BRIDGE MULTI-COLUMN PIERS RATED BY THE STRUT AND TIE METHOD

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ABSTRACT

The recently rehabilitated Countyline Bridge carrying SR 119, over the Jacobs Creek in Fayette/Westmoreland Counties, Pennsylvania represented a unique challenge for rating severely deteriorated multi-column piers. The strut and tie method was used to obtain acceptable ratings so the bridge did not have to be closed. Substructure units were rehabilitated under traffic to extend the bridge service life several years until the bridge can be replaced. The Pennsylvania Department of Transportation is adopting a method for rating substructures as a result of this project.

The Countyline Bridge is 1122 feet long, concrete spread box beam structure consisting of 20 spans and nineteen multi-column piers with leaky expansion joints over every pier. Throughout the years, the piers have developed severe deterioration to the concrete and reinforcing steel to the point that the bridge requires replacement. Replacement of the bridge is scheduled for several years from now to allow adequate time for completing the environmental process, studying candidate alignments and bridge types, and final design. Analyzing the bridge piers with PennDOT's LRFD pier program yielded unacceptable results for keeping the heavily used bridge in service. Unique strut and tie models were subsequently developed, that eventually resulted in acceptable rating factors to keep the bridge open to traffic. The piers were then rehabilitated under traffic by concrete pier cap encasement to extend the service life of the bridge until the new bridge is constructed. PennDOT is changing their philosophy on substructure ratings to extend the service life of structures that otherwise may require closure.

Keywords: Load Rating, Bridges, Strut and Tie, Rehabilitation, Inspection

INTRODUCTION

The S.R. 0119 Countyline Bridge over Jacobs Creek, Southwest Pennsylvania Railroad and an abandoned railroad is located in East Huntington Township, Westmoreland County, Pennsylvania and Upper Tyrone Township, Fayette County, Pennsylvania. Constructed in 1963, the prestressed concrete spread box beam structure is composed of a series of twenty (20) simple spans ranging from 34 ft-10 1/8 in to 63 ft-0 in for a total length of 1122 ft and supported by multicolumn pier bents founded on deep drilled shaft foundations. The multiple simple span configuration of the bridge resulted in transverse expansion joints over every pier. Over time, the failure of these joints has led to leakage of water and deicing chemicals directly onto the pier caps and their eventual deterioration. The general condition of the piers is shown in Figures 1 and 2.



Fig. 1 General Condition of Piers



Fig. 2 Condition of Pier Caps (Pier 7 shown between columns 1 and 2)

Due to the flat grade on the structure metal curb drains were placed along the barriers. Water contaminated with deicing chemicals, draining from the structure was subsequently blown against the exterior columns causing the deterioration of the outer faces of the end columns as shown in Figure 3.



Fig. 3 Typical Column Condition

INSPECTION

PennDOT officials contacted Michael Baker Jr., Inc. in February 2007 after being alerted of the deteriorating condition of the Countyline Bridge piers. Baker immediately sent inspection crews to the site to assess the situation. A visual inspection of all nineteen piers was performed on February 16, 2007 to determine the most severely deteriorated piers for analysis. Piers 2, 7, 9, 13 and 16 were identified as the five most severely deteriorated piers. The deterioration is attributed to a combination of cracked, loose and spalled concrete in the pier caps and columns. Exposed and deteriorated reinforcing steel was noted at all of the piers. An in-depth inspection of the five piers began on the afternoon of February 16, 2007 and concluded on February 21, 2007. The extent of concrete spalling (locations and dimensions) and section loss of reinforcing steel (locations and approximate loss) were A subsequent inspection was conducted on March 26, 2007 to thoroughly detailed. determine the number of piers that needed emergency repairs to extend the service life of the bridge for several years, to when the bridge is replaced. This process was tedious but nonetheless would prove critical in determining the structure's capacity.

