LRFD LIVE LOAD DISTRIBUTION USING REFINED ANALYSIS

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ABSTRACT

The AASHTO LRFD Bridge design specifications, Art. C4.1 encourages the use of refined methods of analysis that may lead to more accurate understanding of the structural behavior of bridges. However, the demands of the fast-paced design environment and the production deadlines are rarely conducive to the practicing engineer in regular practice and day-to-day designs. Calculation of live load distribution factors by refined analysis can produce more accurate results and extend their range of applicability for longer bridges, skewed support conditions, and for load rating bridges.

Using experience and guidance from previous research, analytical models are generated without the need for practicing engineers to get into the time-consuming and intricate details of setting up the finite element model and modeling the load paths and combinations. LEAP Software's prestressed concrete bridge design program, CONSPAN®, is now able to build the model, process either a grillage model or a beam and plate model (depending on the absence or presence of bridge skew), and automatically run a standard HS20 truck to generate accurate live load distribution factors.

Preliminary studies have shown that design moments are reduced by approximately 7-8 percent and, in some cases, lead to an 11 percent reduction in the number of strands for prestressed concrete girders, thereby providing great improvement in the overall efficiency of the design. Sample case studies showing the benefits of using refined methods of analysis to compute distribution factors vs. using the code-specified equations are presented as well as a discussion on the relevance of using the Lever Rule in computing distribution factors, effect of skew angles, and extending span ranges.

Keywords: LRFD, Refined Analysis, Distribution Factors, Lever Rule

INTRODUCTION

The AASHTO LRFD Bridge Design specifications[1], Art. C4.1 encourages the use of refined methods of analysis that may lead to more accurate understanding of the structural behavior of bridges. These specifications allow the use of both refined methods and approximate methods of structural analysis, and the designers are expected to use an appropriate method depending on the size, complexity, and importance of the structure. First introduced and adopted in 1994, and revised a few times since, these specifications have introduced considerable advances in the computation of live load distribution factors, compared to the AASHTO Standard specifications[2]. These new factors are now more consistent and accurate and are based on a wider range of parameters that are likely to affect the distribution of live loads when compared to the simplified and conservative S/D (i.e. spacing/constant) type equations in the Standard specifications.

Using the Approximate method, distribution factors for girders can be obtained based on LRFD Tables 4.6.2.2.2b-1 through 4.6.2.2.2g-1 and 4.6.2.2.3a-1 through 4.6.2.2.3c-1, if the following conditions are satisfied:

- The width of deck is constant.
- The number of beams is not less than four, unless otherwise specified.
- Beams are parallel and have approximately the same stiffness.
- The roadway part of overhang d_e does not exceed 3 ft.
- The curvature in plan is less than the limit specified in Art. 4.6.1.2[1].
- The cross-section is consistent with one of the cross-sections specified in LRFD[1] Table 4.6.2.2.1-1.

If any of these conditions are not satisfied, then the distribution factors, or perhaps the analysis itself, may have to be done using a refined method of analysis, such as a finite element or grillage.

If all the above conditions are met, but any of the ranges of applicability criteria in Tables 4.6.2.2.2b-1 through 4.6.2.2.2g-1 and 4.6.2.2.3a-1 through 4.6.2.2.3c-1 are not, the distribution factor is typically computed using the Lever Rule. This involves summing moments about one support/beam to find the reaction at another support/beam by assuming that the supported component is hinged at interior supports.

Unlike the Standard specifications, not only are there different equations depending on the number of lanes, but the moment distribution factors are quite different from the shear distribution factors. Furthermore, depending on the section type, there are other modifiers or checks that must be performed to accurately compute the distribution factors for the girder. The LRFD code lists skew angle correction factors separately for moment and support shear in Arts. 4.6.2.2.2e and 4.6.2.2.3c respectively. AASHTO Standard requires use of the lever rule to calculate the shear distribution for wheels adjacent to the support and use of the factor from Moment formula for other wheels, although this refinement is often ignored in practice. When working with section type k, i.e., I-girders, an additional check is required for rigid cross section behavior of exterior beams as specified in LRFD Art. 4.6.2.2.2d for Moment and Art. 4.6.2.2.3b for Shear.

Table 1. Typical Distribution Factors in Standard and LRFD Specifications (Interior Beams)

Distribution Factors	**Standard Specifications (Wheel Load DF)**	**LRFD Specifications (Lane Load DF)**
Moment I-girders (type-k)	For two or more lanes loaded: DF = S/5.5, or Lever Rule if S exceeds 14 feet	For two or more lanes loaded: $DFM = 0.075 + \left(\frac{S}{9.5}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1}$
	For one lane loaded: DF = S/7, or Lever Rule if S exceeds 10 ft	For one lane loaded: $DFM = 0.06 + \left(\frac{S}{14}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1}$
Shear I-girders (type-k)	same as for Moment above*	For two or more lanes loaded: $DFV = 0.2 + \left(\frac{S}{12}\right) - \left(\frac{S}{35}\right)^{2}$
	same as for Moment above*	For one lane loaded: $DFV = 0.36 + \left(\frac{S}{25}\right)$
Moment Adjacent Box Beams (type g with post-tensioning)	S/D, with K = 1.0 and $D = (5.75 - 0.5N_L) + 0.7N_L(1-0.2C)^2$ When $C \leq 5$, else $D = (5.75 - 0.5N_L)$ $C = K(W/L)$	For two or more lanes loaded: $DFM = k\left(\frac{b}{305}\right)^{0.6}\left(\frac{b}{12.0L}\right)^{0.2}\left(\frac{I}{J}\right)^{0.06}$
		For one lane loaded: $DFM = k\left(\frac{b}{33.3L}\right)^{0.5}\left(\frac{I}{J}\right)^{0.25}$
Shear Adjacent Box Beams (type g with post-tensioning)	same as for Moment above*	For two or more lanes loaded: $DFV = \left(\frac{b}{156}\right)^{0.4}\left(\frac{b}{12.0L}\right)^{0.1}\left(\frac{I}{J}\right)^{0.05}$
		For one lane loaded: $DFV = \left(\frac{b}{130L}\right)^{0.15}\left(\frac{I}{J}\right)^{0.05}$

