SIMPLIFIED ANALYSIS OF WEB SPLITTING IN PRETENSIONED CONCRETE GIRDERS

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ABSTRACT

This paper presents a simple approach to estimating the vertical tension that can develop in the end of a pretensioned bridge girder at release. This tension, which results from the concentrated forces applied by the transfer of prestress from pretensioned strands into the concrete, may lead to horizontal splitting at the ends of girders. In some cases, these splitting cracks may be unacceptable and may require repair. Therefore, this type of cracking should be reduced. Development of a simple strut-and-tie model for estimating the vertical tension will be discussed and then applied to several girder designs. Use of this model indicates that current AASHTO design requirements intended to control web cracking may not be adequate for some design situations. The analysis will be compared to other methods used to estimate web splitting. Strategies for reducing potential web splitting are discussed.

Keywords: Web Splitting, Cracking, Strut-And-Tie Model, Girder, Design, Reinforcement.

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INTRODUCTION

Web splitting cracks may develop in the prestress transfer zones at the ends of pretensioned concrete bridge girders. In some cases, repair of these cracks may be required, or steps may need to be taken during fabrication to reduce or prevent cracking in subsequent girders of similar design. It is therefore desirable to control web splitting to maintain durability and to avoid the additional costs and time required to repair these cracks and to modify girder details to control their growth.

To maximize the economy of prestressed concrete girders, an effective method must be developed for predicting which girders are susceptible to web splitting, and to have an effective means for reducing cracking. To this end, a simple strut-and-tie model has been developed for estimating splitting forces in the end regions of a girder, which will enable the design of reinforcement to limit the extent and severity of cracking.

DESCRIPTION OF WEB SPLITTING

At prestress release in pretensioned concrete girders, or within a few days of release, cracks extending from the end face into the girder may appear. These cracks are generally horizontal, although in some cases they may be inclined, following the trajectory of the draped strands.

The cracks often occur at the juncture of the web with the bottom flange, but they may also occur at other locations along the depth of the web. In some cases, only one crack forms, while in other cases, several cracks may form.

SIGNIFICANCE OF WEB SPLITTING

These cracks may open to widths great enough to allow penetration of water. However, after the girder is erected and the composite deck placed, the cracks tend to close under the load of the deck, and the cracks may be protected by the deck. If the end of the girder affected by cracking is at an expansion joint, the crack may continue to be exposed to water. However, the authors are not aware of any instances where cracks of this type have led to corrosion of reinforcement in a girder.

Repair of girders with web splitting cracks can be expensive. In some cases the effectiveness of the repair may be questionable because the cracks may reopen with time. The appearance of the girders may also be affected if epoxy is applied to the sides of the girder to seal the cracks, as is done in some states. Production and delivery of girders may also be delayed as repairs are performed or approvals are obtained.

Preventative measures may also be taken when web splitting occurs. These actions may include the addition of reinforcement or debonding of strands, and will also increase the cost of production of the girders.

FACTORS AFFECTING WEB SPLITTING CRACKING

Research has been conducted by several investigators regarding web splitting. The research has revealed that factors affecting web splitting include: the magnitude of the prestress force, the distribution of the prestress force between the top and bottom of the girder, the depth and shape of the girder, and the quantity and arrangement of (vertical) reinforcement crossing the potential cracks.

The two main methods for controlling web splitting are discussed below.

Reinforcement

Reinforcement is provided at the ends of girders to control cracking that may occur if tensile stresses in the concrete are high enough to cause cracking. Mild reinforcement cannot eliminate cracking, but can control it by distributing the cracking and limiting its size. Current design provisions require a minimum area of reinforcement for this purpose. However, field experience indicates that the reinforcement currently provided is sometimes inadequate to control cracking.

Since the mild reinforcement required by the current specifications is not prestressed, cracking cannot be eliminated, but can only be controlled and distributed. While a vertical prestress force could be applied at the ends of the girders to completely eliminate the possibility of web splitting, this approach is not an economical solution for pretensioned girders.

Debonding

Since web splitting is caused by the application of a high concentrated force at the end of the member, the distribution of the force over a greater distance would reduce the potential for cracking. Debonding may be used to distribute the transfer of the prestress force over a greater length of the girder, thereby reducing the total force acting at the end of the girder.

This approach is often applied during girder fabrication after web splitting cracks have been found. An increasing number of strands is debonded until web splitting is eliminated. The *AASHTO LRFD Specifications*[1] limits the number of debonded stands to 25% of the total number of pretensioned strands in a girder.

Because no method currently exists to estimate the amount of debonding required to eliminate web splitting, this trial-and-error approach can consume significant time and effort during the fabrication process. Furthermore, debonding of strands affects the shear capacity of pretensioned members, so debonding should not be performed without reevaluating the shear capacity of the girder.

RESPONSE OF OWNERS TO WEB SPLITTING

Owners of prestressed concrete girders that experience web splitting have responded in several ways, including the following:

- Requiring that web splitting cracks be repaired prior to installation in the bridge. The required repair may involve digging out cracks and filling them with epoxy; pressure injecting the cracks with epoxy; or exterior sealing of the cracks with a silane sealer.
- Requiring that measures be taken to eliminate cracking in subsequent girders, including the addition of reinforcement and debonding of strands. Girders with cracking may be accepted with a repair.
- Not requiring any repair.

