Reliability Assessment of Multigirder Concrete Highway Bridges Designed By LRFD

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ABSTRACT

The reliability index is examined for prestressed high performance concrete girder highway bridges designed by AASHTO *LRFD* Strength I limit state. Based on the extensive stochastic finite element method (SFEM) study of steel girder bridges, the reliability index is very sensitive to the lateral distribution of live loads. The reliability analysis using the advanced first-order second-moment (AFOSM) method can achieve a good accuracy when combined with finite-element-based consideration of the lateral distribution of live load. In this study, the lateral distribution factors are computed by the 3-dimensional finite element analysis program BRUFEM in lieu of AASHTO specified values. Basic design variables include sectional properties as well as dead and live loads. The statistical data of these variables are mainly chosen from existing studies in the profession.

The objective of this study is to investigate the reliabilities of prestressed concrete girder highway bridges using the extensive consideration of live load distribution. As numerical examples, high strength concrete bridges with various span lengths and girder spacings are designed in accordance with AASHTO *LRFD* (1998) Strength I limit state for flexure. Using live load distribution factor obtained from 3-dimensional finite element analysis, the study indicates that: (1) The reliability indices based on the AFOSM are generally much higher than the target, especially in the case of the large girder spacing of 12 ft; (2) When using different concrete strengths $f_c^{'}$, reliability index remains almost the same; and (3) Reliability index varies significantly with span length and girder spacing.

Keywords: highway bridges, reliability, concrete structures, random variables, girders, stochastic models

INTRODUCTION

The commencement of AASHTO *LRFD Specifications* (1998) is an important development in bridge design philosophy. The AASHTO *LRFD* (1998) introduced a design conception and approach different from AASHTO *Standard* Specifications (1996) for highway bridges. The advantages of a design based on load and resistance factor design (LRFD) method include the consideration of variability in both resistance and load, obtainable uniform levels of safety for different limit states and bridge types without performing complex probability or statistical analysis, and rationality and consistence in the design (Barker and Puckett 1997). The basic design expression in the AASHTO *LRFD* (1998) Specifications that must be satisfied for all strength and serviceability limit states, both global and local, is given as

$$\eta \sum \gamma_i Q_i \leq \phi R_n \tag{1}$$

where Q_i = the force effect; R_n = the nominal resistance; γ_i = the statistically based load factor applied to the force effects; ϕ = the statistically based resistance factor applied to nominal resistance; and η = a load modification factor. The load and resistance factors in Eq. (1) were determined by transferring design experience inherent in the *Standard* code.

In the development of new code, Nowak (1993) demonstrated a uniform safety margins among the highway bridges of various types, span lengths, and girder spacings. The concept of girder distribution factor was used and the bridge structure was simplified into an isolated single girder. The reliability analysis is based on the advanced first order and second moment (AFOSM) approach. Recently, a more advanced reliability analysis, namely stochastic finite element method (SFEM), has been used for the structural reliability analysis (Kleiber and Hien 1993). The SFEM approach was employed to validate the steel structures designed in accordance with AISC *LRFD* (1986) approach (Mahadevan and Haldar 1991; Haldar and Mahadevan 2000). The SFEM is capable of involving the consideration of the element in an actual structural configuration and the effect of statistical correlation among the random parameters of the system. Based on the extensive stochastic finite element method (SFEM) study of steel girder bridges (Liu 2002), the reliability index is very sensitive to the lateral distribution of live loads. The reliability analysis using the advanced first-order second-moment (AFOSM) method can achieve a good accuracy when combined with finite-element-based consideration of the lateral distribution of live load.

The objective of this study is to investigate the reliabilities of reinforcement and prestressed concrete girder highway bridges using the extensive consideration of live load distribution. As examples, there are 60 bridges designed in accordance with AASHTO *LRFD* (1998) Strength I limit state for flexure. The actual safety margins corresponding to the specified load modification factor, η, are studied.

Reliability Analysis Using AFOSM

According to the AFOSM approach, one performance criterion of a structural system can be defined as a limit state in the following form:

$$g(\mathbf{X}) = g(\mathbf{R}(\mathbf{X}),\mathbf{S}(\mathbf{X})) = 0 \tag{2}$$

where $\mathbf{X} = \{X_1, X_2, \ldots, X_n\}^T$, the vector of the basic parameters of the system and external loads, usually nonnormal and correlated; $\mathbf{R}(\mathbf{X})$ = the vector of resistance variables; and $\mathbf{S}(\mathbf{X})$ = the vector of load effects.