LOAD RATING

A "Load Rating Factor" is the measure of the capacity of a structure to carry live loads. A rating factor is expressed as a ratio of reserve capacity to live load demand, when this value exceeds unity the structure is deemed safe to support the prescribed loading. When a load rating factor is less than unity, the metric indicates that the structure has insufficient load carrying capacity. A structure's load rating factor is given by the following equation:

$$Rating = \frac{Capacity - \gamma_{DL}DL}{\gamma_{LL}LL}$$
(1)

Inventory ratings (IR) represent a level of the full design live load that can safely utilize the structure for an indefinite period of time, whereas operating ratings (OR) represent the maximum permissible live load to which the structure may be subjected such as occasional permit loads. PennDOT manages their bridge inventory by examining operating ratings; this is the metric by which a structure's load limit is determined and eventually, if necessary, posted.

A load rating analysis often times presents a somewhat different kind of problem to the engineer, a problem of determining where the reserve capacity actually is in a structure that otherwise would not "rate out". At times, successful load rating demands that the engineer examine several methods of analysis to evaluate the structure. The conservatism that structural engineers customarily build into their designs, sometimes can be stripped away to find the true reserve in the structure.

Most load rating analyses concentrate exclusively on the superstructure portion of the bridge; however, as was discovered by the examination of the Countyline Bridge, it is probable the load carrying capacity of a structure can be limited by its substructure. Substructures, often massive in size, heavily reinforced and overall shielded from environmental effects by the superstructure can withstand quite a bit of torture. However, severe deterioration as seen on the Countyline Bridge eventually takes it toll to where the structure's capacity may be in jeopardy.

ANALYSES

The load rating analysis for the Countyline Bridge was carried out using the AASHTO LRFD Bridge Design Specifications (LRFD), Second Edition, 1998¹ as supplemented by PennDOT Design Manual, Part 4, April 2000 Edition², the AASHTO Guide Specification for Condition Evaluation of Existing Bridges³ and PennDOT Publication 238⁴. Analysis of the piers began by evaluating the capacity of the structure to carry today's live loads in its current state with the deterioration measured during the inspection. The live loads evaluated consisted of PHL-93² (PennDOT's version of HL-93)¹, P-82 (PennDOT's Operating Level Permit vehicle)², H-20¹, HS-20¹, ML-80 (PennDOT's maximum legal loading)², and TK527 (PennDOT rating vehicle)^{1.4}. Load combinations Strength-I¹ and Strength-IA², were the primary focus of the investigation. Service level distress was already present and as such service load combinations were not examined. Little could be done to produce ratings greater than unity for PHL-93 and P-82 loadings; therefore, the focus shifted towards evaluating HS-20 and ML-80 as these loads would likely be more probable to occur before the structures eventual replacement.

As a first "level of analysis", Baker engineers utilized PennDOT's PAPier Program⁵ to rate the structures. The PAPier Program performs basic frame analysis of the pier using beam

theory and performs resistance calculations in accordance with AASHTO⁵. Initial evaluation revealed multiple problem areas in both the pier caps and columns. The pier caps all had adequate flexural capacity. This excess capacity facilitated the development of a secondary load path as will be discussed later. The pier caps were deficient in shear capacity due primarily to the level of deterioration while the columns in flexure due in large part to the presence of longitudinal loads. It was unclear whether or not these loads were considered in the original design of the columns. Moreover, little guidance was found as to whether longitudinal forces from braking or wind loading should be included for rating computations. This remains a subject of open debate and requires further research. For the pier caps, the ratings were refined by accounting for the beneficial contribution of the inclined compression block in a tapered member contributing to the shear resistance in the cap overhangs⁶. This increase or decrease for compression block sloped in the direction of the design shear is given by (2):

$$V_{u,adj} = V_u \left(1 - \frac{M_u}{V_u \cdot d} \cdot \tan \psi \right)$$
(2)

where M_u = absolute value of design moment.

 V_{μ} = absolute value of design shear.

- d = effective depth.
- ψ = sum of angles of compression face and of centroid of flexural reinforcement in tension relative to member axis. Angle ψ is taken as positive if magnitude of moment and depth of member increase or decrease in same sense, negative otherwise.

