RELIABILITY OF PRESTRESSED CONCRETE GIRDERS

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ABSTACT

In the new generation of design codes, structural performance is measured in terms of reliability index. Load and resistance (load carrying capacity) are treated as random variables, represented by statistical parameters such as the mean value, bias factor or ratio of mean-to-nominal, coefficient of variation and type of cumulative distribution function (CDF). The development of the current AASHTO LRFD Code was based on the state-of-the-art from 1980's. Recently, an extensive material test data was provided by the industry in conjunction with the calibration of ACI 318 Code. It is clear that material properties have changed and considerably improved over the last 30 years. The objective of this study is to present the up-dated statistical parameters for prestressed concrete girders and the resulting reliability indices. The analysis is performed for AASHTO type girders and NU girders developed in Nebraska. The new material test data include compressive strength of concrete, yield strength of reinforcing steel and tensile strength of prestressing strands. The cumulative distribution functions are plotted on the normal probability paper for an easier interpretation of the results. The statistical parameters of the flexural capacity of bridge girders are obtained by Monte Carlo simulations. For an easier comparison, the load parameters are taken as the same as in original calibration of the AASHTO LRFD Code in early 1990's. The results indicate that the reliability indices are higher than in the original calibration and, therefore, the resistance factors can be increased by 5-10%.

Keywords: Prestressed Girders, Strength of Materials, Reliability Index Statistical Parameters,

INTRODUCTION

Prestressed concrete girders are commonly used in the highway bridge design, especially for long spans. The advantages of prestressed concrete include crack control and lower construction costs, reduction of the dead weight of the superstructure and increased span lengths. This type of construction is very cost-effective since the girders can be erected in one piece without work on site. Therefore, an accurate prediction of load carrying capacity of prestressed concrete girders is a growing need.

New generation of load and resistance factor design (LRFD) codes requires the knowledge of safety reserve involved in loads and resistance. In particular, it is important to evaluate the difference between the nominal (design) capacity and mean capacity (Nowak et al. 1994).

This paper documents the up-dated statistical parameters for prestressed concrete girders and the resulting reliability indices. Load and resistance factors depend on the statistical parameters of the design parameters: load components and their combinations, material strength, dimensions, and modelling uncertainties The present study is based on a new large and representative number of data of compressive strength of concrete (Nowak et al 2011) and prestressing strands.

The main objective of this study is to determine the statistical parameters for prestressed concrete girders, based on new material properties, and the resulting reliability indices. The analysis is performed for AASHTO type girders and NU girders developed in Nebraska.

STATISTICAL PARAMETERS OF MATERIAL

The cumulative distribution functions (CDF) of the ultimate strength of prestressing strands are plotted on the normal probability paper, where the vertical axis is the inverse normal probability, and it is equal to the distance from the mean value in terms of standard deviations. The construction and use of the normal probability paper can be found in textbooks on probability e.g. Nowak and Collins (2000). The probability paper allows for an easy evaluation of the statistical parameters as well as type of the distribution function. If the CDF curve is close to a straight line, then the variable can be considered as a normal random variable. For normal variable, the mean value and a standard deviation can be read directly from the CDF. The mean value is the horizontal coordinate of intersection with the CDF and the slope of the CDF is the inverse of standard deviation. Bias factor is defined as the ratio of the mean to nominal value.

The prestressing strands data include about 50,000 samples of strands with 0.5 in and 0.6 in diameter. The data was submitted by five sources and number of samples is presented in Table 1.

Table 1 Number of samples submitted by different manufacturers

Source \ Strands Diameter	0.5	0.6
Source 1	3908	700
Source 2	1158	785
Source 3	268	212
Source 4	9795	3442
Source 5	18258	8895
Total	**33387**	**14034**
Grand total	**47421**	

The obtained test results are processed to determine cumulative distribution functions (CDF). In addition, the statistical parameters such as the mean value, bias factor and the coefficient of variation are determined by fitting a normal distribution to the lower tail of the CDF.

Figures 1 and 2 present the CDF's and the approximating straight lines for the ultimate strength of prestressing strand. From the reliability analysis point of view, for material strength, the most important part of the CDF is the lower tail. Therefore, the statistical parameters are determined by fitting a straight line to the lower tail of the. The statistical parameters determined by fitting the lower tail with straight lines are summarized in Table 2.