CURRENT DESIGN APPROACH

The *AASHTO LRFD Bridge Design Specifications*[1] and the AASHTO *Standard Specifications for Highway Bridges*[2] address web splitting in Articles 5.10.10.1 and 9.22, respectively. The requirements are essentially the same in both specifications. The provisions require that a minimum area of reinforcement be provided near the ends of all girders to resist a force equal to 4 percent of the prestress force. This reinforcement must be provided within a distance of no more than 20% of the girder height from the end of the girder. The distribution of the pretensioning force across the end of the girder and the girder cross-section type are not considered. The area of reinforcement is computed from the force equal to the specified percentage of the prestress force using a working stress of 20 ksi. The use of a higher working stress of 24 ksi has been proposed, but this has not been approved. Higher working stresses in this reinforcement would result in increased crack widths if tensile stresses at the end of the girder are high enough to cause cracking. The specifications do not provide any method to determine the potential for web splitting.

STRUT-AND-TIE MODEL FOR WEB SPLITTING

The strut-and-tie model has been developed for use in the design of concrete members within so-called “disturbed regions”, or locations where conventional beam theory is not strictly applicable. It is generally used at the strength limit state, but can also be used effectively in many situations for the service limit state. Perhaps the most comprehensive presentation of the basic concepts of strut-and-tie modeling is found in an article by Schlaich, et al.[3] The *AASHTO LRFD Bridge Design Specifications*[1] contain provisions for the use of strut-and-tie modeling as a general design approach.

A strut-and-tie model provides a method for visualizing the flow of forces through a concrete component. It can be assumed that the flow of forces through the component is resisted by a truss-like model. The elements of the model are:

- compression struts
- tension ties
- nodes

The compression struts and tension ties are uniaxial elements that resist only compression or tension, respectively. The nodes are the connections between struts and ties. The most effective models closely follow the flow of elastic forces.

LOAD PATH METHOD

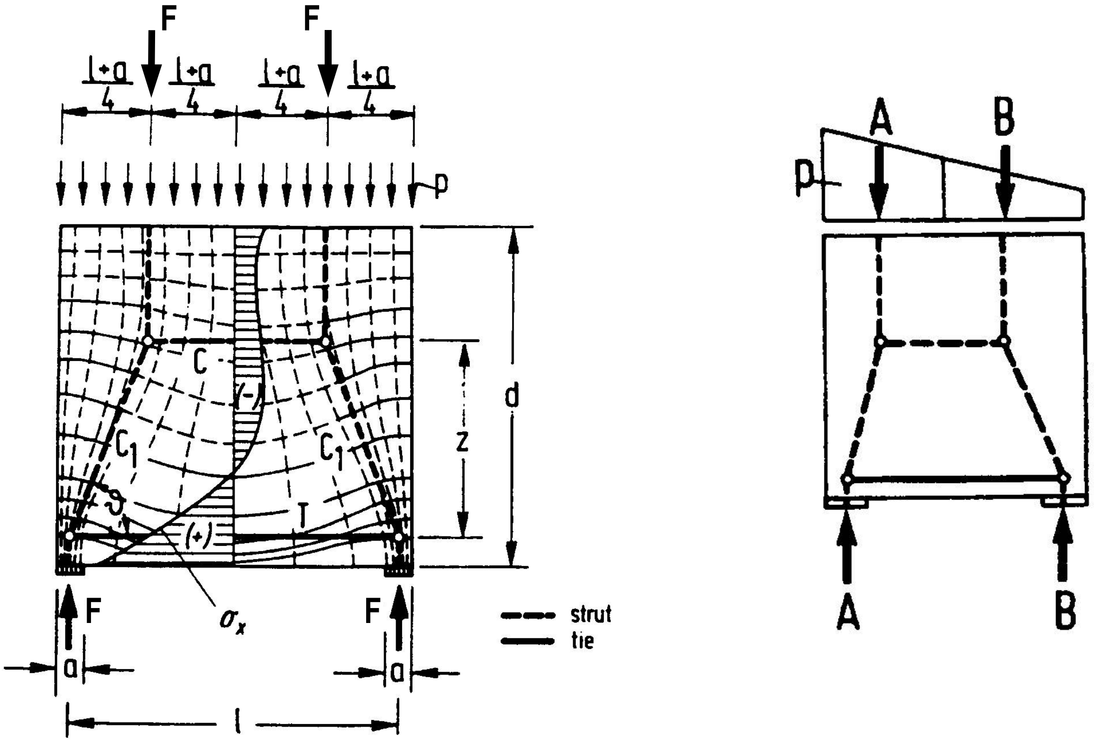
A strut-and-tie model can be developed based on several approaches, including the use of elastic analyses, such as finite element analysis. However, the simplest approach is the load path method.

The load path method for developing a strut-and-tie model for a particular component and loading can be illustrated using Figure 1, which is given in Schlaich, et al.[3] In Figure 1a), a simple strut-and-tie model is shown for a member with equal concentrated loads placed near the edges of the member. Stress contours from an elastic analysis are overlaid on the strut-and-tie model. Stresses acting across the midline of the member are also shown on the figure. Tensile stresses occur close to the end of the member (bottom) and compressive stresses occur within the member. The transverse elements of the strut-and-tie model (“struts” indicating compression and “ties” indicating tension) are positioned near the centroids of these areas. The longitudinal struts exit the face of the member opposite the applied loads where stress resultants equal to the applied loads are located. Since the applied loads are equal and symmetrically placed, the resisting stress at the opposite face is uniform and the two stress resultants are located at the quarter points.

