OPTIMAL DESIGN AND PERFORMANCE OF SPLICED PRECAST/PRESTRESSED GIRDER BRIDGES

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ABSTRACT

Design of spliced precast/prestressed concrete girder bridges through post-tensioning has gained renewed interest. However, design and analysis tools to assess their feasibility during the preliminary design phase are limited. Nonlinear structural optimization procedures were integrated with time-dependent analyses to evaluate the performance of single- and two-span continuous spliced girder bridges and preliminary design aids were developed. Optimal design solutions for pre- and post-tensioning requirements and maximum span lengths satisfying a common objective of minimum cost were obtained and developed in the form of design charts. The studies provided a better understanding of this bridge design type and the developed design aids can be a valuable asset to expedite their design and potentially promote their use.

Keywords: Spliced girders, Optimization, Analysis, Design, Precast, Prestressed, Post-tensioned, Design aids.

INTRODUCTION

Full advantage of the spanning capabilities of spliced girder bridges will only be achieved if the interaction between the many parameters controlling their design and behavior is better understood and design aids become available for their consideration during preliminary design. While several methods have been proposed to achieve continuity of and increase the spanning capability of precast prestressed girders[1,2,3], longitudinal splicing of girder segments through post-tensioning, commonly referred to as girder splicing, appears to have the greatest potential[4]. While not new, interest in the development of standard design procedures and guidelines for these systems has recently gained wide attention in bridge engineering practice.

Spliced girder bridges have a proven track record, with more than 250 having been constructed in the US, some of them dating back as early as 1952[4]. Yet, the application of this technique is not widespread. A reason seems to be the ambiguity in their design and analysis. While their design requirements are not significantly different from conventional prestressed concrete design, the analysis procedure must take additional considerations rendering it more complex. Among the most relevant additional issues are: staged construction, multiple stressing stages, and combined pre-tensioning and post-tensioning. While analysis tools for spliced girder bridges have recently become more available[5,6], design guides, aids, and design examples of spliced girder bridges to help designers are still limited, or address only limited portions of the design, as they tend to be job-specific[4].

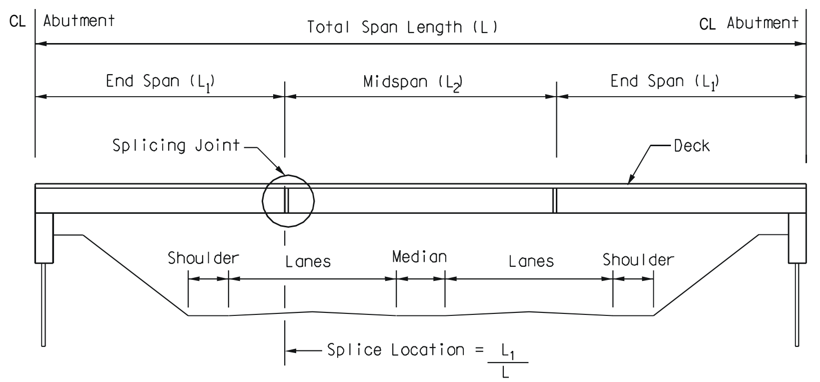
The design of spliced girder bridges depends on many parameters that significantly influence performance and cost, such as time dependent effects, splicing locations, construction sequence, girder segment geometries, number of beams, and number or profiles of pre-tensioned and post-tensioned reinforcement. Thus, the traditional design process, based on the designer's judgment and iteration, is prone to fail in obtaining an economical design and requires considerable effort to explore. On the contrary, numerical optimization methods can provide a systematic approach to arrive at appropriate design solutions.

Optimization techniques[7] can thus be used as a tool to develop the optimum system configuration and girder sectional design of spliced girder bridges and serve as a tool to produce design aids for the different design features of these systems[8]. Bridge engineers would find benefits in design charts and tables based on optimal solutions to expedite the design process. In addition, such studies can provide improved understanding on the interdependence of variables so that design experience and engineering judgment with these systems is improved.

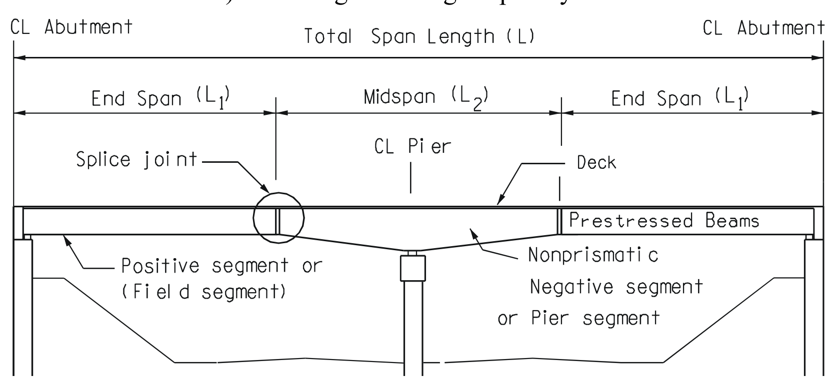
Based on the above motivation, this paper presents an overview of a comprehensive study on the design optimization and behavior of spliced girder bridges[9]. Nonlinear structural optimization procedures were integrated with time-dependent analyses for the optimum design of single-span and two-span continuous spliced girder bridges. The designs follow the AASHTO LRFD Specifications[10] and the latest NCHRP recommendations[4] for spliced girder bridges. An overview of the implemented design and analysis procedure, the developed structural optimization approach, and representative results for both systems are provided.