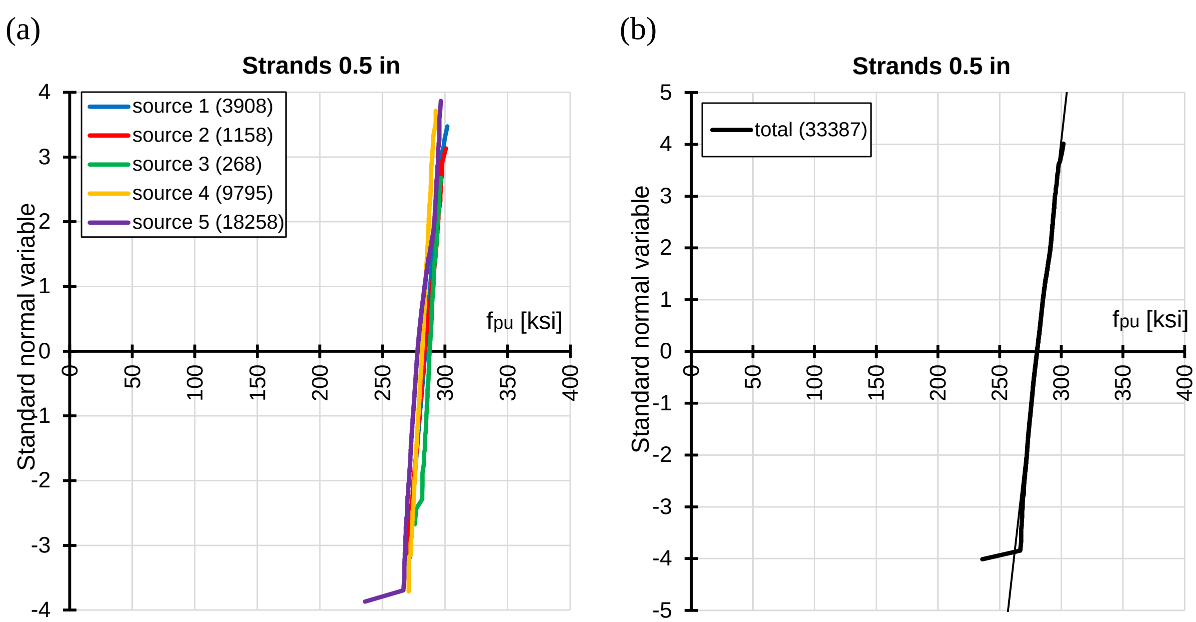

Fig. 1 CDF of Tensile Strength of Prestressing Strands, 0.5 in diameter, (a) separately for each source, (b) all data plotted together with a straight line representing the best fit to the lower tail.

(a) (b)

