result of studies[28] similar to those illustrated in Figs. D and E, it was concluded that for the vast majority of practical cases the design method outlined in this discussion will lead to results slightly more conservative than those obtained from the design method of the authors. For very high ratios of torsion to shear (say $T/(Vb) > 1$), the results of the new method will often be considerably more conservative. For these special cases it may be worthwhile to design the reinforcement by the more complex procedures suggested by the authors.

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Emory L. Kemp*

The torsion provisions in ACI 318-71 are the result of a sustained effort by the ACI Torsion Committee. In formulating these provisions the committee concentrated on developing a method for determining the torsional strength of reinforced concrete beams subjected to pure torsion and torsion in combination with bending and shear. The committee is currently attempting to develop additional provisions dealing with torsional stiffness, designing for compatibility and equilibrium torsion situations and to extend the present provisions to include prestressed concrete. Thus, it is most gratifying to see the comprehensive paper by Drs. Zia and McGee which develops design criteria for prestressed concrete beams subjected to torsion.

The ACI Code sets forth a method for torsional design based on the same kind of philosophy which designers are familiar with in analyzing sections for flexural shear. The total member capacity for the case of pure torsion is assumed to be the sum of a portion of the concrete capacity of a companion plain member plus the contribution of the reinforcing cage.

The concrete contribution is based on a rational skew bending theory whereas the reinforcement factor has been obtained empirically. The behavior of beams subjected to combined loadings is accounted for by empirical interaction curves. Despite the fact that the entire ACI Code has been formulated on an ultimate strength basis, Chapter 11 presents an integrated treatment of the flexural shear and torsion in terms of nominal stresses. This tends to obscure the entire method and reduce it to the solution of a series of rather complicated equations which have apparently little relationship to member behavior. The entire chapter is now under review by the Shear Committee and hopefully a revised version would be in terms of stress resultants rather than nominal stresses.

The proposed design method by Drs. Zia and McGee follows the basic philosophy of the ACI method for reinforced concrete beams and can be easily incorporated in the present code provisions. It is developed clearly with sufficient detail and experimental verification to be given serious consideration by both the designer and appropriate technical committees. If the Code is changed to a stress resultant format this method could be recast with little difficulty.

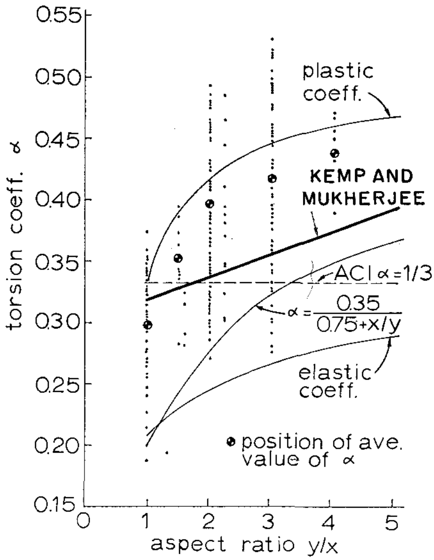

Fig. F. Variation of torsion coefficients with aspect ratio.

As pointed out in their paper the addition of prestressing causes a dramatic increase in the cracking torque of a concrete beam with a diminished range over which unstressed web reinforcement is effective. In fact, it is possible to increase the prestress in the concrete so that the cracking and ultimate torques occur at the same value as a result of a compression failure in the concrete. Thus, special provisions must be established limiting the intensity of prestress and recognizing the increased importance of the concrete contribution to the total torsional capacity. Both of these effects are considered in the paper.

In the case of the torsion function, α, the simplified constant of one-third is probably not sufficiently accurate for pre-

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stressed concrete. The authors support representing α as a function of the depth to breadth ratio. In an earlier study by Mukherjee and Kemp[29] this function was studied for plain, reinforced, and prestressed concrete. A linear regression analysis indicated that the following expression correlated very well for all the specimens studied and could be used for design purposes.

$$\text{Torsion factor} = 0.412(1 - 0.2333\ b/d)$$

This expression is a function of the breadth to depth ratio and is plotted on Fig. 1 shown in the paper (see Fig. F). In the case of prestressed concrete the expression is also a function of the prestress to f_c' ratio as shown in the following formula:

$$\text{Torsion factor} = 0.4124(1 - 0.2333\ b/d) + (1/6)\sigma_p/f_c'$$

This added term could be incorporated in code formulas with little difficulty.