For the convenience of computation, the nonnormal design variables X_i (i = 1, …, n) are transformed into the space of uncorrelated standard normal variables $\mathbf{Y} = \{Y_1, Y_2, \ldots, Y_n\}^T$. When transformed into the space of $\mathbf{Y}$, the performance criterion in Eq. (2) becomes:

$$g(\mathbf{X}) = G(\mathbf{Y}) = 0 \tag{3}$$

The point $\mathbf{Y}^*$ on the limit-state surface with the minimum distance to the origin is the most probable failure point or the design point. The minimum distance is a measure of reliability, i.e., the reliability index denoted by $\sqrt{\mathbf{Y}^{*T}\mathbf{Y}^*}$. The following efficient formula is used to compute the design point $\mathbf{Y}^*$ (Madsen et al. 1986):

$$\mathbf{Y}_{i\ 1} \qquad \mathbf{Y}_i^T \quad {}_i \quad \frac{G(\mathbf{Y}_i)}{\|\ G(\mathbf{Y}_i)\|} \quad {}_i \tag{4}$$

where $\nabla G(\mathbf{Y}_i) = \{\partial G(\mathbf{Y}_i)/\partial Y_1, \ldots, \partial G(\mathbf{Y}_i)/\partial Y_n\}^T$, the gradient vector of the performance function at $\mathbf{Y}_i$; $\mathbf{Y}_i$ = the design point in the ith iteration; and $\alpha_i = -\nabla G(\mathbf{Y}_i)/\|\nabla G(\mathbf{Y}_i)\|$, the unit vector normal to the limit-state surface away from the origin.

In this study, the transformation from $\mathbf{X}$ to $\mathbf{Y}$ is accomplished by the following approximate approach (Ang and Tang 1984):

(1) The variables X_i (i = 1, …, n) are transformed into standard normal variables $\mathbf{Z}$:

$$\text{For normal variables } X_m,\ Z_m = (X_m - \mu)/\sigma \tag{5a}$$

$$\text{For lognormal variables } X_n,\ Z_n = (InX_n - \lambda)/\zeta \tag{5b}$$

where μ = the mean value of X_m; σ = the standard deviation of X_m; λ = the mean of $\ln(X_n)$; ζ = the standard deviation of $\ln(X_n)$.

(2) The standard normal variables $\mathbf{Z}$ are transformed into independent standard normal variables Y_i:

$$Z_i = \sum_{j=1}^{i} \alpha_{ij} Y_j \tag{6}$$

where

$$\alpha_{11} = 1.0$$

$$\alpha_{i1} = \rho_{Y_i Y_1}$$

$$\alpha_{ik} = \frac{1}{\alpha_{kk}}\left(\rho_{Y_i Y_k} - \sum_{j=1}^{k-1} \alpha_{ij}\alpha_{kj}\right) \qquad 1 < k < i$$

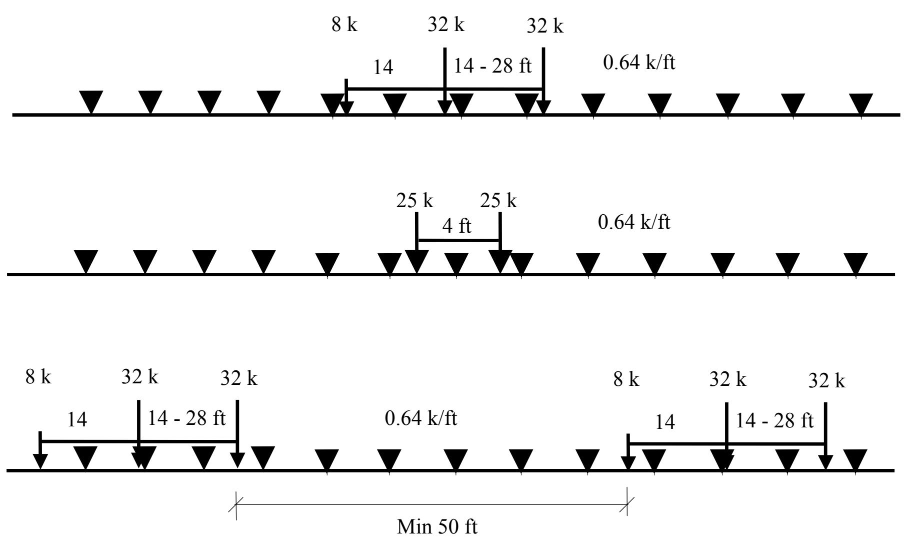
$$\alpha_{ii} = \sqrt{1 - \sum_{j=1}^{i-1} \alpha_{ij}^2}$$

$\rho_{Y_i Y_k} \approx \rho_{X_i X_k}$, coefficient of correlation between Y_i and Y_k.

