SHEAR STRENGTH OF FULL-SCALE, HIGH-PERFORMANCE SAND-LIGHTWEIGHT CONCRETE PRECAST COMPOSITE GIRDERS

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ABSTRACT

Although lightweight concrete has been viewed as a suitable structural material for nearly 50 years, design codes tend to penalize its use by reducing its design shear strength relative to normal weight concrete. One of the penalties given in AASHTO's LRFD Bridge Design Specifications (2012) is a reduced splitting tensile strength for shear design. As part of a wider study on high-performance, high-strength sand-lightweight concrete, twelve tests on six, full-scale, prestressed girders with composite decks were conducted to investigate the validity of this reduction. Variables included concrete density, concrete compressive strength, effective shear depth, shear span-to-effective depth ratio, shear reinforcing index, and composite cross-sectional area. The analysis evaluated the effects of these variables on shear strength as well as the influence of the lightweight modification factor. Results show that the sand-lightweight concrete girders had shear strengths that exceeded those predicted by the LRFD Bridge Design Specifications. Furthermore, there were no distinct shear behavior differences between girders constructed with sand-lightweight and normal weight concrete. Additionally, calculations show that the splitting tensile strength reduction for sand-lightweight concrete has only a small effect on the calculated shear strength. Therefore, this modification for sand-lightweight concrete in shear design is not recommended.

KEYWORDS: Lightweight, Concrete, Prestress, Shear

INTRODUCTION

Although the Romans are the first known engineers to use lightweight aggregate on a massive scale about 2000 years ago[1], modern construction industries did not use the material until the early 20th century[2-4]. As for prestressed bridges, the first application of lightweight concrete was in the early 1960's[5]. Today, the material cost of lightweight concrete is typically greater than normal weight concrete due to the manufacturing of the lightweight aggregate. The production process involves heating the raw material (usually shale, clay, slate), causing the gases trapped within to expand the aggregate. After cooling, the aggregate retains most of its expanded shape, resulting in a density that is roughly one-half that of traditional gravel[6,7]. With the less dense aggregate, structural lightweight concrete typically has an equilibrium density ranging from 105 to 120 pcf[5], which can yield a 17 to 24% reduction in the structure's weight. This weight reduction can result in construction cost savings that generally outweigh the added material cost[6].

However, one obstacle to using this material is that the splitting tensile strength of lightweight concrete has been known to be as much as 30% below the strength of a comparable normal weight specimen[5,7]. Thus, there is even more uncertainty when calculating the shear strength of lightweight concrete. Regardless of the type of concrete, there are many other factors that can affect the shear strength of concrete beams, such as: the other material properties of both the concrete and the reinforcing steel, the cross-sectional geometry, the amount and arrangement of the longitudinal and shear reinforcement, the shear span-to-effective depth ratio, aggregate interlock, aggregate size, prestressing conditions, as well as loading and support conditions[8-12]. Some of these factors are interdependent[13], and all of these combined influences complicate the understanding of shear interaction.

In light of these factors, there has been a great deal of research on the shear strength of lightweight concrete beams. Over the course of more than 50 years, there have been more than 400 shear tests on members ranging from reinforced concrete beams without shear stirrups to high-strength, high-performance prestressed lightweight concrete girders containing shear reinforcement. Although the research results tended to vary, the overall consensus appeared to indicate that reinforced lightweight concrete beams tended to have shear capacities that were 70% to 100% of the strength of normal weight beams with comparable parameters. On the other hand, the larger prestressed girders exhibited shear capacities that exceeded calculated strengths using design codes.

AASHTO LRFD BRIDGE DESIGN SPECIFICATIONS FOR SHEAR DESIGN

Thus, there is still some degree of uncertainty and even disagreement as to how to treat lightweight concrete relative to normal weight concrete. The 2012 American Association of State Highway and Transportation Officials (AASHTO) LRFD Bridge Design Specifications[14] dictate that the nominal shear resistance, V_n, shall be the lesser of

$$V_n = V_c + V_s + V_p \qquad (1)$$

$$V_n = 0.25 f'_c b_v d_v + V_p \qquad (2)$$

where the concrete contribution to the nominal shear resistance, V_c, shall be calculated as

$$V_c = 0.0316\beta \sqrt{f'_c\,(\text{ksi})}\,b_v d_v \quad (3)$$

if the *General Procedure* is used for prestressed sections or the lesser of

$$V_{ci} = 0.02\sqrt{f'_c\,(\text{ksi})}\,b_v d_v + V_d + \frac{V_i M_{cre}}{M_{max}} \geq 0.06\sqrt{f'_c\,(\text{ksi})}\,b_v d_v \;(\text{kip}) \quad (4)$$

$$V_{cw} = \left(0.06\sqrt{f'_c\,(\text{ksi})} + 0.3 f_{pc}\right) b_v d_v + V_p \quad (5)$$

if the *Simplified Procedure for Prestressed and Nonprestressed Sections* is used. However, when designing with lightweight concrete, the term $\sqrt{f'_c}$ in Eqs. (3) through (5) is replaced by

$$4.7 f_{ct} \leq \sqrt{f'_c\,(\text{ksi})} \quad (6)$$

if the splitting tensile strength, f_{ct}, is specified or $0.85\sqrt{f'_c}$ if f_{ct} is not specified for sand-lightweight concrete, where sand is the fine aggregate.

OBJECTIVE

As a part of National Cooperative Highway Research Program (NCHRP) Project 18-15[15], the objective of this study was to alleviate some of the uncertainty regarding calculated shear strength of prestressed, sand-lightweight concrete beams by augmenting the existing database of full-scale tests, comparing various models for shear design with the shear test results, examining the need to modify the tensile strength calculation when designing for shear in prestressed, sand-lightweight concrete bridge girders, and recommending changes to the AASHTO LRFD Bridge Design Specifications for designing prestressed, sand-lightweight concrete girders for shear.

EXPERIMENTAL PROGRAM

GIRDER DESIGN

In order to achieve the objective, six prestressed concrete girders were constructed with composite cast-in-place decks. The variables under consideration amongst these six girders were: unit weight, γ_c; compressive strength, f_c; effective shear depth, d_v; shear span-to-effective shear depth ratio, a/d; composite cross-sectional area, A_{comp}; and the shear reinforcing index, $\rho_v f_{yv}$.

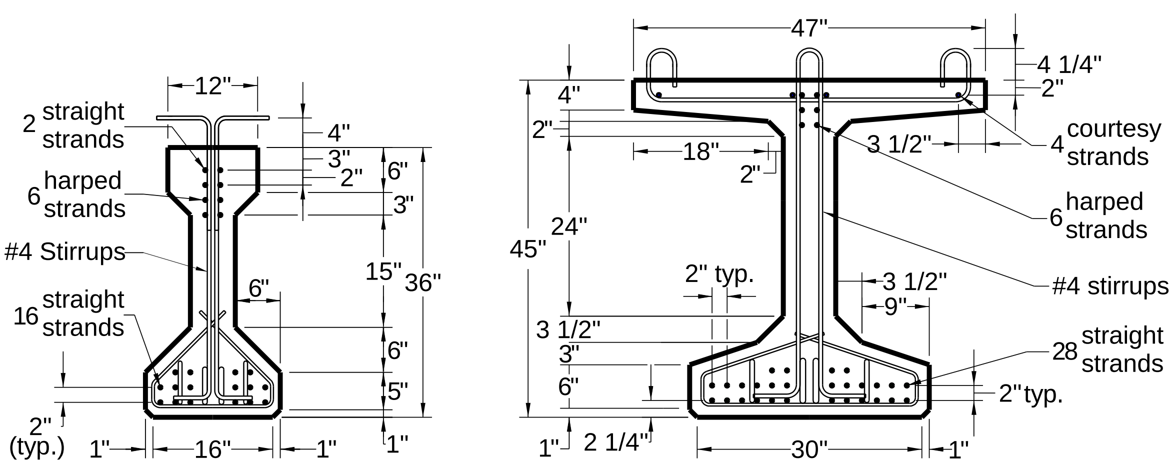
All of the girders were either AASHTO Type II or Virginia PCBT-45 girders designed for both shear and flexure according to the 2007 AASHTO LRFD Bridge Design Specifications[16], assuming the longest possible span length for a given cross-section.

However, because the purpose of this study focused on shear capacity, most of the section of the beam between the two harping points for the prestressing tendons was omitted from the actual test beams. This change resulted in a more manageable girder size in the laboratory. All other aspects of the test beams remained more or less the same compared to the original full-length beam design, including stirrup size and spacing, number of strands, and the number and inclination of harped strands. See Figures 1 and 2 for cross-sections and elevation views of these girders, and see the work by Kassner[17] for additional details regarding the parameters used to design the beams. On top of the beams, there were 8.5-in. deep, sand-lightweight composite concrete decks also designed according to AASHTO specifications, with the exception that the as-built deck width was 7 ft instead of the assumed 8 ft girder spacing due to geometry limitations of the load frame in the laboratory.

