MAXIMUM SKEW ANGLE OF INTEGRAL ABUTMENT BRIDGES

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ABSTRACT
The use of integral abutment (IA) bridges has been limited by many states to those bridges with a maximum skew angle to 30°. This limitation reduces the potential benefits of utilizing integral abutments for a significant amount of short to medium span bridges with a skew angle larger than 30°. This is particularly true for bridges to be built in urban areas. In some states, where there is no limit on the maximum skew angle in IA bridges, concrete cracking of wingwalls and bridge misalignments have been reported as a maintenance issue because of improper details to accommodate or constrain transverse thermal movements at the ends of the bridges. A rational approach is needed to quantify the maximum skew angle allowed for a specific IA bridge in order to help bridge engineers decide whether integral abutments should be used for a high-skew bridge under certain design conditions. Such an approach needs be simple and effective yet applicable to the general practice of IA bridges. Through an analytical investigation of the mechanism of plane rotation of a skewed integral bridge, this research studies the limit stability condition under thermal movements. This limit stability condition considers many factors including the passive soil pressure, thermal deformation, pile constraints (if applicable), and soil conditions. Based on this limit condition, the effects of several design parameters are discussed including bridge length, allowable lateral movement, and length of equivalent pile cantilever. A simplified design equation is proposed to determine the maximum allowable skew angle based on certain input design parameters for a typical pile-supported IA bridge. The equation has been calibrated by several design variables identified in this paper. To illustrate the use of this proposed equation, a design example is provided.

Keywords: Integral Abutment Bridges, Skew Angle, Passive Soil Pressure, Equivalent Pile Cantilever, Thermal Effects

INTRODUCTION

Integral Abutment (IA) bridges have received a wide acceptance and use in the U.S. over the past twenty years. Benefits include reduction of initial and future maintenance costs, and improvement of bridge durability. Because of soil resistance under thermal expansion, integral abutment bridges are often limited to short to medium span structures with small skew angles. In 1996, a survey was conducted by the New York State Department of Transportation to investigate the validity of different design assumptions of IA Bridges[1]. In the survey, twenty nine states responded with the maximum skew angles used in design. Fig. 1 shows a histogram of the number of states and the maximum skew angle limits. From the figure, skew angle constraints vary from 0^o to no limit (referred to as 90^o in the figure), but most states limit skew of an integral structure to 30^o or less.

The limitation of skew angles restrains the application of integral abutments to those bridges with skew angles larger than 30^o which often exist in urban areas. Regarding the angle limit, some questions are often asked by bridge engineers, i.e., whether a limit is needed and how the maximum angle should be selected. Another question is whether the angle limit is a constant or a variable which depends on certain bridge conditions. Unfortunately, most states set the limit as a constant angle by local experience instead of scientific proof. With the lack of clarity in choosing the maximum skew angle, some problems may occur. During a survey conducted by Greiman[2], the Idaho Department of Transportation reported that in a skewed three-span steel girder bridge with integral abutment, rotation forces from lateral earth pressure on the end wall caused failure in the pier anchor bolts on the exterior girder.

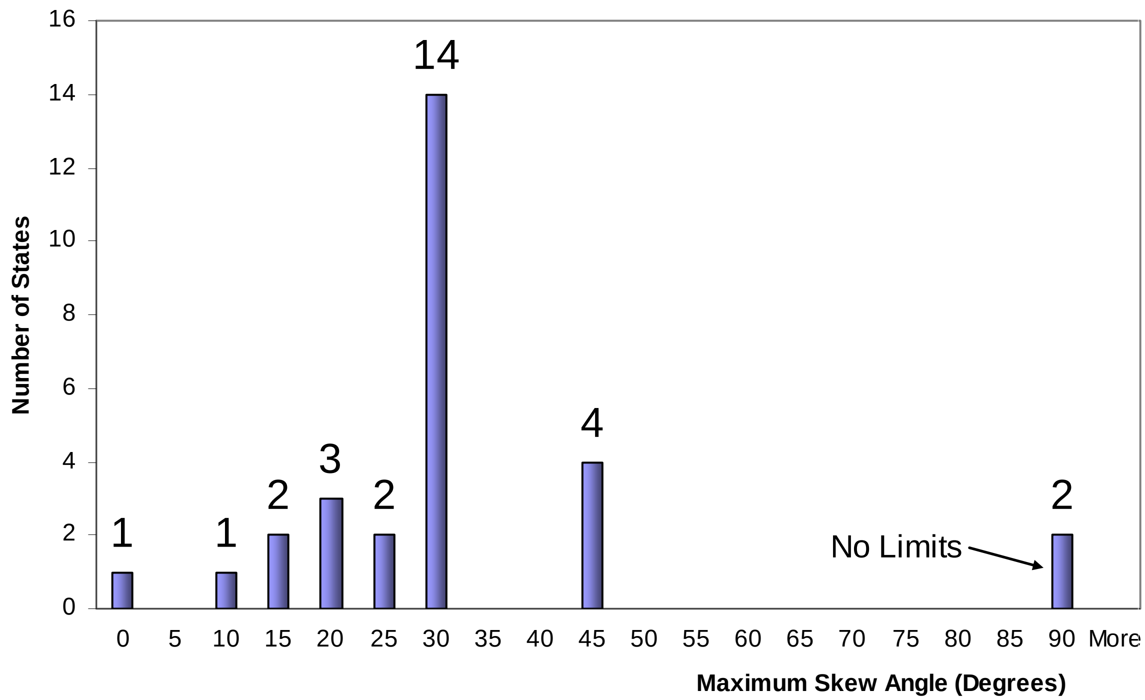

Fig. 1 Histogram of Maximum Skew Angle of IA Bridges in 29 States
(Based on data published by Kunin[1])

The main reason to limit the skew angle is to avoid the rotation of skewed IA bridges caused by external forces, i.e., passive soil pressure behind abutments. The passive soil pressure could be caused due to the expansion of bridge girders during the summer months and they may not distribute uniformly behind abutments. Burke[3] investigated the rotation of semi-integral abutment bridges during a temperature rise. Because a semi-integral abutment bridge is supported directly on pile caps without constraints of lateral movements, the bridge became unstable when rotation occurred. Assuming the friction angle between backfill soil and abutment concrete was 22^o, for a safety factor of 1.5, the proposed maximum skew angle was 15^o in order to keep the bridge stable. For general IA bridges supported on pile foundations, the maximum skew angle was not discussed in the research.