*Except for wheels near support, which should be distributed by lever rule

Table 2. Typical Distribution Factors in Standard and LRFD Specifications (Exterior Beams)

Distribution Factors	Standard Specifications (Wheel Load DF)	LRFD Specifications (Lane Load DF)
Moment I-girders (type-k)	For two or more lanes loaded: Lever Rule	For two or more lanes loaded: $g = e\ g_{interior}$ $e = 0.77 + d_e/9.1$
	For one lane loaded: Lever Rule	For one lane loaded: Lever Rule
Shear I-girders (type-k)	same as for Moment above*	For two or more lanes loaded: $g = e\ g_{interior}$ $e = 0.6 + d_e/10.0$
	same as for Moment above*	For one lane loaded: Lever Rule
Moment Adjacent Box Beams (type g with post-tensioning)	S/D, with K = 1.0 and $D = (5.75 - 0.5N_L) + 0.7N_L(1-0.2C)^2$ When $C \leq 5$, else $D = (5.75 - 0.5N_L)$ $C = K(W/L)$	For two or more lanes loaded: $g = e\ g_{interior}$ $e = 1.04 + d_e/25.0$
		For one lane loaded: Lever Rule
Shear Adjacent Box Beams (type g with post-tensioning)	same as for Moment above*	For two or more lanes loaded: $g = e\ g_{interior}$ $e = 1.02 + d_e/50.0 \geq 1.0$
		For one lane loaded: Lever Rule

*Except for wheels near support, which should be distributed by lever rule

PRACTICAL BRIDGE DESIGN

Calculation of live load distribution factors using refined analysis produces more accurate results and extends the range of applicability for longer bridges and skewed support conditions. Many studies have shown the benefits of using refined methods of analysis for the complete modeling and design of bridges.

Aswad and Chen[3] used finite element analysis to show that using refined methods significantly reduces the amount of prestressing reinforcement by 11 to 14 percent. Depending on the structure type and location of beams, their study showed reduction in the live load moment for spread box beams by 13 to 17 percent and 18 to 24 percent for I-beams.

Similarly Barr, Eberhard and Stanton[4] showed that if distribution factors had been computed using finite element models, the live load could have been increased by 39 percent for the same prestressed concrete girder bridge designed using code values. The *PCI Bridge Design Manual*[5] recommends using finite element or grillage analysis for designing prestressed concrete bridges with high span-to-depth ratios because they allow a significant reduction in the required release strength or alternatively stretching of the span capability.

Today, the demands of the fast-paced design environment and the production deadlines are rarely conducive to the practicing engineer to employ advanced Finite Element methods in regular practice and day-to-day designs. The effort of building a finite element model (FEM) for a bridge design from ground-up is non-trivial and the savings in strands or the structural efficiency improvements may not justify the effort for common bridges. Software providers are increasingly stepping up to provide customized solutions that address the specific problem at hand while hiding the complexity of the modeling behind the scenes. LEAP Software's CONSPAN®[6] software application for the design of prestressed concrete bridge beams, is an effort in this direction.

CONSPAN now features an analysis engine that computes these refined distribution factors, by utilizing a rib-stiffened plate model (for solving skewed cases) or a plane grillage model (for solving non-skewed cases). The program generates the appropriate FEM based on the geometry input into the program, then generates various load case responses and calculates the corresponding refined load distribution factors. By modeling the entire bridge, the program computes the distribution factors at every tenth point along all beams. This variation along the span is generally much more accurate than a single distribution factor computed for the entire length of the beam. In addition to the positive moment distribution factors, the program also computes the negative moment distribution factors and separate shear distribution factors for both single lane and multi-lane cases at every tenth point along every beam in the bridge as shown in Fig. 1.

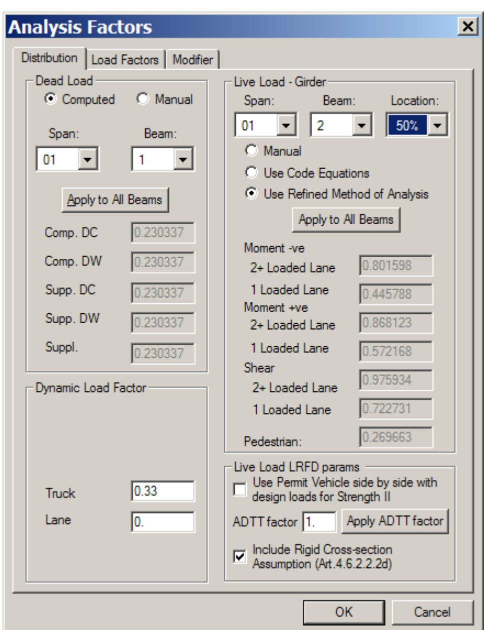

Fig. 1 Screen Shot from CONSPAN® Showing Options for Live Load Distribution Factor Computation (in LRFD Specification mode)

LRFD Art. 4.6.3.1 specifically requires providing a table of live load distribution coefficients for extreme force effects in each span to aid in permit issuance and rating of the bridge as shown in Fig. 2.

Print Screen

DISTRIBUTION FACTORS:

For Live Load, computed using Refined Methods of Analysis:

Location	+ve Moment		Shear	
	1Lane	2+Lane	1Lane	2+Lane
0.0L	0.689	0.689	0.706	0.706
0.1L	0.689	0.689	0.693	0.693
0.2L	0.659	0.659	0.668	0.668
0.3L	0.638	0.660	0.664	0.664
0.4L	0.629	0.661	0.676	0.676
0.5L	0.625	0.662	0.742	0.742
0.6L	0.625	0.661	0.693	0.693
0.7L	0.629	0.661	0.686	0.686
0.8L	0.638	0.660	0.670	0.670
0.9L	0.659	0.659	0.691	0.691
1.0L	0.689	0.689	0.706	0.706

Pedestrian : 0.147 (Calculated)

Comp. DC	:	0.147	(Calculated)
Comp. DW	:	0.147	(Calculated)
Suppl. DC	:	0.147	(Calculated)
Suppl. DW	:	0.147	(Calculated)
Suppl.	:	0.147	(Calculated)

Fig. 2 Screen Shot from CONSPAN® Output Showing Distribution Factors Computed Using Refined Methods of Analysis (in LRFD Specification mode)

The detailed theory and background of the modeling techniques used in the computation of the refined distribution factors can be obtained from the CONSPAN® User Manual and the NCHRP Report 12-26(2)[7]. Toorak Zokaie, one of this paper's authors, developed the FEM-based software (LDFAC) as part of the NCHRP study and the same algorithm is used to produce the refined distribution factors within CONSPAN. LDFAC itself has been calibrated and verified against numerous real bridges and forms the basis for the development of many of the live load distribution factor equations in the LRFD specifications.