The paper is a thoughtful attempt to develop design criteria for prestressed concrete members subjected to torsion. Thus, it deserves the serious attention of design engineers. It is hoped that a lively discussion will result from its publication.

Reference

29. Kemp, E. L., and Mukherjee, "A Study of the Strength and Stiffness of Reinforced concrete Members Subjected to Torsion," *Proceedings*, International Conference on Shear, Torsion, and Bond Reinforced and Prestressed Concrete, January, 1969.

G. S. Pandit*

In view of the present unsettled status of torsion design of prestressed concrete structural members, the significant contribution made by the authors should be particularly welcome. The authors' approach closely follows the current ACI Code[1] and seeks to extend the provisions of this Code to prestressed concrete. Torsion in prestressed concrete is being studied intensively at the Malaviya Regional Engineering College at present. This discussion highlights the main findings. It is hoped that the authors' interesting paper and the comments from the discussers will form a useful source material for the formulation of the Code provisions for torsion of prestressed concrete.

The basic ultimate strength equation of a reinforced concrete beam in torsion can be written as:

$$\frac{T_u}{T_{up}} = \frac{T_o}{T_{up}} + \Omega \frac{R}{T_{up}} \qquad \text{(G)}$$

in which R is the effective reinforcement factor.

A careful study[30] of all available test results, particularly those carried out by Hsu and the writer, showed that if a suitable allowance is made for the discrete location of the stirrups and the commonly occurring unsymmetrical distribution of the longitudinal reinforcement, the equation for the ultimate torsional strength could be written as

$$T_u = 2\,x^2 y \sqrt{f_c'} + R_1 R_2 x_1 y_1 A_t f_{sy}/s \qquad \text{(H)}$$

where

R_1 = stirrup effectiveness factor, and
R_2 = longitudinal steel effectiveness factor

On the basis of their tests carried out at the Malaviya Regional Engineering College, Pandit and Mawal[31] extended Eq. (H) to prestressed concrete:

$$T_u = (1/3)\,x^2 y (f_c'/10) \sqrt{1 + 10\sigma/f_c'} + R_1 R_2 x_1 y_1 A_t f_{sy}/s \qquad \text{(I)}$$

In agreement with Eq. (6), Eq. (I) implies that the contribution of nonprestressed reinforcement is not affected by prestress. A study[32] of all available test results showed that Eq. (I) gave a better correlation compared to two other expressions presented earlier, one by Hsu and Kemp[33] and the other by Bishara and Peir.[34] Although the basic form of Eqs. (6), (I), and those presented in References 33 and 34 is the same, there are important

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differences. These differences have been discussed in detail in Reference 32.

The writer believes that the authors' Eq. (3) is applicable only to concentrically prestressed beams without web reinforcement subjected to pure torsion. Pandit and Sharma[35] have derived equations based on skew bending theory for the ultimate strength of eccentrically prestressed beams without web reinforcement subjected to combined bending and torsion. Graphical plots as design aids have also been included in Reference 35.

Certain curved prestressed members, particularly in bridge structures, may be subjected to biaxial bending moments and torsion. For such members Pandit and Sharma[36] have used the skew bending theory for the derivation of the ultimate strength expressions. It is interesting to note that the expressions presented in References 35 and 36 reduce to the Authors' Eq. (3) for the case of a concentrically prestressed beam subjected to pure torsion.

The writer has also initiated tests on the basic philosophy of the torsion problem. Tests to investigate the true failure mechanism in torsion have been carried out. These tests included rectangular beams with deep longitudinal slits on all four faces to inhibit the truss action but at the same time not to interfere with the failure mechanisms envisaged in the skew bending and equilibrium theories. A comparison of the ultimate strength of the slotted beams with the companion unslotted ones showed that beams without web reinforcement failed by skew bending. In the case of beams with adequate web reinforcement, a supporting shell forms as the torque exceeds the cracking torque. Thus, for a beam with adequate nonprestressed reinforcement, the space truss action seems to represent the true internal mechanism as the applied torque approaches the ultimate strength.