PROTOTYPE STRUCTURES

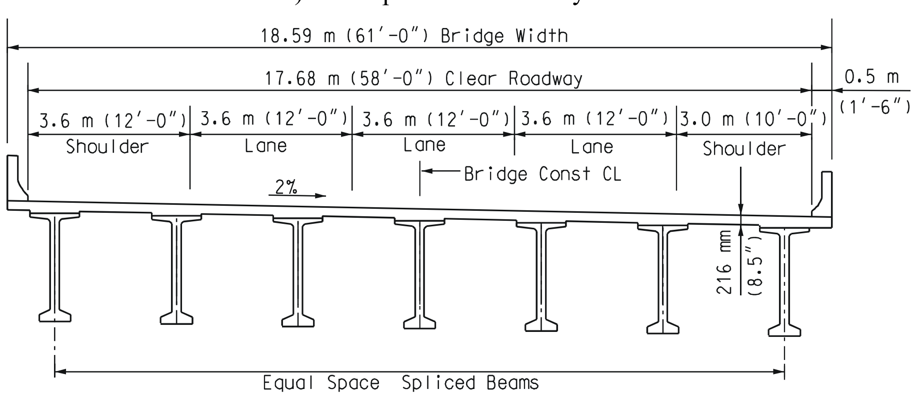
Two prototype structures were considered in this study: a single-span system composed of three segments (Fig. 1a) and a two-span continuous system with equal spans about a middle pier (Fig. 1b). The girder splices, which are usually placed in regions of low moment demands toward the end spans, were assumed to be symmetrically located for both bridge systems, leading to equal end girder segments. The superstructures (Fig. 1c) consisted of prestressed girders with variable spacing of 6 to 9 ft, an 8.5 in. composite cast-in-place concrete deck and a width of 61 ft to accommodate three traffic lanes and two shoulders.



a) Three-segment Single-Span System



b) Two-Span Continuous System



c) Typical Cross-Section

Fig. 1 Overall Geometry and Parameters for Prototype Spliced Girder Bridges in Study

DESIGN AND ANALYSIS APPROACH

The independent girder segments are taken to be spliced by cast-in-place concrete joints between segments and continuous post-tensioning along the entire bridge length. Temporary supports are assumed to be provided underneath the splice locations before the girders are connected. The composite deck, splice joints, and end diaphragms have the same concrete compressive strength. No permanent intermediate diaphragms are used.

Of particular importance to the design of spliced girder is the construction process, i.e., the post-tensioning and construction stage sequence. Studies on the effect of different construction sequences have been done by the authors[9,11] but only the results for a single-stage post-tensioning construction procedure are presented here as summarized in Table 1.

Table 1 Construction Sequence for Single-Stage Post-Tensioning in Spliced Girder Bridges

Stage	Time (days)	Action Description	Tensioning	
			Pre	Post
1	-	Pre-tensioning strands are stressed		
2	0	Girder segments are cast		
3	1	Pre-tensioning strands are released	X	
4	50	Girder segments are erected		
5	60	Deck and Splice concrete are placed	X	
6	75	Post-tensioning strands are stressed	X	X
7	100	Barriers are added		X
8	140	Live load is applied to the bridge		
9	15000	Future wearing surface is added		
10	27500	At final condition after all prestress losses	X	X

The studies were conducted using the simplified design method of the AASHTO LRFD Bridge Design Specifications[10] and the recommendations from the NCHRP 12-57 study[4]. Four limit states were considered. Two follow the service limit states defined in the LRFD specifications[10], namely, Service Limit State I (SLS-I), used to check concrete compression stress limits, and Service Limit State III (SLS-III), used to check concrete tension stress limits. The two additional limit states recommended by the NCHRP 12-517 report[4] deal with compression stress limits for partial service load at the final construction stage.

Non-composite dead loads include the weights of girders, deck, stay-in-place deck forms, and construction loads[9]. Composite dead loads include splice and spliced diaphragms, removal of temporary support, barrier weight, and future wearing surface. Composite loads are distributed equally to all girders in the superstructure cross-section. Loads from splice joints and temporary diaphragms are applied to the composite beam after splicing due to post-tensioning is complete and the temporary supports are removed. Design live loads consist only of vehicular loads as specified in AASHTO LRFD[10]. Full service loads included girder self-weight, deck load, non-composite dead load, temporary pier removal, construction dead load, and live load.

For the reported study, the assumed concrete properties for the girders, deck and splice joints are given in Table 2. The pre- and post-tensioning strands are assumed to be 0.6-in. in diameter and made from low-relaxation steel, with properties as per the AASHTO LRFD specifications[10] (Table 2).

Table 2 Assumed Material Properties

Girder Concrete	Deck & Splice Concrete	Pretensioning Strands	Post-Tensioning Strands
f'_c = 6500 psi	f'_{cd} = 4500 psi	A_{ps} = 0.217 in.^{2}	A_{ps} = 0.217 in.^{2}
f'_{ci} = 5000 psi	f'_{cdi} = 3500 psi	f_{pu} = 270 ksi	f_{pu} = 270 ksi
E_c = 4888 ksi	E_{cd} = 4067 ksi	f_{py} = 243 ksi	f_{py} = 243 ksi
E_{ci} = 4287 ksi	E_{cdi} = 3587 ksi	f_{po} = 202.5 ksi	f_{pj} = 218.7 ksi
		E_p = 28500 ksi	E_p = 28500 ksi

All pre-tensioning strands are straight without any draping or debonding. The strands are placed in the top and bottom flanges of girder sections and they are fully stressed to $0.75f_{pu}$. A parabolic post-tensioning strand layout is assumed to counteract the bending moment demand for the simple supported bridge system, while three connected parabolic profiles are assumed for the continuous bridge systems. Multiple post-tensioning strand tendons are assumed and the distance between each duct varies along the girder length. Post-tensioning design requirements are evaluated at four critical locations: at the middle of both end segments, at the center of the middle segment, and at the center of the splice joint.