Some situations where the computation of load distribution factors using more refined methods is more appropriate than the code formulae are listed below:

- When the parameters are outside the range of applicability specified in the code.
- When working with the older, conservative approximate distribution factors in the Standard specifications.
- When skew angles are varying for the beginning and end of span.
- When more accurate DF computation is needed during the bridge rating process.

- When an alternative to the conservative Lever Rule approach is desired. Currently, the Lever Rule method gives very conservative results for the design of the exterior beam (when used with an LRFD multiple presence factor of 1.2 for single lane).

There may be several other situations where a refined analysis (using finite element methods) is not required and perhaps a more accurate analysis and distribution for Live Load effects is all that is necessary prior to a typical girder design and subsequent design code checks for prestressed concrete.

The next few sections discuss some general metrics related to LRFD distribution factors.

MULTIPLE PRESENCE FACTORS

The LRFD Bridge Design specifications have introduced a more accurate distribution factor in many cases. However, the concept of using the Lever Rule with the multiple presence factor of 1.2 for single lane loads is quite conservative. These lane reduction factors or multiple presence factors take into account the probability or lack thereof that adjacent lanes will be loaded simultaneously. In the Standard, even though the use of the Lever Rule for computing distribution factor is recommended in certain cases, the lane load reduction factor for the case of single lane loaded was at 1.00 compared to 1.20 in LRFD. Some states have developed state specific criteria that avoid the more stringent 1.20 value for single lanes and use a 1.00 factor for single loaded lane case as well.

The multiple presence factors should not be used with the code provided live load distribution formulas, since these effects are already built into the various equations. However, they must be used when computing distribution factors based on the Lever Rule or when using refined analysis.

Table 3 Multiple Presence Factors (LRFD Table 3.6.1.1.2-1) and AASHTO Standard Specifications (Art 3.12.1) Reduction in Load Intensity Factors

Number of Loaded Lanes	AASHTO LRFD Table 3.6.1.1.2-1	AASHTO Standard Art. 3.12.1
1	1.20	1.00
2	1.00	1.00
3	0.85	0.90
>3	0.65	0.75

THE LEVER RULE

The LRFD Bridge Design specifications recommend the use of the Lever Rule in quite a few cases for computation of the distribution factors. According to Art. 4.6.2.2.1, when the beam spacing exceeds the range of applicability, the live load shall be the reaction of the loaded lane. Example illustrations for both interior and exterior girders are shown in Figs. 3 and 4. Due to the use of Lever Rule along with the multiple presence factor of 1.2 for single lane loaded case, there are instances where the two-lane load distribution factor is less than the single lane distribution factor. Therefore, it is always prudent to check both cases and take the governing value, irrespective of the number of lanes actually on the bridge. Some sample hand calculations showing the computation of Distribution Factors using the Lever Rule are presented below. This method is applicable to both I-girders and adjacent box beams.

It is noted that the Lever Rule was initially proposed as a conservative solution for uncommon cases (such as S>16′) and for the cases where, for any reason, a more accurate formula was not developed (such as shear distribution to exterior girders of multi-beam bridges).

EXAMPLE: COMPUTATION OF LEVER RULE FOR EXTERIOR BEAM

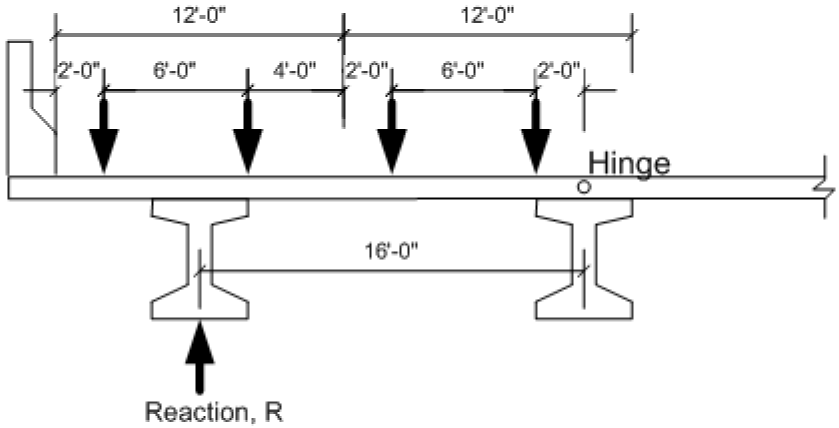

Fig. 3 Lever Rule, Exterior Beam

The distribution factor is the Reaction, R, about the Hinge.

Case (A). If Only One Lane is Loaded

The first axle is placed 2 feet from the face of the curb. The reaction is computed by summing moments about one support to find the reaction at the other support by assuming that the supported component is hinged at interior supports.

If P is lane (axle) load, then

$$R \times (16) = 0.5 \times P \times (20) + 0.5 \times P \times (14)$$

$$R = 0.5 \times P \times (34) / (16)$$

$$R = 0.5 \times P \times (2.125)$$

Multiple Presence Factor for single lane loaded = 1.20

$$R = 0.5 \times P \times (2.125) \times 1.20$$

$$R = 1.275\ P$$

Case (B). If Two Lanes Are Loaded

If P is lane (axle) load, then

$$R \times (16) = 0.5 \times P \times (20) + 0.5 \times P \times (14) + 0.5 \times P \times (8) + 0.5 \times P \times (2)$$

$$R = 0.5 \times P \times (44) / (16)$$

$$R = 0.5 \times P \times (2.75)$$

Multiple Presence Factor for two lanes loaded = 1.00

$$R = 0.5 \times P \times (2.75) \times 1.00$$

$$R = 1.375\ P$$

The distribution factor using Lever Rule is taken as the larger of these two cases, which in this example, is 1.375.

