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NOTATION

CURVED AND SKEWED BRIDGES

A = area

A = area of cross-section of the precast beam

A_c = total area of the composite section

A_{cp} = total area enclosed by outside perimeter of concrete cross-section

a = a length defined in **Figure 12.4.1.1-1**

B = width

b_v = effective web width

C = coefficient to compute centrifugal force

DC = dead load structural components and nonstructural attachments

DW = dead load of wearing surfaces and utilities

E = modulus of elasticity

E_c = modulus of elasticity of concrete

E_{ci} = modulus of elasticity of the beam concrete at transfer or at post-tensioning

E_{cs} = modulus of elasticity of concrete slab

E_p = modulus of elasticity of prestressing tendons

E_s = modulus of elasticity of reinforcing bars

e = eccentricity

f_b = concrete stress at the bottom fiber of the beam

f'_c = specified compressive strength at 28 days

f'_{ci} = concrete strength at transfer or at post-tensioning

f_{pc} = compressive stress in concrete at the centroid after prestress losses have occurred

f_{pc} = compressive stress in concrete after prestress losses have occurred either at the centroid of the cross-section resisting transient loads or at the junction of the web and flange where the centroid lies in the flange

f_{pe} = effective stress in the prestressing steel after losses

f_{pj} = jacking stress (maximum)

f_{pu} = specified tensile strength of prestressing steel

f_{py} = yield strength of prestressing steel

f_y = specified minimum yield strength of reinforcing bars

g = gravitational acceleration

H = elevation difference between ends of a beam

h = overall depth of a member

h_c = total height of composite section

I = moment of inertia

I_c = moment of inertia of composite section

I_{clat} = moment of inertia of composite section for lateral bending

I_{lat} = moment of inertia for lateral (weak axis) bending

I_p = polar moment of inertia

I_{pc} = polar moment of inertia of composite section

IM = dynamic load allowance (impact factor)

J = torsional constant

J_c = torsional constant for composite section

L = live load
L = overall beam length or design span
L_a = arc length
L_c = chord length
M = bending moment
M_c = moment applied to cross beam
M_i = moment in inside beam
M_o = moment in outside beam
M_t = torsional moment
M_u = factored bending moment
$M(-)$ = negative moment
m = multiple presence factor
n = modular ratio between deck slab and beam materials
P = effective prestressing force
p_c = the length of the outside perimeter of the concrete section
R = radius of curvature
R_i = reaction of inside beam
R_o = reaction of outside beam
S = section modulus
S_b = section modulus for the extreme bottom fiber of the non-composite precast beam
S_{bc} = composite section modulus for extreme bottom fiber of the precast beam
S_t = section modulus for the extreme top fiber of the non-composite precast beam
S_{tc} = composite section modulus for top fiber of the deck slab
S_{tg} = composite section modulus for extreme top fiber of the precast beam
s = sagitta, arc-to-chord offset
T = unfactored torsional moment
T_u = factored torsional moment
T_{cr} = torsional cracking moment
t_s = cast-in-place deck thickness
V = shear
v = highway design speed
W = weight
w = clear width of roadway
w_c = unit weight of concrete
w_g = beam self weight per unit length
x = a length
y = distance
y_b = distance from centroid to the extreme bottom fiber of the non-composite precast beam
y_{bc} = distance from the centroid of the composite section to extreme bottom fiber of the precast beam
y_{max} = maximum distance, used in computing section modulus

y_t = distance from centroid to the extreme top fiber of the non-composite precast beam

y_{tc} = distance from the centroid of the composite section to extreme top fiber of the slab

y_{tg} = distance from the centroid of the composite section to extreme top fiber of the precast beam

γ = grade angle expressed as a decimal

ϕ = resistance factor

θ = skew angle

ψ = an angle

Curved and Skewed Bridges

12.1 SCOPE

This chapter deals with the geometric and structural challenges for bridges with curvature in plan, or with skewed supports, and on a grade. The effects of skew and grade are primarily geometric, with some effect on shears and moments. Larger skew angles also have some effect on live load distribution. The effects of curvature are both structural and geometric. This chapter primarily describes the design of curved bridges. Structures with very sharp curvature (say 300-ft radius) such as freeway on or off ramps, may require the use of specially made box beams that are also described in ABAM (1988). Straight AASHTO I-beams and box beams are normally used on curved bridges with shorter spans, or on longer spans with larger radii, because the offset between arc and chord is small.

Curve and skew effects are described in more detail in the *LRFD Specifications* than in the *Standard Specifications*. Even if one is designing using the *Standard Specifications*, it would be prudent to consult the *LRFD Specifications* for guidance on curve and skew effects. This chapter is based on the *LRFD Specifications*.

12.2 SKEW AND GRADE EFFECTS

12.2.1 General

A skewed bridge is one in which the major axis of the substructure is not perpendicular to the longitudinal axis of the superstructure. For the work at most agencies, the skew angle (usually given in degrees) is the angle between the major axis of the substructure and a perpendicular to the longitudinal axis of the superstructure. Some agencies use a different convention. Usually, different substructure units have approximately the same skew angle.

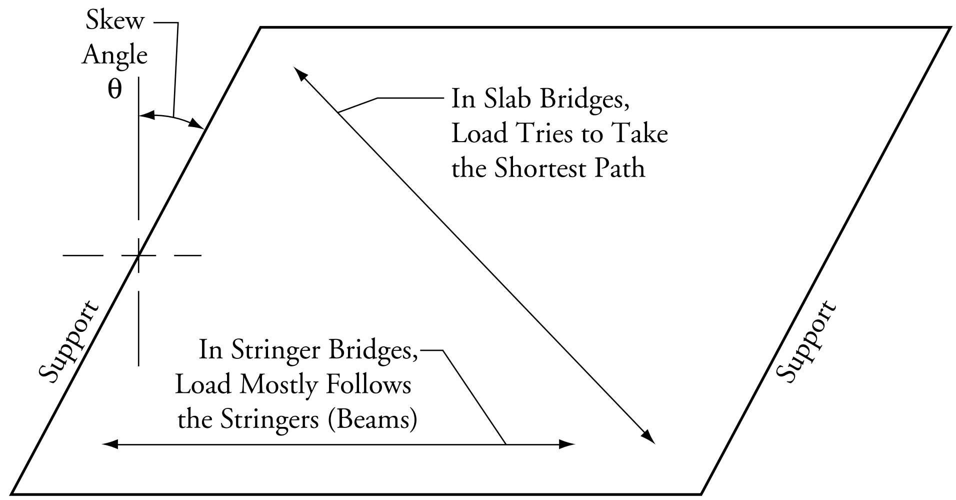
The presence of skew affects the geometry of many bridge details. Skew angles greater than 20 degrees also have an effect on bending moment, and on shear in the exterior beams. The structural response of a skewed bridge to seismic loads can be significantly altered by the skew angle of the substructure.

The effects of grade are geometric.

12.2.2 Superstructure Behavior

In stringer bridges (bridges supported by longitudinal I- or bulb-tee beams), the load tends to flow along the length of the supporting beams, and the effect of skew on the bending moments is minimized. In solid slab bridges and other bridges with high torsional rigidity, the load tends to take a “short cut” between the obtuse corners of the span, as shown in **Figure 12.2.2-1**. This reduces the longitudinal bending moments, but it increases the shear in the obtuse corners. The same effect occurs in stringer bridges, but is less pronounced. The modification factors due to skew for shear and moment are given in Section 7.5.4.

Figure 12.2.2-1
Load Distribution in Skewed Spans

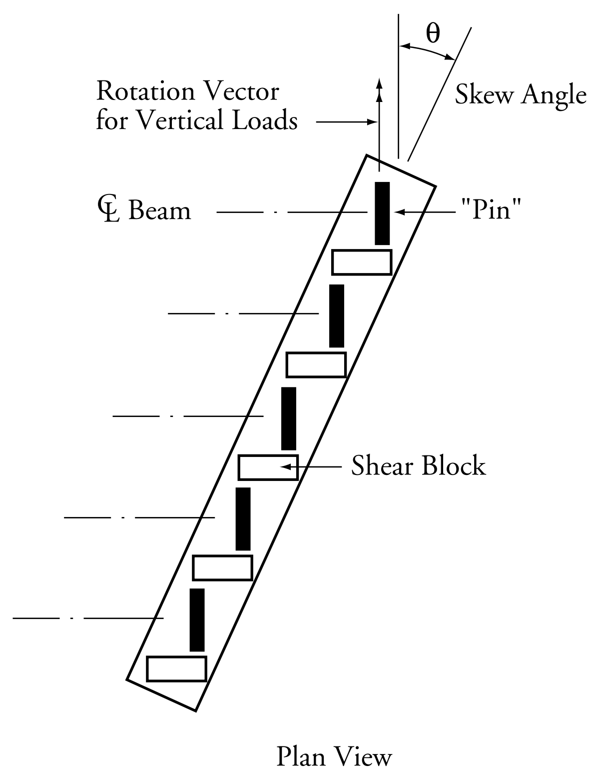


12.2.3
Substructure Behavior

The relative stiffness of the substructure about its major and minor axis is important. A substructure consisting of round columns and a cap beam is about four times as stiff when acting as a frame resisting loads along its major axis, compared to its stiffness acting as a cantilever for loads along its minor axis. For rectangular cantilever piers, the ratio of major-to-minor-axis stiffness is proportional to the dimension ratio squared. For wall piers, the major axis stiffness is almost infinite compared to the minor axis stiffness.