Oesterle and Lotfi[4] conducted analytical and experimental studies of the transverse movements and restraint forces of skewed IA bridges. The authors reported that with increasing the skew angle, the transverse force required to resist transverse moment could be a sizable portion of the passive soil pressure and it might not be feasible to design sharply skewed abutment for relatively short bridges and for bridges in locations with small effective annual temperature ranges. This statement is based on the use of plumb piles under the abutments and the authors didn't further investigate other possible techniques (such as battered piles, etc,) used to resist the transverse loads and the impact of the use of those techniques on the applications of skewed IA bridges.

The purpose of this research is to investigate the skew angle limit for general IA bridges, especially for those supported on piles. During the preliminary investigation, many design parameters were identified to have an impact on the rotation of the skewed bridge in the slab plane including soil properties, wingwall types, pile stiffness, etc. To simplify the study, the restraint of the bridge rotation at piers as well as at wingwalls was neglected. The bridge is further assumed to have the same skew angle at the two abutments. The backfill behind abutment was assumed to be loose sand with an internal friction angle of 30^o. The results of this research could be easily extended to other conditions not conforming to this assumption.

ROTATION OF SKEWED IA BRIDGES

The rotation of skewed IA bridges refers to the rotation of the bridge slab by assuming the slab is a rigid body. The slab rotation is primarily driven by the thermal changes in the bridge superstructure. The amount of rotation is affected by many factors, such as temperature range, coefficient of thermal expansion, backfill material properties, bridge length and width, etc. In the following discussion, two cases were considered. Case I presents the situation where backfill soil pressure was ideally removed. In engineering practice, such a situation exists when a gap was set between the abutment and backfill. Case II is the normal case with backfill soil pressure applied to back faces of abutments.

CASE I: WITHOUT BACKFILL SOIL PRESSURE

For a symmetric skewed IA bridge with both abutments built in the same soil conditions, the thermal movements of the bridge under uniform temperature changes was illustrated as **Fig. 2**. Each pile head moves along the direction passing through the center of the bridge, Point "O", which has zero movement. Because the movements of each pair of piles (*Ai* and *Bi*, i = 1, 6) are along the same line passing through the bridge center, Point "O", the reaction of external thermal forces applied to the bridge slab will have a zero force arm relative to the center of the bridge. This results in zero force couples to drive the bridge to rotate.

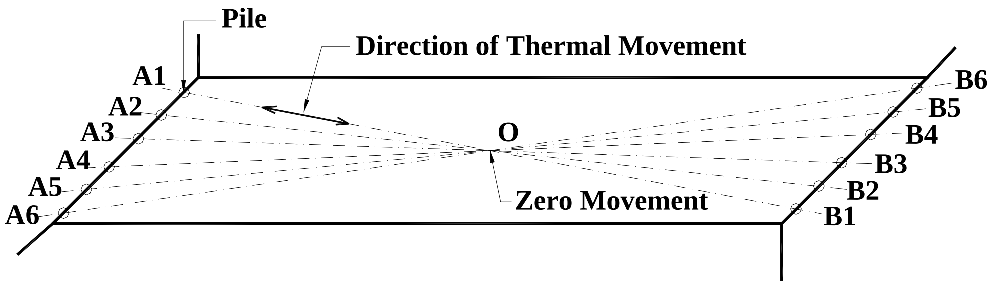

Fig. 2 Thermal Movement of Skewed Bridge without Backfill Soil Pressure

For such an ideal case, the skew angle causes no rotation of the structure and IA bridges could be designed for any skews. It should be reminded however that removing the backfill soil pressure through setting up a gap between the backface of the abutment and the backfill may increase both initial and maintenance costs and some benefits of IA bridges may be harmed such as improved seismic behavior.

CASE II: WITH BACKFILL SOIL PRESSURE

With backfill existing behind the abutments, thermal movements of the bridge superstructure result in passive soil pressure for temperature rise, acting perpendicular to the abutment. When the bridge is skewed at the abutments, a pair of passive soil forces from each abutment form a force couple which causes the slab to rotate in the horizontal plane. Fig. 3a illustrates the external forces applied to the slab of the IA bridge during uniform temperature rise. An anti-clockwise force couple, M_p is caused by the passive backfill soil pressure, expressed as

$$M_p = P_p \cdot L_p = P_p L \cdot \sin\theta \tag{1}$$

where θ is the skew angle of the bridge, P_p is the total passive backfill soil force acting against the abutment and L is the total bridge length.