Prestress losses are computed using the simplified AASHTO-LRFD method[10] with consideration of the effects of combined pre-tensioning and post-tensioning on the girder segments, as recommended by the recent NCHRP study on spliced girder bridge design[4]. Design lifetime of the spliced girder bridge system is taken to be 75 years after all prestress losses have occurred. For a single-stage construction procedure there are four critical construction stages for evaluation of prestress losses as noted in Table 1. Prestress losses for the pre-tensioned strands are calculated at the mid-span of each girder segment, while prestress losses for the post-tensioning tendons are calculated at both girder ends and at mid-span. The prestress loss values are then used along the entire length of the girder segment.

Analysis of the spliced girder bridge structures is based on 2-D elastic beam theory including the effects of time-dependent material behavior and staged construction. A custom frame analysis and design program coded in the MATLAB[12] environment was used. The analysis module solves for the beam-line model subjected to the demands of an interior girder according to the load distribution factors given in the AASHTO-LRFD Specifications[10]. Girder section design is performed using an algorithm which enforces compliance with service limit states as previously discussed. The analysis and design program was verified against Design Example 1 from the NCHRP 517 report[4] as discussed later in this paper.

DESIGN OPTIMIZATION OF SPLICED GIRDER BRIDGES

A numerical optimum design process differs from the conventional design process by taking into account all the design criteria simultaneously and improving the design iteratively. The objective of an optimal bridge design problem is to determine the structural system design variables that satisfy the behavior and constraints of the problem. Constraints can be assembled directly from bridge design code specifications, while the design problem behavior can be the cost of the bridge structure or other merit functions.

The standard mathematical form of an optimization problem is normally written as:

$$
\begin{array}{lll}
\text{Objective function:} & \text{Min } f(\boldsymbol{x}) & \\
\text{Subject to:} & g_i(\boldsymbol{x}) \leq 0 & i = 1 \text{ to } m \\
 & h_j(\boldsymbol{x}) = 0 & j = 1 \text{ to } p \\
 & \boldsymbol{X}^L \leq \boldsymbol{x} \leq \boldsymbol{X}^U &
\end{array}
$$

where $\boldsymbol{x}$ is the design variables vector, $\boldsymbol{X}^L$ and $\boldsymbol{X}^U$, are the lower and upper bound vectors for the design variables, $f(\boldsymbol{x})$ is the objective function, and $g_i(\boldsymbol{x})$ and $h_i(\boldsymbol{x})$ are the inequality and equality constraints, respectively.

Typically, there can be several design solutions that satisfy all requirements of safety and serviceability imposed by a design code and a number of specified merit criteria. Thus, the designer makes a difficult decision in selecting the best design among the possible alternative solutions that adequately satisfy all the governing design criteria. A common criterion of interest in bridge design is the minimization of total structural cost. Thus, in this study, the objective function is the minimization of superstructure cost, which included costs due to concrete, pre- and post-tensioning strand, temporary supports, beams, and passive reinforcement.

General cost information for post-tensioned spliced girder bridges is difficult to obtain since most states only track cost data on a project basis. In addition, costs vary significantly between regions. Cost estimates for use in the cost minimization objective function were thus based on values used for conventional prestressed girders with supplemental costs for temporary supports and post-tensioning[9]. While this type of incremental approach is rarely valid[4], it is considered adequate for the comparative study being reported here.

The design variables for the optimization procedure are a mixed set of integer and continuous geometric parameters that define the dimension and shape of the precast/prestressed girder segments, the material layout, and the span arrangement of the spliced girder bridge system. The variables for the reported prototype structures are given later in their respective sections.

The constraints to the optimization problem consisted on enforcing service limit state requirements. Only service limit states were used since most of the time they govern over the strength limit criteria. Constraints for the service limit state included: concrete girder stress limits, prestressing stress limits, deflection limits, and concrete slab stress limits. The stress constraints were formulated and enforced for each of the critical construction stages.

A gradient-based algorithm using the constrained steepest descent method with Pshenichny's descent function was used to solve the optimization problem[7]. This method involves a sequence of major iterations, each requiring the solution of a linearly constrained sub-problem where the nonlinearities are confined to the objective function. A design point is chosen to initiate the process and iterative designs and evaluations are continued until a final design point is reached, that is when there are no further changes in the value of the objective function. Gradient-based logarithms may get trapped in local minimum solutions. To avoid this problem, the optimization problems were done several times starting at different points and in different directions. While this strategy does not guarantee finding the global optimum, this scheme was repeated for multiple initial designs until it was found that the solution was essentially converging to the same point.

The optimization procedure was implemented in a custom automated program written in Matlab[12] making use of this program's optimization tool box together with the previously discussed custom analysis and design procedures. A flow diagram of the design optimization program is shown in Fig. 2.