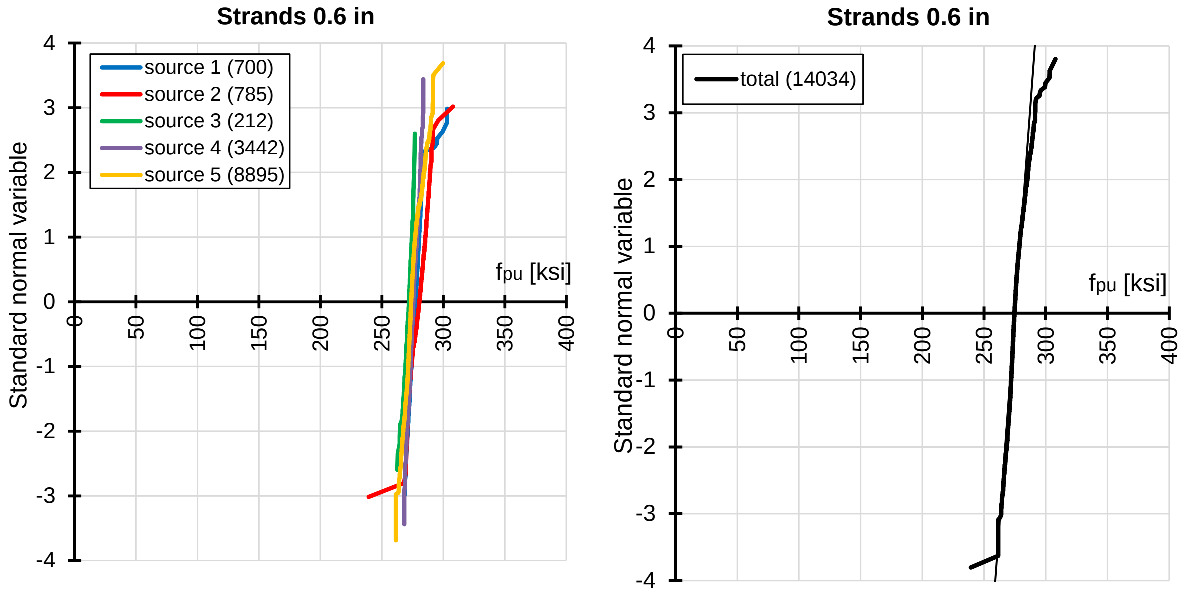

Fig. 2 CDF of Tensile Strength of Prestressing Strands, 0.6 in diameter, (a) separately for each source, (b) all data plotted together with a straight line representing the best fit to the lower tail.

Table 2 Summary of the Statistical Parameters for Prestressing Strands

Strands diameter	**Number of samples**	**Bias factor, λ**	**Coefficient of variation, V**
0.5 in	33387	1.04	0.017
0.6 in	14028	1.02	0.015

The new statistical parameters for concrete strength were presented by Nowak et al in 2011. The data of 28 day compressive strength of concrete for standard cylinders, 6 in x 12 in (150 mm x 300 mm) included ordinary concrete with nominal strength of 3000-6500 psi (21-45 MPa) and high strength concrete with nominal strength of 7000-12,000 psi (49-84 MPa).
The cumulative distribution functions (CDF) of compressive strength, fc', are plotted on the normal probability paper in Figure 3. The CDF's include all the available samples, from different sources. The recommended statistical parameters (the bias and coefficients of variation) of compressive strength fc' are listed in Table 3 for compressive strength and shear strength. The bias factor, λ, is calculated as the ratio of mean and nominal values and it is also shown in the table (Nowak et al., 2011).

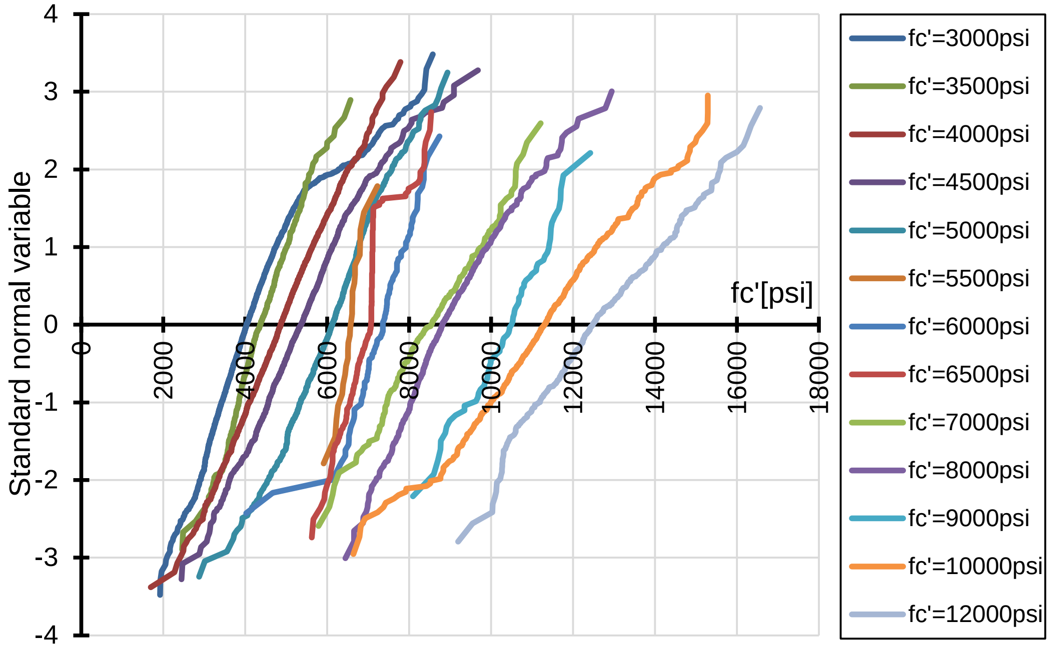

Fig. 3 CDF of Compressive Strength of Concrete, (Nowak et al. 2011)