Figure 1b) shows a slightly different model for a member, with unequal loads applied at the end (bottom). For this model, the transverse tie and strut are located at about the same place as for the model with equal loads, but the longitudinal struts at the end of the member opposite the applied loads are shifted to align with the position of the stress resultants equal to the applied forces A and B.

When rotated clockwise 90º, the strut-and-tie models shown in Figure 1 are similar to prestressed concrete girders with concentrated forces applied at the location of the strands. Since the number of strands differs between the top and bottom of the girder depending on the number and arrangement of draped strands and straight strands, the applied forces will be unequal in models developed for the ends of pretensioned girders.

There are several basic considerations in constructing the strut-and-tie model using this approach. Concentrated forces applied at one end of a member are resisted by the member in a nonlinear manner. This region of nonlinear behavior, which is referred to as a "disturbed" zone or "D Region", is assumed to extend into the member for a distance equal to the depth of the member. The complete strut-and-tie model must be developed within this region. The nonlinear flow of forces (stresses) within the D region can be seen in the superimposed stress contours in Figure 1a).



a) Equal Loads b) Unequal Loads

Figure 1 Examples of the Load Path Method (from Ref. 3)

At the interior boundary of the D region, stresses in the member caused by the applied loads can be determined using the usual assumptions of elastic analysis. The locations of stress resultants equal to the applied loads are determined considering the trapezoidal stress distribution and the shape of the girder cross section. The flanged shape of pretensioned concrete girders results in a force distribution across the depth of the girder that differs from the simple trapezoidal shape of the stress diagram.

At the ends of the member, where the concentrated forces are applied to the component, the model is assumed to provide an equal and opposite reaction in the form of a strut. The strut extends into the model a short distance to a node, where the force is redirected by the action of an intersecting tension tie that is oriented perpendicular to the axis of the member for the models shown in Figure 1. An equal and opposite force, provided by a compression strut that is also oriented perpendicular to the axis of the member, must be located further into the member to complete the model and provide equilibrium across the section. The vertical strut and tie represent the center of an area of concrete in compression or a group of reinforcing bars, respectively. Therefore, the entire area contributing to the strut and tie must be fully contained within the D region, and not at its boundaries. In Figure 1a), the locations of the tie and strut for the component are shown to correspond to the approximate centroids of areas of tension and compression that the elastic analysis indicates form across the midline of the component. The tension tie is located in the triangular region of tensile stress near the face of the member, while the compression strut is located near the center of the more uniform compression stress.

Perhaps the most important feature of the load path method is that the longitudinal struts are located by determining the locations of stress resultants equal to the applied forces at the interior boundary of the D region, as shown for both situations in Figure 1. Once the locations of these resultants are found, the model can be constructed.

MODEL FOR WEB SPLITTING – GIRDER WITH DRAPED STRANDS

For web splitting analysis, a strut-and-tie model can be developed using the above approach to represent the end of a pretensioned girder with draped strands (Figure 2). In the case of a girder, the two applied forces are unequal. While the force corresponding to the draped strands will occur at a small slope, this slope is neglected to simplify the analysis. The reaction at the end of the girder is also neglected in this analysis.

The force from the pretensioned strands is applied to the end of the member as a concentrated load. Since the prestress force is actually transferred from the strands to the concrete over a distance (the transfer length), this is not an accurate assumption. While the AASHTO design specifications indicate that the transfer length may be 60 strand diameters or 50 strand diameters, this is an upper bound. Depending on the concrete strength and the surface condition of the strands, the transfer length can be much shorter. Therefore, it seems reasonable to make the assumption that the prestress force is applied to the end of the member as a concentrated force. This may result in an overestimation of the effect of the prestress in the analysis, but it is expected that this overestimation is not large.