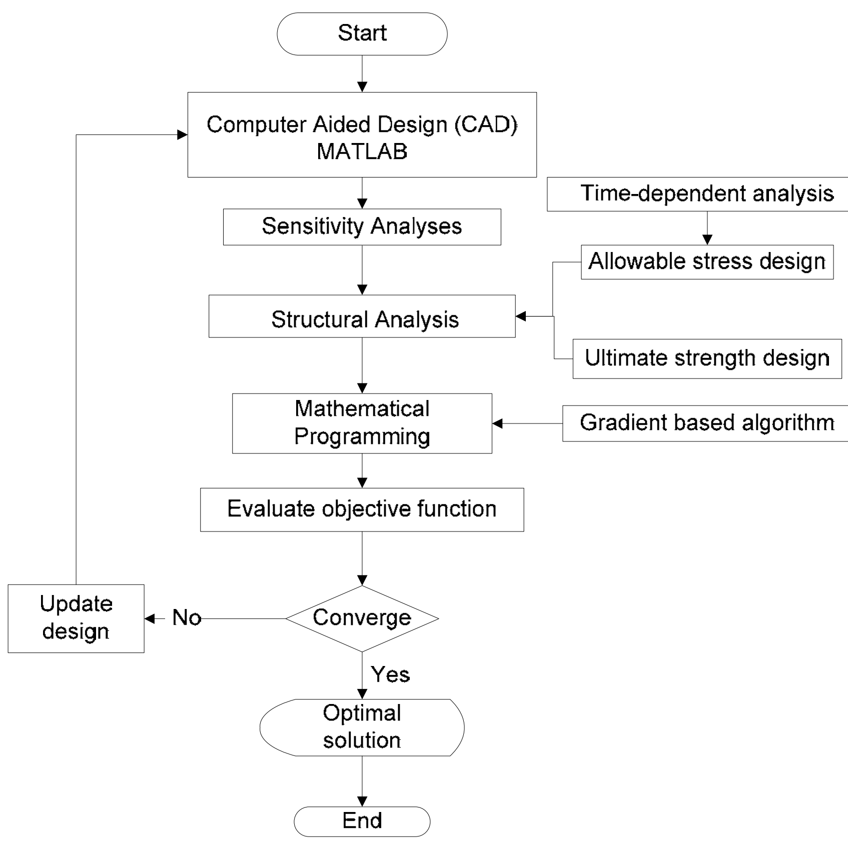


Fig. 2 Flow Chart of Computer Program for Optimization of Spliced Girder Bridge Systems

OPTIMUM DESIGN OF SINGLE-SPAN SYSTEMS

The optimum design of single-span spliced girder bridges using standard precast/prestressed beams was investigated by considering AASHTO I-beam types III through type VI, Michigan's DOT MI-1800 I-beam, PCI's PCI-BT 72 and PCI-BT 96 bulb-tee beam. The design variables for the optimization problem were those defining the amount and layout of the pre-tensioning and post-tensioning strands in the girders, as described in Fig. 3.

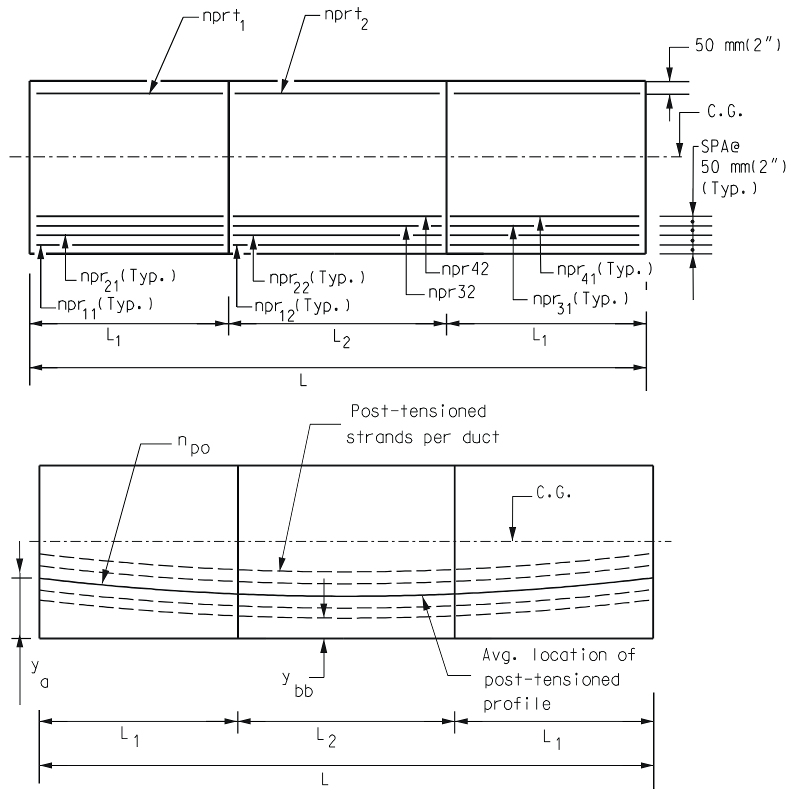
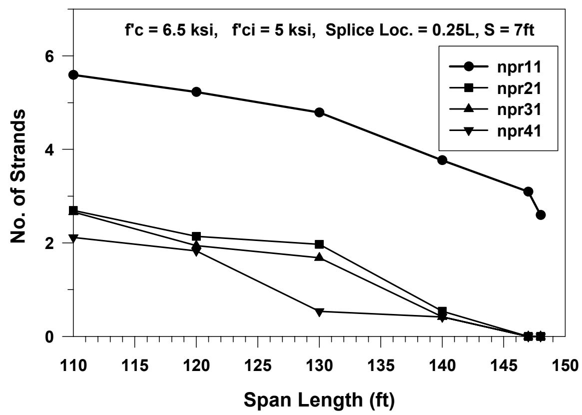