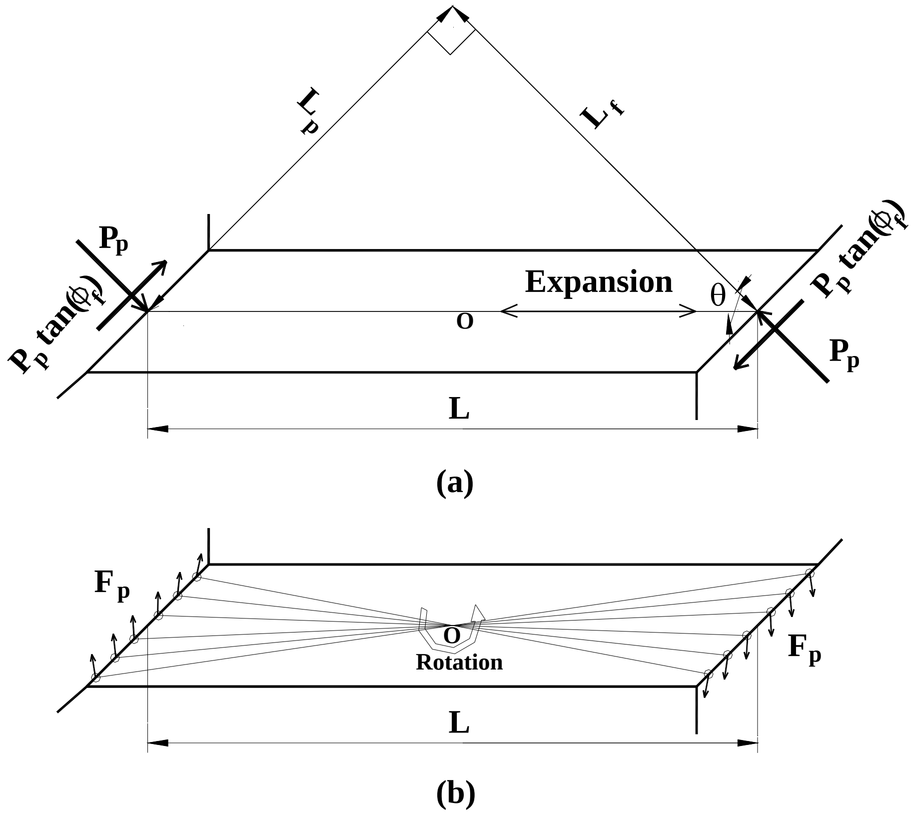

Fig. 3 Thermal Movement of Skewed Bridge without Backfill Soil Pressure

The static friction force between the abutment and the backfill forms a force couple to resist the horizontal rotation of the slab, which is expressed as

$$M_f = P_p \tan(\phi_f) \cdot L_f = P_p L \cdot \tan(\phi_f) \cos\theta \quad (2)$$

where ϕ_f is the friction angle,

To prevent horizontal rotation of the superstructure during temperature rise, the following criteria must be satisfied:

$$M_p \le M_f \quad (3)$$

Substituting equations (1) and (2) into Equation (3), the skew angle is evaluated by

$$\tan\theta \le \tan(\phi_f) \text{ or } \theta \le \phi_f \quad (4)$$

Based on the research of Potyondy[5], the friction angle, ϕ_f, between the back face of the abutment and the backfill may be conservatively evaluated as $0.76\,\phi_b$, where ϕ_b is the internal friction angle of backfill. When ϕ_b was equal to 30^o (for loose sand), the maximum skew angle was computed as 23^o to prevent the bridge superstructure from rotation.

For semi-integral abutment bridges (soil bearing foundations), Equation (4) becomes a stability equation because no pile will constrain the rotation of the structure. However, for general IA bridges supported on piles, Equation (4) is only the determination condition for bridge rotation. With the increase of the rotation angle, pile constraints become larger; therefore, the structure is still stable. The limit state for the maximum skew angle becomes the service limit state (i.e., allowable lateral movement) instead of stability.

When Equation (4) is not satisfied, the rotation of the bridge superstructure causes the reaction lateral pile forces, F_p, as illustrated in Fig. 3b. The force couple due to the pile forces can be approximated as

$$M_r = \sum_{i=1}^{n} F_{pi} r_i \approx n\overline{F}_p \cdot L \equiv R_L L \tag{5}$$

where n is the number of piles under the single abutment, $\overline{F}_p$ is the average lateral force at the pile heads and R_L is the total lateral pile resistance which is defined as $n\overline{F}_p$.

The maximum skew angle is determined by the bridge rotation condition:

$$M_p \le M'_f + M_r \tag{6}$$

where M'_f is the force couple due to the movable friction force which was assumed the same as the force couple, M_f due to the static friction force.

Substituting equations (1) (2) and (5) into (6), Equation (6) is simplified as

$$\sin(\theta) - \tan(\delta)\cos(\theta) \le \frac{R_L}{P_p} \equiv \beta \tag{7}$$

where β is defined as the ratio between the total lateral soil resistance and the passive backfill soil pressure.

Equation (7) indicates that the skew angle, β, could be increased through increasing the ratio β. The maximum skew angle increases with the increase of the lateral resistance force, R_L, and decreases with the increase of the passive backfill soil force, P_p. When P_p is equal to zero, Equation (7) is always satisfied for all the skew angles. This verifies the results shown in Case I (without the backfill soil pressure). When R_L is equal to zero, Equation (7) turns to Equation (4), which corresponds to the stability equation of the semi-integral abutment bridge.

To determine the maximum skew angle using Equation (7), the ratio, β , needs to be determined first, which relies on different structure configurations and soil conditions. In the following discussion, design parameters of an examined IA bridge will be used to determine the maximum skew angle.

MAXIMUM SKEW ANGLE FOR AN EXAMINED BRIDGE

The examined bridge is a straight IA bridge, Br. 55555 in Rochester, Minnesota. The bridge is 40 ft wide and 216 ft long with an abutment depth of 11 ft. Six HP 12x53 piles under a single abutment were oriented in the weak axis bending in the longitudinal direction. The effect of wingwall on the rotation of the bridge was conservatively neglected.

BACKFILL SOIL FORCE

Backfill soil pressure was evaluated using an interaction curve of passive soil pressure developed by Duncan and Mokwa[6], which is expressed as

$$P_b = \frac{\Delta}{\frac{1}{k_{\max}} + R_f \frac{\Delta}{P_{ult}}} \tag{8}$$

where P_b is the backfill soil pressure at any deflection Δ, P_{ult} is the ultimate soil pressure, k_{max} is the initial stiffness, and R_f is the failure ratio which is defined as the ratio between the actual failure pressure and the hyperbolic ultimate pressure.