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K. S. Rajagopalan*

The writer compliments the authors for their clear exposition of a design procedure and for the design example that is very useful to designers. The writer offers the following comments:

The proposed procedure is for the design of prestressed concrete members under torsion combined with shear and bending. Of the total 394 results analyzed by the authors, how many tests were in combined bending, shear, and torsion? Based

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on the proposed method, what is the margin of safety available in these tests? The writer realizes that this analysis would be time consuming and difficult; some of the data may not fit in with the suggested procedure. However, such an analysis would provide empirical evidence on the soundness of the proposed procedure, and may even provide clues for further simplification and economy.

In a simple beam subject to combined shear, torsion, and bending generally three regions of shear design are recognized:

1. The region near the midspan where both shear and torsion are well within the concrete capacity. Only a minimum web reinforcement (ACI 318-71, Eq. 11-2) would be required here. In this region, providing web reinforcement for the development of cracking torque of section would be unnecessary and uneconomical; torsion should be neglected, as stated by the authors. Also the minimum web reinforcement could be the open type here.

2. A region near the supports, perhaps up to one-quarter the span, where both shear and torsional stresses could be larger than the concrete capacities, each requiring web reinforcement. The design example considers this region where minimum web reinforcement for development of cracking torque may be essential.

3. A region between Regions 1 and 2 above. In this region the torsional stress may be less than the concrete capacity, but the shear stress may be larger than the concrete capacity or vice versa. The excess concrete capacity available for either torsion or shear could reduce the web reinforcement required for the other. If torsional web reinforcement is required, should one have to provide the minimum required for the development of cracking torque? Or should something less, say a minimum of 1.2 times the required web reinforcement be adequate? The writer would like to hear the authors' opinion.

Another aspect of the design that is of particular interest to this writer is how to reduce the design time required for the proposed procedure. A computer solution may be necessary if the time-consuming calculations involving v_{ci}, v_{cw}, and the various torsional constants are to be performed for different sections in each beam. Could there not be shortcuts for at least Regions 1 and 3 cited above?

The authors suggest the addition of required web reinforcements for shear and torsion each separately calculated. The present state of the art may justify this as a prudent conservative design procedure. However, a vectorial addition of required web reinforcements would be more economical and this approach should be studied, in view of the circular interaction suggested for the concrete capacities v_c and τ_c.

The writer agrees with the authors that the value of α should be dependent on the aspect ratio y/x, and be not a constant.

Charles H. Raths*

The authors are to be commended for filling an obvious design void not covered by ACI 318-71 regarding torsion of prestressed concrete members. Their presentation of design requirements is straight forward, and will be of great value to the designing structural engineer.

All engineers differ in their design approach depending upon their experience and philosophy. The following comments, which are based on the writer's design and practical experience, are offered for consideration:

1. Loads to beams by stemmed members are concentrated and not uniform which results in stepped shear and torsion diagrams.

2. Concentrated loads slightly less than d from the support should be considered and not neglected.

3. The effective prestress stress, σ, varies from zero at the end of the member to its effective value at about 50 strand diameters from the end.

4. L-shaped members that have relatively large leg thicknesses have an elastic shear center that does not coincide with

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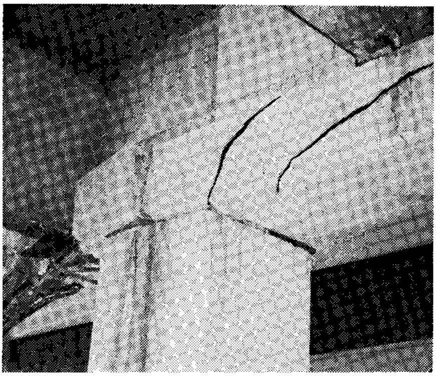

Fig. G. Torsion distress within distance "d". (Note: cracks have been darkened for emphasis.)