The contribution of torsion caused by unequal span loads on the ahead and back station bearings was accounted for in the PAPier analysis and examined. PAPier assumes an additional eccentricity of two inches for new designs and analysis of existing piers to account for construction tolerances. Since the actual eccentricity for the cap was determined from field measurements, the additional two inch eccentricity was removed were appropriate. Determined to minimally impact the loads on the pier cap overhangs, torsion was thereby neglected in the strut and tie modeling done later which greatly simplified the analysis.

This initial analysis using PAPier revealed that the pier cap shear and column biaxial bending operating rating factors for all piers were significantly below unity for the HS20 and ML80 vehicles. The cap flexure and column shear ratings were all greater than unity for the ML80 and HS20 vehicles. Because the first method did not yield acceptable ratings for all of the ratings, a second analysis was performed using the Load Factor Design (LFD) method by which the original structure was designed, AASHTO Standard Specifications for Highway Bridges, 15th Edition, 1992 and Interim Specifications 1993 and 1994 (LFD code)⁷ as supplemented by PennDOT Design Manual, Part 4, 1993 Edition⁸. In cases where load ratings are unsuccessful, PennDOT permits evaluation of the structure by the code the structure was originally designed^{2, 4}. The pier cap shear operating rating factors for Piers 2, 7 and 9 were still less than unity for HS20 and ML80 vehicles utilizing LFD method.

The column biaxial bending operating rating factors for the piers were all greater than unity for the HS20 and ML-80 vehicles. The LFD analysis for HS20 and ML-80 was used to determine the bi-axial bending/axial load Inventory and Operating ratings of the deteriorated columns. The ratings were based only upon gravity loads as traditionally performed for superstructures. Currently, longitudinal loads such as braking, thermal or wind are not considered for ratings. There is a need for further research to determine if longitudinal loads should be considered for rating of columns.

STRUT AND TIE MODELING

Strut and tie models can be an expedient and rational way of designing structural concrete members as has been demonstrated in the literature^{6, 9}. However, heretofore the authors are not aware of a study or practical application of the strut and tie method to load rating for a bridge. As the strut and tie method will typically result in lower ratings, it would not generally be employed. However, for the cases where an alternate load path can or will be established, the strut and tie method may produce more acceptable ratings than sectional methods. Therefore, the Countyline Bridge piers presented the authors with a unique opportunity to employ the strut and tie method to load rating. The primary load path through the vertical stirrups was insufficient to carry the applied loads due to reinforcing steel corrosion. Traditional sectional models indicated insufficient load carrying capacity; however, through the use of the strut and tie method it was possible to engage the flexural reinforcement in the pier cap and produce acceptable ratings.

A LRFD strut and tie model was developed for the pier cap cantilevers on Piers 2, 7, 9, 13 and 16 in accordance with AASHTO¹ and supplemented by the provisions of ACI¹⁰ and other research^{6,9}. Moreover, the overhang portion of the cap was the focus of the analysis as other locations on the cap either rated out satisfactorily using sectional methods or did not lend themselves to a benefit by analysis with the strut and tie method. Locations between columns would not benefit from examination by the strut and tie method because of complete loss of shear reinforcing steel and/or insufficient flexural reinforcing steel to present an alternate load path to redistribute forces. In locations between columns without stirrups, direct struts became too flat to employ and the strut and tie method resulted in either lower or similar load ratings to sectional methods as generally would be the case as previously discussed. Since the strut and tie method is a lower bound plasticity model, the predicted capacity in nearly all cases, is conservative¹¹. However, caution by the strut and tie method can over predict the collapse load in extreme circumstances¹¹.

A view of the deterioration of one of the pier cap overhangs is shown in Figure 4. The ends of the pier caps suffered a twofold penalty as the superstructure drainage contaminated with deicing chemicals leaked directly onto them through the failed expansion joints and also blew against them as it drained from the metal curb drains in the barriers. Many of the reinforcing bars suffered from significant section loss.



Fig. 4 Pier 7 Overhang Deterioration

The cap overhangs on the Countyline Bridge had excess flexural steel in the tops of the cap. This additional steel made for the development of a secondary load path for the overhang loads by way of a direct strut from the bearings reaction to the column or a combination of a direct strut and a truss. The presence of this secondary load path was the impetus for analyzing these substructures with the strut and tie method. Due to the loss of cover around the 180° hooks of the main tension reinforcing steel in the pier cap cantilevers, several reinforcing bars were discounted in the analysis because it is questionable if they were effective or could be developed beyond the compression node under the bearing plate.