One of the beams, BT.8N.Typ, contained normal weight concrete for comparison with its lightweight counterpart, BT.8.Typ. Also, two of the beams, T2.8.Min and BT.10.Min, were intended to have the minimum amount of shear reinforcement. Rearrangement of Eq. 5.8.2.5-1 of the 2012 AASHTO LRFD Bridge Design Specifications results in the minimum shear reinforcing index being 0.089 ksi and 0.100 ksi for design concrete compressive strengths of 8 ksi and 10 ksi, respectively. However, the philosophy behind the design of all of the test subjects was to create beams that could reasonably be constructed in an actual bridge. Thus, double-legged No. 4 reinforcing bars were used for stirrups instead of a smaller diameter or single-legged stirrup. Therefore AASHTO's maximum spacing limits of 12 in. and 24 in. for beams T2.8.Min and BT.10.Min, respectively, controlled the shear design instead of the minimum area of reinforcement.

INSTRUMENTATION

The primary sensors used on the test beams were electrical resistance strain gages placed on the transverse reinforcement. Because the location of cracks in the concrete could not be known a priori, and in order to maintain the number of gages at a reasonable level, the gages



(a) (b)

Figure 1. Cross-sectional geometry and reinforcement details for (a) AASHTO Type II girder and (b) PCBT-45 girder.

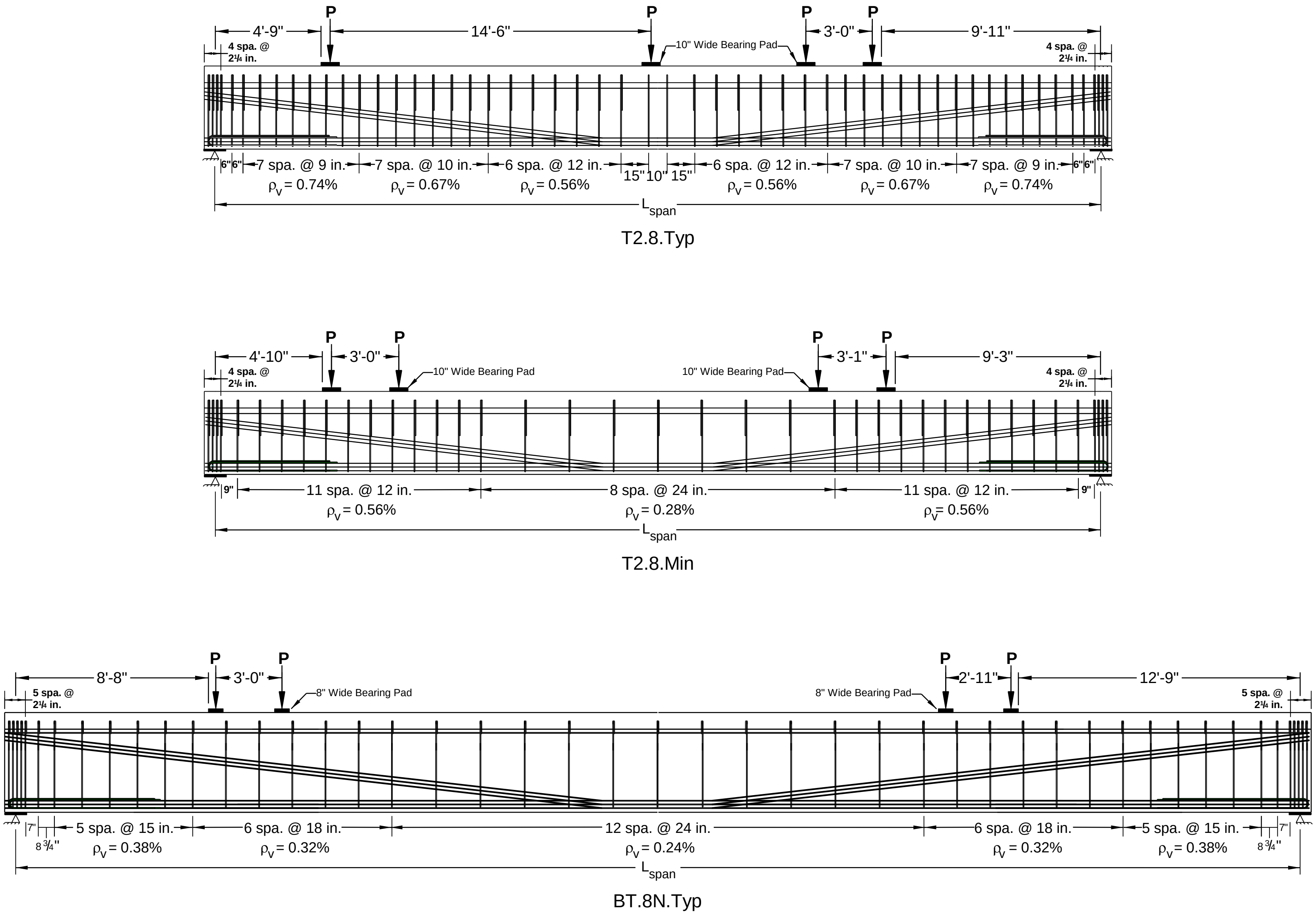
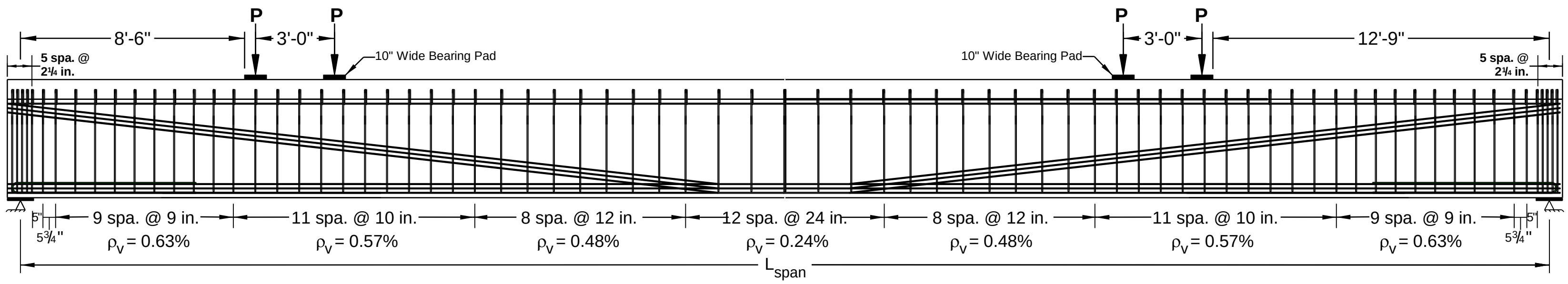
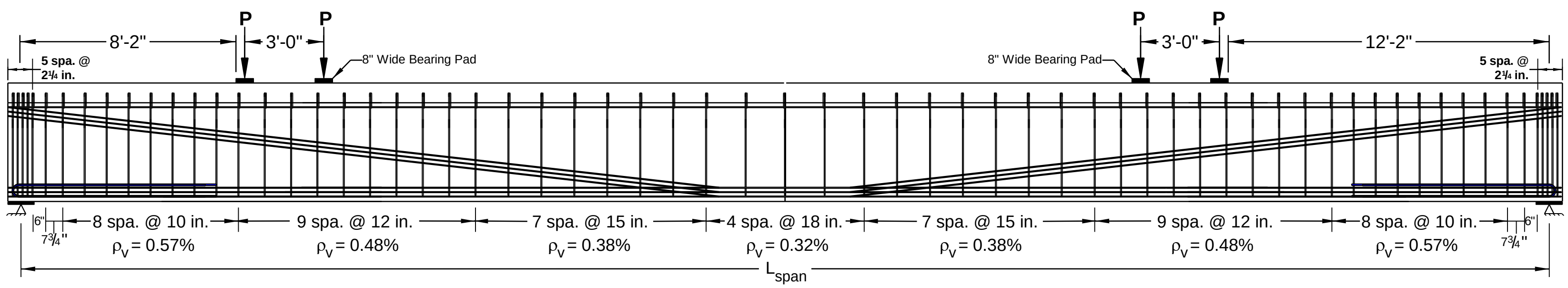


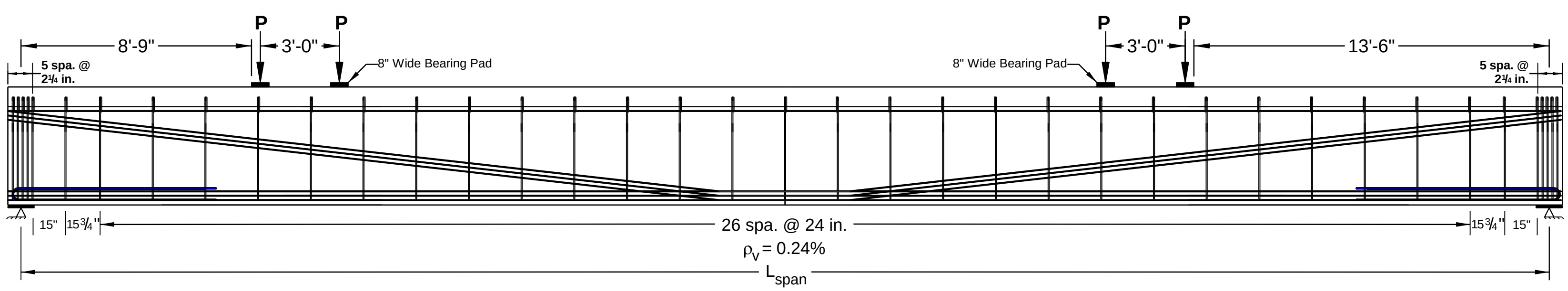
Figure 2. Elevation view of test girders showing stirrup spacing, harped prestress tendons, and concentrated load locations.