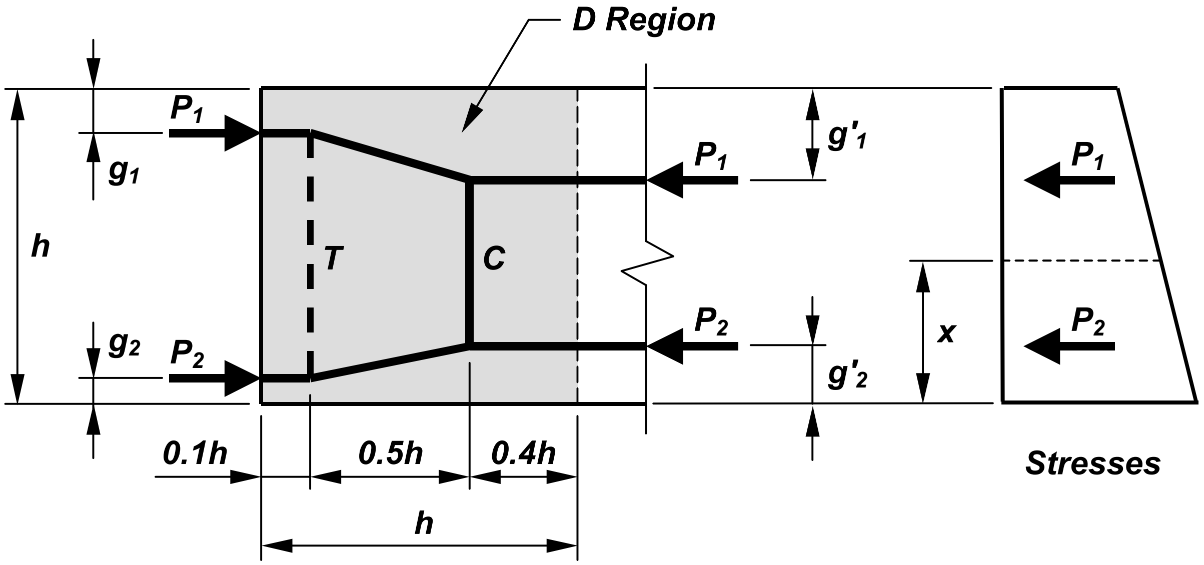


Figure 2 General Strut-and-Tie Model for Web Splitting in a Pretensioned Girder with Draped Strands

For the purpose of this model, the tension tie near the end of the member is assumed to be located at 0.1h from the end of the member, where h is the full depth of the precast girder. This location corresponds well with the location of the centroid of the elastic tensile force in Figure 1 and the requirement of the *AASHTO LRFD Specifications*[1] that the reinforcement for web splitting be placed within h/5 (or 0.2h) from the end of the girder. This requirement means that the center of this reinforcement is located at 0.1h.

Assuming the region of tension extends 0.2h from the end of the member, the remainder of the length of the D region is in compression. The compression strut is centered in that region. Therefore, the compression strut is assumed to be located at 0.4h from the end of the D region, or 0.6h from the end of the girder. This means that the moment arm between the strut and tie is equal to 0.5h. While this distribution of forces is not theoretically correct, especially for elastic conditions where the end of the girder is not cracked, it is appears appropriate for a simplified analysis.

Once the geometry of the model is determined, the forces in the crossing strut and tie can be determined. These forces will be equal but opposite. The tensile force in the tie near the end of the member is used to determine the required area of reinforcement to resist splitting. It may also be used to give an indication of the potential for web splitting cracking.

A model must be developed for each strand configuration and type of girder. It does not appear that the procedure can be reduced to a system of one or more equations. However, the procedure has been implemented in a spreadsheet that greatly simplifies the solution of the model.

MODEL FOR WEB SPLITTING – GIRDER WITH STRAIGHT STRANDS AT BOTTOM

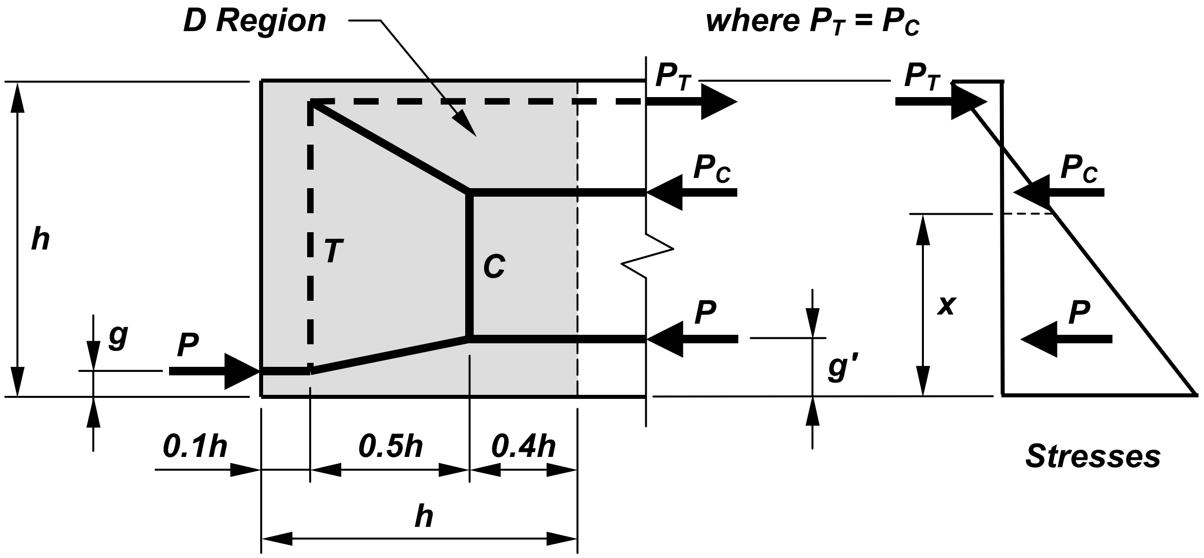


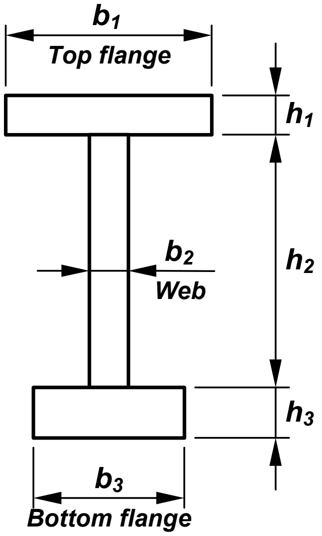
Figure 3 General Strut-and-Tie Model for Web Splitting in a Pretensioned Girder with Straight Strands in the Bottom Flange

Using the same type of approach and assumptions as used above for girders with draped strands, a strut-and-tie model can be developed for a pretensioned girder with only straight strands in the bottom flange as shown in Figure 3.