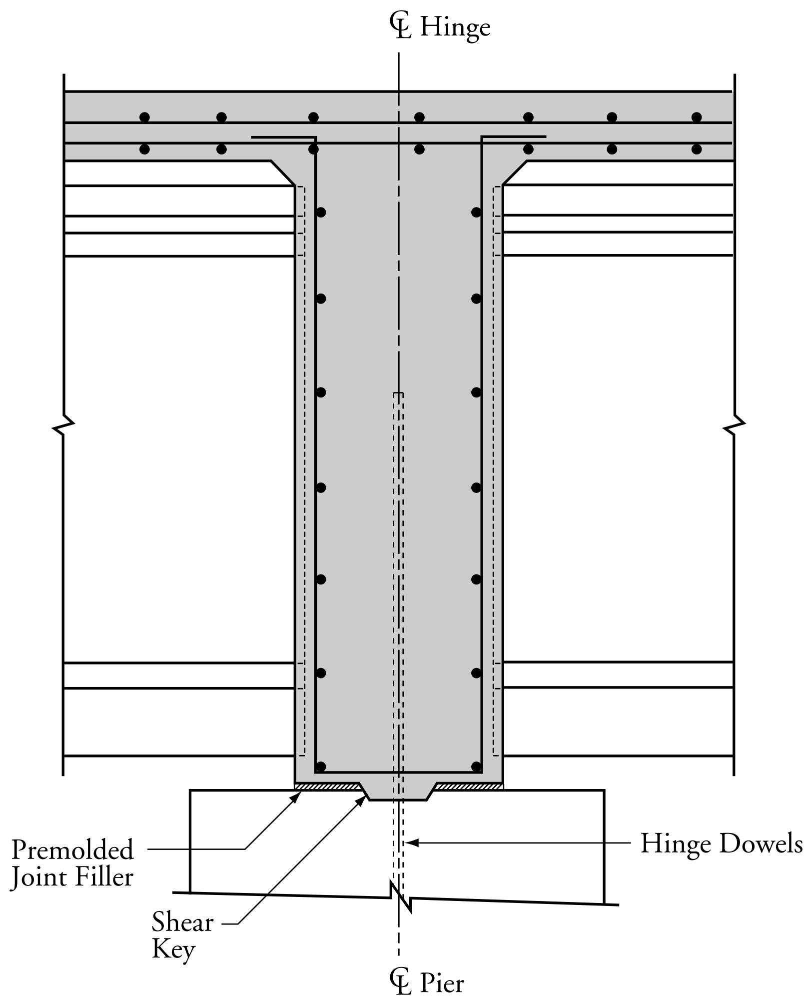
When a substructure unit deflects due to horizontal loads or superstructure deformations, the deflection is primarily along the minor axis, and the rotation vector at the top is along the major axis. When the superstructure deflects due to vertical loads, the rotation vector at the support is perpendicular to the axis of the beams, as shown in **Figure 12.2.3-1**.

Figure 12.2.3-1
Rotation Vector for Vertical Loads



This raises a question of how to orient the "pin" between superstructure and substructure. Concrete bridges are seldom supported by real pins. Bearings consisting of elastomeric pads can provide rotation capacity about all axes; this solves the problem of how to orient the pin. Continuous bridges are sometimes constructed using a concrete hinge between superstructure and substructure, as shown in **Figure 12.2.3-2** (also, see Section 3.2.3.2.2). This forces the rotation vector to lie along the major axis of the substructure, which is inconsistent with the end rotation of the superstructure beam. However, live load rotations at an interior support of a continuous bridge are small, and structures so constructed seem to perform satisfactorily.

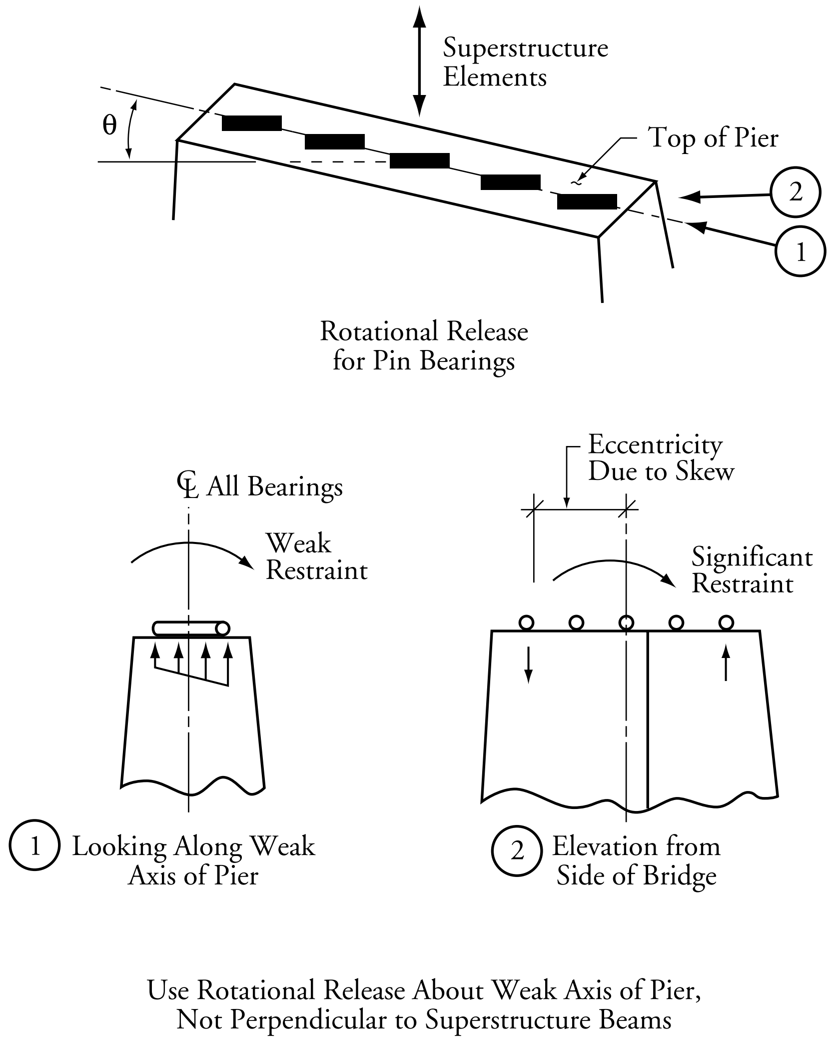
Figure 12.2.3-2
Typical Hinge Section



NOTE: Reinforcement from beams extending into diaphram and other details not shown for clarity.

A sidelight to this discussion concerns computer modeling. Orienting the rotational release vector with respect to the superstructure axes may force a component of rotation about the major axis of the substructure. This will create a fictitious moment at the top of the substructure in the computer model, as shown in **Figure 12.2.3-3**. In general, a rotational release between superstructure and substructure should be oriented with respect to the substructure axes.

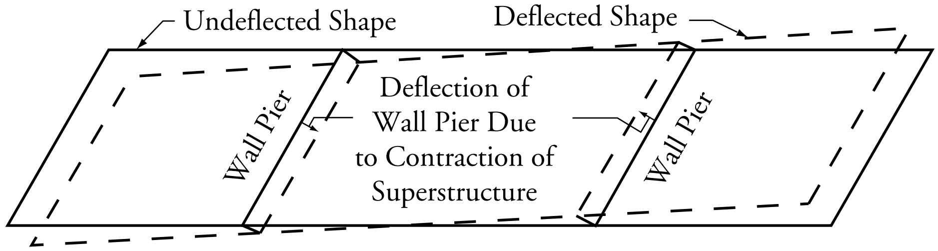
Figure 12.2.3-3
Orientation of Pins in Computer Model



12.2.4 Temperature and Volume Change Effects

The shortening of a skewed span due to shrinkage and temperature will cause the supporting substructure units to deflect, if they are connected longitudinally to the superstructure. The substructure units will tend to deflect about their minor axes, causing a rotation of the superstructure, as shown in **Figure 12.2.4-1**. If transverse shear blocks are provided at the abutments, transverse forces at the abutments can develop, as well as forces along the major axis of the piers.

Figure 12.2.4-1
Bridge Rotation Caused by Shrinkage Deflection



12.2.5
Response to Lateral Loads

Wind and seismic loads transverse to the major axis of the bridge cause both transverse and longitudinal deflection of the superstructure, as the substructure elements deflect about their weak axes. Similarly, longitudinal loads also cause both longitudinal and transverse deflections of the superstructure. This can lead to a coupling of transverse and longitudinal modes in a dynamic seismic analysis. This subject is more fully discussed in FHWA (1996 A and B).