ACI¹⁰ recommends strut angles no flatter than 25 degrees and AASHTO¹ penalizes the capacity of the struts severely in this range. Typically, the authors do not recommend strut angles below 30 degrees as this penalty generally becomes rather great. This is done to ensure strain compatibility of the model and in a design problem to prevent premature service level cracking in the member. The objective of a load rating is to determine the most actual usable capacity at the strength level, adherence to this limitation was attempted but in some cases, models with strut angles flatter than preferable were employed.

AASHTO¹ requires strict adherence to a minimum steel ratio of 0.003 to ensure that the member has sufficient ductility to reach its ultimate load. For illustration, Pier 2 cap overhang had a transverse reinforcement ratio of approximately 0.002 and a longitudinal reinforcement ratio of approximately 0.01. As AASHTO¹ considers these directions exclusive of one another this would generally preclude the use of strut and tie modeling. However, ACI¹⁰ permits a more liberal approach. Combining the reinforcing steel crossing a strut in both directions, which for a strut angle of 21.3° (see Figure 8, Model 4), results in a ratio of approximately 0.006 that, exceeds the ACI minimum of 0.003. Satisfaction of the ACI criteria was deemed sufficient evidence to employ the strut and tie method, because the reinforcing steel detailing had followed industry practice. This limit deserves further research for employing the strut and tie method to rating problems.

SKEWED END PIERS 1, 2, 18 AND 19

As previously discussed, the analysis of Pier 2 cap by sectional methods resulted in low ratings, as shown in Table 1.

Vehicle	Inventory Rating Factor	Operating Rating Factor	Inventory Rating (Tons)	Operating Rating (Tons)	
HS-20	0.61	0.79	21	28	
ML-80	0.54	0.70	20	26	

Table 1 Minimum Ratings for Pier 2 overhang using sectional method

The strut and tie models for Pier 2 overhang were formed using a combination of a two panel trusses and a simple two member systems consisting of a tension tie for the main flexural reinforcement and a shallow strut joining the column and the bearing location. Due to the long overhang, some beam action is anticipated in the cantilever (shear span to depth ratio of 2.1). Therefore, some force demand on the stirrups is present. The analysis was generally performed using graphic statics, which is well suited for the analysis of strut and tie models. Where possible the force diagrams are shown with the models employed below. For illustration purposes only the results of HS20 loading are shown.

Initially, the overhang was modeled with just a direct strut from the bearing reaction to the column results in unacceptable ratings (see Table 2) as shown in Figure 5: Pier 2 Model 1. Negative ratings indicate that the model is insufficient to carry even the dead load.

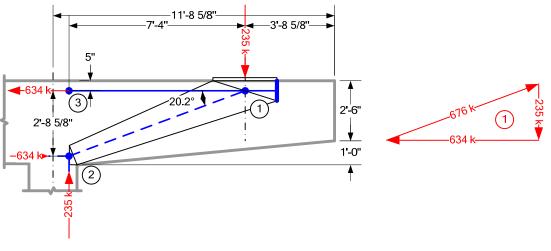


Fig. 5 Pier 2 Model 1

Member	Inventory Rating Factor	Operating Rating Factor	Inventory Rating (Tons)	Operating Rating (Tons)	
1-2 top	-0.43	-0.56	0.00	0.00	
1-2 bot	1-2 bot -0.59		0.00	0.00	
1-3	1-3 1.97		71.00	92.00	

Table 2 Pier 2 Model 1 S&T Ratings

Refinement of the model using Pier 2 Model 2 as shown in Figure 6 engages the vertical stirrups using a strut angle conforming to the recommended minimum. The loads on the members however were greater than they could withstand (See Table 3).