BT.8.Typ



BT.10.Typ



BT.10.Min

Figure 2 (cont.). Elevation view of test girders showing stirrup spacing, harped prestress tendons, and concentrated load locations.

were placed at select locations in a region where cracking was most likely to occur. Figure 3 shows the location of these gages for typical specimens. Each group of strain gages on a given stirrup was centered on a straight line connecting the edges of the bearing plates located at the support and at the load closest to the support. The dotted lines in the figure indicate an average approximated maximum bound of diagonal cracking observed in the literature from previous full-scale tests. Note that the planned *a*/*d* ratio for test BT.10.Typ.2 indicated in Figure 3 changed after girder construction, as discussed in the Testing Procedure section below. Also note that only one leg of the double-legged stirrup was instrumented, which unfortunately did not provide any redundancy when a gage failed to work properly. Nevertheless, there were about 30 gages installed in each beam. All strain gages were shunt calibrated using the data acquisition system just prior to the start of testing.

Four linear variable differential transformers (LVDTs) were placed on the ends of strands in the bottom row of prestressing steel extending out from the end of the beam. The purpose of these instruments was to measure strand end-slip as a sign of bond failure between the concrete and the strands. In this case, bond failure was defined as the strand being pulled into the girder by a distance greater than 0.02 in. Additionally, wire potentiometers were placed underneath the test beam to measure vertical displacements. If a relatively small increase in load resulted in a large increase in vertical displacement, then the girder was deemed to be nearing flexural failure. There were also load cells located in between the crossbeams of the load frame and the hydraulic actuators to measure the load being applied to the beam. Other instrumentation included vibrating wire gages, located at midspan at the top and bottom layer of prestressing strands, as well as strain rosettes, centered horizontally at the critical section and then $0.5d_v$ vertically on the concrete surface. However, the measurements from these devices are not pertinent to the current discussion.

TESTING PROCEDURE

There were two tests per girder, one on each end of each beam, resulting in a total of 12 tests. All of the beams were simply supported 6 in. away from the ends, with the pin support located at the end of the girder that was being tested in order to minimize longitudinal movement of the actuators during the experiment. The locations and spacing of the loads for each test are indicated by the “P” in Figure 2. Note that these distances are different from

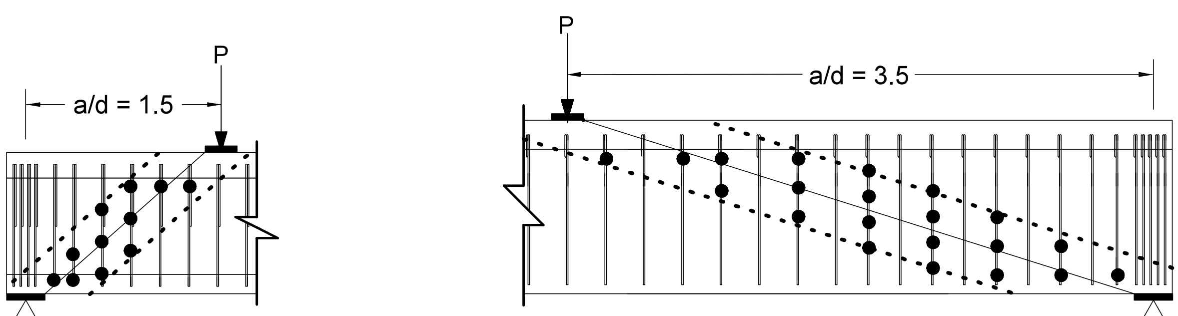


Figure 3. Locations of electrical resistance strain gages for two tests that are typical of other tests.

what was originally planned for two reasons. First, the intent was to simulate the loading from the two rear axles of an AASHTO HL-93 design truck; therefore, the two point loads were planned to be 14 ft apart. However, the first experiment resulted in flexure being the dominant failure mode. Thus, subsequent tests were conducted with the two concentrated loads being as close together as possible, or 3 ft. Even with the closer load spacing, some of the earlier tests results suggested that the second tests of the PCBT-45 girders with longer shear spans were likely to reach their flexural capacities prior to achieving their experimental shear capacities. Therefore, the shear span-to-depth ratio for the second test of the PCBT-45 girders was decreased from 3.5 to about 3.0 in the hopes of achieving a shear failure. The a/d ratios for all of the other tests remained as planned.

Before conducting the second test on each beam, the experimental team moved the support at the first end of the beam to the position of the load closest to the first end of the beam. The reason for the relocation was the concern that substantial damage had occurred in the concrete and steel reinforcement in the end region such that there might not be sufficient shear strength for the subsequent test. Because the amount of shear force during the first test dropped dramatically the concentrated load closest to the first end of the beam, the assumption was that the structural integrity would be relatively intact beyond this location. The resulting length of beam overhanging the support farthest away from the concentrated loads during the second test was about 5 ft and 9 ft for the AASHTO Type II and PCBT-45 girders, respectively. During this support repositioning, the researchers also switched the pin and roller supports so that once again the pin was located at the beam end being tested. Photos of typical test setups are shown in Figure 4.

For the actual testing, two 400-kip capacity hydraulic actuators applied load to the girders, where the load was distributed through either a squat spreader beam with stiffeners or a series of stacked plates that formed a 45^{o} pyramid. Underneath the bearing steel, there was either a reinforced or unreinforced bearing pad, resulting in a bearing area that approximated the design tire contact area specified in the AASHTO code. Initially, the researchers applied load to the girders in 20-kip increments, but those load increments decreased as each test neared capacity. Up to an arbitrary point during testing, and while the situation was deemed

Figure 4. Photos of typical setups for load testing girders.

safe, the experimental team marked the progress of crack growth in the concrete in between each load step. Material tests of both the girder and deck concrete were generally conducted at 7, 28, and 56 days from the time of casting as well as on the day of testing for each particular beam end. The researchers also performed tension tests on samples of vertical reinforcement and prestressing strands.

RESULTS AND ANALYSIS

EXERIMENTAL SHEAR STRENGTH VERSUS DESIGN EQUATIONS

Results from some of the material testing and geometry measurements are listed in **Error! Reference source not found.**. Note that the unit weights for the girder and deck concrete, γ_c and $\gamma_{c\ deck}$, respectively, are for the fresh concrete and do not include the 5 pcf assumed for the weight of the reinforcing steel in dead load calculations. See the work by Kassner[17] for more detailed material test results. Also note that the effective shear depth, d_v, was calculated using the AASHTO code, and the composite cross-sectional area is the transformed area assuming an uncracked section.

Table **Error! Reference source not found.** lists the failure modes and shear strengths from the twelve tests, along with the ratios of the experimental-versus-calculated shear strength ratios. The calculated strengths were computed using the three design methods given in the 2012 AASHTO LRFD Bridge Design Specifications: the *Appendix B5* method using tables from Appendix B5.2, the *General Procedure* using equations from Article 5.8.3.4.2, or the *Simplified Procedure* from Article 5.8.3.4.3. These three different calculations are listed as *A.B5*, *Gen*, and *Sim*, respectively, in the table. The basic steps for calculating the shear strength of an existing girder using *Appendix B5* and the *General Procedure* parallel the process originally outlined by Kulicki et al.[18], where all load and resistance factors were set equal to 1.00. Also, the calculated strengths assumed that f_{ct} was not specified for the sand-lightweight concrete; therefore, $\sqrt{f'_c}$ was multiplied by 0.85 when used in equations

Table 1. Material and geometric properties for individual tests.