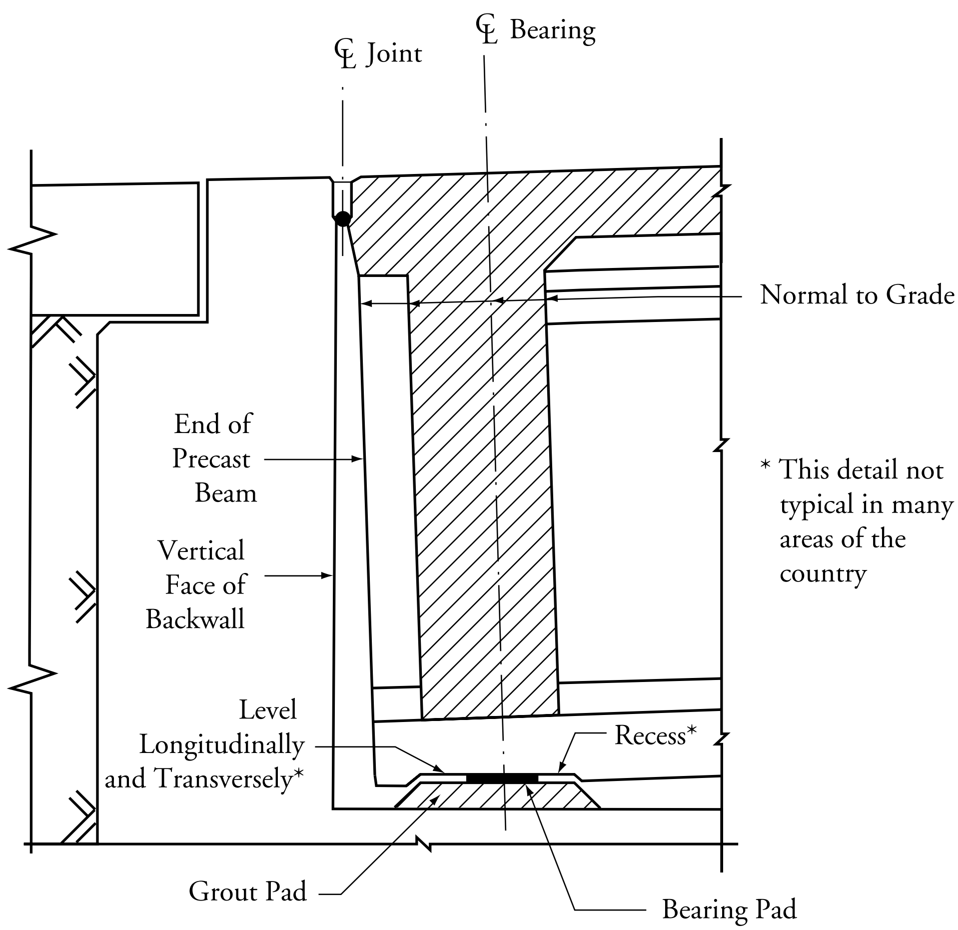
12.2.6
Detailing

12.2.6.1
Effects of Grade

Grade affects the geometry of the precast beams. The slant length is increased over the plan length by an amount $\gamma^2L/2$, where γ is the grade, expressed as a decimal. The precast beam is normally made in the shape of a rectangle, as seen in elevation. That is to say, the ends of the beam are usually square with the long axis of the beam, rather than being vertical in the final position of the beam. Similarly, the diaphragms are normally square with the axis of the beam.

Cast-in-place substructures are normally cast with vertical surfaces. This needs to be considered in the abutment detail (see **Figure 12.2.6.1-1**) in which the beam end is not vertical. The bearing pad is set on a level, horizontal surface. Recesses, shims, sloped risers or grout pads are used to compensate for the difference in planes between the beam soffit and the top of the cap beam. Sometimes, on moderate grades, the bearing pads and bearing surfaces on the abutment and on the underside of the beam are set parallel to the grade.

Figure 12.2.6.1-1
Section at Abutment
Showing Effects of Grade



12.2.6.2 Skewed Beam Ends

Skewed beam ends (in plan) are sometimes provided at expansion joints. Skew angles should be grouped into standard increments because each skew angle will require a special end bulkhead to form it. At interior ends in continuous beams, the beam-ends are normally made square in plan. Some end diaphragm details may require that the ends of continuous beams be skewed. In the latter case, when using precast, prestressed box beams, a maximum skew angle of 30 degrees avoids warping or racking of the beams.

12.2.6.3 Diaphragms

Intermediate diaphragms, if used, may be perpendicular to the beam axes, or parallel to the skew. Making them parallel to the skew can have the advantage of making interior beams identical. Making them perpendicular to the beams simplifies their construction in the field.

12.2.6.4 Deck Reinforcement

For skew angles of 10 degrees or less, deck reinforcement is normally placed parallel to the skew. This simplifies detailing and the placement of reinforcement. For skew angles exceeding 20 degrees, transverse deck reinforcement should be placed normal to the axis of the beams, and shorter bars should be used in the acute corners of the deck. For skew angles between 10 and 20 degrees, practice varies concerning orientation of the deck reinforcement.

12.2.6.5 Plans

The detailing recommendations made in Section 12.2.6 are by no means universal. The plans must show the geometric effects of skew and grade. It is important to indicate which surfaces are parallel or normal to the skew, and which surfaces are parallel or normal to the beam axis. Similarly, the plans should indicate which surfaces are truly vertical and horizontal, and which surfaces are parallel or normal to the inclined beam axis for beams on a grade.

12.3 CURVED BRIDGE CONFIGURATIONS

12.3.1 Use of Chords

Curved bridge beams are usually made as a series of short straight segments, or chords, to approximate the theoretical arc. Forms for the beams are made in straight segments, with a small angle at the form joints. The exception is monorail beams in which the concrete beam surface is the running surface for the wheels of the monorail vehicle. Such monorail beams are made in an adjustable form that can be bent to form a smooth arc.

The maximum offset between an arc and its chord is equal to $L_c^2/8R$, where L_c is the chord length and R is the radius of curvature. Although this is an approximation, it is a good one. Because it is an approximation, the length may be either the arc length, L_a, or the chord length, L_c, whichever is known. The formula shows that the offset varies as the square of the chord length. For practical curve radii encountered in bridges, a curve approximated by 20-ft chords will appear to the eye as a smooth continuous curve.

The simplest way to support a curved roadway is to use straight beams beneath a curved deck. If the offset between chord and arc is too large, the appearance will be poor, and the exterior beam on the outside of the curve will be required to support too much additional load. It is desirable that the arc-to-chord offset be limited to 1.5 ft, and that the edge of the top flange of the beam be no closer than 0.5 ft to the slab edge. **Table 12.3.1-1** shows the minimum curve radii that satisfy the 1.5 ft maximum offset criterion. This limit is often exceeded, but each case should be examined for acceptability.

Table 12.3.1-1 Radii that Provide Offsets Shown for Various Straight Beam Lengths

Beam Length, ft	Radius, ft			
	Offsets, ft			
	0.5	1.0	1.5	2.0
70	1,225	613	408	306
80	1,600	800	533	400
90	2,025	1,013	675	506
100	2,500	1,250	833	625
110	3,025	1,513	1,008	756
120	3,600	1,800	1,200	900
130	4,225	2,113	1,408	1,056
140	4,900	2,450	1,633	1,225
150	5,625	2,813	1,875	1,406
160	6,400	3,200	2,133	1,600

Straight beams are by far the simplest and most cost-effective way to use precast, prestressed beams in a curved bridge; they should be used whenever appropriate. This solution is not discussed further in this chapter because the analysis is almost identical to that for a straight bridge. The only difference is in the computation of loads on the exterior beams. The "lever rule" [LRFD Art. C4.6.2.2.1] may be used in the same manner as for a straight bridge, as long as the variable overhang is accounted for. In addition, the extra span length on the outside of the curve must, of course, be used in the design of these beams.

For situations in which the offset exceeds 1.5 ft, the number of chords may need to be increased. One method is to splice I- and bulb-tee-beam segments together in the field using methods described later in this chapter and in Chapter 11. With two chords, the offset will decrease by a factor of 4; and with three chords, the offset will decrease by a factor of 9.

12.3.2 Beam Cross-Section

12.3.2.1 Box Beams Versus I-Beams

Full-span-length, chorded, curved beams may be made in the plant, using post-tensioning. Torsional stresses and handling considerations will usually cause a closed box section to be preferred for full-length curved beams.

Segmental construction may be used with conventional I-beams. Two or three straight segments may be supported on temporary shores, and post-tensioned in the field after constructing diaphragms at the segment joints. Refer to details in Chapter 11, as well.

12.3.2.2 Box Section Configuration

Box sections will often require a new form, as standard box sections of the size needed do not exist in many localities. The precast box beam needs to be closed at the top, in order to have sufficient torsional resistance.

The sides of box beams may be vertical or sloped. Vertical sides are somewhat easier to form. Sloping sides are generally thought to have a better appearance.

The maximum span of box beams is often limited by shipping weight. Field-splicing of shorter segments may be used to minimize weight of individual segments. In order to minimize the thickness of webs and flanges, consideration should be given to the use of "external" post-tensioned tendons inside the box section.