EXAMPLE: COMPUTATION OF LEVER RULE FOR INTERIOR BEAM

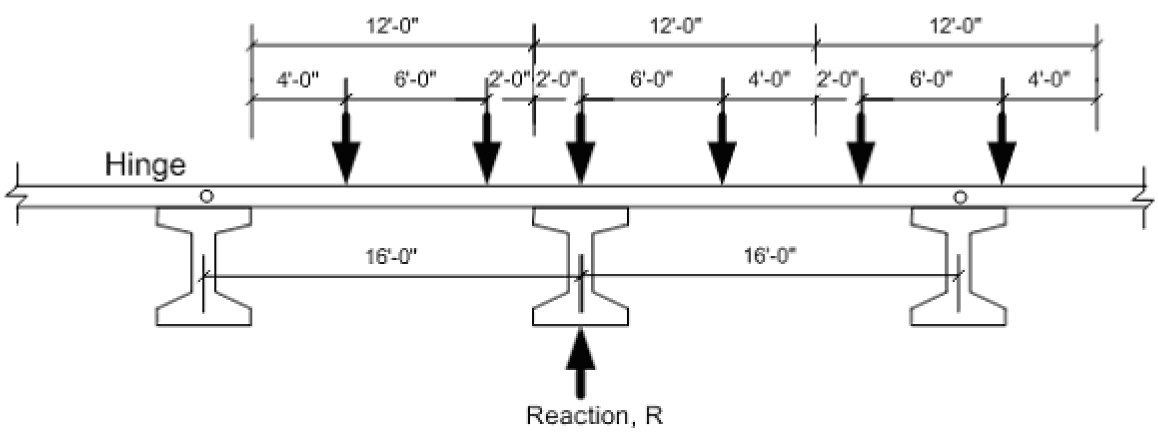

Fig. 4 Lever Rule, Interior Beam

The distribution factor is the Reaction, R, is independently computed about the Hinge on both the right and the left sides.

Case (A). If Only One Lane is Loaded

The first wheel line is placed directly over the interior girder of interest to generate the largest reaction. Since the girder spacing is only 16 feet, it cannot accommodate an additional two axles from a second truck on the right hand side. The reaction is computed by summing moments about the right hinge.

If P is lane (axle) load, then

$$R \times (16) = 0.5 \times P \times (10) + 0.5 \times P \times (16)$$

$$R = 0.5 \times P \times (26) / (16)$$

$$R = 0.5 \times P \times (1.625)$$

Multiple Presence Factor for single lane loaded = 1.20

$$R = 0.5 \times P \times (1.625) \times 1.20$$

$$R = 0.975\ P$$

Case (B). If Two Lanes are Loaded

In addition to the axles placed to the right of the interior beam, two other wheel lines can be placed to the left of the interior beam being studied. The reaction is computed by summing moments about the hinges independently on the right side and then the left side as shown below.

If P is lane (axle) load, then

$$R \times (16) = 0.5 \times P \times (10) + 0.5 \times P \times (16) + 0.5 \times P \times (6) + 0.5 \times P \times (12)$$

$$R = 0.5 \times P \times (44) / (16)$$

$$R = 0.5 \times P \times (2.75)$$

Multiple Presence Factor for two lanes loaded = 1.00

$$R = 0.5 \times P \times (2.75) \times 1.00$$

$$R = 1.375\ P$$

Case (C). If Three Lanes are Loaded

In addition to the axles placed for the two-lane loading, two more axles are placed, if they fit, to the right. The reaction is computed by summing moments about the hinges independently on the right side and then the left side as shown below.

If P is lane (axle) load, then

$$R \times (16) = 0.5 \times P \times (10) + 0.5 \times P \times (16) + 0.5 \times P \times (6) + 0.5 \times P \times (12) + 0.5x\ P \times (4)$$

$$R = 0.5 \times P \times (48) / (16)$$

$$R = 0.5 \times P \times (3.0)$$

Multiple Presence Factor for two lanes loaded = 0.85

$$R = 0.5 \times P \times (3.0) \times 0.85$$

$$R = 1.275\ P$$

The distribution factor using Lever Rule is taken as the largest of these three cases, which in this example is 1.375.

EFFECT OF RIGID CROSS-SECTION

When computing the live load distribution factors using the approximate methods, for beam-slab bridges with diaphragms or cross frames, the LRFD specifications require an additional investigation for multi-girder cross-section types. Arts. 4.6.2.2.2d and 4.6.2.2.3b require that both Moment and Shear distribution factors in such cases be checked to ensure that the DF computed using the tables is not less than that which would be obtained by assuming that the cross section deflects and rotates as a rigid cross section.

$$R = \frac{N_L}{N_b} + \frac{X_{ext} \sum^{N_L} e}{\sum^{N_b} x^2} \tag{1}$$

LRFD Commentary states that the additional investigation is required because the distribution factors for the above cases were not determined by taking into consideration diaphragms or cross-frames and the recommended procedure is an interim provision. Note that, in a wide bridge with stiff superstructure loaded primarily on one side of the bridge, the torsional rotation of the superstructure can cause higher deformation (and, thus, flexure) in the exterior beams, which is not predicted by the formulas. Some states have chosen not to consider these criteria unless the effectiveness of diaphragms on the lateral distribution of truck loads is investigated.

A sample calculation is provided below.

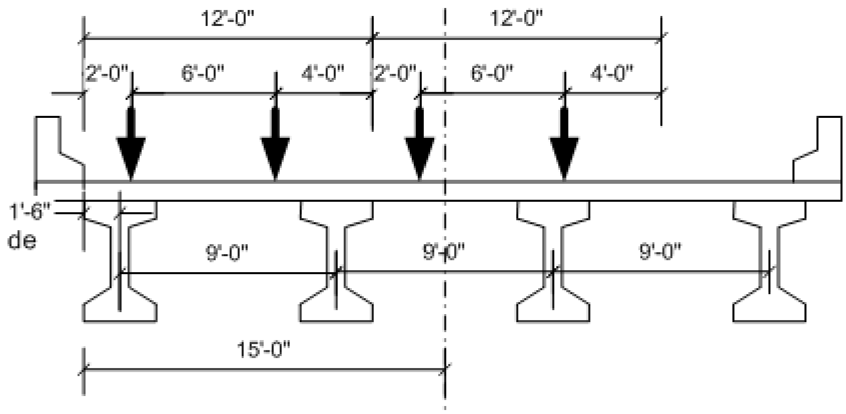

Fig. 5. Rigid Cross Section Example

X_{ext} = horizontal distance from the center of gravity of the pattern of girders to the exterior girder.