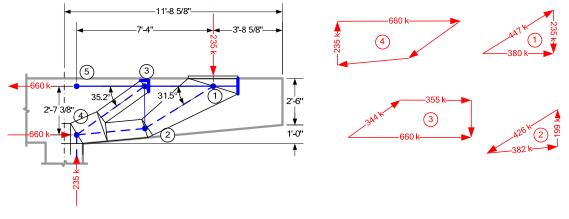


Fig. 6 Pier 2 Model 2

Member	Inventory Rating Factors	Operating Rating Factors	Inventory Rating (Tons)	Operating Rating (Tons)
1-2 top	2.85	3.69	102.00	132.00
1-2 bot	1.16	1.51	41.00	54.00
1-3	4.35	5.64	156.00	202.00
2-3	0.04	0.05	1.00	1.00
3-4	1.17	1.52	42.00	54.00
3-5	1.83	2.38	65.00	85.00
2-4	2.59	3.36	93.00	121.00

Table 3 Pier 2 Model 2 S&T Ratings

To alleviate this, the strut angle between the bearing and the top tie was decreased until the force in the stirrups is equal to the resistance of the vertical tie; see Figure 7 and Table 4: Pier 2 Model 3. This "transition" from models 2 to 3 demonstrates the development of the secondary load path.

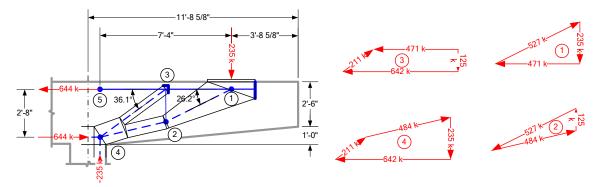


Fig. 7 Pier 2 Model 3

Table 4 Pier	2 Model 3	3 S&T Ratings
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Member	Inventory Rating Factors	Operating Rating Factors	Inventory Rating (Tons)	Operating Rating (Tons)
1-2 top	1.02	1.33	36.00	47.00
1-2 bot	1.09	1.42	39.00	51.00
1-3	3.20	4.15	115.00	149.00
2-3	1.00	1.30	36.00	46.00
3-4	1.52	1.98	54.00	71.00
3-5	1.93	2.50	69.00	90.00
2-4	1.33	1.73	47.00	62.00

Further refinement of the model is attempted by examining a fourth model; see Figure 8 and Table 5: Pier 2 Model 4. It was recognized that additional load carrying capacity could be attributed to the overhang by the superposition of the two models. Under service conditions, it is likely that both load paths are present and contribute to the overall load resistance. Superimposing the results of models 1 and 2 gives similar results as models 3 or 4. Model 2 only resists the maximum force that can be resisted by the vertical ties and the remaining force resisted by model 1. The shallow strut does little to improve the ratings since it takes a low load. This method of superimposing results, while not generally recommended by the authors, appears to be a valid approach for at least a first approximation.