12.3.2.3 I-Beam Configuration

The use of post-tensioning requires webs thicker than the 6-in. webs of AASHTO-PCI Bulb-tees and other standard I-beams. To accommodate post-tensioning ducts and reinforcement, the minimum web thickness should be 7 to 8 inches. Thicker webs can often be obtained by spreading the side forms of standard shapes by 1 or 2 in.

12.3.2.4 Continuity

Continuity is very desirable in curved bridges. In addition to the benefits that continuity provides for straight bridges, there are two additional benefits for curved bridges. Continuity greatly reduces torsion resulting from applied loads, and it reduces the excess load on the exterior beam on the outside of the curve.

12.3.2.5 Crossbeams

Transverse members spanning between beams within a span (intermediate diaphragms) are often omitted on straight bridges (see Section 3.7). However, in curved bridges, the transverse members, which will be referred to as crossbeams in this chapter due to their unique role, are required to counteract both the effects of torsion and the lateral forces resulting from curvature. The crossbeams should also be deep enough to brace the bottom flange.

12.3.2.6 Superelevation

Standard practice is to keep the beam cross-section vertical, and provide a "haunch" or "pad" of cast-in-place deck concrete to fill the space between the sloping deck and the horizontal top flange.

12.4 PRELIMINARY DESIGN

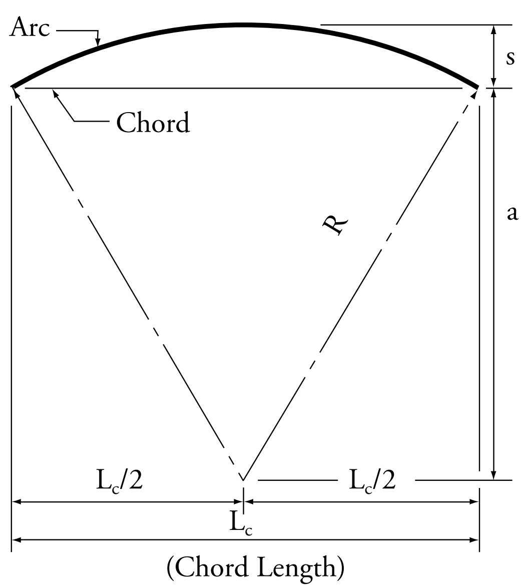
12.4.1 Useful Geometric Approximations

Despite the immense computing power now available, simple approximations remain very useful for preliminary design. They are quick to use, and they give the designer a "feel" for how a change in one parameter affects other parameters.

12.4.1.1 Arc Offset from Chord

The maximum offset between arc and chord is called the middle ordinate or the "sagitta" (sagitta is Latin for "arrow") and represented by the symbol, s. As noted in Section 12.3.1, the sagitta is approximately equal to $L_c^2/8R$. The derivation is simple and is shown in **Figure 12.4.1.1-1**. Once again, since these are approximations, it is unimportant whether the arc length or chord length is used.

Figure 12.4.1.1-1
Arc Offset from Chord



By Pythagorean Theorem:

$a^2 + (L_c/2)^2 = R^2$

Also:

$a = R - s$

$\cancel{R^2} - 2Rs + s^2 + L_c^2/4 = \cancel{R^2}$

But s is small compared to R and L_c. Therefore, ignore the term s^2 and solve for s.

$$s = \frac{L_c^2}{8R}$$

The formula slightly underestimates the distance, s. The approximation is slightly better if the length is taken as the arc length, L_a.

12.4.1.2 Excess of Slant Length over Plan Length

The slant length of a beam on a grade is longer than the plan length by an amount $H^2/2L$, where H is the difference in elevation of the two ends of the beam. This is a well-known formula, and is identical to the $\gamma^2L/2$ formula given in Section 12.2.6.1 (γ is equal to H/L). The derivation is similar to that for the arc-chord offset. The Pythagorean theorem is used, neglecting a small second-order quantity.

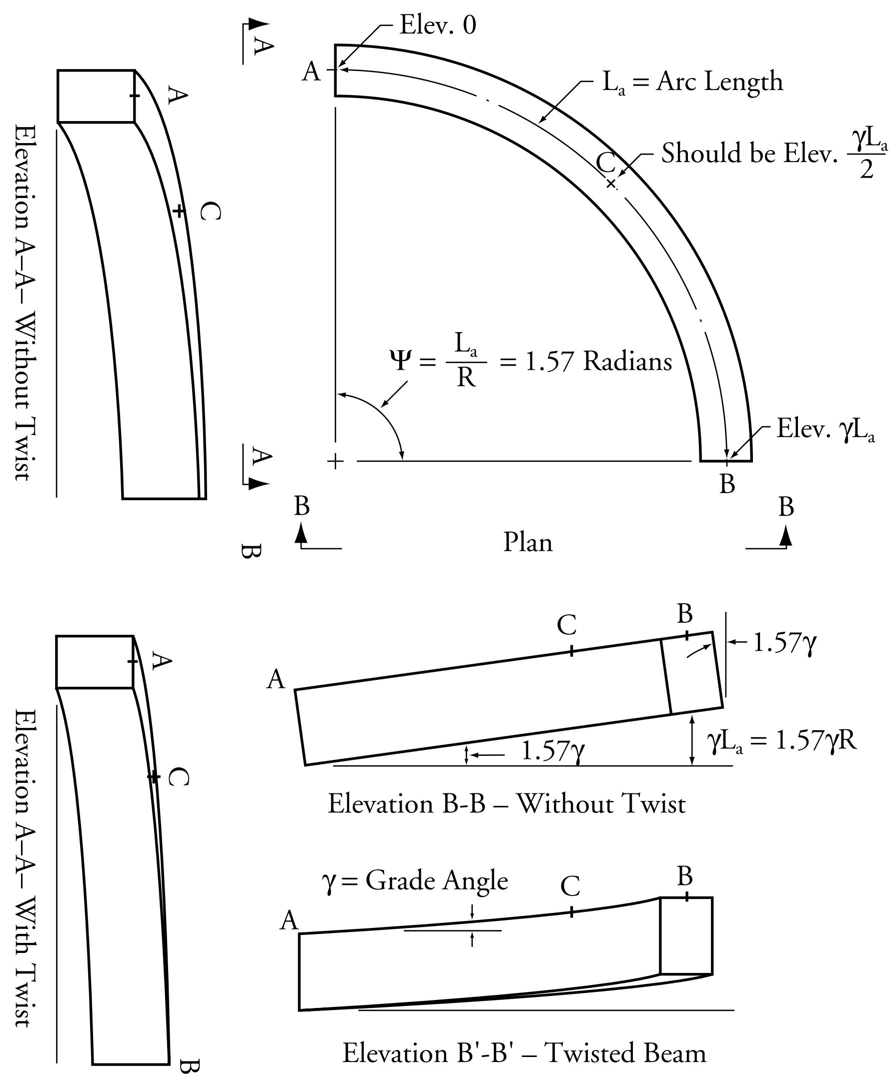
12.4.1.3 Excess of Arc Length over Chord Length

The length of an arc is longer than its chord by an amount $8s^2/3L_c$, where s is the arc-chord offset and L_c the chord length. The excess length may also be expressed as $L_c^3/24R^2$. This formula is derived by approximating the arc length as a series of short chords, then taking the limit as the chord length approaches zero.

12.4.1.4 Twist Resulting from Grade

The shape of a curved beam on a grade is a helix. It has the same shape as the railing on a “spiral” (more correctly, helical) stair. Such a railing is twisted. If a section were cut out of the railing and laid flat, the twist would be apparent.

Figure 12.4.1.4-1 Twist Resulting from Grade Change



To understand more fully the twist in a curved beam caused by grade, consider a beam curved 90 degrees (1.57 radians) in plan, made without twist, with square ends as illustrated in the Plain View of **Figure 12.4.1.4-1**. The bearing at Point B is elevated higher than at Point A by an amount $1.57\gamma R$ as shown in Elevation B-B. Therefore, the beam will be tipped by an angle of 1.57γ. At Point B, the sides of the beam will not be plumb; they will be tipped by an angle 1.57γ. Also, note that at Point C, the midpoint of the beam, the elevation of the beam will not be half of $1.57\gamma R$, as it should be.

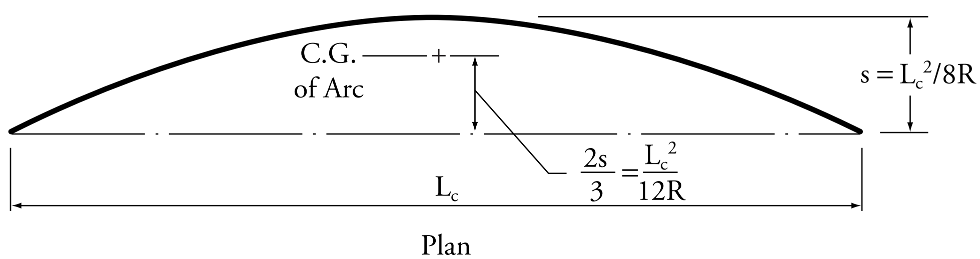
Elevation B'-B', **Figure 12.4.1.4-1**, shows the elevation of the beam fabricated to a true helix. The ends and sides of the beam will be plumb at Points A and B, and the elevation at C will be correct. The beam must be twisted by an amount 1.57γ. Generalizing for angles other than 1.57 radians, the amount of twist is $\psi\gamma$, or $(L_a/R)\gamma$ where L_a is the arc length.