The reliability analysis is performed by removing the load and resistance factors in Eq. (1).

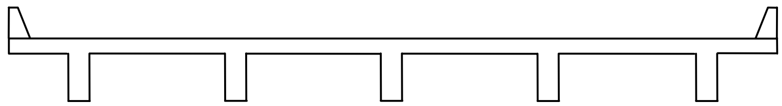
BRIDGE MODEL

To study the reliability of concrete beam/girder bridges, 60 simply supported T-beam bridges are designed according to the Strength I limit state for flexure of AASHTO *LRFD* (1998). Span length ranges from 30 ft to 120 ft with a 30-ft interval. For 120 ft span, the prestressing steel is used, while the reinforcing steel is used in other spans. Girder spacing varies from 4 ft to 12 ft with a 2-ft interval. Concrete compressive strength $f_c^{'}$ = 6, 8, and 10 ksi, respectively. These bridges are of five Tee beam sections and are designed on the basis of HL-93 loading as shown in Fig. 1. The concrete deck thickness is 8 inches. The deck overhang is assumed to be 0.45*S* in width, *S* =girder spacing. The standard traffic barrier width is 1.25 ft. These bridges have a roadway width of 17.1 ft to 56.3 ft with the number of lanes of 1, 2, 3, 3, and 4, respectively. The typical cross section of the bridge is shown in Fig. 2. All the beams/girders have identical section.

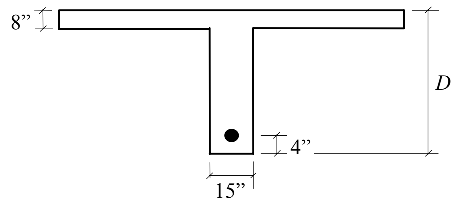


Note: The 2-truck live load is used for negative moment only

Fig. 1. HL-93 Model



(a) Cross Section Layout



(b) Tee Beam Section

Fig. 2. Typical cross section of the bridge

DESIGN VARIABLES AND STATISTICAL DATA

In the reliability analysis of a system, a probabilistic description of the basic variables, X_j, ($j = 1, \ldots, n$) is necessary. There are a total of 6 design variables chosen in this study: dead load *DL*, future wearing surface *WS*, lane load *LNL*, truck load *TL*, dynamic load allowance *IM*, and nominal resistance M_n. The components of the vector **X** in Eq. (2) consist of these random variables. Table 1 gives the statistical data on loads, as well as on nominal resistance. The distribution type of loads is assumed to be normal, while those of sectional and material properties are assumed to be lognormal. Since the correlations among the design variables are not available in the literature, the coefficient of correlation is assumed to be zero. These statistical data are chosen from the state-of-the-art research conducted by Nowak (1993). However, it should be mentioned that these available statistical data do not reflect the change in member nominal resistance with different values of $f_c^{'}$.

Table 1. Statistical Data of Loads and Nominal Resistance

Load Types	**Dead Load** DL_i	**Wearing Surface** *WS*	**Lane Load** *LNL*	**Truck Load** *TL*	**Dynamic Load Allowance** *IM*	**Nominal Resistance Moment** [a] M_n
Bias Factor	1.03	1.0	1.15	1.15	1.15	1.12 (1.05)
COV	0.08	0.25	0.12	0.18	0.18	0.135 (0.075)
Distribution	Normal	Normal	Normal	Normal	Normal	Lognormal
Note: a. Values in parentheses are for prestressed concrete.						

DESIGN EXAMPLES AND RESULTS

Design Examples

All bridges are designed according to AASHTO *LRFD* Strength I limit state:

$$\text{for flexure} \quad M_n = \phi_f M_n \geq M_u \tag{7}$$

where M_n = the nominal resistance moment; M_u, = the ultimate moment induced by live and dead loads; and ϕ_f = the resistance factor for flexure (ϕ_f = 0.9 for reinforced concrete beam and ϕ_f = 1.0 for prestressed concrete beam, respectively).