Analysis will not be performed in this paper using this model. It has been found that, for practical designs, the prestress forces must be relatively low and the force developed in the tension tie is not as large as for girders with draped strands.

SIMPLIFIED CROSS SECTIONS FOR ANALYSIS

To simplify application of the model to the solution of the web splitting situation, the shape of the precast prestressed concrete girder is represented by three rectangular areas: the top flange, web, and bottom flange. This is illustrated in Figure 4. A table of dimensions for the simplified representation of several popular girder cross sections accompanies the figure. The simplified dimensions have been selected to provide section properties as close as possible to the original section properties. The cross-sectional area, moment of inertia and distance from the bottom of the girder to the centroid of the cross section are within ±0.70% of the correct values for these properties using the dimensions listed.



	b_1 (in.)	h_1 (in.)	b_2 (in.)	h_2 (in.)	b_3 (in.)	h_3 (in.)
Type II	12.00	7.50	6.00	19.60	18	8.90
Type IV	20.00	11.00	8.00	30.70	26	12.30
Type VI	42.00	7.30	8.00	51.90	28	12.80
PCI BT-54	42.00	4.70	6.00	41.10	26	8.20
PCI BT-72	42.00	4.70	6.00	59.10	26	8.20
NEBT 1400	47.24	4.80	7.09	39.52	31.89	10.80
NEBT 1800	47.24	4.80	7.09	55.27	31.89	10.80
NU 1350	49.20	3.85	6.90	40.90	39.40	8.35
NU 1800	49.20	3.90	6.90	58.75	39.40	8.25
NU 2400	49.20	3.90	6.90	82.30	39.40	8.30

Figure 4 Simplified Girder Cross Section Dimensions

Using the simplified girder cross section dimensions in Figure 4, the locations of the resultant forces at the interior boundary of the D region are determined. A spreadsheet is used to solve for this location, although a closed form solution is possible. The moments resulting from the offset between the force location at the end of the member and location of the corresponding force at the interior boundary of the D region are equal for the upper and lower forces.

REINFORCEMENT TO CONTROL WEB SPLITTING

The area of reinforcement required to resist the force in the tension tie is computed using a working stress. A working stress of 20 ksi has been used in the specifications for years. It has been proposed that a higher working stress of 24 ksi be used, but this has not been approved. Higher working stresses in this reinforcement will lead to increased crack widths, where tensile stresses at the end of the girder are high enough to cause cracking. For this paper, a working stress in the web splitting reinforcement of 20 ksi is used.

PREDICTION OF WEB SPLITTING

The distribution of tensile stresses at the end of the girder is generally triangular, as shown in Figure 1a). Assuming that the area of tensile stress extends into the girder a distance 0.2h, the maximum tensile stress at the end of the girder can be estimated as two times the average stress over the assumed tensile area in the D region. Therefore, the maximum tensile stress can be computed as:

$$f_{tmax} = 2T / (0.2h\ b_w), \text{ where } b_w = \text{ the thickness of the web.}$$

This assumption has not been experimentally verified and is expected to be conservative, that is, it is expected that this equation will predict a higher tensile stress than actually occurs. However, the stress computed using this equation can be used to provide a relative tendency for cracking between different sections and strand patterns.

This stress would be compared to the modulus of rupture, f_r, for the girder concrete at release, which is generally computed using the relationship from the *AASHTO LRFD Specifications*[1]:

$$f_r = 0.24 \sqrt{f'_{ci}} \text{ (ksi)} \qquad \text{LRFD Art. 5.4.2.6}$$

Some investigators have reported that this relationship may be conservative (underestimating the tensile strength of concrete), especially for high strength concrete. This may introduce additional conservatism into the prediction of cracking by the strut-and-tie model.

APPLICATION OF THE STRUT-AND-TIE MODEL TO SELECTED GIRDERS AND STRAND PATTERNS

The strut-and-tie model developed above has been applied to two strand patterns for each of the types of girders for which dimensions are given in Figure 4. The results of the analyses are given in Tables 1 through 4. All strands are 0.6 in. diameter low relaxation strands, except 0.5 in. diameter strands are used for the AASHTO Type II. All strand patterns begin at 2 in. from the bottom or top of the girder, except for the NEBT girders, for which the pattern begins at 70mm from the bottom and 50mm from the top.

Case 1 (Tables 1 and 2) represents a strand pattern in which the bottom flange has been filled with strands (except for positions taken by draped strands at midspan) and strands have been draped to the full height. The number of rows of draped strands is equal to the number of rows of strands in the bottom flange plus two. This case represents a strand pattern close to the maximum number of strands possible for the section. Some strand patterns shown are not practical because of the very large prestress forces required.