Test ID	γ_c (pcf)	f_c (ksi)	f_{ct} (ksi)	E_c (ksi)	$\gamma_{c\ deck}$ (pcf)	$f_{c\ deck}$ (ksi)	E_{deck} (ksi)	f_{yv} (ksi)	ρ_v (%)	A_{comp} (in^2)	d_v (in)	L_{span} (ft)	a/d
T2.8.Typ.1	116.5	8.9	0.690	3610	118.4	5.8	3570	67.3	0.74	1046	38.7	40.0	1.5
T2.8.Typ.2	116.5	8.9	0.690	3610	118.4	6.1	3210	67.3	0.74	978	38.8	35.1	3.1
T2.8.Min.1	116.5	8.9	0.690	3610	120.0	5.4	3240	67.3	0.56	985	38.2	40.0	1.5
T2.8.Min.2	116.5	8.9	0.690	3610	120.0	5.4	3240	67.3	0.56	984	38.6	34.8	2.9
BT.8.Typ.1	120.8	9.1	0.705	3590	123.4	5.6	3600	67.3	0.63	1468	48.8	58.0	2.0
BT.8.Typ.2	120.8	9.1	0.705	3590	130.8	6.7	4050	67.3	0.63	1558	49.1	49.5	3.1
BT.8N.Typ.1	145.0	8.9	0.815	4820	122.0	4.9	2940	69.8	0.38	1185	48.5	57.9	2.1
BT.8N.Typ.2	145.0	8.6	0.735	4590	122.0	5.0	3110	69.8	0.38	1234	48.6	49.0	3.1
BT.10.Typ.1	119.6	8.9	0.610	3910	121.6	4.1	3160	67.3	0.57	1328	47.9	57.9	2.0
BT.10.Typ.2	124.0	9.7	0.620	4060	124.0	4.9	3270	67.3	0.57	1326	48.4	49.5	2.9
BT.10.Min.1	124.0	9.7	0.620	4060	121.6	5.2	3200	67.3	0.24-0.38	1313	48.5	58.0	2.1
BT.10.Min.2	125.6	10.3	0.765	4140	122.8	5.9	3410	67.3	0.24-0.38	1339	48.7	49.5	3.0

Table 2. Experimental shear capacities along with comparisons to AASHTO-calculated shear strengths.

Test ID	**Failure Mode(s)**	V_{Exp} **(kip)**	**App. B5** $V_{n\ A.B5}$ **(kip)**	**App. B5** $^{Exp}/_{A.B5}$	**General** $V_{n.Gen}$ **(kip)**	**General** $^{Exp}/_{Gen}$	**Simplified** $V_{n.Sim}$ **(kip)**	**Simplified** $^{Exp}/_{Sim}$
T2.8.Typ.1	Flexural / Bond	361	217	1.66	204	1.77	333	1.08
T2.8.Typ.2	Flexural	294	196	1.50	185	1.59	305	0.96
T2.8.Min.1	Web Shear / Bond	382	189	2.02	178	2.15	278	1.38
T2.8.Min.2	——	308	182	1.69	170	1.81	271	1.14
BT.8.Typ.1	Web Shear / Bond	500	306	1.63	289	1.73	373	1.34
BT.8.Typ.2	Flexural	408	269	1.52	269	1.52	339	1.20
BT.8N.Typ.1	Web Shear / Bond	431	259	1.66	248	1.74	317	1.36
BT.8N.Typ.2	Web Shear / Bond	382	241	1.58	239	1.60	290	1.32
BT.10.Typ.1	Web Shear / Bond	518	285	1.82	277	1.87	339	1.53
BT.10.Typ.2	Web Shear / Flex. / Bond	428	268	1.60	260	1.65	328	1.30
BT.10.Min.1	Web Shear / Bond	475	224	2.12	224	2.12	241	1.97
BT.10.Min.2	Web Shear	371	226	1.64	226	1.64	236	1.57
Mean $^{Exp}/_{Calc}$ ratio for Lightweight				**1.72**		**1.78**		**1.35**
CoV $^{Exp}/_{Calc}$ ratio for Lightweight				**0.12**		**0.12**		**0.22**
Mean $^{Exp}/_{Calc}$ ratio for Normal Weight				**1.62**		**1.67**		**1.34**

in Articles 5.8.2 and 5.8.3 of the

CoV $^{Exp}/_{Calc}$ ratio for Normal Weight		**0.03**		**0.06**		**0.02**
Average $^{Exp}/_{Calc}$ ratio for all tests		**1.70**		**1.76**		**1.35**
CoV $^{Exp}/_{Calc}$ ratio for all tests		**0.11**		**0.11**		**0.20**

Specifications. The analysis divided the shear spans into ten equal lengths; the segment with the lowest shear resistance became the predicted shear strength for the given test in Table **Error! Reference source not found.**.

Broadly speaking, the girders performed exceptionally well compared to the expected shear strengths, regardless of the shear design procedure. In some tests, the beams failed in flexure or bond prior to achieving a true shear failure. In all of these cases, there were signs of concrete powdering or light flaking in the web, indicating that the girder was nearing its shear capacity. Thus, the total shear force in the beam at the time any alternate failure modes occurred was deemed to be the experimental shear capacity, V_{Exp}, given in **Error! Reference source not found.**. Note that V_{Exp} includes the shear due to the dead load of the beam and composite deck.

The average ratio of experimental versus calculated shear strength for all of the tests ranged from 1.35 to 1.76 amongst the three design methods. Thus, the AASHTO LRFD Bridge Design Specifications was fairly conservative in forecasting the shear strength of prestressed, sand-lightweight concrete girders, regardless of which shear design method was used. The *Simplified Procedure* was the least conservative, yet most accurate design calculation, while the *General Procedure* proved to be the most conservative. The most complicated method, the *Appendix B5* calculations, tended to be marginally less conservative than the *General Procedure*, with an average strength ratio of 1.70 for all of the specimens. Note that the strength ratios containing the *General Procedure* and *Appendix B5* computations had smaller coefficients of variation compared to the *Simplified Procedure*. One reason why the *Simplified Procedure* was the least conservative predictor is that the angle of shear cracking in this method tended to be lower relative to the angle calculated in the other two methods. Therefore, the *Simplified Procedure* generally resulted in a larger force resisted by the vertical steel, and thus a less conservative shear strength prediction. There was one test, T2.8.Typ.2, where the *Simplified Procedure* determined that the shear capacity would be greater than the experimental result. Granted, this particular test did experience a flexural failure. Again, however, the powdering and flaking that was evident at termination of the experiment indicated that the girder was close to shear capacity. On the opposite extreme, the largest ratio of V_{Exp} versus calculated shear strength was 2.15 for test T2.8.Min.1 using the *General Procedure*. In any event, the range of experimental-versus-calculated strength ratios is similar to those reported by other researchers[19-21].

EFFECTS OF VARIOUS PARAMETERS ON SHEAR CAPACITY

In order to investigate the effect of different parameters on the overall shear capacity, the results from this study were combined with earlier research performed by Malone[22], Kahn et al.[21], and Dymond et al.[19] The calculated shear strengths for this set of previous research are based on the reported geometry and material characteristics. This set of data contained prestressed concrete girders with concrete compressive strengths ranging from 6.5 ksi to 11.0 ksi. The cross-sectional areas ranged from 468 in^2 to 1629 in^2 while the shear span-to-effective depth ratios, *a*/*d*, ranged from 1.3 to 3.0. Some girders had no shear reinforcement, while others had minimal ($\rho_v \approx 0.003$) and typical ($\rho_v \approx 0.019$) shear reinforcement ratios, resulting in reinforcing indices ranging from 0 psi to 1180 psi. Only the one test by Dymond et al. contained harped tendons. The calculated shear strengths from the earlier research are detailed in Table 3.

Table 3. Calculated shear strengths and corresponding experimental-versus-calculated strength ratios from previous research on large-scale prestressed lightweight concrete beams.

Author	Specimen ID	V_{Exp} (kip)	App. B5		General		Simplified	
			$V_{n\ A.B5}$ (kip)	$^{Exp}/_{A.B5}$	$V_{n\ Gen}$ (kip)	$^{Exp}/_{Gen}$	$V_{n\ Sim}$ (kip)	$^{Exp}/_{Sim}$
Malone	PC6N	80	50	1.61	41	1.96		
Malone	PC6S	117	72	1.63	72	1.62	80	1.45
Malone	PC10N	105	59	1.76	47	2.21		
Malone	PC10S	120	77	1.56	79	1.52	90	1.33
Kahn et al.	G1A-East	363	240	1.51	153	2.37	344	1.05
Kahn et al.	G1A-Center	258	135	1.92	131	1.97	179	1.44
Kahn et al.	G1B-East	312	432	0.72	263	1.19	592	0.53
Kahn et al.	G1B-Center	234	125	1.87	121	1.94	181	1.30
Kahn et al.	G1C-East	289	238	1.22	154	1.88	349	0.83
Kahn et al.	G2A-Center	256	136	1.88	133	1.93	182	1.41
Kahn et al.	G2B-Center	246	127	1.93	122	2.02	183	1.34
Dymond et al.	web-shear	658	514	1.28	421	1.56	690	0.95
Average $^{Exp}/_{Calc}$ ratio				**1.57**		**1.85**		**1.16**
CoV $^{Exp}/_{Calc}$ ratio				**0.23**		**0.18**		**0.27**

The ratios of experimental-to-calculated shear strengths in Table 3 are included in Figures 5 and 6, which show the effect of the various parameters for the *Appendix B5* and *Simplified Procedure*, respectively. The parametric results for the *General Procedure* are not displayed because that data was generally slightly more conservative than that presented for the *Appendix B5* results in Figure 5. Also note that because there was no apparent relationship or trend between the $V_{Exp}/V_{n\ calc}$ ratio and the transformed cross-sectional area, graphs for that particular parametric variable were excluded from Figures 5 and 6.