The factored moment by the dead and live loads are as follows:

$$M_u = \eta \cdot [1.25 \cdot M_{DL} + 1.5 \cdot M_{WS} + 1.75 \cdot DF_m \cdot M_{LL} \cdot (1.0 + IM)] \tag{8}$$

in which $\eta = \eta_D \cdot \eta_R \cdot \eta_I \geq 0.95$, a load modifier; η_D = a factor relating to ductility; η_R = a factor relating to redundancy; η_I = a factor relating to operational importance; M_{DL} = the moment caused by self-weight of structural components and nonstructural attachments; M_{WS} = the moment caused by self-weight of wearing surfaces and utilities; M_{LL} = the moment caused by design loading HL-93; DF_m = the load distribution factor of moment; and IM = the vehicular dynamic load allowance.

The nominal resistance moment strength is in the following form:

$$M_n = A_{ps} f_{ps}(d_p - a/2) + A_s f_y (d_s - a/2) \tag{9}$$

where A_{ps} = area of prestressing steel; f_{ps} = average stress in prestressing steel at nominal bending resistance; d_p = distance from extreme fiber to the centroid of prestressing tendons; A_s = area of nonprestressed tension reinforcement; f_y = specified yield strength of reinforced bars, 60 ksi is used herein; d_s = distance from extreme fiber to the centroid of nonprestressed tensile reinforcement; $a = c\beta_1$, depth of the equivalent stress block; β_1 = stress block factor; c = distance between the neural axis and the compressive face.

In this study, all the design examples have rectanglular section behavior, therefore,

$$c = (A_{ps} f_{ps} + A_s f_y)/0.85 f_c' \beta_1 b \tag{10}$$

where b = width of the compression face of the member.

The average stress in prestressing steel, f_{ps}, is taken as:

$$f_{ps} = f_{pu}(1 - kc/d_p) \tag{11}$$

where f_{pu} = specified tensile strength of prestressing steel (270 ksi); $k = 2(1.04 - f_{py}/f_{pu})$, see LRFD *Specifications* Table C5.7.3.1.1.-1; f_{py} = yield strength of prestressing steel.

For load lateral distribution factor, the distribution of live load per lane for moment is used as follows:

$$DF_m = 0.06 + (S/14)^{0.4} \cdot (S/L)^{0.3} \quad \text{one design lane loaded} \tag{12a}$$

$$DF_m = 0.075 + (S/9.5)^{0.6} \cdot (S/L)^{0.2} \quad \text{two or more design lane loaded} \tag{12b}$$

in which S = girder spacing (ft); L = span length (ft). Neglecting the longitudinal stiffness-related term of $(K_g/Lt_s^3)^{0.1}$, Eq. (12) is usually used in the preliminary design. Eq. (12) is used for interior girders in this study. If the design is controlled by exterior girder, the lever rule is used.

Tables 2, 3, and 4 give the design examples using AASHTO *LRFD* Strength I limit state in the case of $f_c^{'}$ = 6, 8, and 10 ksi. For simplicity, a reinforcement ratio of 0.6% and 0.15% is, respectively, used for reinforcement concrete and prestressed concrete. The relative differences between nominal strength (M_r) and factored load effect (M_u) are generally controlled less than ±0.1%. In these designs, load modifier, η, is taken as 1.0. It can be seen that Tee beam dimension vary very little with concrete strength $f_c^{'}$.

Table 2. Tee Beam Section Dimension *D* (inches) in Design Examples ($f_c^{'}$ = 6 ksi)

***S* (ft)**	**4**	**6**	**8**	**10**	**12**
L = 30 ft	23.90	22.65	23.60	25.40	28.35
L = 60 ft	41.70	39.05	39.12	39.10	43.35
L = 90 ft	60.20	55.00	54.36	53.90	59.60
L = 120 ft	71.90	64.70	63.20	62.40	68.70

Table 3. Tee Beam Section Dimension *D* (inches) in Design Examples ($f_c^{'}$ = 8 ksi)

***S* (ft)**	**4**	**6**	**8**	**10**	**12**
L = 30 ft	23.82	22.55	23.50	25.30	28.23
L = 60 ft	41.55	38.85	38.96	38.90	43.15
L = 90 ft	59.90	54.80	54.10	53.68	59.30
L = 120 ft	71.40	64.25	62.75	62.00	68.25