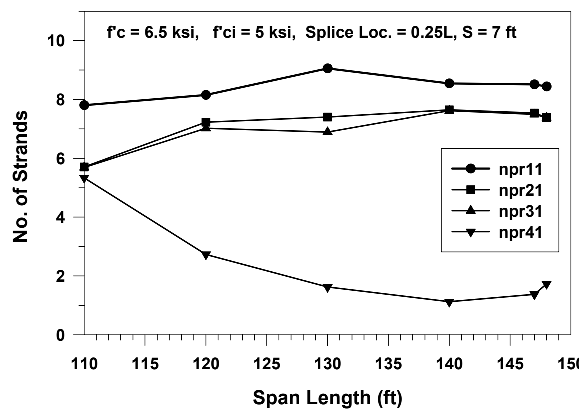
Fig. 3 Variables for Design Optimization of Single-Span Spliced Girder Bridges

Results of single span spliced girder bridges with AASHTO Type-IV I-beams are shown in Fig. 4. These results are direct output from the optimization procedure with the span length kept as a design parameter. Optimum designs were developed by incrementing the span length until no feasible solution was obtained. Thus, these results also provide the maximum achievable span length for this girder type.

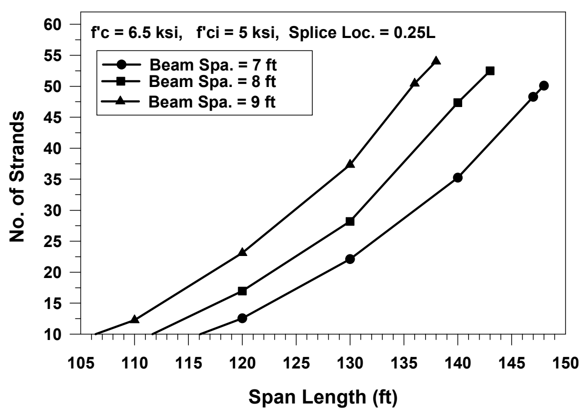
Several splice locations (L_1/L – see Fig. 1a) were investigated, however, only the results for the most efficient splice location at $0.25L$ are shown in Fig. 4. The plots provide traces that relate the optimal requirements of pre-tensioning strands, in the end and middle girder segments, and the amount of continuous post-tensioning to the achievable span length, both with reference to Fig. 3. The information in the plots of Fig. 4 makes them suitable to serve as preliminary design aids[11], as they permit determining pre-tensioning strand requirements per row for each girder segment, as well as the strand requirements and eccentricity locations of the effective, or average, post-tensioning duct at the bridge ends and at mid-span (Fig. 3).



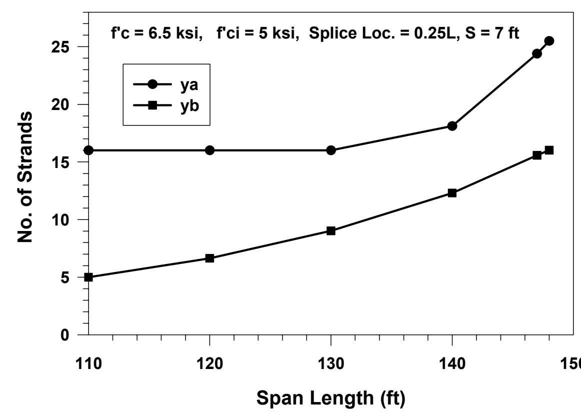
a) Pre-tensioning Strands per row in end-segment



b) Pre-tensioning strands per row in mid-segment



c) Post-tensioning strands

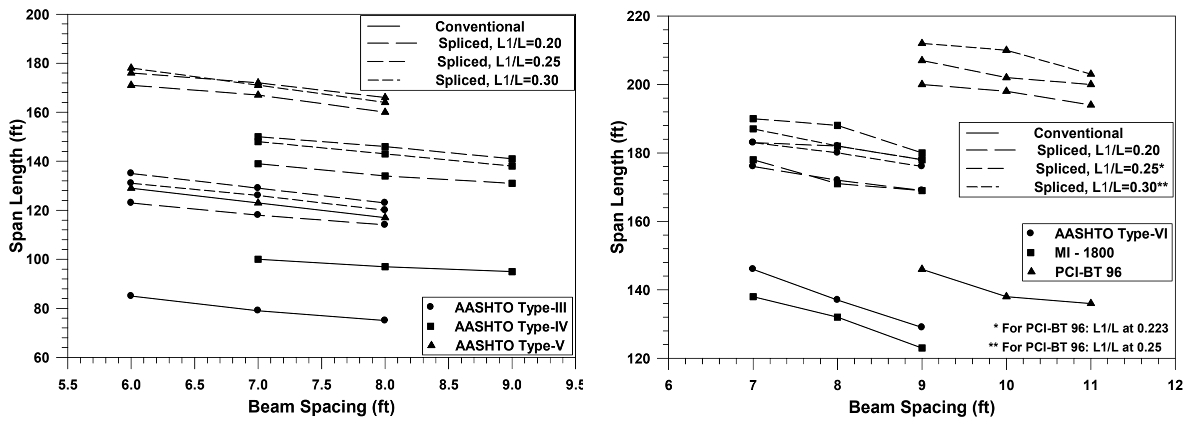


d) Post-tensioning profile at end and mid segment

Fig. 4 Stressing Requirements for AASHTO Type IV Beams in Single-Span System

The data in Fig. 4 can also be used to explore the achievable spans lengths of spliced girder bridge systems and the gains that are made with respect to conventional designs. For comparison purposes, the maximum span length of conventional pre-tensioned girder designs were determined by using the software CONSPAN[13].