The approximation is this: The twist angle is normally small enough to be ignored in beam fabrication, except for monorail beams. If the twist is ignored in beam fabrication, it should be realized that when the beam is set in the field, it will not be possible for both ends to be perfectly plumb. If the apparent twist is large enough to be measurable, the beam should be set "splitting the difference" of the out-of-plumbness at the two ends. This will also result in the midpoint of the beam being at proper elevation (not including the effects of camber).

12.4.1.5 Center of Gravity of an Arc

The center of gravity of an arc (and of a load applied along the arc) is offset from the chord by 2s/3, or $L_c^2/12R$. See **Figure 12.4.1.5-1**.

Figure 12.4.1.5-1
Center of Gravity of Arc



12.4.1.6 Curved Surfaces

The area of a curved surface with radial ends, such as a bridge deck, is equal to BL_a, where B is the width and L_a is the arc length along the centerline. See **Figure 12.4.1.6-1**.

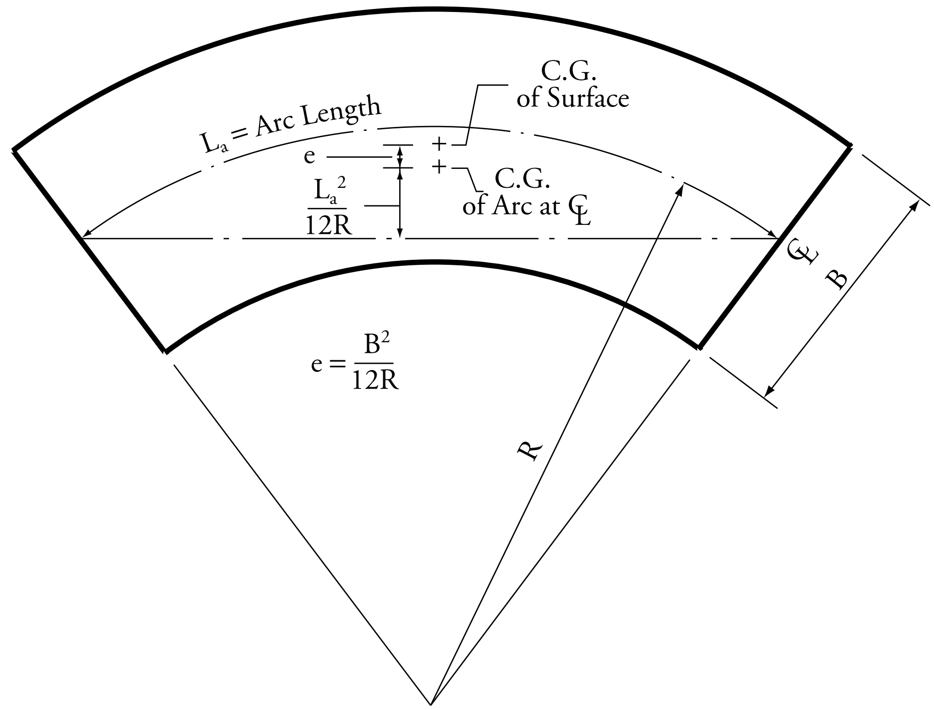
The center of gravity of a curved surface lies outside of the center of gravity of the centerline arc, because there is more area outside of the centerline than inside. This additional eccentricity, e, is equal to $B^2/12R$. The total offset from the chord to the center of gravity of the surface is therefore $(L_c^2 + B^2)/12R$. Where ends of the bridge are not radial, a more detailed calculation is required for area and centroid of the surface.

12.4.2 Useful Structural Approximations

12.4.2.1 Analysis as a Straight Beam

The bending moments in a curved beam due to vertical loads may be analyzed by considering the beam to be a straight beam of span equal to the arc length of the curved beam. This approximation is very good, and sufficiently accurate for preliminary design.

Figure 12.4.1.6-1
Properties of a Curved Planar Surface



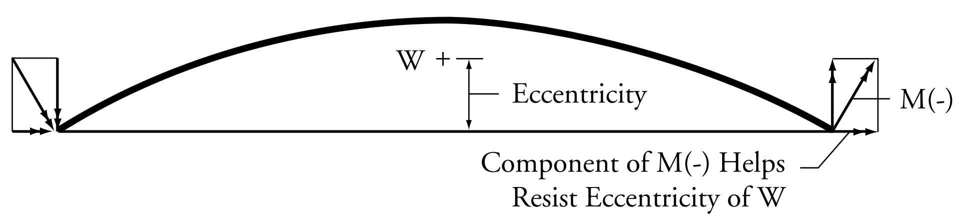
12.4.2.2
Torsion

Although flexural moments may be estimated by analyzing a straight beam of length equal to the arc length of the curved beam, the same cannot be said for torsional moments. Torsional moments are necessary for equilibrium of a curved beam. **Figure 12.4.1.5-1** shows that, as noted in Section 12.4.1.5, the center of gravity of an arc (and of loads applied along that arc) is offset from a line through the supports of a simple span beam by an amount equal to $L_c^2/12R$. The moment of weight, W, about the supports is $WL_c^2/12R$. This is resisted by torsional moments at each beam end, approximately equal to $WL_c^2/24R$. Again, because these are approximations, a known value of L_a may be used in lieu of L_c.

12.4.2.3
End Moments and Torque

The presence of end moments in continuous beams significantly reduces the torsional moments at the support. As shown in **Figure 12.4.2.3-1**, end moments have a component that helps resist the eccentricity of the weight, W, applied to the arc.

Figure 12.4.2.3-1
Negative End Moments Counteract Torsion in Continous Beams



For a uniformly loaded, fixed-ended beam, the end moment of $WL_a/12$ reduces the torsional moment at the support to (approximately) zero. For continuous beams, the torsional moment at the support will not be zero, but it will usually be less than half of the simple span torsional moment at the support. This is discussed in more detail in Section 12.5.2.

12.4.3 Design Charts

Design charts for continuous, curved box beams are given in ABAM (1988). These charts are useful for preliminary sizing of curved box beams.

12.5 STRUCTURAL BEHAVIOR OF CURVED-BEAM BRIDGES

12.5.1 Longitudinal Flexure

12.5.1.1 Analysis as a Straight Beam

As previously noted, the bending moments from longitudinal flexure are virtually the same as those for a straight beam of the developed length. However, the distribution of loads to the beams will be different in a curved bridge.

12.5.1.2 Loads on Outside Beam

The shears and moments in the exterior beam on the outside of the curve are substantially larger than for other beams in the bridge. This is caused by the following factors:

- The arc length on the outside of the curve is longer than the nominal length at the centerline of the bridge. This increases bending moments in the outer beam by (approximately) the square of the ratio of the arc lengths.
- The overhang at mid-arc may be increased by an amount equal to the arc-to-chord offset.
- Other beams will shed some of their torsional moment by shifting load toward the next beam to the outside. The outermost beam is the final resting place for this shifted load.

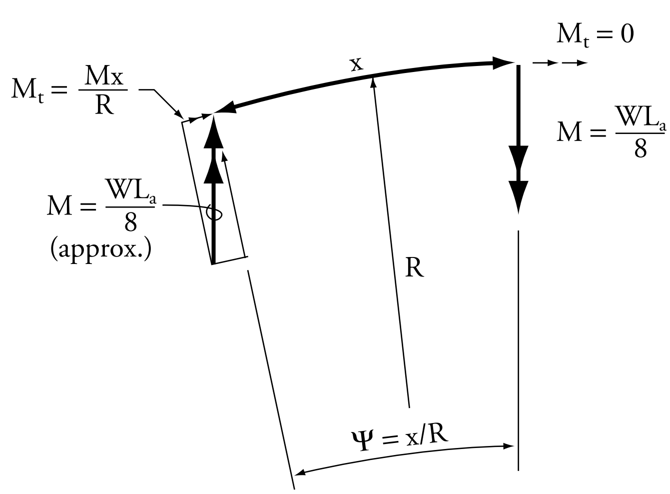
12.5.2 Torsion

It is useful to look in more detail at how torsional moments develop in a curved beam. It will be shown that torsional moments are related to the flexural moment M divided by the radius of curvature R.