Table 3 Recommended Statistical Parameters for Compressive Strength of Concrete fc'

f_c'	Compressive strength		Shear strength	
	Bias factor λ	Coefficient of variation, V	Bias factor λ	Coefficient of variation, V
4000 psi (27.6 MPa)	1.24	0.150	1.24	0.180
5000 psi (34.5 MPa)	1.19	0.135	1.19	0.160
6000 psi (41.4 MPa)	1.15	0.125	1.15	0.150
7000 psi (48.3 MPa)	1.13	0.115	1.13	0.140
8000 psi (55.2 MPa)	1.11	0.110	1.11	0.135
9000 psi (62.0 MPa)	1.10	0.110	1.10	0.135
10000 psi (69.0 MPa)	1.09	0.110	1.09	0.135
12000 psi (82.7 MPa)	1.08	0.110	1.08	0.135

The reinforcing steel data for f_y = 60 ksi (420 MPa) and a wide range of bar sizes from #3 through #14 was analysed by Nowak et al in 2011. The CDF of yield strength, f_y, are plotted on the normal probability paper in Figure 4. The bias factors for reinforcing steel bars vary from λ = 1.12 to λ = 1.14 for all diameters except No.3 rebars for which λ = 1.18. Therefore, the recommended bias factor for fy is λ = 1.13. The coefficient of variation of fy varies from

V = 0.02 to V = 0.04. The recommended coefficient of variation of fy is V = 0.03 (Nowak et al., 2011).

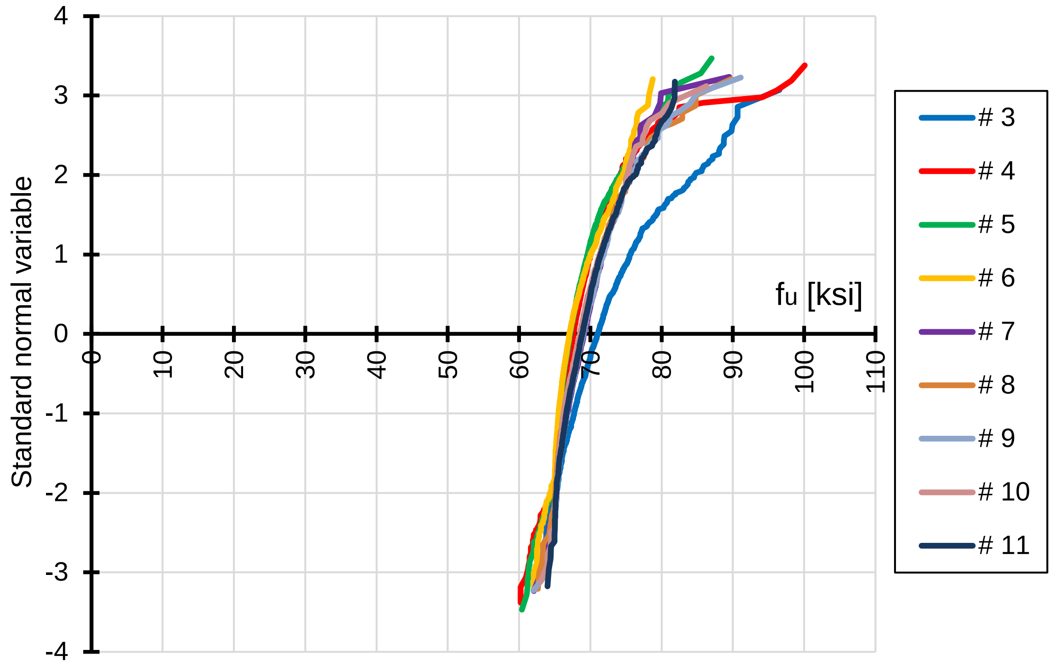

Fig. 4 CDF of Yield Strength of Rebars, (Nowak et al. 2011)

LOAD AND RESISTANCE MODEL

Resistance of a structural component, R, is a function of material properties and dimensions. R is a random variable due to various categories of uncertainties. It is convenient to consider R as a product of three factors: material factor (M), fabrication factor (F) and professional factor (P):

$$R = R_n \cdot M \cdot F \cdot P \tag{1}$$

The statistical parameters for material improved over the years. Therefore, the new material test data, described in previous section, served as a basis for determining material factor, M.