In Equation (8), $k_{\max}$ is a constant obtained from an elastic analysis of a plate in soil; the ultimate soil pressure F_{ult} is computed using log spiral numerical analysis. Using a spread sheet PYCAP developed by Duncan and Mokwa[6], $k_{\max}$ and F_{ult} were computed as 1420 kips/in and 2051 kips, respectively. The failure ratio, R_f, is always smaller than unity, and varies from 0.75 to 0.95 for most soils, in the following discussion, R_f is assumed to be 0.85. The soil-structure interaction curve for the examined bridge is shown in Fig. 4.

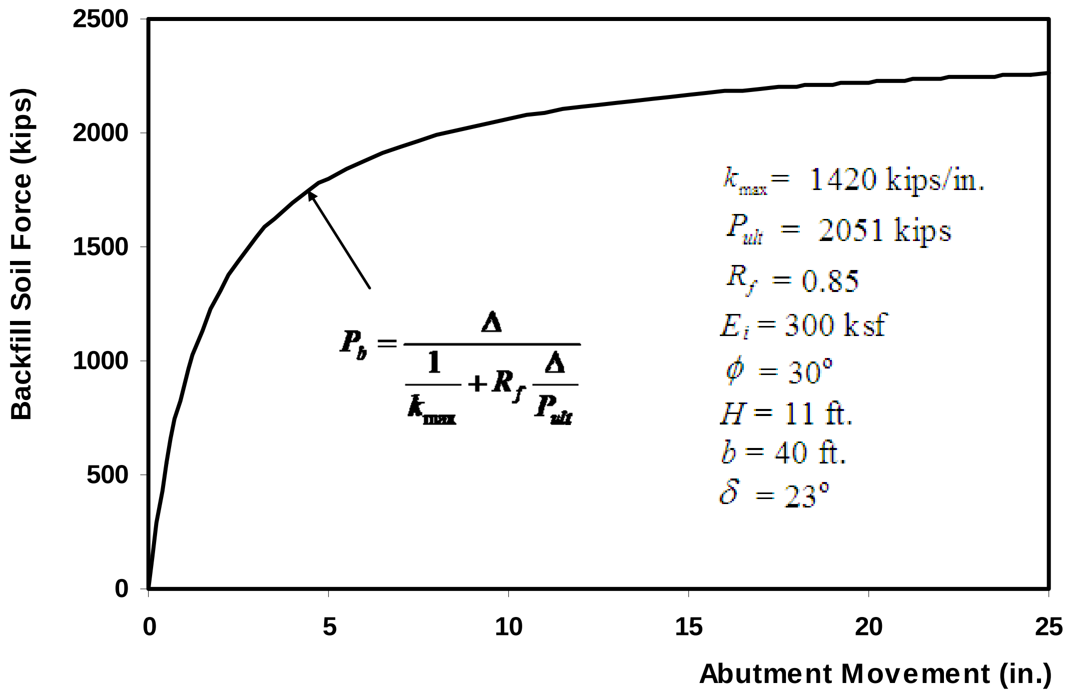

Fig. 4 Thermal Movement of Skewed Bridge without Backfill Soil Pressure

The movement of abutment toward the backfill was computed as

$$\Delta = \alpha \Delta TL \cdot \cos(\theta) \leq \alpha \Delta TL \quad (9)$$

where α is the coefficient of thermal expansion, 6 $\mu\varepsilon/^{o}F$, and ΔT is the temperature rise which is equal to 35^{o}F for cold area specified in the AASHTO[7].

In Equation (9), $\cos(\theta)$ corresponds to the effect of skew angle; however, involvement of θ in equations (8) and (9) will cause iterative solving for the maximum skew angle using Equation (7). To simplify the computation, the movement, Δ, was conservatively computed using $\alpha \Delta TL$, which was 0.272 feet for the examined bridge. Using Equation (8), the backfill soil force was calculated to be 333 kips.

In Equation (7), R_L affects the value of the maximum skew angle; however, its value depends on different substructure configurations. In the following discussion, three types of details were studied: using vertical piles, battered piles and shear keys.

DETAIL I: USING VERTICAL PILES

For the examined bridge, six HP 12x53 piles were located under each abutment. The average lateral resistance of piles is evaluated by

$$\overline{F}_p = \frac{\Delta_{allow} 6EI_X}{L^3_{ep}} \tag{10}$$

where Δ_{allow} is the allowable lateral movement, E is the modulus of elasticity of piles, I_x is the lateral moment of inertia of piles and L_{ep} is the length of equivalent pile cantilever which could be calculated using the method recommended by Greimann[8].

Equation (10) assumes the pile head is fixed for rotation and free for displacement and the piles are fixed at a distance, L_{ep}, below the pile head neglecting the lateral soil pressure. The length of the equivalent pile cantilever for the examined bridge was calculated to be approximately 160 inches. The allowable lateral movement Δ_{allow} was selected as ½" for this study, regardless of the length of the bridge. From equations (5) and (10), the lateral pile resistance, R_L was computed as 50 kips. By solving Equation (7), the maximum skew angle was obtained as 31^o. It should be remembered that this skew angle is larger than 23^o, which is obtained from Equation (4) due to the effect of pile lateral resistance. The resistance provides approximately 8.3 kips per pile, which is much smaller than the lateral pile capacity, 18 kips, for HP piles specified by Mn/DOT Design Manual[9].

DETAIL II: USING BATTERED PILES

The main purpose in using battered piles is to increase the lateral pile resistance thereby further increasing the maximum skew angle. Considering the lateral capacity of the battered piles, the total lateral resistance could be computed as

$$R_L = n\overline{F}_p + \frac{n_b \sin(\gamma) EA_P}{L_p} \tag{11}$$

where n_b is the number of battered piles, γ is the pile inclined angle, A_p is the pile cross-section area, and L_p is the distance from the pile head to the vertical bearing point which could be conservatively assumed to be the total pile length, 80 ft., for the examined bridge.