12.5.2.1 Torsion in Simple-Span Beams

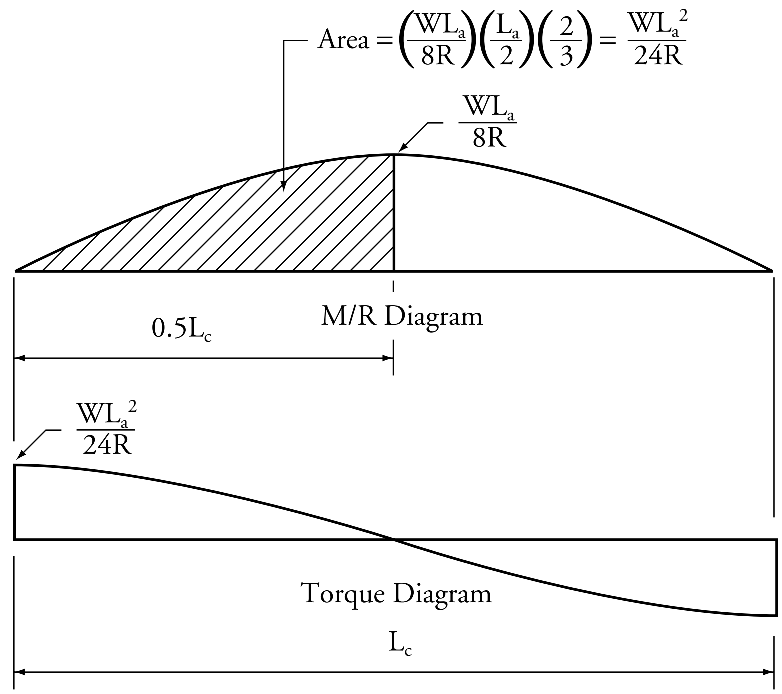
The development of torsional moments in a curved beam may be thought of in the following way. Consider a short segment near midspan of the simple-span curved beam shown in **Figure 12.5.2.1-1**.

Figure 12.5.2.1-1
Torsion and Curvature



At midspan, the bending moment is $WL_a/8$, and the torsional moment is zero (by symmetry). At a small angle, ψ, away from midspan, the bending moment must "turn" through the angle, ψ, and a torsional moment (approximately) equal to $xWL_a/8R$ is necessary for equilibrium. Following around the curve to the support, the torsional moment increases by increments of xM/R. However, M changes between midspan and the support. Integrating the M/R diagram from midspan to support, as shown in **Figure 12.5.2.1-2**, a torsional moment of $WL_a^2/24R$ is obtained. This is identical to that obtained from equilibrium in Section 12.4.2.2.

Figure 12.5.2.1-2
Torque in a Simple-Span Curved Beam



12.5.2.2 Torsion in Continuous Beams

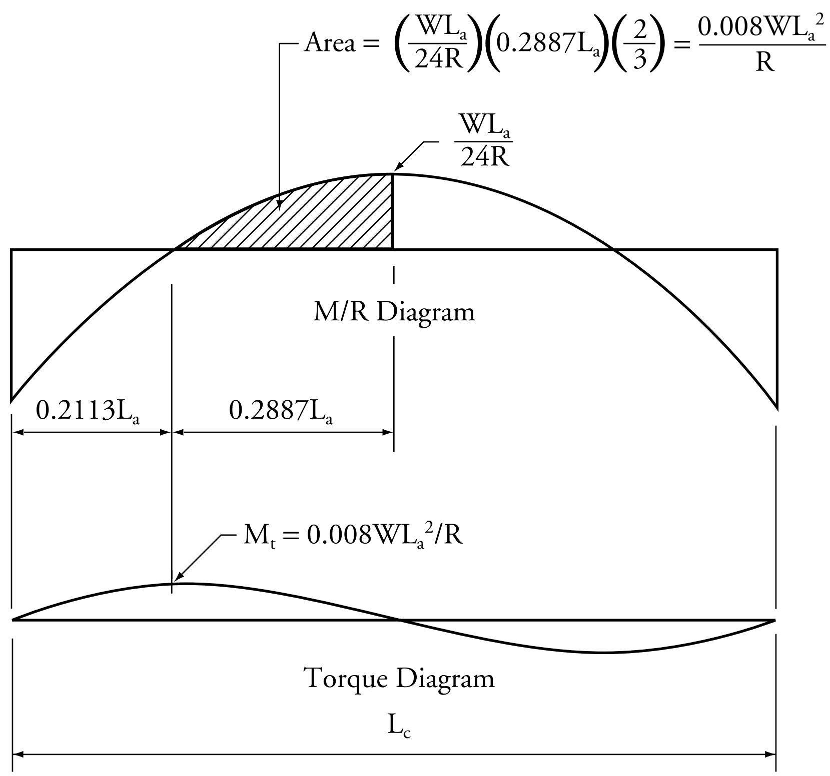
Torsion in continuous beams may be understood by first examining torsion in a fixed-ended beam. **Figure 12.5.2.2-1** shows the M/R diagram for a fixed-ended beam.

Because the area under the M/EI diagram for a fixed-ended beam must integrate to zero, the area under the M/R diagram will also integrate to zero, given constant EI and R. Thus, the torsion at the support will be zero. The maximum torque occurs at the inflection point, and is 19 percent of the maximum torque in a simple-span beam.

Continuous beams are intermediate between simple-span and fixed-ended beams. Interior spans resemble the fixed case more closely, and the free end of exterior spans may be closer to the simple-span case.

Continuity can significantly reduce torsional moments.

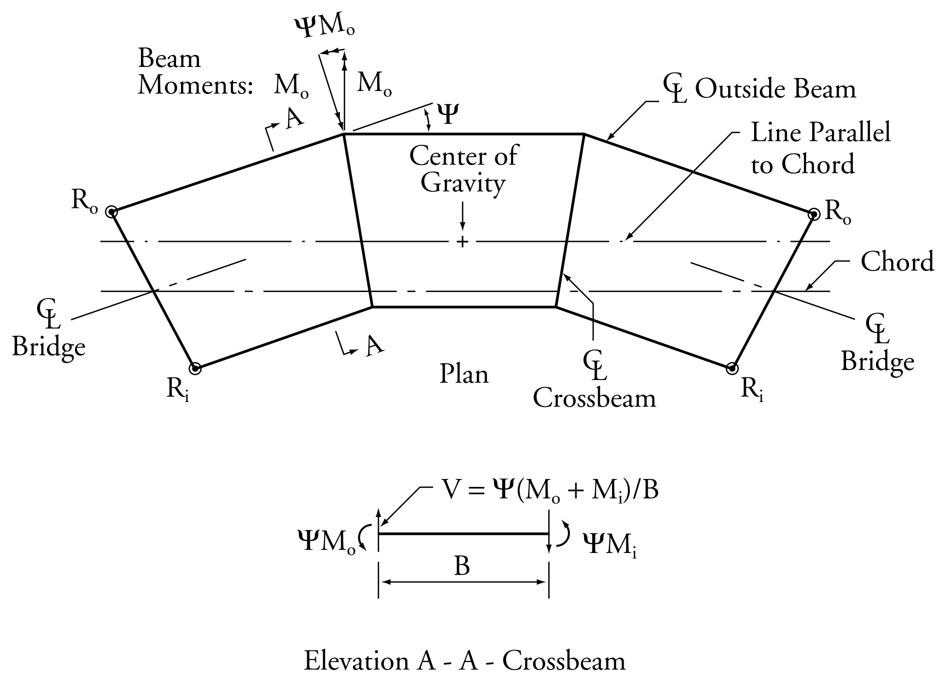
Figure 12.5.2.2-1
Torque in a Fixed-Ended Curved Beam



12.5.2.3
Behavior of Beam Gridworks

Beam gridworks composed of straight beam segments can resist eccentric loads without torsion. **Figure 12.5.2.3-1** shows a simple two-beam, three-segment gridwork.

Figure 12.5.2.3-1
Simple Gridwork



The beam moment at a joint must "turn the corner." In this case, equilibrium is supplied by a bending moment in the crossbeam. This bending moment in the crossbeam is equal to the angle (in radians) between the two beam segments multiplied by the bending moment in the main beam.

An equilibrium sketch of the crossbeam is shown in **Figure 12.5.2.3-1**. The moments at the two ends of the beam are equilibrated by shear forces, which transfer load from the inner to outer beam.

Note that for a two-beam gridwork, the reactions may be determined by statics, because the resultant of the reactions at each end must lie on a line through the resultant location of the loads. For multiple beam gridworks, reactions may be estimated by assuming a straight-line distribution of reactions that produces the correct location of the resultant. A procedure similar to that described in the *LRFD Specifications* Commentary [Article C4.6.2.2.2d], may be used. This is illustrated in the Design Example, Section 12.9.5.2.

After estimating the end reaction of the outside beam, one may estimate the bending moment in the outside beam. This is done by comparison to the bending moment in a straight beam of length equal to the arc (or chord) length of the centerline of the bridge. Two correction factors are then applied to this bending moment. The first correction is the ratio of the estimated end reaction in the beam grid work of the curved bridge to that in a straight bridge. A simplifying assumption is made that the bending moment is proportional to the end reaction multiplied by the length, giving the second correction factor, the ratio of the length of the outside beam to the centerline length. The bending moment of a straight beam of length equal to the centerline length of the bridge is then multiplied by these two factors to obtain the estimate of bending moment in the outer beam.

Loads applied after the gridwork is completed can theoretically be supported without torsion. Although equilibrium could be obtained without torsion, an analysis will show a small amount of compatibility torsion. If the factored compatibility torsion is below that given in the *LRFD Specifications* [Equation 5.8.2.1-3], the torsion may be safely ignored.