Fabrication factor, F, represents the variation in dimensions and geometry. The recommended statistical parameters are based on previous studies by Ellingwood et al. (1980). For the dimensions of concrete components the recommended parameters are: λ_F = 0.92 and V_F = 0.12 for effective depth of cast-in-place slab, λ_F = 1.0 and V_F = 0.025 for effective depth prestressed concrete girder and λ_F = 1.01 and V_F = 0.04 for effective width of cast-in-place beam. For steel components, reinforcing bars and stirrups, the bias factor of dimensions is λ_F = 1.0 and V_F = 0.01. The area of reinforcing steel, As, is treated as a practically deterministic value, with λ_F = 1.0 and V_F = 0.015; similar, area of prestressing strands has a λ_F = 1.0 and V_F = 0.01.

Professional (analysis) factor, P, represents the variation in the ratio of the actual resistance and what can be analytically predicted using accurate material strength and dimension values. Most of the statistical parameters of P are based on the previous study by Ellingwood et al. (1980). Professional factor for prestressing beam in flexure are λ_P = 1.01 and V_P = 0.06 and for shear are λ_P = 1.075 and V_P = 0.10.

In this study, the cumulative distribution functions are generated for the considered structural types using Monte Carlo technique. The cumulative distribution function (CDF) of resistance was obtained by generating one million values of R for each considered design case. This served as a basis to calculate the mean resistance, μ_R, standard deviation, σ_R, the bias factor λ_R and coefficient of variation V_R. The simulations were performed for NU girders and AASHTO girders with different span length and spacing between beams. It was found that all type of girders have a similar parameters of resistance.

The resulting bias factors and coefficients of variation are shown in Table 4, for flexure and in Table 5 for shear.

Table 4 Statistical Parameters of Resistance for P/C Beams, Flexure

Beam Type	**Span, ft**	**Strands size**	**Girder Spacing 8 ft**		**Girder Spacing 10 ft**	
			λ_R	**V_R**	**λ_R**	**V_R**
NU 2000	200	0.6	1.049	0.069	1.051	0.069
	190	0.6	1.048	0.068	1.052	0.069
	180	0.6	1.048	0.068	1.050	0.068
NU 1800	180	0.6	1.049	0.069	1.051	0.069
	170	0.6	1.048	0.069	1.051	0.069
	160	0.6	1.048	0.069	1.051	0.069
NU 1600	160	0.6	1.049	0.069	1.052	0.069
	150	0.6	1.049	0.069	1.051	0.069
	140	0.6	1.048	0.069	1.052	0.069
NU 1350	140	0.5	1.066	0.070	1.069	0.071
	130	0.5	1.064	0.069	1.068	0.070
	120	0.5	1.065	0.070	1.068	0.070
AASHTO IV	120	0.5	1.066	0.070	1.068	0.069
	110	0.5	1.066	0.070	1.068	0.069
	100	0.5	1.067	0.070	1.069	0.070
AASHTO III	100	0.5	1.066	0.071	1.068	0.070
	90	0.5	1.067	0.071	1.070	0.071
	80	0.5	1.068	0.071	1.072	0.071

Table 5 Statistical Parameters of Resistance for P/C Beams, Shear

Beam Type	**Span, ft**	**Girder Spacing 8 ft**		**Girder Spacing 10 ft**	
		λ_R	V_R	λ_R	V_R
NU 2000	200	1.203	0.114	1.201	0.114
	190	1.202	0.114	1.201	0.114
	180	1.202	0.114	1.200	0.114
NU 1800	180	1.203	0.114	1.200	0.114
	170	1.202	0.114	1.200	0.114
	160	1.194	0.113	1.200	0.114
NU 1600	160	1.203	0.114	1.201	0.114
	150	1.203	0.114	1.200	0.114
	140	1.202	0.114	1.200	0.114
NU 1350	140	1.205	0.114	1.202	0.114
	130	1.204	0.114	1.202	0.114
	120	1.203	0.114	1.201	0.114
AASHTO IV	120	1.198	0.113	1.196	0.113
	110	1.198	0.113	1.194	0.113
	100	1.196	0.113	1.192	0.113
AASHTO III	100	1.201	0.114	1.198	0.113
	90	1.200	0.114	1.198	0.113
	80	1.198	0.113	1.196	0.113