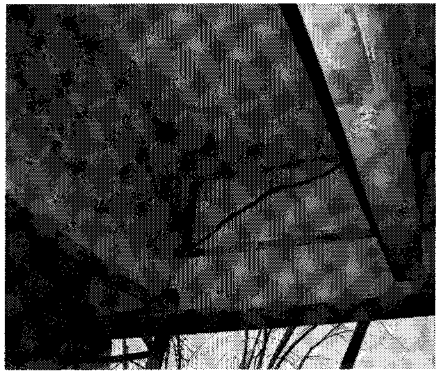

Fig. H. Torsion distress within distance "d" of 6-ft deep beam. (Note: cracks have been darkened for emphasis.)

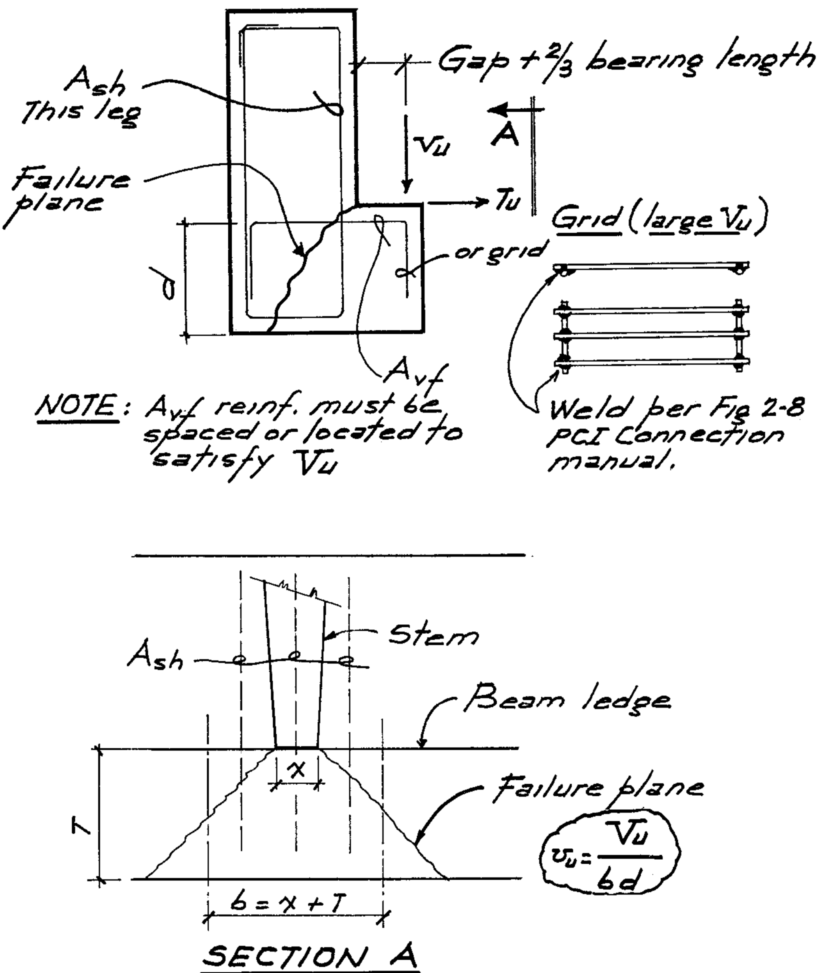

Fig. I. Required ledge reinforcement.