Case 2 (Tables 3 and 4) represents a draped strand pattern with approximately half the strands of Case 1. The number of draped and straight strands used in Case 1 is reduced by half, then rounded down to an even number of strands. Draped strands are still raised to the maximum height. The distances from the top and bottom of the girder to the centroids of the top and bottom groups of strands are reduced because of the decreased number of strands.

In Tables 2 and 4, the column labeled "x_w / web ht." indicates the fraction of the web height that corresponds to the dimension x. The value x_w is computed as x minus the distance from the bottom of the girder to the bottom of the web. For this purpose, the bottom of the web is taken as the top of the radius fillet between the bottom flange and web for the NEBT and NU girders which have curved fillets at the junction of the web and flange. The top of the web for these same girders is taken as the bottom of the radius fillet at the top flange.

The stress resultants above and below this location (at the distance x above the bottom of the girder) are exactly equal to the applied forces, P_1 and P_2. Therefore, this is a plane of zero shear, which is also the location of maximum horizontal moment across the section (see discussion of Libby's Method below). This location is considered by some to be the most likely site for web splitting to occur.

	N_{ps} Top	g_1 (in.)	g'_1 (in.)	N_{ps} Bottom	g_2 (in.)	g'_2 (in.)	F_{po} (kips)	T (kips)	T / F_{po} (%)
Type II	12	7.00	9.22	18	4.22	5.70	930	45.8	4.93
Type IV	18	10.00	14.94	50	6.40	8.18	2,988	144.7	4.84
Type VI	20	11.00	19.58	60	7.33	10.19	3,515	209.4	5.96
PCI BT-54	12	7.00	13.50	28	4.00	6.79	1,758	127.0	7.23
PCI BT-72	12	7.00	19.01	28	4.00	9.15	1,758	175.9	10.01
NEBT 1400	12	6.89	14.30	42	5.29	7.41	2,373	141.8	5.98
NEBT 1800	12	6.89	20.10	42	5.29	9.06	2,373	196.5	8.28
NU 1350	12	7.00	15.17	44	3.88	6.11	2,461	162.3	6.59
NU 1800	12	7.00	21.93	44	3.88	7.95	2,461	222.0	9.02
NU 2400	12	7.00	32.05	44	3.88	10.71	2,461	279.5	11.36

See Figure 1 for definitions of "g" dimensions. N_{ps} = number of strands at location.
F_{po} = total initial prestress force = $A_{ps} N_{ps} f_{po}$, where f_{po} = 202.5 ksi

Table 1 Results of Analyses for Selected Sections and Strand Patterns – Case 1

	x from Bot. (in.)	x_w / web ht.	M (k-ft)	A_s (in^2)	h/5 (in.)	A_s / (h/5) (in^2/in.)	f_{tmax} (ksi)	f_{tmax} / f_r
Type II	15.05	0.41	825.1	2.29	7.20	0.32	2.12	3.47
Type IV	23.93	0.51	3,906.4	7.23	10.80	0.67	3.35	5.47
Type VI	32.20	0.46	7,539.8	10.47	14.40	0.73	3.64	5.94
PCI BT-54	22.73	0.40	3,429.3	6.35	10.80	0.59	3.92	6.41
PCI BT-72	31.02	0.42	6,332.8	8.80	14.40	0.61	4.07	6.65
NEBT 1400	24.42	0.56	3,907.7	7.09	11.02	0.64	3.63	5.93
NEBT 1800	31.37	0.51	6,963.7	9.83	14.17	0.69	3.91	6.39
NU 1350	22.15	0.55	4,308.3	8.11	10.62	0.76	4.43	7.24
NU 1800	30.17	0.52	7,870.6	11.10	14.17	0.78	4.60	7.52
NU 2400	40.30	0.48	13,208.3	13.98	18.90	0.74	4.29	7.01

x = distance from bottom of girder to point dividing the trapezoidal stress into areas with resultants equal to the applied forces.
M = internal moment in D region = 0.5h (T) = $(g'_1 - g_1) P_1$ = $(g'_2 - g_2) P_2$
A_s = area of reinforcement for tension tie = T / f_s, where f_s = 20 ksi
f_r = modulus of rupture (tensile strength) of concrete at release = $0.24 \sqrt{f'_{ci}}$ (ksi)

Table 2 Additional Results of Analyses for Selected Sections and Strand Patterns – Case 1

	N_{ps} Top	g_1 (in.)	g'_1 (in.)	N_{ps} Bottom	g_2 (in.)	g'_2 (in.)	F_{po} (kips)	T (kips)	T / F_{po} (%)
Type II	6	4.00	8.61	8	2.50	5.96	434	47.7	10.99
Type IV	8	5.00	17.48	24	3.50	7.66	1,406	162.5	11.55
Type VI	10	6.00	22.60	30	4.00	9.53	1,758	202.7	11.53
PCI BT-54	6	4.00	13.67	14	2.60	6.74	879	94.4	10.74
PCI BT-72	6	4.00	19.17	14	2.60	9.10	879	111.1	12.64
NEBT 1400	6	3.94	15.71	20	3.54	7.07	1,143	112.6	9.86
NEBT 1800	6	3.94	21.25	20	3.54	8.73	1,143	128.8	11.28
NU 1350	6	4.00	16.39	22	2.52	5.90	1,230	123.1	10.00
NU 1800	6	4.00	23.10	22	2.52	7.73	1,230	142.1	11.55
NU 2400	6	4.00	33.21	22	2.52	10.49	1,230	163.0	13.25