Table 4. Tee Beam Section Dimension *D* (inches) in Design Examples ($f_c^{'}$ = 10 ksi)

***S* (ft)**	**4**	**6**	**8**	**10**	**12**
L = 30 ft	23.75	22.50	23.45	25.25	28.15
L = 60 ft	41.40	38.75	38.86	38.80	43.05
L = 90 ft	59.70	54.60	53.95	53.52	59.15
L = 120 ft	70.90	63.90	62.40	61.60	67.90

Calculated Reliability Indices Versus Target

In this study, a reliability analysis using the AFOSM combined with finite-element-based consideration of the lateral distribution of live load is used to validate actual reliability levels of the designed T-beam concrete bridges. The lateral distribution factors are computed by the 3-dimensional finite element analysis program BRUFEM (Hays et al 1994), which has demonstrated a good accuracy (Shahawy and Huang 2001). In BRUFEM, four-node PLATE and SHELL elements are used to model deck slab. Girders, stiffeners and beam type diaphragms are all modeled using standard beam elements. A rigid link is assumed to connect the centroid of the eccentric members to the midsurface of the slab. The rigid link does not exist physically, but it represents the manner in which the stiffness of eccentric members is mathematically formulated in finite element analysis. Figure 3 shows the three dimensional finite element cross section described above.

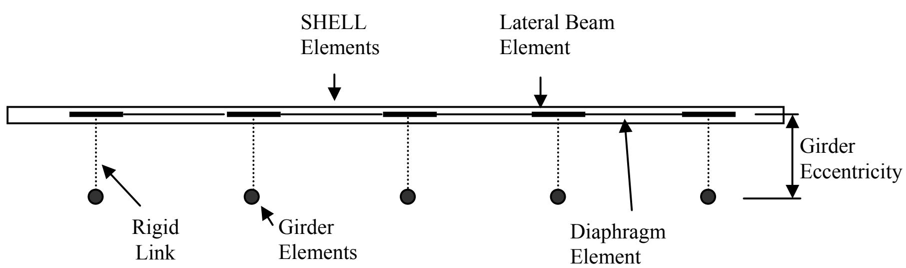
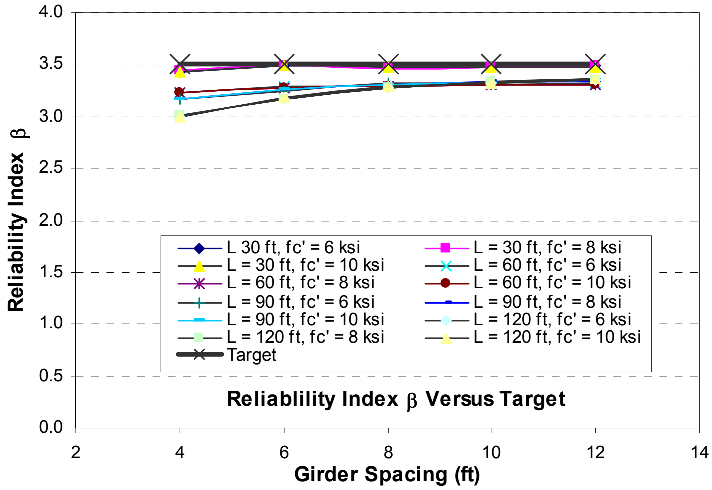


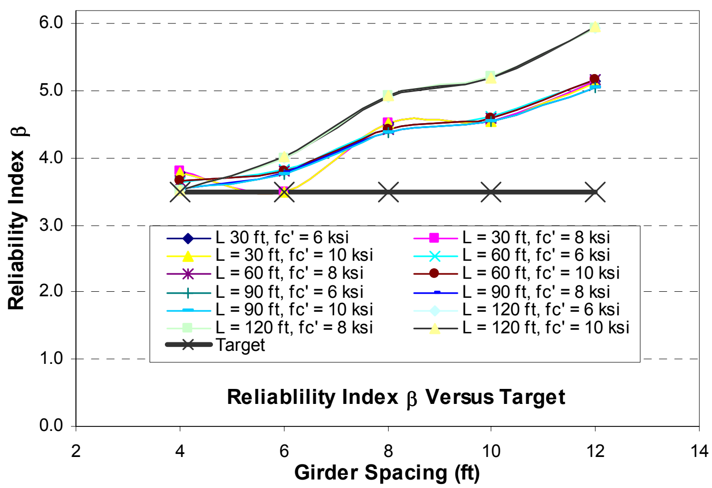
Fig. 3. Three Dimensional Finite Element Cross Section