Comparison of maximum achievable span lengths and percent span increase between conventional and spliced girder systems for the studied I- and bulb-tee beam types are shown graphically in Fig. 5. These results show that the splice location at 0.25*L* is the most efficient for single-span spliced girder bridges with equal end-segment lengths. The results also show that longitudinal splicing is not only efficient for deep beam sections but also for beams of shallow to moderate depths, such as the AASHTO Type III. In fact, the results show that shallow girder sections have higher gains in percent increase of the span length than deeper girder sections.



a) AASHTO Beam Types III, IV and V b) AASHTO Type VI, MI-1800 and PCI-BT-96 Beams

Fig. 5 Maximum Span Length of SGBs and Conventional I- and Bulb-Tee Beams

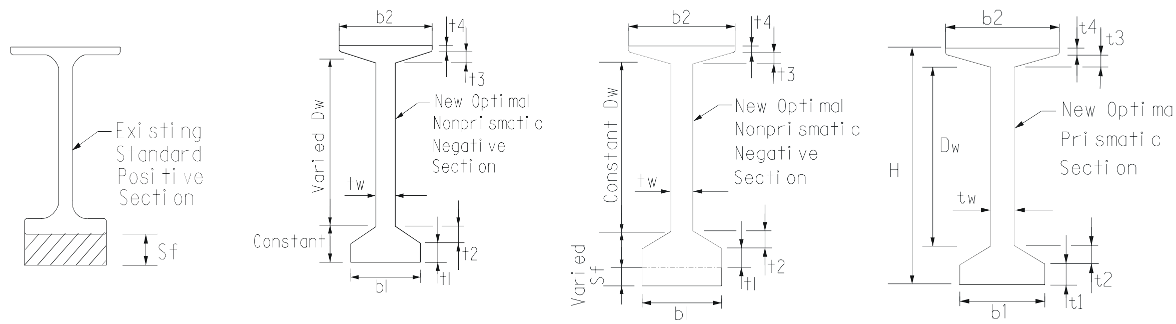
OPTIMUM DESIGN OF TWO-SPAN CONTINUOUS SYSTEMS

The design of spliced girder bridges in continuous systems (such as that in Fig. 1b) requires custom-designed pier, or negative, segments to carry the high negative bending moment and shear demands over the intermediate supports. Consequently, deep sections are typically used for these segments. The availability of standard negative sections could reduce the cost of fabrication of pier segments. Thus, the optimization approach previously presented was coupled with a sizing optimization process to study and develop optimal girder cross-section geometries at the pier locations[9].

Four options for negative pier segments were studied[9] (Fig. 6). The first option explored the use of existing standard sections with a linearly varying bottom soffit to make up a negative pier segment. In option two, a new optimal non-prismatic section having a uniform flange thickness and a linearly varying web height along the negative segment was developed. The third option for a pier segment was a new optimal non-prismatic section having a uniform web height and a linearly varying bottom flange. Finally, as a fourth option, a new optimal prismatic section for use in both positive and negative segments was also developed.

The design optimization of two-span continuous spliced girder bridges requires more variables that those considered for a single-span system. Additional considerations include design variables associated with the sectional optimization options (Fig. 6) and the definition of additional variables to define the parabolic post-tensioning profile over the haunched negative segment. The system design variables are shown in Fig. 7.

The efficiency of using a standard girder for both positive and negative spans (Option 1) was evaluated by considering AASHTO's Type IV I-girder. The maximum soffit thickness (over the pier center line) was varied from 10 to 50% of the standard girder height (54 in.). The maximum feasible span lengths for different soffit thickness and different splice locations for a beam spacing of 6 ft are shown in Fig. 8a. Results show that the achievable span length increases about 1 ft for every 5.4 in. of added soffit thickness (maximum value at pier), which is about 10% of the total beam height. The percent increase in span length compared to the total span length is thus less than 1%, which is very small.



a) Option 1 b) Option 2 c) Option 3 d) Option 4

Fig. 6 Geometry and Design Variables for Pier Segments Options

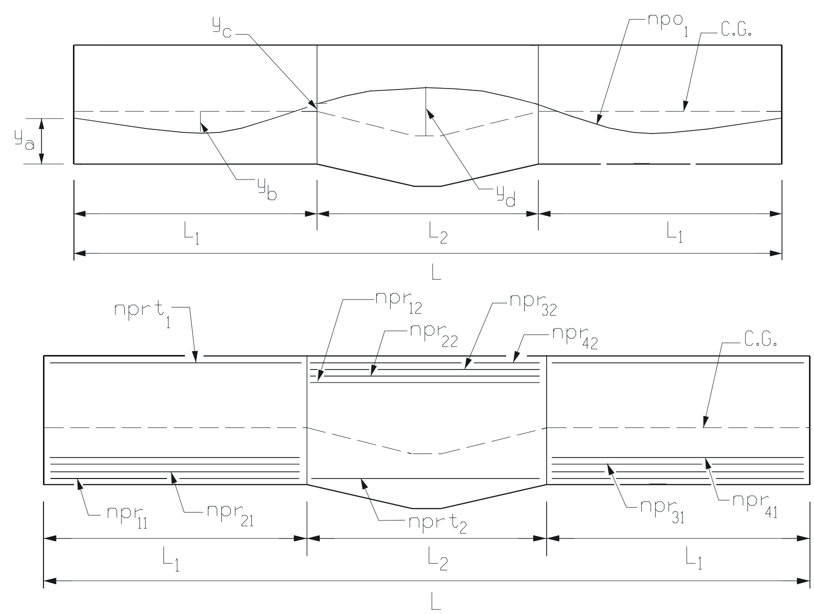
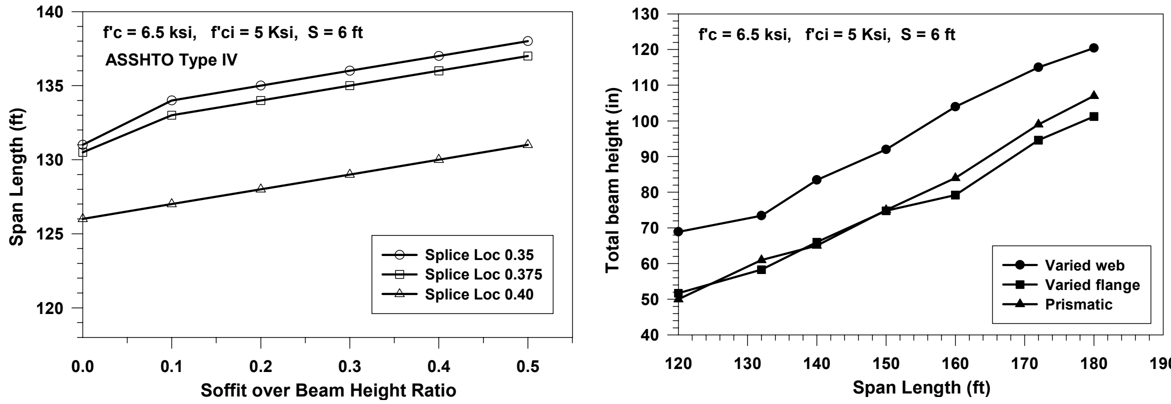