Using Equation (11), the lateral soil resistance was computed to be 198 kips when two battered piles were designed with a batter ratio of 1:3. Further, the maximum skew angle was computed as 56^o by solving Equation (7).

DETAIL III: USING SHEAR KEYS

Another option to increase the lateral resistance is to add shear keys behind abutments as shown in Fig. 5. For the examined bridge, two 2.5 ft wide shear keys were designed to be located behind the abutment. The depth of the shear keys was the same as the abutment.

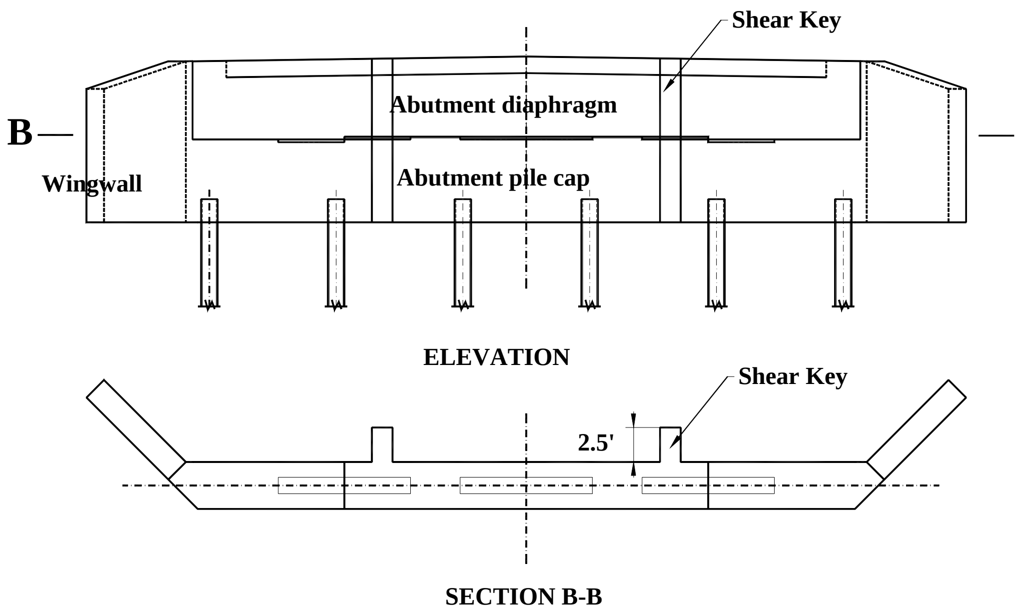

Fig. 5 Details of Shear Keys behind Abutment

The total lateral resistance of the shear keys is calculated to be:

$$R_L = n\overline{F}_p + \frac{n_s \Delta_{allow}}{\frac{1}{k'_{\max}} + R_f \frac{\Delta_{allow}}{P'_{ult}}} \tag{12}$$

where n_s is the number of shear keys, $k'_{\max}$ and P'_{ult} (similar as $k_{\max}$ and P_{ult}) are the parameters based on the width of the shear keys.

Using the spread sheet PYCAP, $k'_{\max}$ and P'_{ult} were calculated as 414 kips/in. and 205 kips, respectively. From Equation (12), R_L was computed as 273 kips when two shear keys were designed. By solving Equation (7), the maximum skew angle was computed as 72^{o}.

The results using the three details above predicted different maximum skew angles. Through battering the external piles or adding the shear keys, the lateral resistance can be greatly increased, resulting in larger maximum skew angles.

EFFECT OF DIFFERENT DESIGN PARAMETERS ON MAXIMUM SKEW ANGLE

In the discussion above, some parameters were fixed to those of the examined bridge, i.e., the bridge length, the allowable lateral movement and the length of the equivalent pile cantilever. In fact, those parameters vary with different bridge structures. Their effects on the maximum skew angle are discussed as follows.