According to AASHTO *LRFD* (1998) Commentary - C1.3.2.1, the target reliability index, β_T, is 3.0, 3.5, 3.8, and 4.0, respectively, for load modifier: η = 0.95, 1.0, 1.05, and 1.1. In this study, load modifier η = 1.0 is examined by AFOSM approach. Figure 4(a) shows the calculated reliability indices using live load distribution factor computed by LRFD *Specifications* (1998) but neglecting girder stiffness-related term. Figure 4(b) shows the calculated reliability indices using live load distribution factor obtained from 3D finite element analysis. From Figure 4(a), it can be seen that the reliability indices computed by AFOSM are distributed around the target β_T = 3.5. That some points locate below the target is mainly due to the statistical data selected for the design variables. For example, the bias factor for live load suggested by Novak (1993) ranges from 1.10 to 1.20. In this study it is taken as 1.15. Using the live load distribution factor obtained by BRUFEM, it can be seen from Fig. 4(b) that the reliability indices turn out to be sparsely distributed and generally above the target β_T very well. Reliability index β tends to increase with girder spacing.

Figure 5 shows the variation of reliability index by SFEM with span length and concrete compressive strength f_c'. It can be seen that the reliability index β almost remains constant with different values of f_c', while it significantly varies with span length.

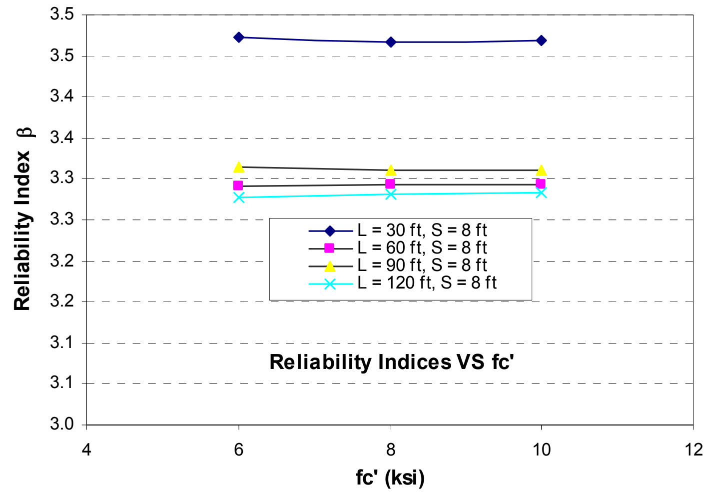


(a)

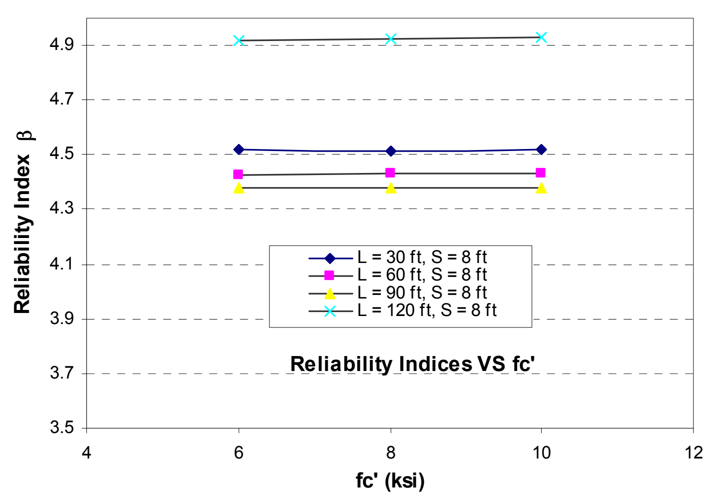


(b)

Fig. 4. Reliability Indices β VS Target



(a)



(b)

Fig. 5. Reliability Indices β VS L and f_c'

CONCLUSIONS

Using live load distribution factor obtained from 3-dimensional finite element analysis, the reliability indices based on the AFOSM are generally much higher than the target, especially in the case of the large girder spacing of 12 ft. The reliability indices of the example bridges are sparsely distributed. When using different concrete strengths $f_c^{'}$, reliability index remains almost the same. However, reliability index varies significantly with span length and girder spacing.

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