$X_{ext} = 15.0 - 1.5 = 13.50$ ft.

N_b = Number of beams, 4

$\sum^{N_b} x^2$ = Sum of the squares of the horizontal distance from the cg of the pattern of girders to each girder.

$= (13.50)^2 + (4.50)^2 + (4.50)^2 + (13.50)^2$

$= 405.00$ ft.^{2}

Case (A). N_L = Number of Loaded Lanes = 1

e = eccentricity of design truck from the cg of the pattern of girders

Since the first wheel is placed 2 feet away from the face of the curb, the cg of the truck from the edge of the curb is 3 + 2 = 5 feet. The distance between the cg of the pattern of girders and the edge of the curb is 15 feet. Therefore e is computed as shown below.

$e = 15 - 5 = 10$ ft.

Therefore, R according to equation C4.6.2.2.2d-1 is

$R = 1 / 4 + 13.50 \times 10 / 405 = 0.5833.$

Applying the single lane multi-presence factor of 1.20, the value becomes 0.70.

Case (B). NL = Number of Loaded Lanes = 2

Once again, the cg of the second lane from the edge of the curb is 12 + 2 + 3 = 17 feet.

The distance between the cg of the pattern of girders and the edge of the curb is 15 feet. Therefore e is computed as shown below.

$$e = 15 - 17 = -2 \text{ ft.}$$

For the first lane, we calculated e = 10. This value must be added to the e of the second lane positioned as shown in the figure above.

$$e = 10 + (-2) = 8 \text{ ft.}$$

R is now computed as

$$R = 2 / 4 + 13.50 \times 8 / 405 = 0.7667.$$

The multiple presence factor for two lanes loaded is 1.0. Therefore, the value remains 0.7667.

Therefore, the controlling distribution factor is from case B, i.e. 0.7667.

EFFECT OF HIGH STRENGTH CONCRETE

When the original distribution factor formulas were studied for introduction into LRFD, all analyses were done based on average strength (modulus) with the stiffness ratio (Ec-girder/Ec-deck) of approximately 1.2 (6,000 psi girder and 4,000 psi deck). Although they are not specifically tested against high-strength concrete, either at the time of development or later, the formulas should work for both steel and prestressed concrete and are, therefore, assumed to be valid for high-strength concrete as well.

WHEN THE RANGE IS EXCEEDED

Many of the distribution factor formulas for use with the approximate methods in the LRFD specifications have specific ranges of applicability for various parameters such as girder spacing, beam width, span length, number of beams, thickness of slab, etc. Although most of these ranges are suited for common bridge types, there are many situations when the range of applicability is exceeded. Under this scenario, the user may use the Lever Rule to get a conservative DF, or use the maximum values from the prescribed range with the formula or choose to perform a refined analysis to compute a more accurate distribution factor based on the specific situation. Each parameter has a different effect on the distribution factor and an appropriate alternative must be picked carefully. For example, you can extend the span length, and use the limiting value if span limit is 200', but if your bridge has a 300' span, you can use 200' in the formula to be conservative. However, girder spacing has an increasing effect, and becomes unconservative at higher values. If you use a high spacing, say 25', in the formula, the results may be unconservative. The other alternative, lever rule, on the other hand is very conservative. These situations can best be addressed using refined methods of analysis as well.

CASE STUDIES

Examples from the *PCI Bridge Design Manual* were chosen as a convenient reference. This study focuses primarily on moment effects only and not on shear. Also, both examples are for simple span and not continuous spans. The most typical concrete bridge girder sections were selected, I-girders and adjacent box beams. The effect of skew (45 degrees) and the effect of the use of high-strength concrete (11 ksi) were studied. Using different span lengths, the refined methods of analysis were compared in both the Standard specifications and the LRFD specifications. The cross-sections of these two bridges are reproduced in Figs. 6 and 7 from the reference for the convenience of the readers.

In the LRFD mode, the live loading used was the HL93 loading and in the Standard specifications mode, the live loading was the HS25 truck and HS25 lane loading. Instead of directly comparing the distribution factors, the approach was to compare the effect on the bridge design by actually completing the girder design and studying the difference in the number of prestressed strands required. Also, since the refined methods of analysis produced different distribution factors along the length of the girder rather than one value for the entire beam, a direct comparison of distribution factors was avoided.

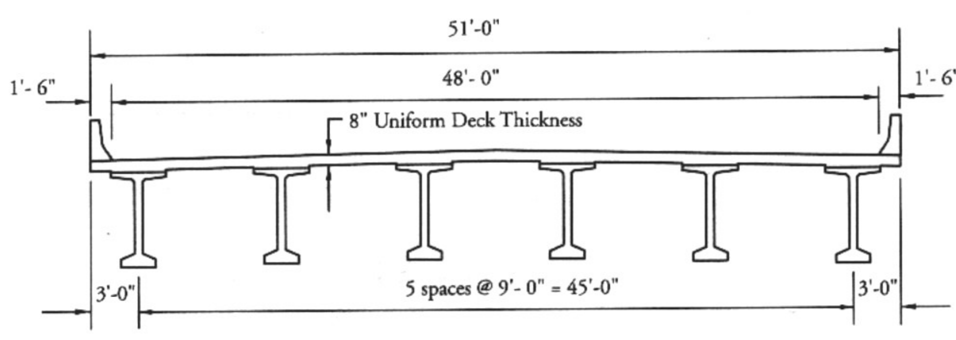

Fig. 6 AASHTO PCI BULB TEE, BT-72 Bridge (courtesy: *PCI Bridge Design Manual*)