Both Figures 5 and 6 show that the reinforcing index appears to influence the relative accuracy of the predicted strengths, where the experimental strength decreases relative to the expected strength as $\rho_v f_{yv}$ increases. Paczkowski and Nowak[23] reached a similar conclusion for lightweight reinforced concrete members. This observation is particularly true for the *Simplified Procedure*. In fact, most of the test specimens where $\rho_v f_{yv}$ was greater than

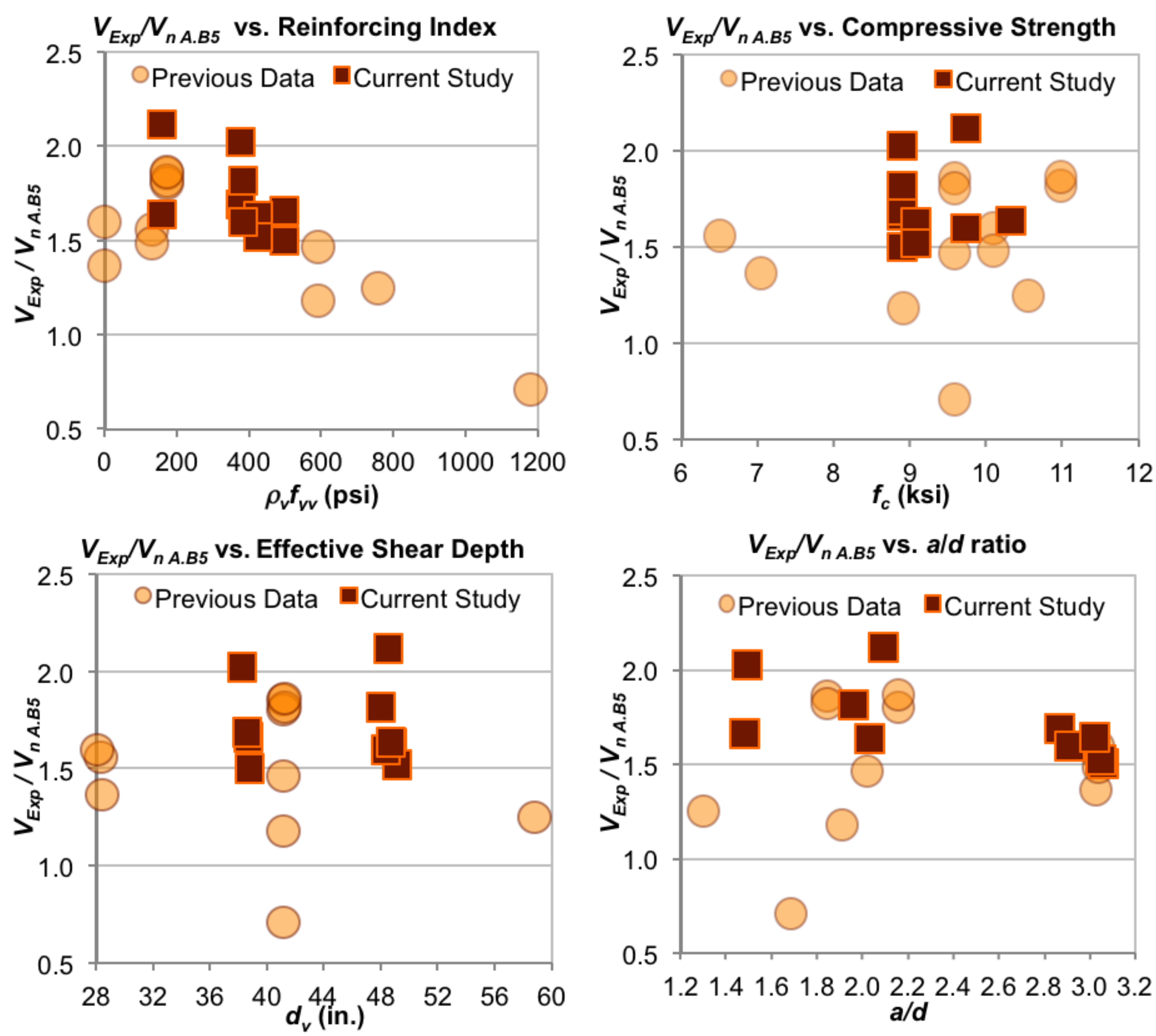


Figure 5. Effects of various design parameters on the ratio of the experimental-to-calculated values using *Appendix B5*

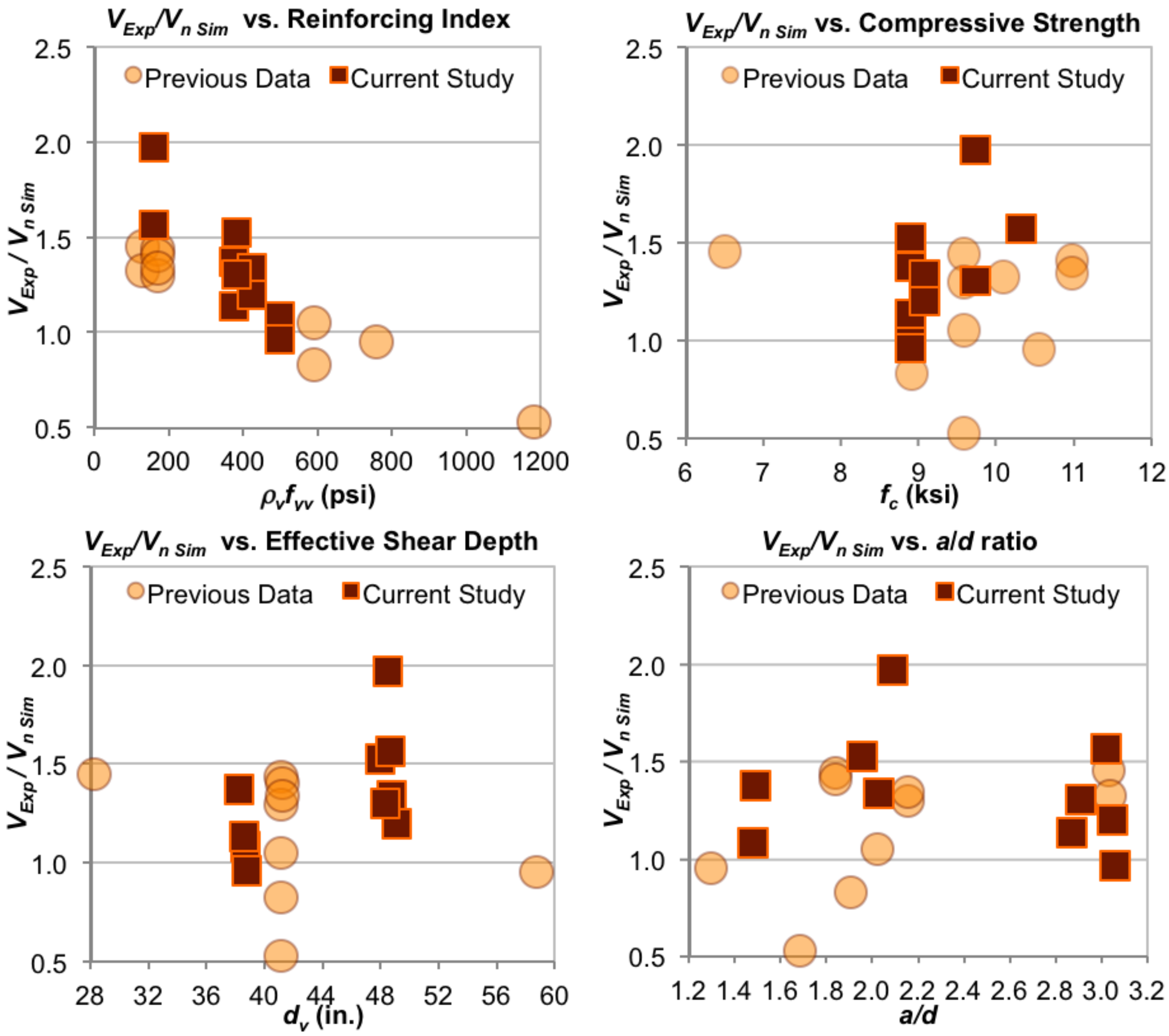


Figure 6. Effects of various design parameters on the ratio of the experimental-to-calculated values using the *Simplified Procedure*.

498 psi had an experimental-to-predicted shear strength ratio of less than 1.0 when considering the *Simplified Procedure*. This fact suggests that there should be some sort of limit on the predicted concrete girder strength based on $\rho_v f_{yv}$ when designing with the *Simplified Procedure*. Although AASHTO does limit the shear strength of concrete girders, that limit is intended to ensure that the transverse steel yields prior to the concrete crushing in the girder web. In the case of test T2.8.Typ.2, the vertical stirrups certainly had started to yield prior to web crushing, yet the ratio of actual-to-predicted failure was 0.96 when considering the *Simplified Procedure*.

Like the shear reinforcing index, results from the previous research on prestressed, lightweight concrete indicated that the compressive strength of concrete does have an inverse influence on the performance of the strength prediction when using the *Simplified Procedure*. However, Figures 5 and 6 indicate the results from the current study had an opposite trend regardless of which design procedure was used, where the $V_{Exp} / V_{n\ calc}$ ratio increases slightly as f_c increases from 6 ksi to 11 ksi. These results were counter to what Kawaguchi et al.[24] found, but are in line with findings by Paczkowski and Nowak[23] and Elzanaty et al[9].