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INPUT: JØB NØ., DESIGN MK, DESIGN LENGTH(FT), (1-RF ØR 2-PS)
 ?ZIA-MCGEE,L-BEAM,30,2
INPUT CØNCRETE DATA: F'C(PSI), WT.(PCF)
 ?5000,150
INPUT REINF. DATA: FY(AT-KSI), FY(AL-KSI), STIR.-#, CØVER(IN)
 ?40,40,4,1.25
INPUT STRD DATA: DIA(16TH), AS/STD(IN2), GRADE(KSI), #STD, R(0.4-1)
 ?8,.153,270,13,1
INPUT NØ. ØF CRØSS-SECTIØNS
 ?1
INPUT EACH SECTIØN,(IN): B', B, H, T, W
SECTIØN 1  ?12,18,30,12,6
INPUT STRAND PRØFILE: ESC(IN), ESE(IN), C-DEP(FT), L-DEP(FT)
 ?6.2,6.2,0,0
INPUT: NØ. LØADS, LØAD FACTØR
 ?8,1.5
INPUT: LØAD(KLF ØR K), A(FT), B(FT-0 FØR PØINT LØAD), ECC.(IN)
LØAD 1  ?.45,0,30,0
LØAD 2  ?12.2,2,0,9
LØAD 3  ?12.2,6,0,9
LØAD 4  ?12.2,10,0,9
LØAD 5  ?12.2,14,0,9
LØAD 6  ?12.2,18,0,9
LØAD 7  ?12.2,22,0,9
LØAD 8  ?12.2,26,0,9
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07/05/74   20:15
 P/S-SHEAR AND TØRSIØN ANALYSIS PER PCI JØURNAL MARCH 1974
 RATHS, RATHS & JØHNSØN**STRUCTURAL ENGINEERS**HINSDALE,ILL

JØB NØ.=ZIA-MCGEE   DESIGN MK=L-BEAM

     DESIGN DATA         LEFT REACT=  52 K   RIGHT REACT=  47 K (WØRK)
=====================
DESIGN LENGTH= 30.00 FT
FC= 5000 PSI (150 PCF)
STIR NØ.= #4 (FY=40 KSI)
FY(AL)  = 40 KSI

 DIST    MU     VU     TU    VU    TU     AV     AT     AL   MIN-AV+2AT
  FT    K-FT    K     K-FT   PSI   PSI  IN2/FT IN2/FT  IN2    IN2/FT
=====================================================================
  0.00     0   78.4   51.2   320   503   0.45   0.38   2.25      1.20
  3.00   216   58.1   37.5   237   368   0.12   0.15   0.91      0.52
  6.00   387   46.9   30.6   192   301   0.12   0.16   0.95      0.52
  9.00   497   35.7   23.7   146   233   0.12   0.13   0.80      0.52
 12.00   563   15.4   10.0    63    99   0.00   0.00   0.00      0.00
 15.00   589   -4.9   -3.7    20    36   0.00   0.00   0.00      0.00
 18.00   571  -16.1  -10.6    66   104   0.00   0.00   0.00      0.00
 21.00   493  -27.3  -17.4   111   171   0.12   0.07   0.43      0.52
 24.00   369  -47.6  -31.2   195   306   0.12   0.16   0.94      0.52
 27.00   207  -67.9  -44.9   278   441   0.12   0.23   1.35      0.57
 30.00    -0  -70.0  -44.9   286   441   0.31   0.31   1.86      0.93