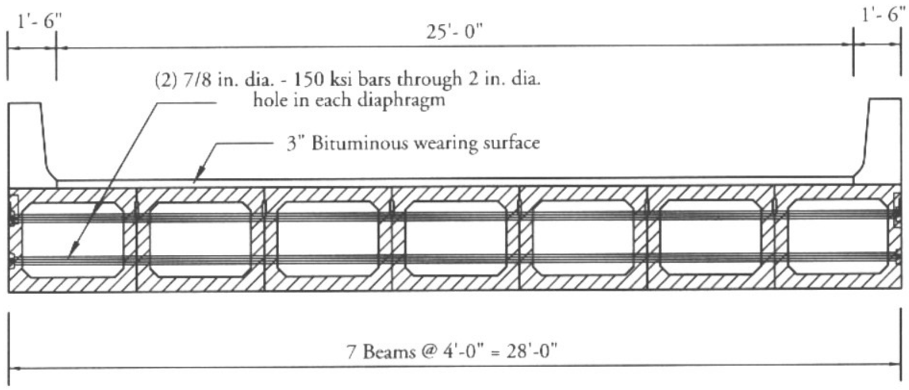

Fig. 7 Adjacent Box, BIII-48 Girder Bridge (courtesy: *PCI Bridge Design Manual*)

For exterior beams, on the BT-72 beam bridge, using the refined methods generally resulted in a lower number of strands when compared to using the equations for both the normal concrete as well as the high-strength concrete. The difference seemed to be more pronounced for longer spans as shown in Fig. 8. For interior beams, a similar trend was seen as shown in Fig. 9.

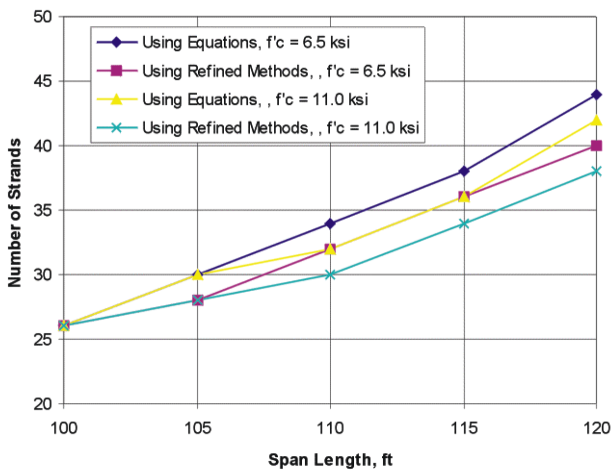

Fig. 8 BT-72 Beams, LRFD Specifications, Exterior Beams

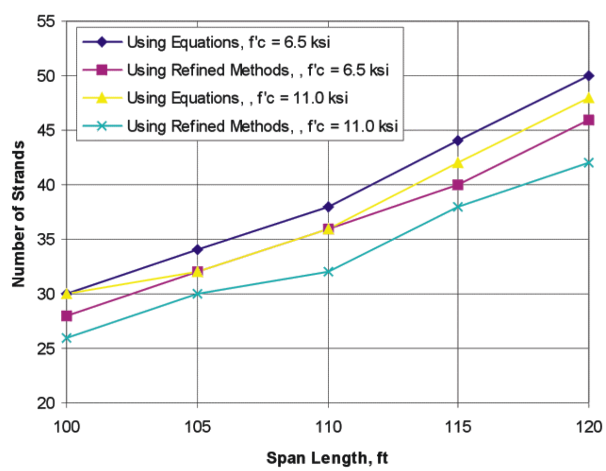

Fig. 9 BT-72 Beams, LRFD Specifications, Interior Beams

When the same bridge models with the BT-72 beams were studied under the Standard specifications with a HS25 live loading, it can be seen that the refined methods of analysis actually give a higher number of strands for the exterior beams when compared to the equations, as shown in Fig. 10. This can be attributed to the Lever Rule giving smaller values for the specific exterior beam configuration with a relatively small overhang distance and curb. Fig. 11 shows that for interior beams, a lower number of strands are required as expected using the refined methods of analysis.

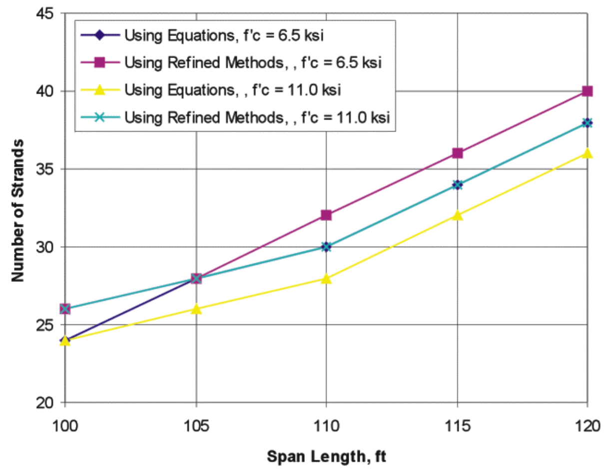

Fig. 10 BT-72 Beams, Standard Specifications, Exterior Beams

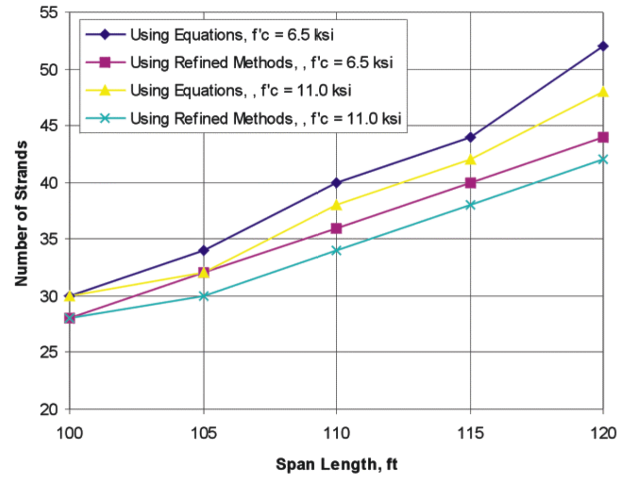

Fig. 11 BT-72 Beams, Standard Specifications, Interior Beams

Figs. 12 and 13 show the effect of a constant skew angle of 45 degrees on exterior and interior beams respectively. For the cases using equations, the skew correction factors (reduction factors for Moment) from Table 4.6.2.2.2e-1 have been applied. The general trend once again is the refined methods requiring fewer strands than when using the code equations. The same trend is also noticed for high-strength concrete. Also, compared to the case where there is no skew (Figs. 8 and 9), these graphs show a lower number of strands.