12.5.3 Crossbeams

Diaphragms in straight bridges, if used at all, are usually designed empirically, i.e., the design is not based on calculated shears and moments. In curved bridges, crossbeams must be designed for the shears and moments resulting from the change in direction of the primary bending moment in the stringer at the location of the crossbeams. The longitudinal forces in the bottom flange have a transverse component at the location of the crossbeam. The crossbeam must be deep enough to brace the bottom flange to resist this component.

12.6 DETAILED DESIGN

The loading stages given in Sections 12.6.2 and 12.6.3 are for simple spans. For continuous spans, more complex loading stages may be required.

12.6.1 Validity of Approximations

Detail design is done using a beam gridwork computer model. For mathematical consistency, it is better to use "exact" plan geometry instead of the approximations used in preliminary design. The computer model may be created in a horizontal plane, ignoring grade and superelevation. The extra weight in the "haunch" (or "pad") caused by superelevation should be taken into account, however.

12.6.2 Loading Stages – Box Beams

12.6.2.1 Bare Beam

An initial stage of plant post-tensioning is applied to the bare simple span beam to assemble the beam segments into a curved beam. This effectively applies the post-tensioning and the self-weight bending moment at the same time. After erection, crossbeams are cast, and their weight is applied to the bare beam.

12.6.2.2 Non-Composite Gridwork

The weight of the deck is applied to a non-composite gridwork, assuming unshored construction.

12.6.2.3 Composite Gridwork

The weights of future wearing surface, barriers, live load plus impact, and centrifugal force are applied to the composite gridwork. The simplifying assumptions for distribution of these loads in straight bridges cannot be used for curved bridges.

Additional field post-tensioning could be applied after casting the deck to partially compensate for the weight of the deck. This should not be done if future replacement of the deck is anticipated.

12.6.3 Loading Stages – I-Beams

A three-segment, simple-span I-beam is considered.

12.6.3.1 Individual Segments

The segments are pretensioned in the plant to compensate for self-weight bending of the individual segment.

12.6.3.2 Shoring Loads

The individual segments are erected in the field, supported by final bearings and by shores at intermediate locations. Post-tensioning ducts are spliced and crossbeams are cast.

During this loading stage, stresses in the beams do not change. Loads are added to the shoring.

12.6.3.3 Non-Composite Gridwork

Post-tensioning is applied to the non-composite gridwork after the crossbeams have cured sufficiently. This lifts the beams from the shores. The load that was present in the shores becomes a load applied to the non-composite beam gridwork.

The post-tensioning is best modeled as a set of external loads. That is, all the forces applied to the concrete by the tendons and their anchors are applied as external loads to the model. It is important that the transverse forces at the crossbeams not be overlooked. These forces are caused by the tendons that change direction (in plan) at the crossbeams.

The weight of the deck and haunch is also applied to the non-composite gridwork.

12.6.3.4 Composite Gridwork

The weights of the future wearing surface, barriers, live load plus impact, and centrifugal force are applied to the composite gridwork. See **Section 12.6.2.3** for additional considerations.

12.6.4 Other Design Checks

Checking allowable stresses, deflection and camber, prestress losses and ultimate strength is generally similar to that for a straight bridge, keeping in mind the differences between post-tensioning and pretensioning.

Torsion is an additional consideration. For segmental I-beam curved bridges, the torsion will often be below the limit for which the *LRFD Specifications* [Eq. 5.8.2.1-3] permit torsion to be neglected. Full-span box beams have higher torsion from self-weight, and a torsion analysis will be needed. Fortunately, for box sections, the torsional shear may be added directly to the vertical shear [*LRFD Specifications*, Eq. 5.8.3.6.2-3], and the analysis is similar to that done for vertical shear only.

12.7 FABRICATION

It is generally more economical to ship a full-span beam to the site instead of assembling segments on site. However, curved I-beams seldom have a sufficient torsional strength to permit this; thus, segments are used.

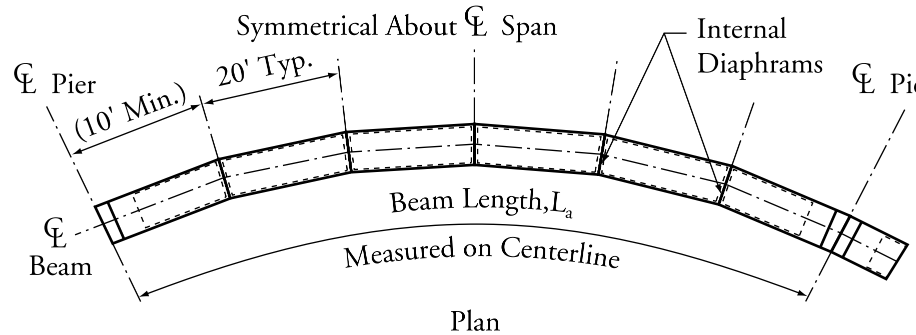
12.7.1 Box Beams

Box beams usually have enough strength to permit shipping a full-span beam. For box beams, segments would only be shipped if the full-span beam is too large to be shipped. The "bathtub" beam segments could be erected on shores, like that which is done for I-beam segments.

12.7.1.1 Chord Lengths

Chord lengths of 20 feet are suggested for curved box beams as shown in **Figure 12.7.1.1-1**. This produces a maximum arc-to-chord offset of 1 in. on a 600-ft radius and 2 in. on a 300-ft radius.

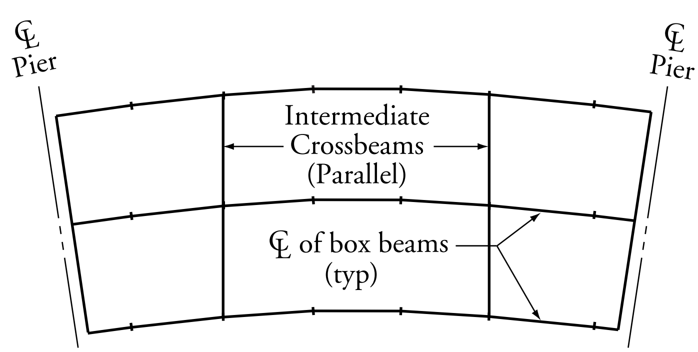
Figure 12.7.1.1-1 Precast Beam Chorded Geometry



12.7.1.2 Bridge Layout

Using 20-ft chords, lay out the bridge so that the chords at each end are between 10-ft and 20-ft long. Lay out the crossbeams parallel to each other, so that they intersect the main beams at a form joint. These considerations will simplify beam forming and fabrication. See **Figure 12.7.1.2-1**.

Figure 12.7.1.2-1 Box-Beam-Bridge Layout



12.7.1.3 Forms

In some localities, prestressed concrete "bathtub" beams of open trapezoidal shape are used. The forms for such beams could possibly be adapted, but the top would need to be closed in order to provide a torsion box.

Side forms can be erected on a steel table, as shown in **Figure 12.7.1.3-1**. The table must be wide enough to accommodate the curvature. The 20-ft chorded side forms are secured to the table, to the desired geometry.

Another forming method is the use of form sections that form both the sides and the soffit. This is described in ABAM (1988). See **Figure 12.7.1.3-2**.

Figure 12.7.1.3-1
Chorded Forms on Flat Table

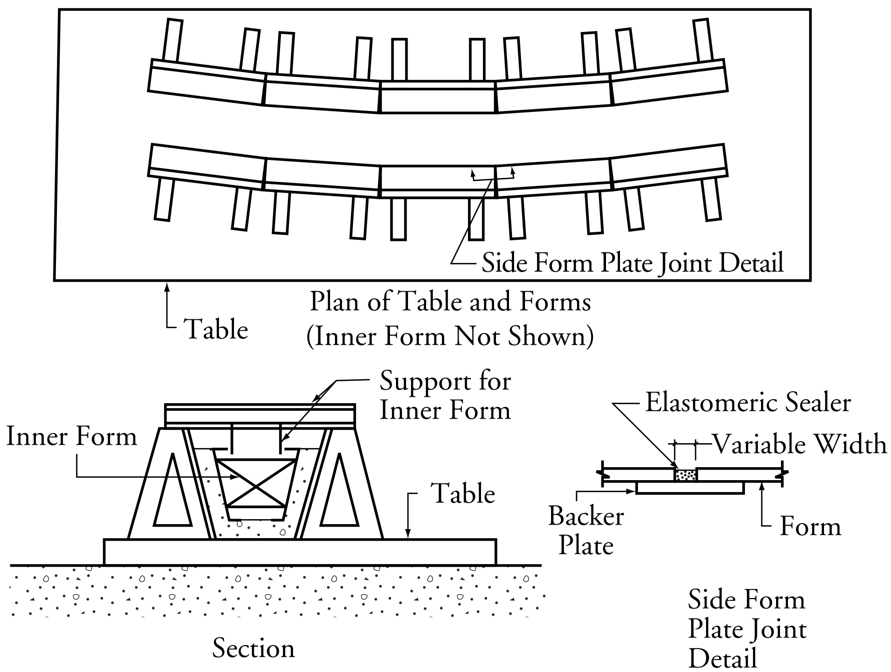
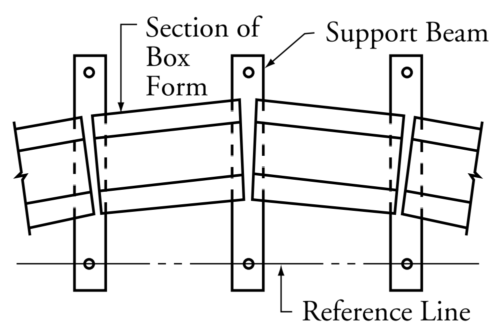


Figure 12.7.1.3-2
Plan View of Forming



12.7.1.4 Casting

Using an inner form, the beam is cast up to the underside of the top flange (top of the web). After the concrete hardens, the inner form is removed, and a stay-in-place form is used to form the top flange which is cast in a second-stage operation. Alternatively, stay-in-place void forms can be used if voids are properly anchored to prevent movement during placement of concrete and if thorough consolidation is attained under the form.