The considered load components include dead load and live load. It is assumed that the available database for loads is sufficient for the purpose of this study. The bias factor (ratio of mean-to-nominal) value of dead load is $\lambda = 1.05$, and coefficient of variation $V = 0.10$ for cast-in-place concrete, and $\lambda = 1.03$, and coefficient of variation $V = 0.08$ for precast concrete. For wearing surface $\lambda = 1.00$, and coefficient of variation $V = 0.25$. The assumed statistical parameters for dead load are based on the data available in literature (Ellingwood et al.; Nowak).

According to NCHRP Report 368, the statistical parameters of live load depend on the span length. For bending moment bias varies from 1.31 for 80 ft span to 1.23 for 200 ft and for shear from 1.27 for 80 ft span to 1.17 for 200 ft. The coefficient of variation is assumed to be 0.18.

The statistical parameters of the total load depend on the dead load and live load ratio and this depend on the span length. For longer bridges dead load governs and for short bridges the live load is crucial. Therefore, the reliability analysis is performed for a full range of span length varying from 80 to 200 ft.

Total design load including load factor is listed below and include live load, LL, dead load, DL, and DW wearing surface.

$$Q_{design} = 1.75LL + 1.25DL + 1.5DW \quad (2)$$

RELIABILITY INDEX

Structural analysis is performed to determine the reliability index, β (Nowak and Collins 2000). The limit state function is

$$g = R - (D + L) \quad (3)$$

Reliability index is calculated using following formula:

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \quad (4)$$

where μ_R = mean resistance, μ_Q = mean load effect, σ_R = standard deviation of resistance, and σ_Q = standard deviation of load effect.

The analysis was performed for AASHTO girders and NU girders for span length between 80 ft and 200 ft with spacing between girders 8 ft and 10 ft. The results of the reliability analysis are plotted vs. span length in Fig. 5 through 8. Figure 5 and 6 present flexure reliability indices for different span length with girder spacing 10 ft and 8 ft respectively. Whereas, Figure 7 and 8 present reliability indices for shear for different span length with girder spacing 10 ft and 8 ft respectively.

The resulting β's vary, depending on limit state, and span length. In all the considered cases, the reliability indices are low for the shorter spans were the live load governs. With increasing span length the bias factor also increase.

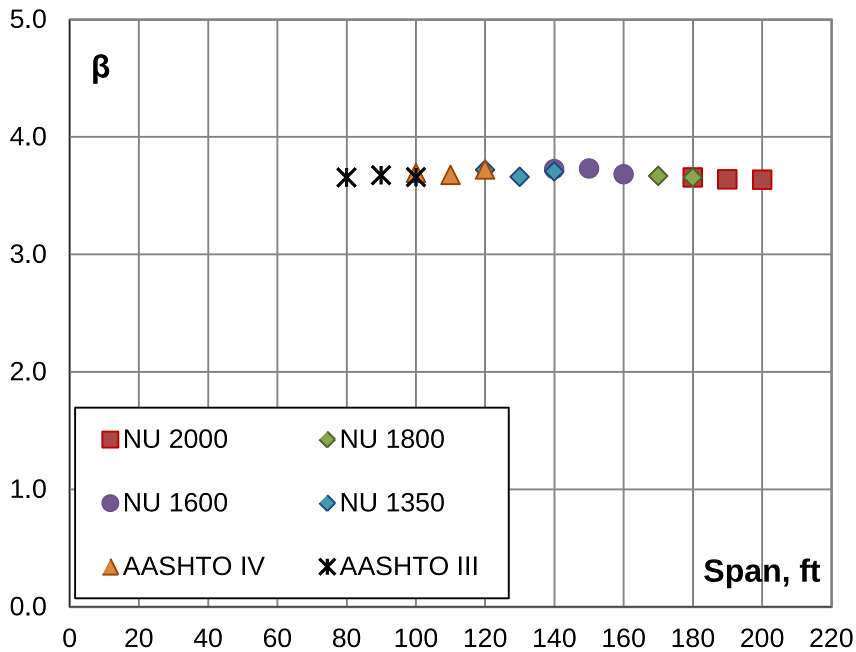

Fig. 5 – Reliability Index for Moment vs. Span, ϕ = 1.0 and Girder Spacing = 10 ft.