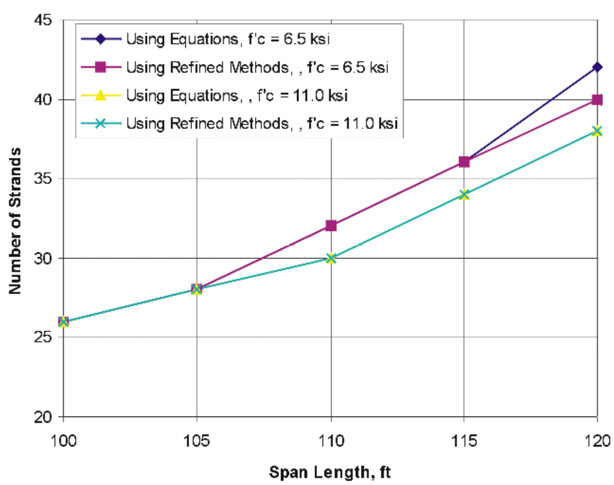

Fig. 12 BT-72 Beams, LRFD Specifications, Exterior Beams, Skew = 45º

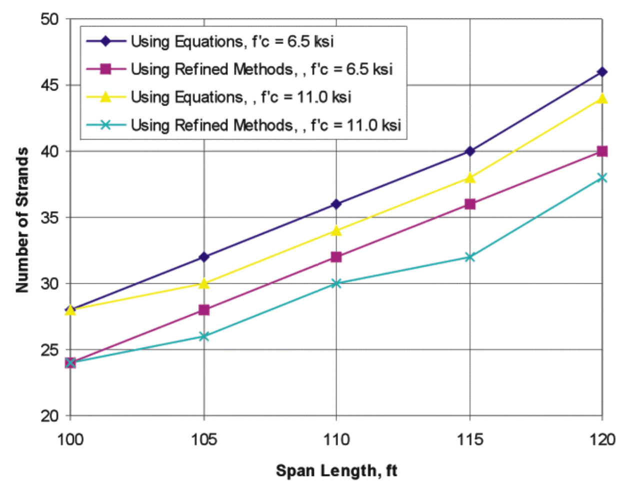

Fig. 13 BT-72 Beams, LRFD Specifications, Interior Beams, Skew = 45º

For the case of the adjacent box beams under LRFD specifications, for exterior beams, the difference between the number of strands computed using code equations and using refined methods is quite significant as shown in Fig. 14. The computation of distribution factors using the Lever Rule and the subsequent application of the multiple-presence factor of 1.2 may be contributing to the large value compared to the refined methods. The values for the interior beams are very close to the refined methods as shown in Fig. 15.

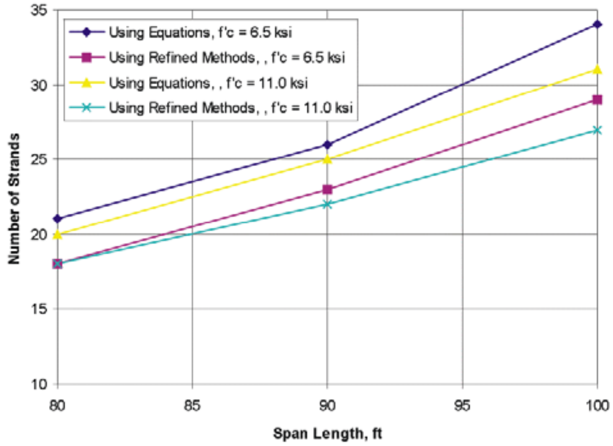

Fig. 14 Adjacent Box, B-III-48, LRFD Specifications, Exterior Beams

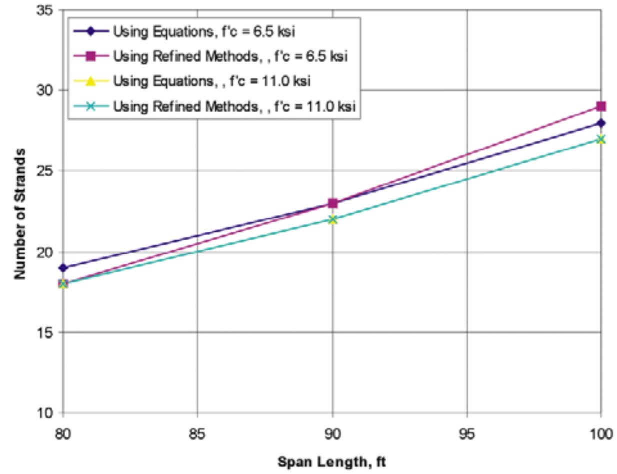

Fig. 15 Adjacent Box, B-III-48, LRFD Specifications, Interior Beams

When investigating the behavior of the box beams under the Standard specifications, for both interior and exterior beams, the general trend of refined methods of analysis is leading to a lower number of strands compared to the code equations as observed in Figs. 16 and 17.