While Ivey and Buth[25] found that the effective shear depth had little if any impact on the shear resistance of a beam, all three of the shear design methods incorporate the d_v into the calculations. However, the results using either *Appendix B5* or the *Simplified Procedure* were mixed. On the one hand, the prior research indicated that the shear strength of prestressed, lightweight girders declined relative to the *Appendix B5* predictions when the effective shear depth was greater than 40 in. Yet, the current investigation showed that beams with d_v measuring about 48 in. had equal or more conservative predictions than other beams with a smaller d_v of 28 in, as indicated in Figure 5. Figure 6 shows similar results for the *Simplified Procedure*. On the other hand, the strength ratios found with the *General Procedure* agreed with Ivey and Buth, showing no clear relationship between the effective shear depth and $V_{Exp}/V_{n\ Gen}$.

With regards to the shear span-to-effective depth ratio, there seems to be the opposite occurrence compared to the observations for the effective shear depth in and of itself. In the case of a/d, the results from previous work showed that the design models are less conservative for smaller values of a/d. However, the lightweight beams in this research that had some of the highest experimental-to-calculated ratios also had values for a/d that were less than 2.0. On the other hand, the results from this study proved to be slightly less conservative when testing at shear span-to-effective depth ratio around 3.0. This observation agrees with other past research for both lightweight and normal weight concrete[9,26].

COMPARISON OF LIGHTWEIGHT VERSUS NORMAL WEIGHT CONCRETE TESTS

When comparing the lone normal weight beam, BT.8N.Typ, with its lightweight counterpart, BT.8.Typ, the two had the same basic design parameters with the exception of the unit weight of concrete. However, because AASHTO penalizes lightweight concrete, the stirrup spacing in the lightweight girder was 9 in. within the testing region versus 15 in. for the normal weight girder. The resulting shear reinforcing index in the lightweight beam was 60% greater than its normal weight companion. Also note that the splitting tensile strength of the lightweight concrete at the time of testing was between 4% and 16% lower than that of the normal weight concrete, as seen in **Error! Reference source not found.**.

With these notes in mind, **Error! Reference source not found.** and Figure 7 show that just prior to the formation of web-shear cracks, the first test with normal weight concrete had about 14% more shear carried by the concrete than the respective lightweight beam. However, the experimental web-shear cracking load, $V_{cw\ Exp}$, in test BT.8N.Typ.2 was 7% lower than the result for BT.8.Typ.2. This last comparison contradicts the earliest lightweight research coming out of the University of Texas[28-33], but agrees with the subsequent work by Hanson[34,35]. Although the beams in this study had shear stirrups while those from Hanson's work did not, research by Moody et al.[36] and Elzanaty et al.[13] showed that vertical reinforcement did not affect the diagonal cracking load.

Table 4. Concrete and steel components of shear strength for lightweight and normal weight concrete comparison tests.

Test ID	V_{Exp} (kip)	$V_{cw\ Exp}$ (kip)	$V_{c\ Exp}$ (kip)	$V_{s\ Exp}$ (kip)
BT.8.Typ.1	500	214	314	168
BT.8.Typ.2	408	207	175	215
BT.8N.Typ.1	431	243	247	168
BT.8N.Typ.2	382	192	189	177

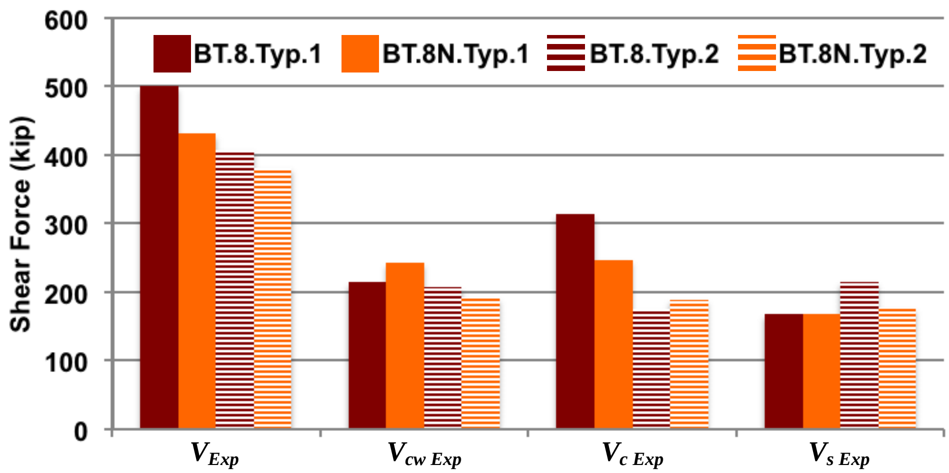


Figure 7. Concrete and steel components of shear strength for lightweight and normal weight comparison tests.

On the other hand, the shear strength provided by the concrete at failure, $V_{c\ Exp}$, was 26% greater in test BT.8.Typ.1 compared to its normal weight companion, BT.8N.Typ.1. When testing the longer shear span, however, $V_{c\ Exp}$ was lower in the lightweight test relative to the normal weight experiment. The results for $V_{c\ Exp}$ are in contrast with the findings by Hamadi and Regan[37], who determined that V_c was lower in reinforced lightweight concrete beams compared to normal weight beams. For the current study, the average ratio of $V_{c\ Exp} / V_{Exp}$ for the two tests on beam BT.8.Typ was the same as girder BT.8N.Typ, although the coefficient of variation was greater for the lightweight beam.

Interestingly, the shear force resisted by the concrete increased between the time of web-shear cracking and the failure load for the test involving the shorter shear span in beam BT.8.Typ. For the similar test in the normal weight beam BT.8N.Typ, V_{cw} and $V_{c\ Exp}$ are relatively equal. On the other hand, the concrete contribution at final load was smaller compared to V_{cw} for the longer shear span in beam BT.8.Typ. Similar results were observed in the other lightweight beams. Because there was only one set of tests involving normal weight concrete, there is no definitive conclusion as to whether or not the results for beam BT.8N.Typ are an anomaly.

Given that beam BT.8.Typ had the higher stirrup density compared to its normal weight counterpart, one might think that the total force in the transverse steel at ultimate capacity, $V_{s\,Exp}$, would be greater in the lightweight beam than the normal weight beam. This thinking is correct when comparing tests BT.8.Typ.2 with BT.8N.Typ.2, where $V_{s\,Exp}$ was 22% greater in the lightweight girder. On the other hand, tests BT.8.Typ.1 and BT.8N.Typ.1 had the same amount of shear force being resisted by the stirrups at the failure load.

Regarding total shear capacity, the lightweight beam tests averaged about 11% greater strength than the experiments on the normal weight girders. Beams BT.8.Typ and BT.8N.Typ in Table **Error! Reference source not found.** had fairly similar ratios of experimental versus calculated shear capacities using *Appendix B5* (1.63 versus 1.66 when $a/d \approx 2.0$ and 1.52 versus 1.58 when $a/d \approx 3.0$, for the lightweight and normal weight beams, respectively). Both the *General Procedure* and the *Simplified Procedure* had similar comparisons for $a/d \approx 2.0$, although the disparity between the two concrete densities was somewhat greater when $a/d \approx 3.0$.

LIGHTWEIGHT MODIFICATION FACTOR

Up to this point, any comparisons between the experimental results and the AASHTO-predicted strengths have included a modification factor for lightweight concrete, which for this research had been designated as λ_v and was taken as 0.85 for unspecified splitting tensile strength of sand-lightweight concrete. In the case of specified splitting tensile strength, λ_v could be calculated by reformulating Eq. (6) as

$$\lambda_v = \frac{4.7 f_{ct}}{\sqrt{f'_c\,(\text{ksi})}} \qquad (7)$$

The result from Eq. (7) would then be combined with $\sqrt{f'_c}$ wherever that term appeared in the equations in Article 5.8.2 and 5.8.3 of the AASHTO LRFD Bridge Design Specifications.

If one were to assume that the concrete mix designs for the test specimens in this study had a specified splitting tensile strength equal to the measured value, the modification for lightweight concrete calculated from Eq. (7) would not have had a large impact on the calculated strengths. Reorganizing some of the data from **Error! Reference source not found.**, **Error! Reference source not found.** indicates that a λ_v factor would have not been applicable in 70% of the tests because the factor would have been greater than 1.00. In the three tests where λ_v would have factored into the calculations, those modification values were fairly close to unity. Note that in the case of **Error! Reference source not found.**, f'_c in Eq. (7) was assumed to be f_c, the measured compressive strength at the time of testing.

The results and analysis of concrete samples taken from the full-scale, prestressed lightweight concrete girders corroborate the material characteristic results in *NCHRP Report 733*[15], where the researchers developed the factor *a* for each of 48 different mix designs containing sand fine aggregate and manufactured slate, shale, and clay coarse aggregate from various locations across the United States. In this study *a* was defined as

Table 5. Calculated modification factors using known material properties from the current study.