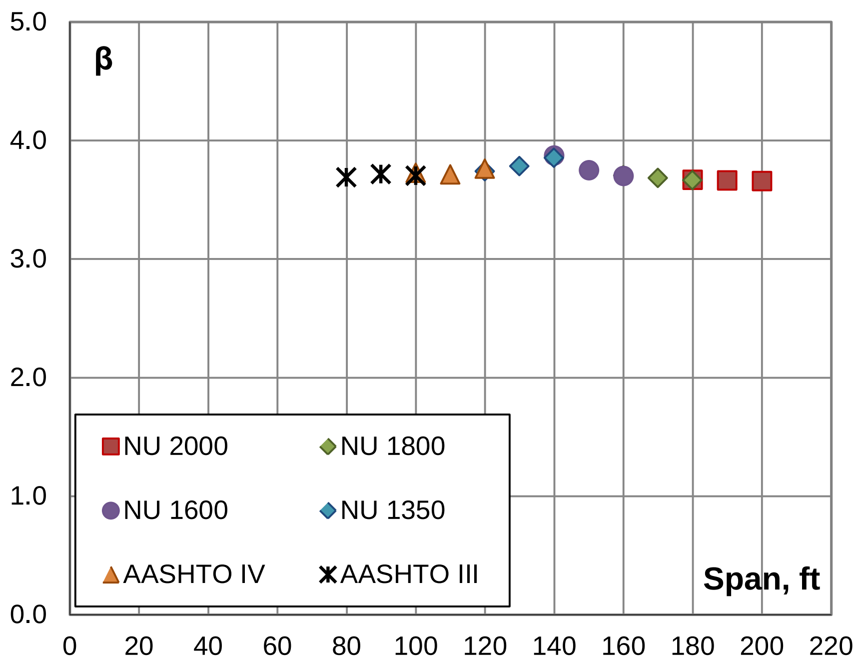

Fig. 6 – Reliability Index for Moment vs. Span, ϕ = 1.0 and Girder Spacing = 8 ft.

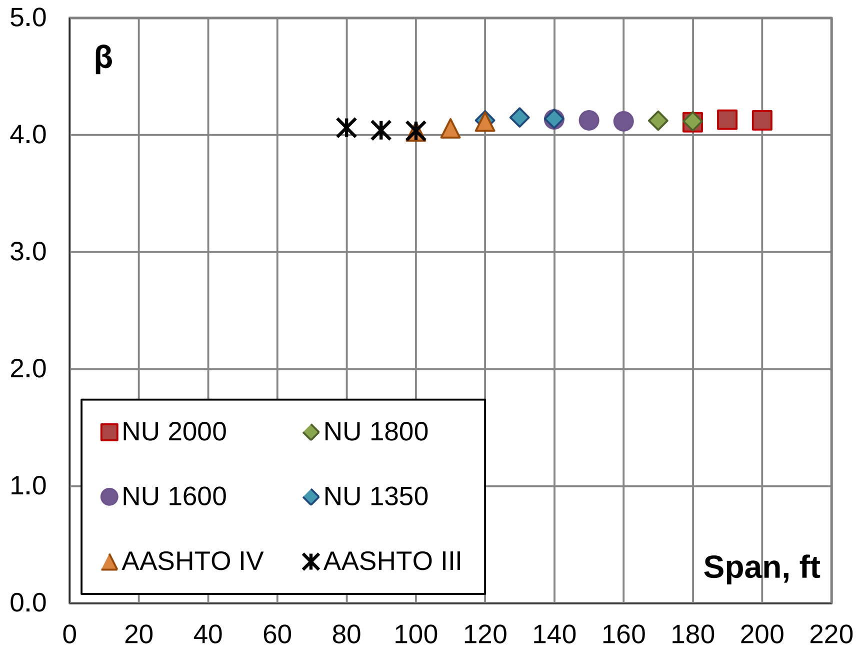

Fig. 7 – Reliability Index for Shear vs. Span, ϕ = 0.9 and Girder Spacing = 10 ft.