TYPE DIST(FT) ØUTPUT ANY PØINT(0 TØ BYPASS)
 ?1.5
  1.50   117   77.4   51.2   316   503   0.14   0.30   1.82      0.75
 ?1.8
  1.80   140   77.2   51.2   315   503   0.12   0.30   1.78      0.71
 ?2.1
  2.10   163   67.8   44.3   277   436   0.12   0.22   1.33      0.56
 ?0
**********************************************************************
TYPE: 0-END, 1-NEW STIR.-#, 2-NEW LØADS, 3-NEXT CASE ?0
```

Fig. J. Typical computer input and output.

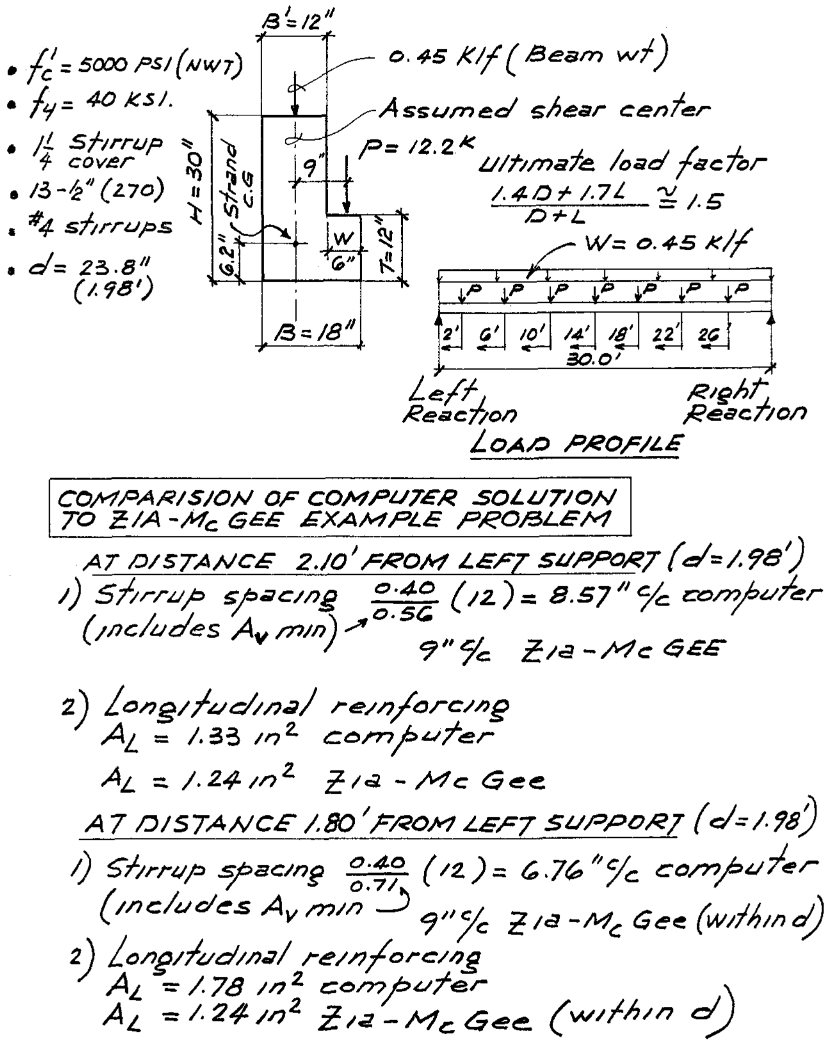

Fig. K. Comparison of Raths computer solution to Zia-McGee example problem.

the centerline of the beam web.

5. Many prestressed or precast members do not have a constant cross section. Thus, a torsional analysis must consider this particularly if the cross section is rectangular within the distance *d* rather than L-shaped.

6. A careful appraisal by the designer is required to determine the location of the stem load centroid on the beam ledge.

7. The design of beam ledge reinforcement must consider the fact that concentrated loads are present, and not uniform.

8. Additional web reinforcement is required, over and above that for shear and torsion, due to "web bending" created by the concentrated load of the stem upon the beam ledge.

Points 1, 2, 5, 6, and 8 are real world concerns. During the past several years the writer's firm has been retained to investigate spandrel L-beam torsional distress in several structures. Based on observations of prestressed members supporting concentrated stem loads, it appears that a torsional analysis cannot stop at the distance

d, but must be carried out all the way to the support of simple span members. Figs. G and H illustrate cases where torsional distress can be observed within the distance d of a prestressed spandrel beam.

Similarly, too often the influence of concentrated loads is neglected in favor of a uniform load approach when designing ledge reinforcement for an L-shaped beam. Typically, the ledge has an l_v/d ratio less than one which indicates a case of shear transfer (shear friction for a dap) rather than a bending and diagonal tension shear analysis. Further, an evaluation should be made as to possible horizontal T_u forces acting outward from the beam ledge due to volume shortenings of the stemmed members supported by the prestressed beam. Fig. I illustrates the reinforcement that must be selected in addition to beam shear and torsion reinforcement.

Examination of the relationships proposed by authors (or by non-prestressed shear and torsion requirements as set out by ACI 318-71) clearly illustrate the problems facing the designer making long-hand calculations. This writer believes that realistic shear and torsion analyses (and most other ACI Code analyses or design) can only be made by use of computers. Staff members of this writer's firm have programmed the proposed procedures set out in the authors' paper. Fig. J shows a computer analysis of the example problem presented in the paper where stem loads are considered concentrated. All other variables are those used in the paper.

Comparison of the computer output and the authors' example problem (see Fig. K) shows basically the same results at or just beyond a distance d from the left support for torsion reinforcement and longitudinal reinforcement. However, for stems on 4-ft centers, at a distance left of the support just less than d, the computer solution indicates a significant jump in torsion reinforcement (an increase of 36 percent). This calculated need for increased torsion reinforcement within the distance d indicates the same requirement that has been observed for actual members suffering distress.

The authors' example problem highlights their comments regarding minimum web reinforcement for torsion. They suggest reinforcement be required to develop the cracking torque of the section. Yet, in their example problem, it is indicated that the minimum $(A_v/2+A_t)$, using parts (f) and (h) of the problem, results in 0.30 sq in. per ft per stirrup leg. Part (g) of the problem requires 0.28 sq in. per ft per stirrup leg. Thus, the writer feels that the required minimum web reinforcement makes it unnecessary to go through the long computations in part (g).

The same argument can be used when dealing with high shear and torsion where $(A_v/2+A_t)$ is available in the other stirrup leg for shear.