Test ID	f_c (ksi)	f_{ct} (ksi)	$4.7f_{ct}$ (ksi)	$\sqrt{f_c}$ (ksi)	λ_v	apply λ_v?
T2.8.Typ.1	8.9	0.690	3.24	2.98	1.09	No
T2.8.Typ.2	8.9	0.690	3.24	2.98	1.09	No
T2.8.Min.1	8.9	0.690	3.24	2.98	1.09	No
T2.8.Min.2	8.9	0.690	3.24	2.98	1.09	No
BT.8.Typ.1	9.1	0.705	3.31	3.02	1.10	No
BT.8.Typ.2	9.1	0.705	3.31	3.02	1.10	No
BT.10.Typ.1	8.9	0.610	2.87	2.98	0.96	Yes
BT.10.Typ.2	9.7	0.620	2.91	3.11	0.94	Yes
BT.10.Min.1	9.7	0.620	2.91	3.11	0.94	Yes
BT.10.Min.2	10.3	0.765	3.60	3.21	1.12	No

$$a = \frac{f_{ct}}{\sqrt{f'_c\,(\text{ksi})}} \tag{8}$$

Error! Reference source not found. shows the average value of *a* based on three splitting tensile cylinder specimens produced from each of two or three batches of various concrete mix designs. These individual averages ranged from above 0.22 to below 0.28, with an overall mean of 0.25. In every case, the mix designs generated an average value that was greater than 0.21, which is the reciprocal of 4.7, which is the coefficient that appears in Eq. (6). In other words, the researchers concluded that typical high-performance, high-strength lightweight concrete mix designs used in prestressed girders have a splitting tensile strength that is greater than $\sqrt{f'_c}/4.7$. Thus, the result from Eq. (6) would normally be limited to $\sqrt{f'_c}$, as if the concrete was normal weight. Therefore, the authors recommended that Article 5.8.2.2 of the 2012 AASHTO LRFD Bridge Design Specifications should be revised such that no modification factor is required for sand-lightweight concrete.

One could consider the effect of excluding the modifier from the calculations entirely, as indicated by the "No λ_v" results in **Error! Reference source not found.**. In this table, the $V_{n\ calc\ No\ \lambda_v}$ results are from the calculations following *Appendix B5*, the *General Procedure*, and the *Simplified Procedure*, respectively, with the exception that λ_v is disregarded, or in other words, is set equal to 1.00. Note that percentage difference shown in this table is calculated as:

$$\frac{V_{n\ calc\ No\ \lambda_v} - V_{n\ calc}}{V_{n\ calc}} \tag{9}$$

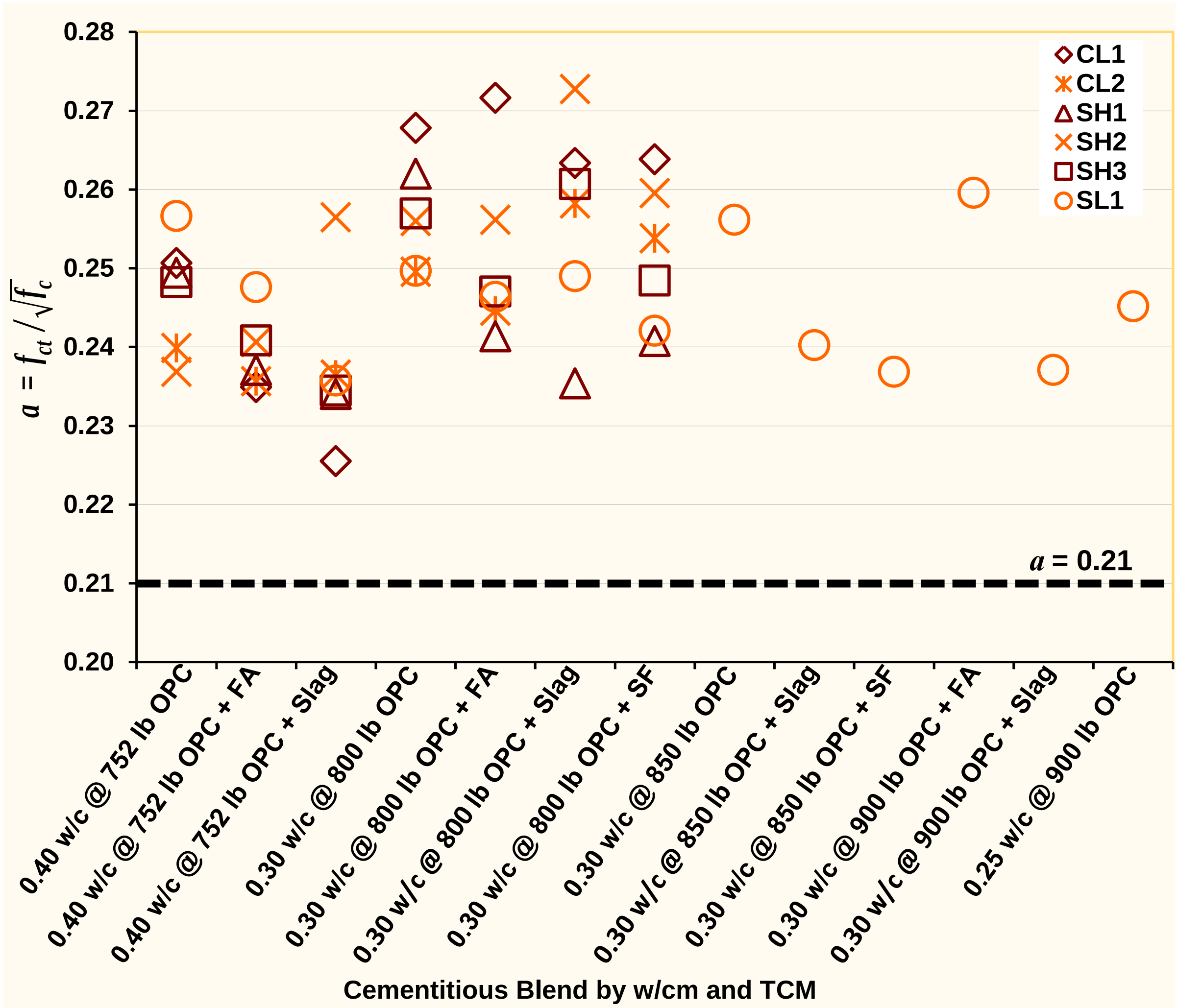


Figure 8. Factor *a* for various concrete mixtures

where $V_{n\ calc}$ are the results from the respective calculations given in Tables **Error! Reference source not found.** and 3.

The difference between the calculations for the nominal shear capacity using the modifier for sand-lightweight concrete and those calculations using no modifier is fairly small. Dymond et al.[19] also found that the modification for sand-lightweight concrete had a relatively minor impact on the calculated overall shear strength. What is interesting is that the two experiments within this data set that did not have any shear reinforcement, tests PC6N and PC10N by Malone[22], are the two largest outliers in terms of the difference between retaining λ_v versus not doing so. If one were to remove these two outliers, then the average percent difference and corresponding coefficient of variation would be 3.0% and 1.2%, respectively, for *Appendix B5*. The results for the *General Procedure* would be 2.2% and 1.1%, respectively. The two outliers had no effect on the *Simplified Procedure* because tests PC6N and PC10N did not meet the minimum reinforcement requirement and thus were not included in the analysis.

One other interesting observation regarding Table 6 is that 30% of the beams being analyzed using the *Simplified Procedure* had a *greater* predicted shear capacity when using the sand-

Table 6. Computed shear capacities disregarding the sand-lightweight modification factor ıg with the percentage difference from the calculated strengths in Tables **Error! Reference source not found.** and 3.