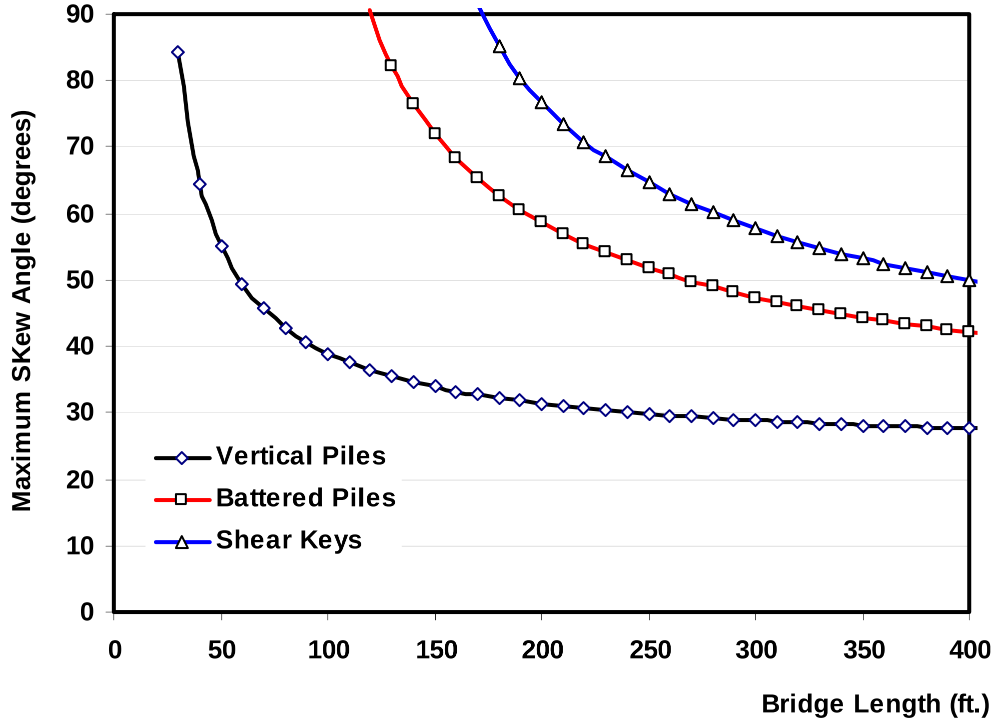

Fig. 6 Effect of Bridge Length on Maximum Skew Angle

EFFECT OF BRIDGE LENGTH

Assuming all the other variables are the same as those of the examined bridge, the relationship between the maximum skew angle and the bridge length is shown in Fig. 6. With the increase of the bridge length, the maximum skew angle was reduced quickly for bridges that were less than approximately 200 ft. long. With only the vertical piles under the abutments, the maximum skew angles were similar for bridges with span lengths larger than 200 ft. For bridges with spans less than 50 ft., the skew angle can be designed for at least 50^o. When a bridge is designed for battered piles or shear keys behind abutments, there is no limit for the skew angle if the span length is less than 125 ft.

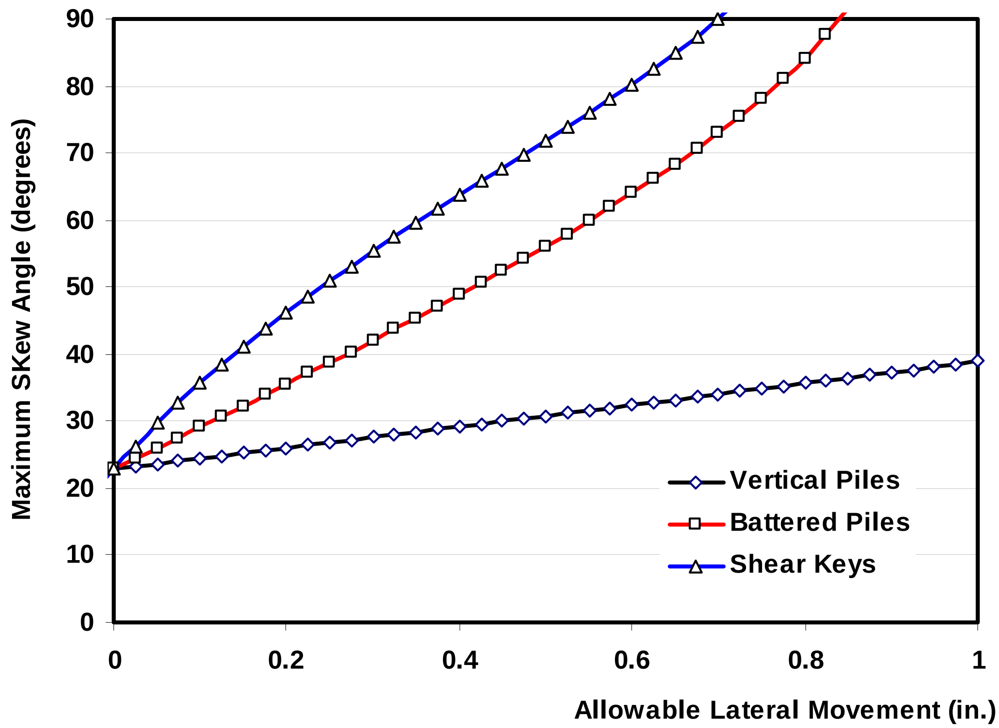

Fig. 7 Effect of Allowable Lateral Movement

EFFECT OF ALLOWABLE LATERAL MOVEMENT

Fig. 7 shows the effect of the allowable lateral movement on the maximum skew angle. The allowable lateral movement has a large impact on the maximum skew angle. When the allowable lateral movement is larger, the maximum skew angle increases correspondingly with a tendency close to a linear relationship. When the allowable lateral movement is zero, the maximum skew angle is 23^o, which is the critical angle to stop the bridge from rotation. When the allowable lateral movement is larger than 0.5 inches, the maximum skew angle can be at least 45^o if battered piles or shear keys are employed.

EFFECT OF LENGTH OF EQUIVALENT PILE CANTILEVER

Fig. 8 shows the maximum skew angles for different lengths of equivalent pile cantilever. With the increase of length of equivalent pile cantilever, the maximum skew angle becomes smaller. Because the increase of equivalent pile cantilever indicates softening of the soil, the lateral soil resistance becomes smaller, thereby resulting in a smaller maximum skew angle. For most soil conditions, battered piles and shear keys are effective measures to keep the maximum skew angle larger than 45^o.