It has been this writer's approach to calculate minimum shear reinforcement using ACI 318-71 Eq. (11-2) and add that amount to the A_t value calculated by Eq. (19). Since A_t is present in both stirrup legs it seems that the sum of $(A_v/2+A_t)$ is available for torsion and shear as just discussed. Additionally, the maximum spacing requirement of $(x_1+y_1)/4$ also appears to establish a minimum torsion requirement. And, the addition of the A_l reinforcement component should be considered when evaluating minimum torsion reinforcement.

Comments by the authors on the points raised in this discussion would certainly add to this writer's understanding, and bring added value to an outstanding paper.

Author's Closure

The authors are grateful to the discussers for their significant contributions. There seems to be an agreement that the torsion coefficient α should be a function of the aspect ratio y/x of a rectangular section rather than a constant ⅓ as is now specified in ACI 318-71. The equations for α developed by Professor Kemp on a statistical basis and the equation presented by Professor Collins based on the tube theory are less conservative than the one suggested by the authors. In view of the scatter of the

Table A. Comparison of theoretically required reinforcement ratios for various specimen groups.

Specimen group	Ratio of theoretical area to actual area			Remarks
	Stirrup	Longitudinal mild steel	Longitudinal prestress steel	
73 rectangular beams	2.22 0.51	1.36 0.99	1.15 0.28	Average standard deviation
34 box beams	1.85 0.47	1.05 0.44	1.46 0.30	Average standard deviation
25 T and I beams	6.25 2.47	6.94 4.24	1.10 0.11	Average standard deviation

experimental data (see Fig. 1), it is felt that the authors' Eq. (4) is preferred. The current efforts of the ACI committees on torsion and shear as described by Professor Kemp are much needed developments that should lead to improved and, hopefully, simplified design procedures.

To account for the effectiveness of the torsion reinforcement, Professor Pandit proposes the use of R_1R_2 as the reinforcement coefficient instead of Ω in Eq. (6). This approach deserves consideration.

The authors' Eq. (3) should be regarded basically as an empirical expression and no claim is made that it is rationally derived. However, in developing the expression, test data of eccentrically prestressed rectangular beams were included. It is interesting to note that Professor Pandit has found that his equations derived from skew bending concept for the cases of combined loadings of eccentrically prestressed members could be reduced to the authors' Eq. (3) as a special case.

Dr. Rajagopalan raised the question as to how many tests of the total 394 results analyzed by the authors were in combined bending, shear, and torsion and the margin of safety available in these tests based on the proposed method. As presented in Reference 2, among the total 394 results examined, 132 tests were in combined loading; of which 73 were rectangular beams, 34 were hollow box beams and 25 were T and I beams. As rightly pointed out by Dr. Rajagopalan, an accurate comparison was very difficult to obtain since almost none of the data could fit in exactly with the suggested design procedure in terms of the arrangement of the reinforcement. Therefore, in making the comparisons, some assumptions had to be made regarding the distributions and the effectiveness of the reinforcement in the test specimens. The theoretically required reinforcement was then calculated for the ultimate torque, shear and moment of each specimen as reported in the literature, and compared with the reinforcement actually provided. The comparisons can be summarized as shown in Table A.

It is apparent from Table A that the proposed procedure is conservative, particularly so with respect to the flanged sections. The authors believe, however, that the apparent over-strength of the flanged sections is due, at least in part, to the conservative assumptions used in assessing the effectiveness and distribution of the reinforcement and the flange bending effect of the section. Judging from the wide scatter of the results, such comparisons should be regarded only as tentative until specimens, designed specifically according to the proposed procedure, can be tested for evaluation. When a more accurate assessment is made, it would appear that the procedure could be improved for more efficient design.