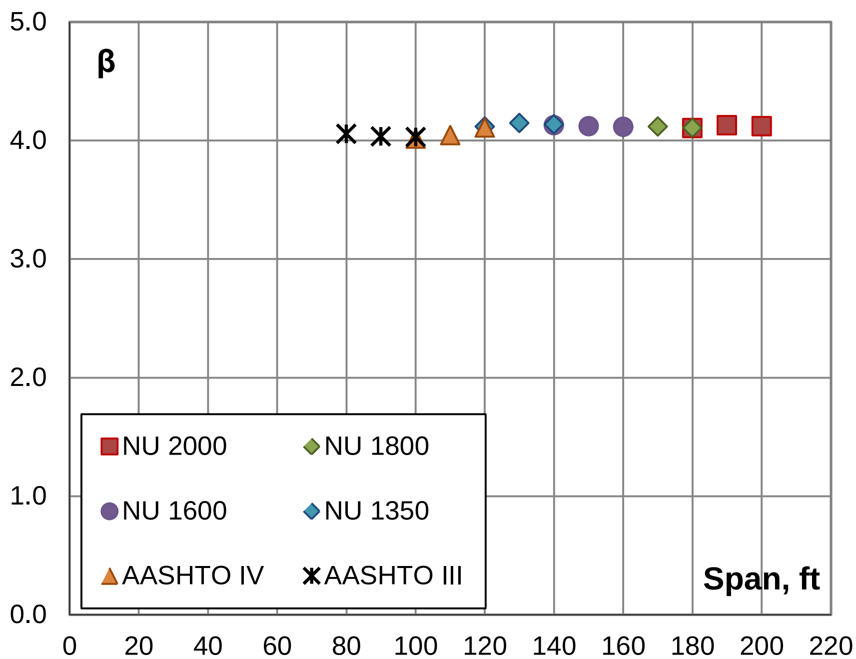

Fig. 8 –Reliability Index for Shear at Different Span, ϕ = 0.9 and Girder Spacing = 8 ft.

CONCLUSIONS

The statistical parameters of resistance were derived for prestressing strands based on new material test data provided by industry. The tests results were plotted on the normal probability paper and the mean values and standard deviations were determined by consideration of the lower tail of the CDF. The results indicate that the bias factors have increased and coefficients of variation have decreased over the last 30 years.
The changes in material properties affect the load carrying capacity of structural components. The statistical parameters of resistance were determined for AASHTO girders and NU girders based on the formula in AASHTO LRFD. The flexural capacity of prestressing beams is mostly affected by prestressing strands and the bias varies from 1.05 for strand 0.6 in to 1.07 for strands 0.5 in. It was found that the geometry of the beam does not have effect on the parameters of beam resistance
The statistical parameters of shear capacity depend on the shear reinforcement ratio and strength of concrete. In this study only one type of concrete f_c' = 10000 psi was considered. Therefore, the bias factor and COV are almost this same for all cases (type of beams and span length).
This paper presents also the reliability analysis for prestressing girders. It was found that the resistance factor for shear equal 0.9 gives reliability index close to 4.0 and resistance factor for flexure equal 1.0 gives reliability index between 3.7 and 3.9.

ACKNOWLEDGMENT

The research reported in this paper was conducted with the sponsorship of the Portland Cement Association (PCA Project Index No. 99-10) and the Precast, Prestressed Concrete Institute. The contents of this paper reflect the views of the authors, who are responsible for the facts and accuracy of the data presented. The contents do not necessarily reflect the views of the Portland Cement Association or the Precast, Prestressed Concrete Institute. The material test data were compiled by: National Ready Mix Concrete Association (NRMCA) staff and Consultant Richard D. Gaynor; Michael F. Pistilli, Prairie Group; Steve H. Gebler, Construction Technology Laboratories, Inc.; David P. Gustafson, Concrete Reinforcing Steel Institute (CRSI); and Roger J. Becker, Spancrete Industries, Inc. Special thanks are due to Maria M. Szerszen, Ewa Szeliga, Aleksander Szwed, and Piotr J. Podhorecki.

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