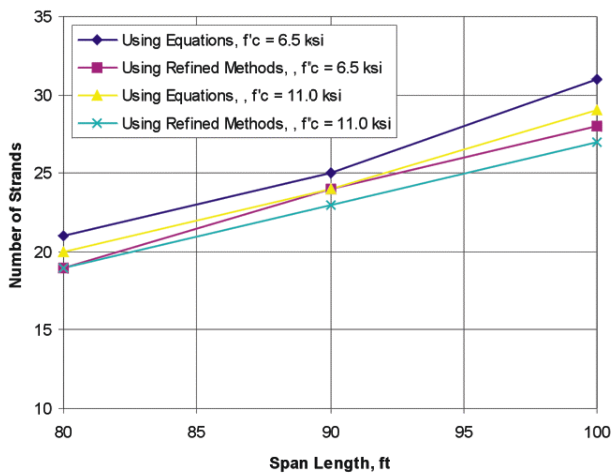

Fig. 16 Adjacent Box, B-III-48, Standard Specifications, Exterior Beams

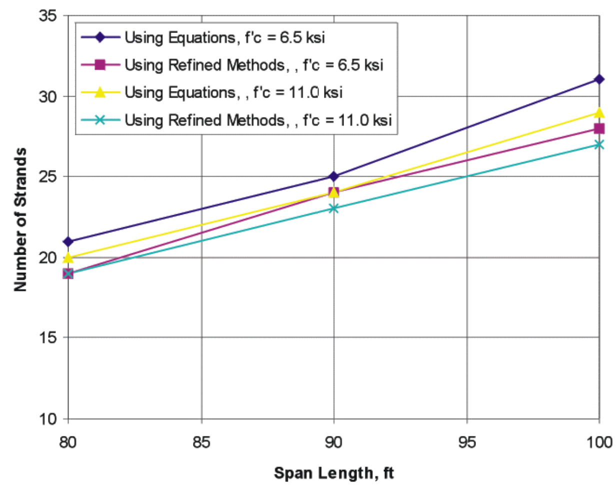

Fig. 17 Adjacent Box, B-III-48, Standard Specifications, Interior Beams

Figs. 18 and 19 show the effect of skew on the adjacent box beam bridge under LRFD specifications. Similar to the BT-72 beams, the refined methods of analysis leads to a lower number of strands compared to the code equations and correction factors for skew. The cases using high-strength concrete show a further decrease in the number of strands compared to cases using regular concrete.

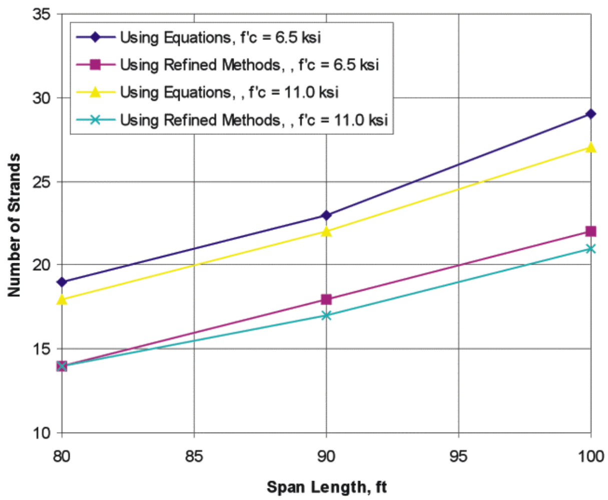

Fig. 18 Adjacent Box, B-III-48, LRFD Specifications, Exterior Beams, Skew =45°

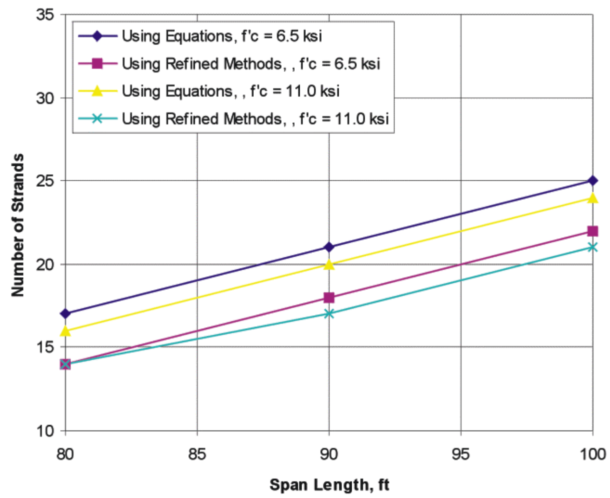

Fig. 19 Adjacent Box, B-III-48, LRFD Specifications, Interior Beams, Skew =45°

CORRECTION FACTORS FOR MULTI-BEAMS

Although the behavior of multi-beams is different than that of beam and slab bridges, the typical Lever Rule calculation still provides an acceptable conservative estimate in most cases, especially when side-by-side beams do not normally exceed 5' in width, and only one or two wheels affect the distribution factors. However, the Lever Rule application was recently reviewed and found to be overly conservative for multi-beam bridges, especially in the case of bending in exterior girders. Alternative formulas were proposed and have been tentatively approved, for inclusion into the next interims of the LRFD specifications.

The proposed revisions are:

Multi-lane:

Shear, Exterior: Proposed new Correction factor:

If $d_e + b - 2 < 0$ then $e = 1$ (2)

If $d_e + b - 2 > 0$ then $e = 1 + \mathrm{sqrt}[(d_e + b - 2) / 40]$ (3)

Single lane:

Moment, Exterior: Proposed new Correction factor:

$e = 1.125 + d_e/30'$ (4)

Shear, Exterior: Proposed new Correction factor:

$e = 1.25 + d_e/20'$ (5)

CONCLUSIONS

The approximate methods of computing distribution factors presented in the LRFD specifications are a vast improvement over the S/D type equations in the Standard specifications. However, there are still some inconsistencies and conservatism built into the LRFD specifications as well, due to the inherent nature of attempting to simplify a diverse set of parameters that generally affect load distribution of slab on stringer bridges.

In a recent survey of state bridge engineers in all 50 states, the live load distribution factors area of the LRFD specifications was stated by some states as producing unusual results and therefore in need of research and clarification. Similarly some states responded that they have either changed distribution factors for live load, deleted the lever rule for distribution factors for exterior beams, or are designing only for the interior beams, etc. in their specific state guidelines for LRFD. An ongoing NCHRP research project, Project 12-62, aims to simplify the LRFD live load distribution factors. This research may lead to some simplification and further consistency in the LRFD distribution factors using the approximate methods.

Numerous studies have already shown the accuracy and benefits of using refined methods for live load distribution factors. Based on the results of the case studies performed as part of this research effort, the authors recommend the use of more refined analysis methods to obtain accurate distribution factors to obtain better understanding of the structural behavior of the bridge, which may in turn lead to savings in material. By hiding the complexity of generation of a refined model and the associated task of modeling a valid finite element model, the CONSPAN program has allowed engineers access to a consistent, verified and easy to use refined methods of analysis to obtain more accurate distribution factors.

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