See Figure 1 for definitions of "g" dimensions. N_{ps} = number of strands at location.
F_{po} = total initial prestress force = $A_{ps} N_{ps} f_{po}$, where f_{po} = 202.5 ksi

Table 3 Results of Analyses for Selected Sections and Strand Patterns – Case 2

	x from Bot. (in.)	x_w / web ht.	M (k-ft)	A_s (in^2)	h/5 (in.)	A_s / (h/5) (in^2/in.)	f_{tmax} (ksi)	f_{tmax} / f_r
Type II	15.66	0.45	857.7	2.38	7.20	0.33	2.21	3.61
Type IV	21.86	0.42	4,386.2	8.12	10.80	0.75	3.76	6.15
Type VI	29.60	0.40	7,296.1	10.13	14.40	0.70	3.52	5.75
PCI BT-54	22.56	0.40	2,548.8	4.72	10.80	0.44	2.91	4.76
PCI BT-72	30.86	0.42	3,999.6	5.56	14.40	0.39	2.57	4.20
NEBT 1400	22.72	0.49	3,104.1	5.63	11.02	0.51	2.88	4.71
NEBT 1800	29.93	0.48	4,565.1	6.44	14.17	0.45	2.56	4.19
NU 1350	21.08	0.51	3,267.0	6.15	10.62	0.58	3.36	5.49
NU 1800	29.24	0.50	5,035.3	7.10	14.17	0.50	2.95	4.81
NU 2400	39.48	0.47	7,700.8	8.15	18.90	0.43	2.50	4.08

x = distance from bottom of girder to point dividing the trapezoidal stress into areas with resultants equal to the applied forces.
M = internal moment in D region = 0.5h (T) = $(g'_1 - g_1) P_1 = (g'_2 - g_2) P_2$
A_s = area of reinforcement for tension tie = T / f_s, where f_s = 20 ksi
f_r = modulus of rupture (tensile strength) of concrete at release = $0.24 \sqrt{f'_{ci}}$ (ksi)

Table 4 Additional Results of Analyses for Selected Sections and Strand Patterns – Case 2

OBSERVATIONS FROM ANALYSIS OF SELECTED GIRDERS

The most significant finding of the analyses appears to be the magnitude of the tensile force that develops at the ends of girders. This is best examined using the T/F_{po} ratio, which indicates the relative magnitude of the tensile force compared to the total prestress force. The T/F_{po} ratio ranges from 4.84% to 13.25% considering both cases. If this model were verified by test to reflect the tension developing in girders, then the higher values of this ratio would indicate that the current requirement of 4% of the prestress force for proportioning web splitting reinforcement is inadequate. However, the strand patterns used for Case 1 are extremes, so the high values may not be representative of normal designs.

However, the ratios for Case 2 are actually higher than those for Case 1. This is surprising, because it would seem that a reduced prestress force would result in a reduced tensile force. This would be true if the reduced forces were placed at the same location as the original forces. But the force locations for Case 2 move closer to the extremes of the section, increasing the distance between the forces. This leads to the increased tensile force.

The actual value of the tensile force is also important, because it defines the area of reinforcement that is required within a short distance from the end of the girder. The designs shown for Case 1 have a significantly higher reinforcement requirement than the designs for Case 2, even though the percentage of the total prestress force may be higher. The required area of reinforcement per inch of girder is given by the A_s / (h/5) ratio. Many values of this ratio indicate that it would not be possible to provide the entire area of reinforcement required using the working stress of 20 ksi. Calibration of the model to actual girder performance is needed before the placement of this quantity of reinforcement is required.

The need for calibration of the model is further reflected in the fact that the computed maximum tensile stress at the end of the girder greatly exceeds the tensile strength of the concrete at release. If accurate, this would indicate that cracking would be unavoidable and would always occur. The fact that cracking does not always occur indicates that the model and/or the equation used to compute the concrete tensile stress from the tension force overestimate the tensile stress at the end of the girder. The assumptions used in the equation for computing the tensile stress may need refinement.

A comparison of different cross sections with approximately the same depth reveals that there is a difference in the magnitude of the tensile force between the cross sections for Case 1, although the larger AASHTO sections have significantly higher prestress forces.

The analyses indicate that shallower sections tend to have lower tensile forces at the end of the girder. Therefore, the tendency for cracking increases for deeper girder sections, when compared to shallower sections.

The x dimension is the location of greatest moment across the section, and is therefore, probably the location of most likely cracking. The computed values of the x_w / web ht. ratio indicate that cracking would be most likely near the middle of the web. This does not seem to agree with observations that web splitting cracks tend to appear near the bottom of the web. An analysis of designs that have exhibited cracking could reveal whether the analysis is accurate regarding crack location.

OTHER DESIGN METHODS

Other methods for computing reinforcement at the ends of pretensioned girders have been proposed. These were briefly reviewed and summarized in an unpublished report of the PCI Ad Hoc Committee on End Reinforcement Requirement for Pretensioned Concrete Beams[4]. The basis of the current specifications provisions was traced to research by Marshall and Mattock[5], whose research was limited to shallow girders. The design recommendations resulting from their research, which were intended to limit the size and length of web splitting cracks, were simplified for inclusion in the AASHTO *Standard Specifications*. Reference 4 indicates that the 4% requirement currently in both bridge design specifications was intended as a rule of thumb, not an exact solution. No other methods were considered for use in the specifications.