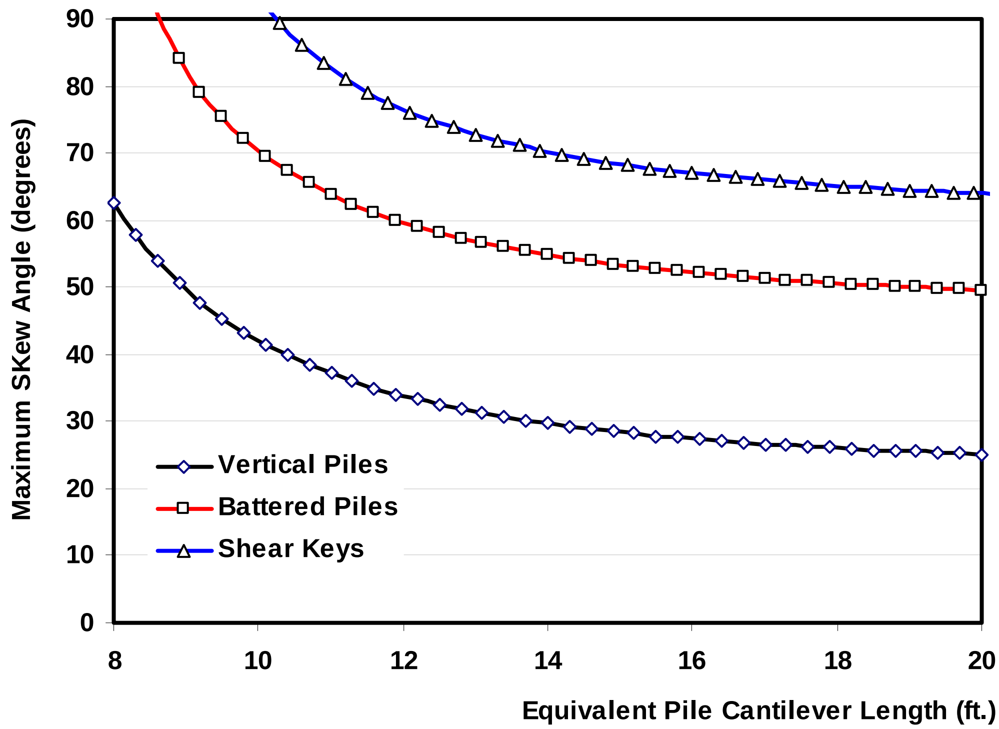

Fig. 8 Effect of Equivalent Pile Cantilever Length

DESIGN RECOMMEMDATION

Based on the investigation of maximum skew angle for IA bridges, the following simplified design equation is recommended:

$$\theta_{\max} = 22.3 + \frac{7000\Delta_{allow}}{L\Delta T}\left(\frac{1.5nI_x}{L^3_{ep}} + \beta_{batter} + \beta_{key}\right) \tag{13}$$

where β_{batter} and β_{key} are computed as

$$\beta_{batter} = \frac{n_b \sin(\gamma) E A_p}{720 L_p}; \beta_{key} = \frac{n_s b_s H}{7} \tag{14}$$

Equations (13) and (14) were calibrated for several design variables. The calibrations for bridge length, allowable lateral movement and length of equivalent pile cantilever were given in figures (9), (10) and (11), respectively. The results obtained from the design equation reflected well and provided a lower bound for all numerical data.