Fig. 7 Variables for Design Optimization of Two-Span Continuous Spliced Girder Bridges

The objective function used to develop new optimal pier segments was to minimize concrete volume, or minimizing girder weight. Minimum construction cost was not chosen as the main objective function since there are no explicit costs available for the modified girders. The relations between maximum span and total beam height, including any added soffit, of negative segments at a beam spacing of 6 ft for all of the new optimized negative sections are shown in Fig. 8b. The total height of the optimal non-prismatic beam with varied web over the negative region (Option 2) is the deepest beam, while the non-prismatic beam with varied bottom flange (Option 3) yields the shallowest beam for a given span length. The height of the new prismatic girder (Option 4) is bounded by those for options 2 and 3. An inverse evaluation Fig. 8b can be done to judge the effectiveness of the new optimal girders in terms of span length. Although the new non-prismatic sections with varied flange thickness were shown to be more efficient, the manufacturing cost of a prismatic beam would most likely be lower. Dimensions for each of the optimized negative pier segments for different span lengths and girder spacing were also obtained. Representative results for spliced girder bridges featuring new optimal pier spans with constant web depth and varying depth bottom flange are shown in Fig. 9. Tabulated values and preliminary design aids are reported elsewhere[9].



a) ASSHTO Type IV Beam with Soffit (Option 1) b) Height and Max. Spans with New Optimal Segments

Fig. 8 Performance of Two-Span Continuous Spliced Girder Systems

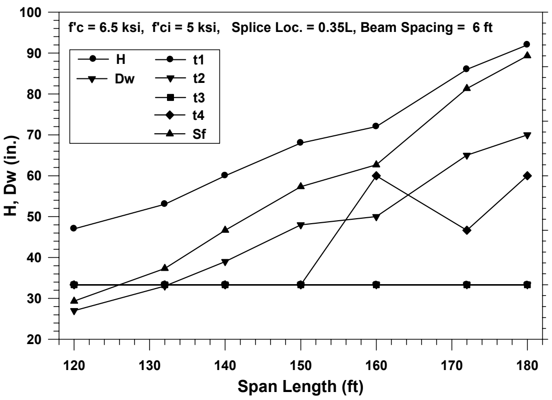
RESULTS EVALUATION AND VERIFICATION

It has been noted that the results obtained from the nonlinear structural optimization approach can be represented in plots that can be used as design aids for spliced girder bridges. These charts can provide guidance on variable sensitivity, facilitate the typical trial and error design process, and expedite the design of this type of bridge systems. The use of these charts as design aids, and essentially the results of the optimization studies, was evaluated and verified[9,11] by comparing them to the results of Example 1 of the NCHRP 517 report[4].

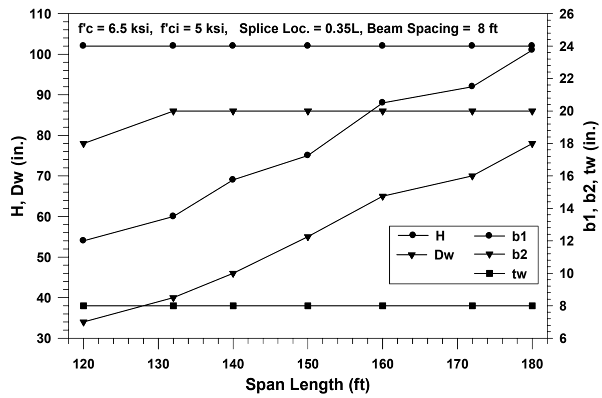
Table 3 Results Comparison between Study and Example 1 of NCHRP 517 Report[4]

	Design Capacities and Demands			*Stressing Requirements*		
	M (kN-m)	V_Splice (kN)	V_h/2 (kN)	n_{po}	npr_1	npr_2
This Study	41,115	5,440	7,980	52	4	32
NCHRP 517[4]	45,990	8,265	NA	60	10	20
Demand	33,858	1,900	3,648			