LIBBY'S METHOD

A method for computing the tensile force acting at the end of a pretensioned girder has been presented in a text by Libby[6]. This method was not discussed in Reference 4.

The applied forces are used to develop a stress distribution based on elastic analysis some distance from the end of the girder. The stress distribution is then converted into a force distribution. The moment across the depth of the section caused by the force distribution can then be computed. The maximum value of this moment is determined as the critical design value. The tension force can then be computed by dividing the maximum moment by an assumed moment arm between the tension reinforcement at the end of the girder and an interior location.

A review of this method reveals that the solution is identical to the strut-and-tie model presented above. The dimension "x" shown in Tables 2 and 4 corresponds exactly to the maximum moment location (since it is the point of zero shear), and the moment "M" computed in these tables is the maximum moment across the depth of the section. Any difference between the results of the two methods depends only on the assumed moment arm.

STRATEGIES FOR REDUCING WEB SPLITTING

There are several strategies for reducing web splitting in pretensioned concrete girders. The strategies include:

- Modifying the strand pattern
    - Lower the height of the draped strands
    - Reduce the number of draped strands (strands may also be debonded)

  - Spread or splay the strands at the end of the girder to a wider than minimum spacing. This also effectively lowers the height of the draped strands
- Reduce the prestressing force at the end of the girder
  - Debond strands at the ends of the girder
- Add mild reinforcement to control the cracking

These methods may also be combined. In many cases, they would have to be combined in order to continue to meet the stress limits at the ends of the girder. For example, if a design required a certain number of strands to be draped to a certain height to satisfy the stress limits, splaying, lowering or reducing the number of draped strands will cause the stress limits to be exceeded. Therefore, debonding may have to be used in conjunction with these other strategies to satisfy design requirements.

The current provisions in the design specifications regarding web splitting cannot be used to compare the effectiveness of these different strategies for reducing web splitting. However, the strut-and-tie model can be used to compare the different strategies.

The following is an example to illustrate the differences in the tension force developed at the end of the girder when the height of the draped strands is changed (or strands are splayed) and when the straight strands are debonded. The example is based on the design for the PCI BT-72 shown in the tables earlier in this paper. The results are summarized in the following table. Cells are highlighted to indicate the values that are changed.

	N_{ps} Top	g_1 (in.)	g'_1 (in.)	g'_1-g_1 (in.)	N_{ps} Bottom	g_2 (in.)	g'_2 (in.)	g'_2-g_2 (in.)	T (kips)	T / F_{po} (%)	A_s (in^2)
PCI BT-72	12	7.00	19.01	12.01	28	4.00	9.15	5.15	175.9	10.01	8.80
PCI BT-72	12	14.00	23.42	9.42	28	4.00	8.04	4.04	138.0	7.85	6.90
PCI BT-72	12	7.00	16.85	9.85	22	4.00	9.37	5.37	144.2	8.20	7.21
PCI BT-72	12	14.00	21.28	7.28	22	4.00	7.97	3.97	106.6	6.06	5.33
PCI BT-72	12	7.00	15.40	8.40	18	4.00	9.60	5.60	123.1	7.00	6.16
PCI BT-72	12	14.00	19.86	5.86	18	4.00	7.91	3.91	85.8	4.88	4.29

Table 5 Comparison of Effectiveness of Different Strategies for Reducing Web Splitting

It can be seen from the data in Table 5 that the tension force, and therefore the required area of vertical reinforcement, is reduced to less than 50% of the original design by the combination of lowering (or splaying) the draped strands and debonding the straight strands in the bottom flange. Using the strut-and-tie model in this way to evaluate the effectiveness of different strategies for reducing the tendency for web splitting is very helpful to both designers and fabricators.

SUMMARY

The use of a strut-and-tie model to define the required tensile force to be resisted near the end of a pretensioned concrete girder at release appears to be a simple approach to solving a complex problem. The solution agrees with a more complex manual method presented by Libby[6]. It can also be used instead of or in conjunction with detailed computerized methods, such as finite element analysis.

The provisions in the current specifications do not account for the section shape and placement of prestressing strands. These important quantities are considered in the proposed strut-and-tie model.

The limited results of analyses presented in this paper using the proposed model indicate significantly higher reinforcement requirements to resist web splitting. This raises concerns that the current provisions may not be adequate, especially for the larger girders and prestress forces that are now being used.

However, some of the results of the analyses using the model indicate that the tensile force and tensile stress at the end of the girder are being overestimated. Therefore, the model needs to be calibrated with field experience and test data before it is implemented. If field results are available, the model can be used to provide relative comparisons between girders, which can identify the potential for web splitting.

The model as presented neglects the transfer length and girder reaction, and does not account for the tensile strength of concrete. Therefore, it is expected to be conservative. It may be possible that the model could be modified to include some or all of these effects without losing its simplicity.

The strut-and-tie model can be used to compare the effectiveness of different strategies for reducing web splitting.

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