Dr. Rajagopalan raised another interesting question regarding the required minimum web reinforcement for torsion in regions where torsion stress is relatively low and shear stress may be less than the concrete capacity so that the excess could help carry torsion. For such a case, it would appear sufficiently safe if the torsion reinforcement provided is at least, say, one-third or one-half greater than that required by analysis rather than what would be required for developing the cracking torque of the beam section.

The authors agree with Dr. Rajagopalan that to expedite the design procedure, a computer program would be a logical approach as it is shown by Mr. Raths in his very interesting discussion. As to the question of how to combine the web reinforcements for shear and torsion, the authors agree that a vectorial addition would be more economical and logical in light of the circular interaction between shear and torsion. It deserves further study, even though such refinement may not be justified at our present state of knowledge.

Professor Collins has presented a different approach to torsion design. Bascially it differs from the current ACI approach in several aspects:

1. Torsional stress is based on the tube theory.

2. The beneficial effects of prestress in resisting torsion is neglected.

3. Ultimate torsional strength is based on the space truss theory.

4. There is no "concrete contribution" to the torsional strength.

5. It is assumed that there is no interaction between torsion and shear.

Professor Collins' approach results in a simpler design procedure but is more conservative, particularly when torsion is a controlling factor in design. However, the method deserves careful study and some of its features may effect simplifications of the ACI design procedures.

In presenting the example problem, the authors have replaced the concentrated reactions from the double tees on the L-girder by an equivalent uniformly distributed load. This was done largely for simplicity to illustrate the torsion design procedure. The discrepancy of such a simplifying assumption may not be too serious for closely spaced concentrated loads. However, the authors fully agree with Mr. Raths' views regarding the various design considerations. With precast prestressed members, the investigation of the stress conditions near the ends often deserves extra care. The stresses can be significantly affected by the loading condition, the transfer of prestress, the distribution of reaction, and the detail of connection, among other things. In the example problem, the authors have checked the torsional stress at the section "d" distance from the centerline of the support since the size of the column supporting the L-beam was not specified. If the column were assumed to be 12 in. square, then the section at which the torsional stress was checked would be essentially at $h/2$ from the face of the support, where h is the overall depth of the L-beam. This is consistent with the ACI Code Section 11.2.2 with regard to shear in prestressed concrete members. However, as Mr. Raths' careful study of the example problem shows, the critical section may be located closer to the face of the support, particularly when there is a concentrated load nearby which causes a sudden change in torque, and when there is a sudden change in cross section. To undertake a thorough design review with speed and economy, computer application is certainly indispensible as illustrated by Mr. Raths' work.

With regard to the web reinforcement, the requirement should be $(A_v/2 + A_t)$ for each stirrup leg, since A_v is the area of both stirrup legs and A_t is defined as the area of one stirrup leg. It is suggested that the minimum amount should be enough to develop the cracking torque of the beam section. If one uses ACI 318-71 Eq. (11-2) or Eq. (11-1) for the minimum shear reinforcement and add that amount to the A_t value calculated by the authors' Eq. (19), as suggested by Mr. Raths, one would over-specify the minimum requirement by an amount of $A_v/2$. The maximum limit on stirrup spacing $(x_1 + y_1)/4$ would not alone establish a minimum torsion reinforcement requirement but rather it may influence the choice of the size of the stirrup.

The required minimum web reinforcement as determined in parts (f) and (h) is nearly the same as that determined in part (g). This is merely a coincidence. It is conceivable that the latter could be greater than the former, and in view of the brittle mode of torsion failure, the authors believe that it is prudent to carry out the computations as shown in part (g).

Finally, a misprint should be pointed out. On page 60 of the JOURNAL paper, 20th line, the last number should be "445 psi" rather than "677 psi."