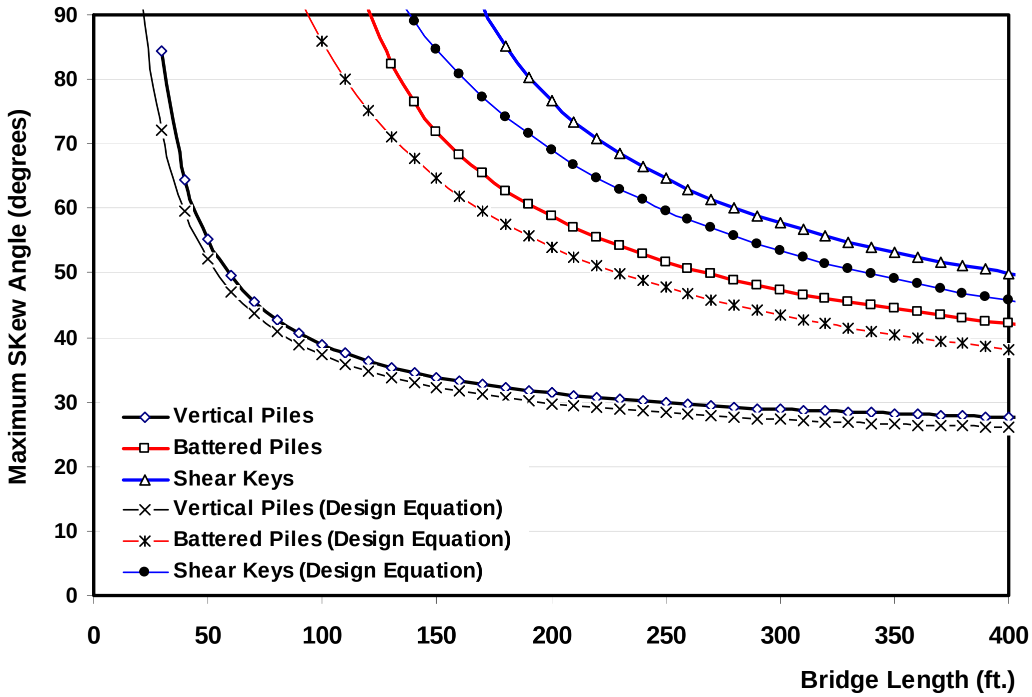

Fig. 9 Calibration Design Equation for Different Bridge Length

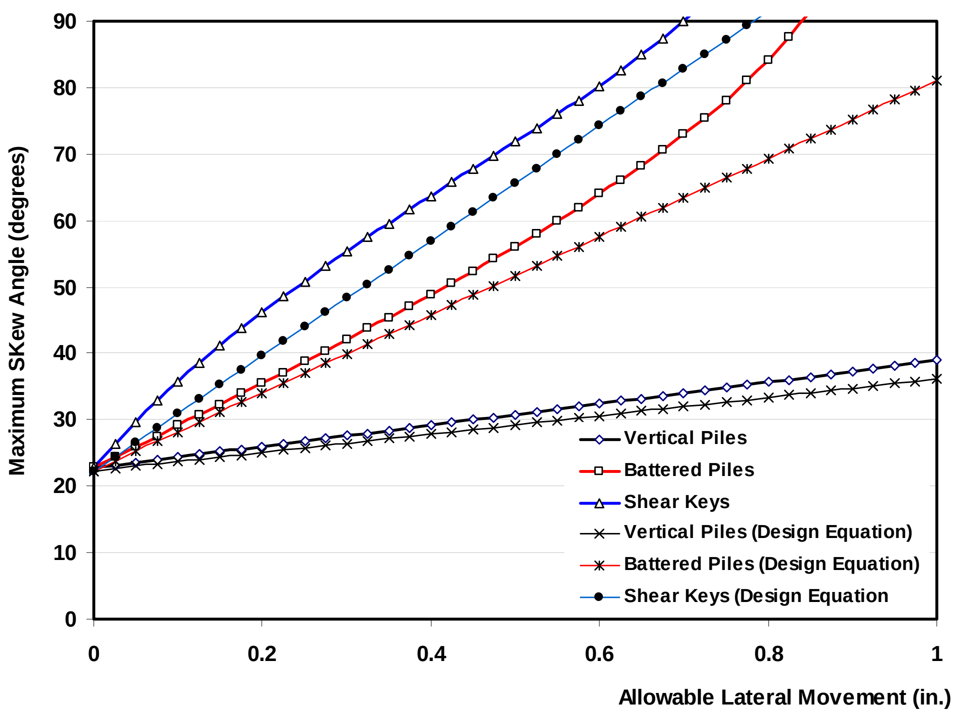

Fig. 10 Calibration Design Equation for Different Allowable Movement

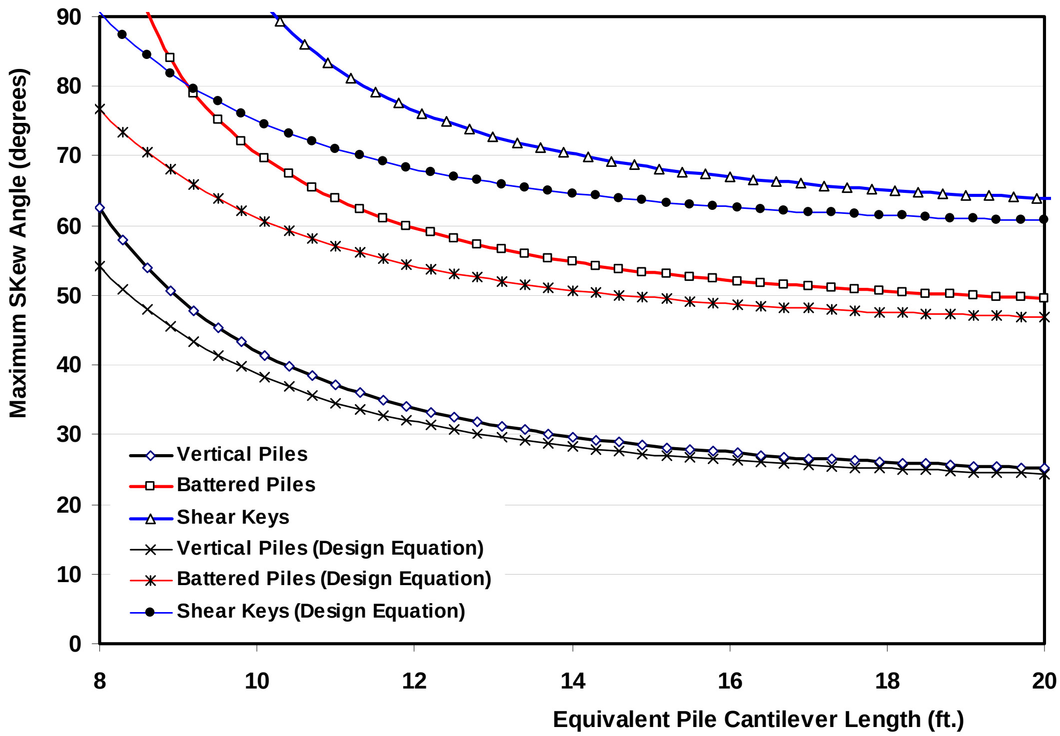

Fig. 11 Calibration Design Equation for Different Equivalent Pile Cantilever Lengths

DESIGN EXAMPLE

To illustrate the use of the design equation, the following design example is presented.
Bridge Length: L = 200 ft.; $\Delta T = 35^{o}$F.
Piles: 8 HP 10x42; I_x = 210 in^4; A_p = 12.4 in^2; Equivalent Pile Cantilever Length, $L_{ep} = 12$ ft.
Allowable lateral movement: Δ_{allow} = 0.25 in.
Detail I (vertical piles):
$\beta_{batter} = \beta_{key} = 0$
Using Equation (13), $\theta_{\max} = 25.9^{o}$
Detail II (battered piles):
Batter the external two piles. $\beta_{key} = 0$.
From Equation (14), β_{batter} = 4.212.
Using Equation (13), $\theta_{\max} = 36.5^{o}$
Detail III (shear keys):
Design for 3 shear keys behind abutment: n_s = 2; b_s = 2.5 ft.; H = 12 ft.
$\beta_{batter} = 0$.
From Equation (14), β_{key} = 12.857.
Using Equation (13), $\theta_{\max} = 68.6^{o}$

CONCLUSIONS

The following conclusions were drawn based on the research of this paper:

1) Through an investigation of the rotation of the skewed IA bridges caused by passive backfill soil pressure, a general equation to determine the maximum skew angle (Equation 7) was derived. The maximum skew angle depends not only on the magnitude of lateral soil pressure, but also on the lateral resistance to bridge rotation.

2) Both battering external piles under abutments and adding shear keys behind abutments are effective measures to increase the lateral resistance to bridge rotation, and further to increase the maximum skew angle.

3) The maximum skew angle decreases with the increase of bridge length and length of equivalent pile cantilever and increases with the increase of allowable lateral movement. A reasonable allowable lateral movement can be specified according to the level of importance of the structure and a more detailed analysis of pile flexural demands.

4) The maximum skew angle can be calculated for new skewed IA bridges through a simplified design equation. The equation forms a lower bound to the numerical data generated based on an examined bridge. A design example was provided to illustrate the use of the design equation.

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