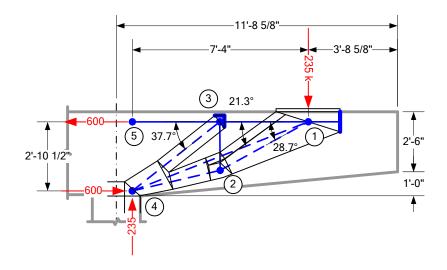


Fig. 8 Pier 2 Model 4

Member	Inventory Rating Factors	Operating Rating Factors	Inventory Rating (Tons)	Operating Rating (Tons)	
1-2 top	1.18	1.54	42.00	55.00	
1-2 bot	0.80	1.03	28.00	37.00	
1-3	3.71	4.80	133.00	172.00	
2-3	0.79	1.03	28.00	37.00	
3-4	2.45	3.17	88.00	114.00	
3-5	2.18	2.82	78.00	101.00	
2-4	0.98	1.27	35.00	45.00	
		0.4.0.4.0		4 4 4 9 9 9 9 9	
<mark>1-4</mark>	<mark>245.64</mark>	<mark>318.42</mark>	<mark>8842.00</mark>	<mark>11463.00</mark>	
<mark>1-4</mark>	245.64	318.42	IR	0R	
1-4 Member	245.64 IR	0R			
			IR	OR	
Member	IR	OR	IR (Tons)	OR (Tons)	
Member 1-2 top	IR 1.19	OR 1.54	IR (Tons) 42.00	OR (Tons) 55.00	
Member 1-2 top 1-2 bot	IR 1.19 0.80	OR 1.54 1.04	IR (Tons) 42.00 28.00	OR (Tons) 55.00 37.00	
Member 1-2 top 1-2 bot 1-3	IR 1.19 0.80 3.70	OR 1.54 1.04 4.80	IR (Tons) 42.00 28.00 133.00	OR (Tons) 55.00 37.00 172.00	
Member 1-2 top 1-2 bot 1-3 2-3	IR 1.19 0.80 3.70 0.80	OR 1.54 1.04 4.80 1.03	IR (Tons) 42.00 28.00 133.00 28.00	OR (Tons) 55.00 37.00 172.00 37.00	
Member 1-2 top 1-2 bot 1-3 2-3 3-4	IR 1.19 0.80 3.70 0.80 2.45	OR 1.54 1.04 4.80 1.03 3.18	IR (Tons) 42.00 28.00 133.00 28.00 88.00	OR (Tons) 55.00 37.00 172.00 37.00 114.00	

Table 5 Pier 2 Model 4 S&T Ratings

Note that "Inf" in Table 5 indicates infinity. Since there is no load in this member an infinte number is produce when calculating the ratings. The difference in the rating values between models 3 and 4 is therefore a result of the changes in the truss geometry as the addition of member 1-4 had little effect.

INTERMEDIATE PIERS 3 THRU 17

The overhangs were much shorter for the intermediate piers; therefore, a simpler two member strut and tie model was more appropriate as shown in Figure 9. For short members where loads are applied within or near a distance, d, from the support, beam action is not anticipated. The load path is that of a direct compressive strut connecting the bearing location to the column and therefore the stirrups are not fully engaged (shear span to depth ratio of 1.25). The stirrups are only necessary to provide a confining effect on the strut. The stirrups in the pier caps do not meet the minimum steel requirements of AASHTO 5.6.3.6; however, due to the nature of the problem at hand the service performance of the structure is not necessarily in question, only the ultimate capacity as discussed previously. No guidance for evaluating the structure at the service level using strut and tie models other than this requirement is given in the literature. Given the assumptions of the analysis, it is believed that the deteriorated stirrups in the overhang are sufficient to provide this confinement. This analysis approach resulted in improvements over beam theory and demonstrated a satisfactory rating for the pier caps at Piers 7, 9, 13 and 16 where advanced deterioration was Again due to the loss of cover around the 180° hooks of the main tension present. reinforcing steel in the pier cap cantilevers, two reinforcing bars were not considered in the analysis because it is questionable if they were effective.

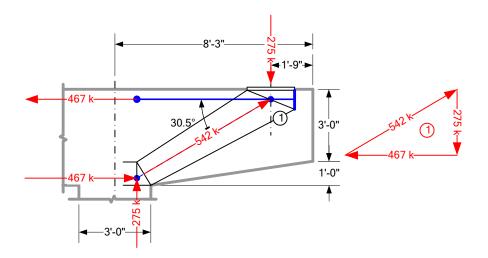


Fig. 9 Strut and Tie model for Pier 7 (HS-20 forces shown)

In the case of Pier 7, enough shear reinforcement was gone that a traditional beam analysis indicated that the overhang would not be sufficient for its own dead loads (OR < 0). The fact the cap remained in service further suggests that an additional load path must be present. After accounting for the effect of the sloping cap bottom, the beam theory results improved to give operating ratings as shown in Table 6.

Vehicle	Vehicle Weight (Tons)	Inventory Rating Factors	Operating Rating Factors	Inventory Rating (Tons)	Operating Rating (Tons)
HS-20	36	0.23	0.30	8	10
ML-80	37.74	0.21	0.27	7	10

Table 6 Minimum ratings for Pier 7 by Sectional methods

The strut and tie model shown in Figure 9 resulted in an operating ratings shown in Table 7 controlled by the capacity of the node under the bearing reaction.