Author	Test ID	Appendix B5			General Procedure			Simplified Procedure		
		$V_{n\,A.B5}$ No λ_v (kip)	$^{Exp}/_{A.B5}$ No λ_v	% Diff.	$V_{n.Gen}$ No λ_v (kip)	$^{Exp}/_{Gen}$ No λ_v	% Diff.	$V_{n.Sim}$ No λ_v (kip)	$^{Exp}/_{Sim}$ No λ_v	% Diff.
	T2.8.Typ.1	222	1.63	2.0	207	1.74	1.7	339	1.07	1.8
	T2.8.Typ.2	199	1.48	1.7	188	1.57	1.3	311	0.94	1.9
	T2.8.Min.1	194	1.97	2.6	182	2.10	2.3	284	1.35	2.1
	T2.8.Min.2	186	1.66	1.9	173	1.78	1.6	277	1.11	2.1
Current Study	BT.8.Typ.1	312	1.60	1.7	293	1.70	1.5	373	1.34	-0.2
	BT.8.Typ.2	284	1.44	5.7	273	1.49	1.7	335	1.22	-1.4
	BT.10.Typ.1	291	1.78	1.9	283	1.83	1.9	348	1.49	2.6
	BT.10.Typ.2	273	1.57	1.9	265	1.62	1.9	334	1.28	1.9
	BT.10.Min.1	232	2.05	3.5	233	2.04	4.0	244	1.95	1.3
	BT.10.Min.2	232	1.60	2.6	235	1.58	4.0	240	1.54	1.6
Malone	PC6N	59	1.37	17.6	47	1.70	15.5			
Malone	PC6S	75	1.56	4.2	75	1.56	3.7	83	1.41	2.7
Malone	PC10N	65	1.60	10.4	54	1.92	15.4			
Malone	PC10S	81	1.48	5.5	83	1.45	4.6	93	1.29	3.2
Kahn et al.	G1A-East	248	1.46	3.1	155	2.35	1.0	335	1.08	-2.8
Kahn et al.	G1A-Center	139	1.86	3.2	134	1.92	2.5	186	1.39	3.8
Kahn et al.	G1B-East	440	0.71	1.8	266	1.17	1.2	574	0.54	-2.9
Kahn et al.	G1B-Center	130	1.80	3.5	124	1.89	2.5	187	1.25	3.8
Kahn et al.	G1C-East	245	1.18	3.0	156	1.86	1.0	338	0.86	-3.1
Kahn et al.	G2A-Center	141	1.82	3.3	136	1.88	2.5	188	1.36	3.5
Kahn et al.	G2B-Center	132	1.87	3.7	125	1.96	2.6	191	1.29	3.9
Dymond et al.	web-shear	527	1.25	2.5	425	1.55	0.9	670	0.98	-2.9
Average			**1.58**	**4.0**		**1.76**	**3.4**		**1.24**	**1.2**
Coefficient of variation			**0.19**	**0.9**		**0.15**	**1.2**		**0.24**	**2.0**
Average without outliers			**1.59**	**3.0**		**1.75**	**2.2**		**1.24**	**1.1**
CoV without outliers			**0.19**	**0.4**		**0.15**	**0.5**		**0.24**	**2.1**

lightweight concrete modifier versus not doing so. The reason for the increase lies within Eq. 5.8.3.4.3-4 of the 2012 AASHTO LRFD Bridge Design Specifications, reproduced below:

$$\cot\theta = 1.0 + 3\left(\frac{f_{pc}}{\sqrt{f'_c}}\right) \leq 1.8 \qquad (10)$$

Note that Eq. (10) applies if and only if $V_{ci} \geq V_{cw}$, which was the case for all of the tests in the current study as well as the twelve large-scale tests from the previous research. However, if the term $\sqrt{f'_c}$ in this equation is substituted by 0.85 $\sqrt{f'_c}$ when the splitting tensile strength of sand-lightweight concrete is not specified, then the value for cot θ will be larger for sand-lightweight concrete compared to when the substitution is not made. The cot θ term figures into AASHTO's Eq. 5.8.3.3-4, or:

$$V_s = \frac{A_v f_{yv} d_v \left(\cot\theta + \cot\alpha\right)\sin\alpha}{s} \qquad (11)$$

Because cot θ is larger when the lightweight concrete modifier is included, the resulting V_s will be larger as well. So, even though V_c decreased when including λ_v, V_s was large relative to V_c for this particular subset of tests in **Error! Reference source not found.**. Therefore, the increase in V_s overcompensated for the decrease in the calculated value of V_c, resulting in a larger theoretical shear capacity when the λ_v "penalty" for lightweight concrete was included in the *Simplified Procedure*. For the eight lightweight tests where $V_{n\,Sim}$ was less than $V_{n\,Sim\ \mathrm{No}\ \lambda_v}$ in the current study, cot θ would have been larger in the latter; however, cot θ was limited by the maximum value of 1.8 in Eq. (10) for both $V_{n\,Sim}$ and $V_{n\,Sim\ \mathrm{No}\ \lambda_v}$. Thus, V_s was the same regardless of whether the lightweight modification factor was included or not. Meanwhile, V_c was lower for $V_{n\,Sim}$, resulting in a lower calculated shear capacity when taking λ_v into account. The same is true for past experiments G1A-Center, G1B-Center, and G2A-Center by Kahn et al.

CONCLUSIONS AND RECOMMENDATIONS

Due to the nature of experimenting with full-scale, sand-lightweight prestressed structures with composite decks, there was little repetition of complete girder designs within this study. Nevertheless, based on the information gleaned from testing in this research and combined with analysis of past literature of full-scale, prestressed lightweight concrete shear tests, the authors offer the following conclusions:

- The *Simplified Procedure* was the least conservative predictor of shear strength. The *General Procedure* was marginally more conservative than *Appendix B5*.
- Even when sand-lightweight concrete had a lower tensile splitting strength relative to normal weight concrete, this characteristic did not necessarily translate into reduced web-shear cracking strength.
- There were no distinct structural differences between sand-lightweight and normal weight concrete when it comes to shear strength of full-scale, prestressed concrete girders, particularly considering the mixed results regarding $V_{cw\,Exp}$ and $V_{c\,Exp}$.
- The ratio of experimental versus calculated shear strengths were marginally lower for the lightweight girders compared to the normal weight beams. Again, however, this conclusion is based on a comparison of two pairs of tests. Analysis of a larger set of tests

suggested that the modifiers for sand-lightweight concrete only have a minor effect on the calculated shear strength in a girder.
- In some cases with the *Simplified Procedure*, the modification factor for sand-lightweight concrete can lead to *greater* predicted shear capacity. Still, the inherent amount of conservatism in the AASHTO LRFD calculations should give sufficient assurance in abandoning the sand-lightweight concrete modifier.

Therefore, the authors recommend that AASHTO should remove the requirement that the $\sqrt{f'_c}$ term be modified when designing for the shear strength of prestressed, sand-lightweight concrete girders.

Unfortunately, making a definitive conclusion or recommendation regarding the sufficiency of the minimum requirements for shear reinforcement for girders constructed with sand-lightweight concrete is not appropriate at this time because the area of vertical reinforcement designed in this investigation was substantially larger than what is dictated in the 2012 AASHTO LRFD Bridge Design Specifications. On the other hand, AASHTO may wish to consider decreasing the maximum limit on the design shear strength of sand-lightweight concrete girders when designing with the *Simplified Procedure*, as the results showed that the strength calculated using this procedure became unconservative as the shear reinforcing index became large.

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NOTATION

a = ratio of the splitting tensile strength versus the square root of the compressive strength of concrete

A. B5 = shear strength calculations following Appendix B5.2 of the 2012 AASHTO LRFD Bridge Design Specifications

A_{comp} = composite cross-sectional area (in^2)

a/d = shear span-to-effective shear depth ratio

b_v = width of the web (in.)

CoV = coefficient of variation

$\cot\theta$ = cotangent of the angle θ, where θ is angle of inclination of diagonal compressive stresses (deg.)

d_v = effective shear depth (in.)

f'_c = specified compressive strength of concrete (ksi)

f_c = concrete compressive strength at time of load test (ksi)

f_{ct} = average splitting tensile strength of lightweight aggregate concrete (ksi)

f_{pc} = resultant compressive stress at the centroid of the composite section or at the junction of the web and flange when the centroid lies within the flange, that results from both prestress and the bending moments resisted by the precast member acting alone (ksi)

f_{yv} = yield strength of the vertical reinforcement (ksi)

Gen = shear strength calculations following the General Procedure in Article 5.8.3.4.2 of the 2010 AASHTO LRFD Bridge Design Specifications

M_{cre} = moment causing flexural cracking at a given section due to externally applied loads (kip-in)

M_{max} = maximum factored moment at a given section due to externally applied loads (kip-in)

Sim = shear strength calculations following the Simplified Procedure for Prestressed and Nonprestressed Sections in Article 5.8.3.4.3 of the 2010 AASHTO LRFD Bridge Design Specifications

V_c = shear resistance provided by the concrete (kip)

$V_{c\ Exp}$ = concrete component of the shear resistance at the experimental failure load (kip)
V_{ci} = shear force at first inclined cracking that develops from a combination of shear stresses and tensile stresses due to the flexural moment, where the shear stresses increase in the region immediately above the flexural cracks (kip)
V_{cw} = nominal shear resistance provided by the concrete when inclined cracking results from excessive principal tensions in the web (kip)
$V_{cw\ Exp}$ = concrete component of shear resistance at first diagonal cracking in the web during experimentation (kip)
V_d = shear force at a given section due to unfactored dead loads (kip)
V_{Exp} = experimental shear capacity (kip)
V_i = factored shear force at section due to externally applied loads occurring simultaneously with M_{max} (kip)
V_n = nominal shear resistance (kip)
$V_{n\ calc}$ = nominal shear strength calculated according to a given shear design method (kip)
V_p = component of the effective prestressing force in the direction of the applied shear (kip)
V_s = shear resistance provided by the shear reinforcement (kip)
$V_{s\ Exp}$ = steel component of the shear resistance at the experimental failure load (kip)
β = factor relating the effect of longitudinal strain on the shear capacity of concrete, as indicated by the ability of diagonally cracked concrete to transmit tension
γ_c = unit weight of concrete (pcf)
$\gamma_{c\ deck}$ = unit weight of the deck concrete (pcf)
λ_v = lightweight modification factor for shear
ρ_v = percentage of vertical reinforcement relative to the gross horizontal area of the web
$\rho_v f_{yv}$ = shear reinforcing index (ksi)