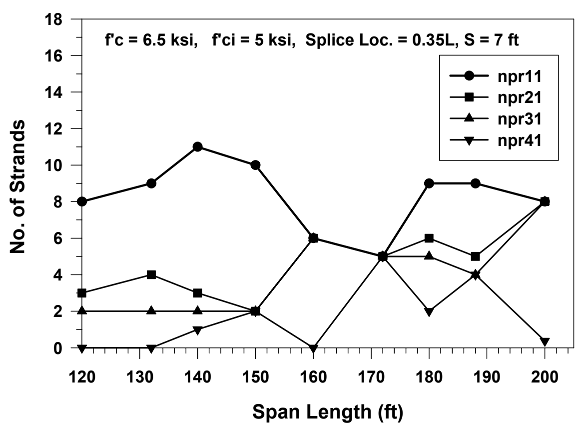
The NCHRP example consists of a three-lane single-span spliced girder bridge with a total width of 61 ft and a span length 198 ft composed of three segments with splices located at 0.223L from both ends (see Fig. 1a). PCI-BT 96 girders with 9 ft spacing are used. The cast-in-place concrete slab has a thickness of 8.5 in. Construction was defined as previously described and summarized in Table 1. A full depth diaphragm cast with the splice was assumed to be provided at the splice locations. Both service and ultimate limit states were considered in the design. A comparison of results is given in Table 3, which shows that the results from the optimization procedure (present study) and their use as design aids are within 5% of the NCHRP example values and thus lead to an acceptable solution. The flexural stresses from each critical construction stage were also compared against the allowable limits and the NCHRP results and found to be adequate[9,11].



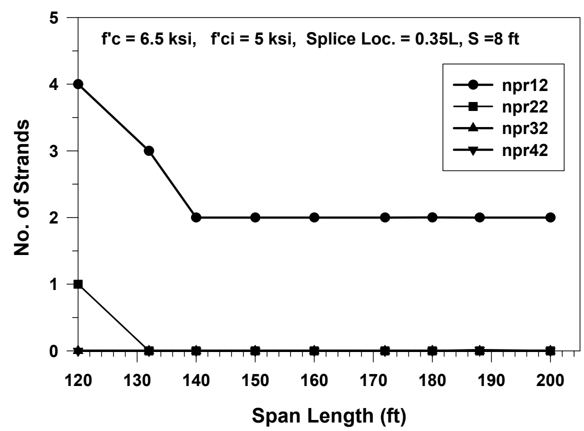
a) Overall section height at pier centerline



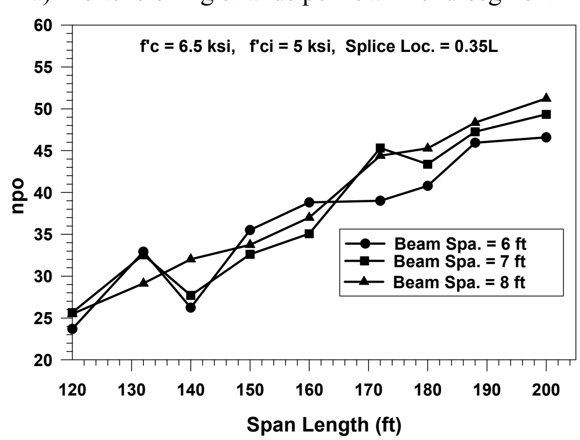
b) Section web height (constant) at pier centerline



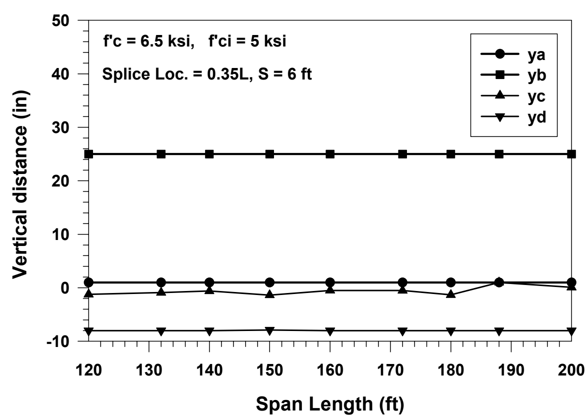
a) Pre-tensioning strands per row in end-segment



b) Pre-tensioning strands per row in mid-segment



c) Post-tensioning strand requirements



d) Post-tensioning profile at end and mid segment

Fig. 9 Two-Span Continuous System with Option-3 Pier Segments Stressing Requirements

CONCLUSIONS

The presented work has utilized structural optimization analyses to perform studies on longitudinally spliced precast/prestressed bridges in single-span and two-span continuous systems. Results based on a single cost minimization objective function and a single post-tensioning construction stage have provided considerable information on the sensitivity and interaction of parameters that influence the design of these systems. Such parameters include girder types, girder spacing and girder splicing location.

The presented studies have thus given insight into the achievable span lengths using standard precast/prestressed sections and a variety of new pier segments for continuous systems have been developed. Studies on single-span systems have shown that the spanning capabilities of shallow girders can be increased significantly through longitudinal splicing. The studies on two-span continuous systems have shown the possibilities of using standard girders as pier segments by providing them with an added soffit. Optimal pier segments with variable bottom flange, variable web, and prismatic geometries were developed. Of these, pier segments with constant web height and a variable bottom flange seem to be the most efficient.

Recognition of the above results and their consequences was possible due to the integration of numerical optimization algorithms and sequential time-dependent analysis tools. The optimization procedure allowed the determination of design solutions for both girder systems in a way that satisfied a common objective. Finally, the optimization results from this study can be very useful as aids for the preliminary design of spliced girder bridges. Given the added complexity in the design and analysis of spliced girder bridges, the availability of design aids can be a great asset to bridge engineers to expedite the design process of this bridge type, which can then result in their wider use by State Departments of Transportation.

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