Vehicle	Vehicle Weight (Tons)	Inventory Rating Factors	Operating Rating Factors	Inventory Rating (Tons)	Operating Rating (Tons)
HS-20	36	0.859	1.113	30	40
ML-80	37.74	0.814	1.055	30	39

 Table 7 Minimum ratings for Pier 7 by Strut and Tie Method

REHABILITATION

Inasmuch as the methods employed to analyze the piers of the Countyline Bridge as described in this paper indicated acceptable ratings for some of the critical components of the bridge substructure, other areas remained deficient. Not withstanding the analysis, it was therefore prudent to ensure the safety of the traveling public; therefore, it was ultimately decided to rehabilitate the substructure units in jeopardy. The complete replacement of the bridge is planned in the near future and as such the rehabilitation aimed at extending the service life by approximately ten years time. This was done using a pier cap encasement for the entire pier cap as shown in Figure 10. The encasement was sized using traditional sectional methods to space the vertical stirrups and shear friction to space the dowels.

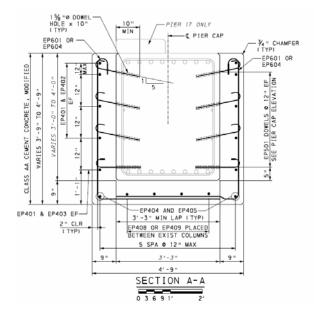


Fig. 10 Typical Pier Cap Retrofit

The completed pier cap rehabilitation is shown in Figure 11 in service. Some reflection cracking shows in the retrofit; but nonetheless, the service performance and ultimate safety of the bridge is no longer in question for the bridge substructure.



Fig. 11 Rehabilitated Pier Cap in Service

PROPOSED REVISIONS TO RATING POLICY

Investigation of the Countyline Bridge piers brought to the attention of Baker and PennDOT engineers the need for load rating of substructures under certain circumstances. Heretofore, it was the general practice of engineers and agencies to concentrate their efforts of load rating on the bridge superstructure with the assumption of great reserve in the substructure. As has been shown in by the case of the Countyline Bridge this is not always the case. PennDOT is proposing revisions to their Bridge Safety Inspection Manual, Publication 238⁴ because the current AASHTO Load Rating Guidelines^{3, 12} do not provide guidance for load rating

substructures. The revisions to Publication 238 will include possible ways of realistically determining the remaining capacities of existing structurally deficient piers by various methods including strut and tie modeling.

CONCLUSIONS

Strut and Tie modeling is another type of analysis method that may be considered by the rating engineer when examining structural concrete for both super and substructures. Strut and Tie modeling will only result in higher rating factors when an alternate load path exists and certain criteria are met as discussed in this paper.

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REFERENCES

- 1. AASHTO Design Division, LRFD Bridge Design Specifications, Second Edition, 1998.
- 2. PennDOT Publication 15M, Design Manual Part 4-Structures, April 2000 Edition.
- 3. AASHTO Manual for Condition Evaluation of Bridges, 1994
- 4. PennDOT Publication 238, Bridge Safety Inspection Manual, 2nd Edition October 2002.
- 5. PAPier, PennDOT Pier Analysis Program Version 1.4.0.0, June 2007.
- 6. Mitchell, D., Collins, M., Bhide, S., and Rabbat, B., "AASHTO LRFD Strut-and-Tie Model Design Examples," Portland Cement Association, 2004.
- 7. AASHTO Standard Specifications for Highway Bridges, 15th Edition, 1992, and Interim Specifications 1993 and 1994.
- 8. PennDOT Publication 15M, Design Manual Part 4-Structures, 1993 Edition.
- 9. Schlaich, J., Schafer, K., and Jennewein M., "Toward A Consistent Design of Structural Concrete," *PCI Journal*, V. 32, No. 3, May-June 1987, pp. 74-150.
- 10. ACI Committee 318 (2002), Building Code Requirements for Structural Concrete, ACI 318-99, American Concrete Institute, Farmington Hills MI, 443 pp.
- 11. Breen, John E., "Plasticity in Structural Concrete, CE 383T", University of Texas at Austin, December 7, 2006.
- 12. AASHTO LRFR Manual for Condition Evaluation of Bridges, 2002.