12.7.1.5 Post-Tensioning

If the complete curved box beam is prefabricated, the beam is post-tensioned and the ducts and anchorages are grouted at the plant. This may be done as a two-stage operation, with the first stage of post-tensioning done at an early age, and the final stage done after concrete design strength is achieved. Where segments of curved beams are spliced in the field due to haul limitations, additional post-tensioning will be required. If curved beams are made continuous over piers, additional post-tensioning near the piers or of the entire structure may be required in the field.

12.7.2 I-Beams and Bulb-Tee Beams

12.7.2.1 Chord Lengths

Chorded segment lengths should be made as long as is permissible (see Section 12.3.1), in order to minimize the field joints in the segmented beam. Generally two, three, or four segments should be used.

12.7.2.2 Bridge Layout

In contrast to the box beam bridge layout, it is recommended that crossbeams be on radial lines. This will result in a more consistent geometry, and the variation in length of beam segments will not cause forming problems.

12.7.2.3 Forms

Standard beam forms may be used. It is usually necessary to widen the webs to accommodate post-tensioning ducts. This can often be done by spreading the side forms. A new pallet or pan, as well as new end bulkheads, may be required.

If post-tensioning tendons are anchored at the ends of the beams, as is frequently done, end blocks will be required. End blocks are often cast with the segment but may be added later as a secondary pour. End blocks will be needed only at one end of each end segment, so odd lengths can be accommodated by adjusting the bulkhead location at the opposite end. In some cases, end blocks may be eliminated by placing post-tensioning anchorages in the end walls or end diaphragms.

12.7.2.4 Casting

The beam segments are cast in the usual manner with the addition of post-tensioning ducts and anchorages. Splices between segments are generally wet cast, so match-casting is not required.

12.7.2.5 Pretensioning

The beam segments may have a small amount of pretensioning to compensate for self-weight bending of the individual segments.

12.8 HANDLING, TRANSPORTATION AND ERECTION

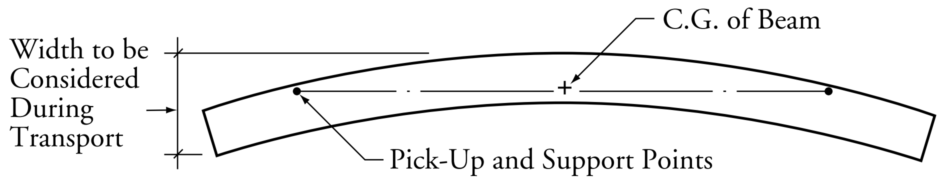
12.8.1 Box Beams

This section addresses handling considerations for curved box beams. Handling for individual straight box beam segments to be assembled in the field is similar to the handling of I-beam segments addressed in the following sections.

12.8.1.1 Handling

Pickup and support points must be located on a line through the center of gravity (in plan) of the curved beam. Pickup and temporary support points may be located inward from the beam ends, if the curvature is too great for the beam to be stable when supported at the ends. Of course, the beam stresses must be checked for the pickup and support point location. See **Figure 12.8.1.1-1**.

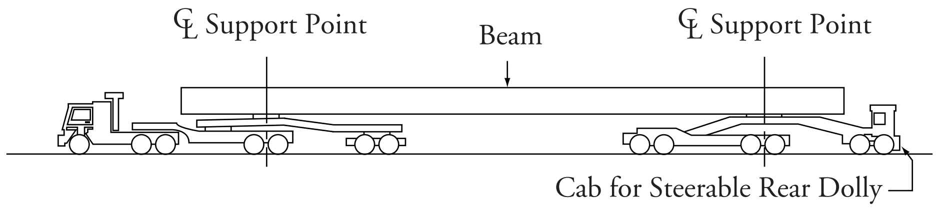
Figure 12.8.1.1-1
Pick-Up and Support Points for Curved Beam



12.8.1.2
Transportation

Long-span box beams are very heavy. The maximum span may be governed by the maximum practical transportable weight or transportable width instead of design considerations. Curved box beams may also be spliced in the field if weight or width limitations restrict transportable length. Special transporters will usually be required, as illustrated schematically in **Figure 12.8.1.2-1**, to accommodate weight and long overhangs from support points.

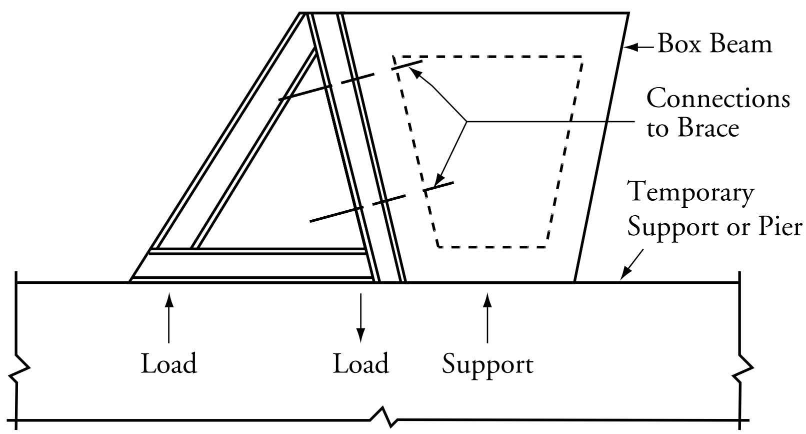
Figure 12.8.1.2-1
Beam Transporter



12.8.1.3
Erection

A temporary brace will probably be needed to stabilize the beam after erection, as shown in **Figure 12.8.1.3-1**. This brace needs to remain in place until the end and intermediate diaphragms are cast.

Figure 12.8.1.3-1
Schematic of Temporary Brace to Stabilize Beam



12.8.2
I-Beams and Bulb-Tee Beams

12.8.2.1
Handling and Transportation

The I-beam segments present no unusual difficulties in handling and transportation.

12.8.2.2
Erection and Post-Tensioning

The I-beam segments are typically erected on shoring located at the interior cross-beam closure pours. The beams generally will rise off of the shores as they are post-tensioned.

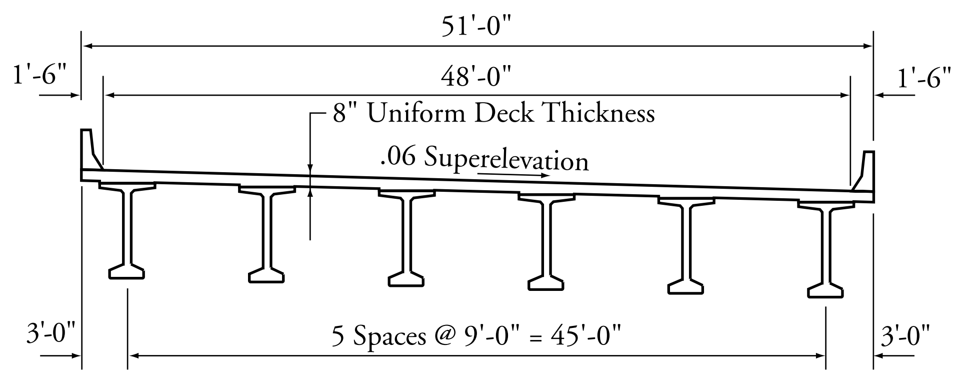
12.9 DESIGN EXAMPLE

12.9.1 Introduction

This design example demonstrates the preliminary design of a 120-ft, simple-span, bulb-tee-beam bridge on a 600-ft radius curve. Except for changes brought about by the curvature, the bridge is the same as that designed in Section 9.4. The 120-ft span is measured along the arc at the centerline of the bridge. The bridge is superelevated 6 percent, and the design speed is 40 mph. The splices, intermediate diaphragms and piers are all arranged radial to the curve.

The superstructure consists of six beams spaced at 9'-0" centers, as shown in **Figure 12.9.1-1**. Beams are designed to act compositely with the 8-in. cast-in-place deck to resist all superimposed dead loads, live loads and impact. A 1/2-in. wearing surface is considered an integral part of the 8-in. deck. The design is accomplished in accordance with the *LRFD Specifications*, 2nd Edition, 1998. Design live load is HL-93.

Figure 12.9.1-1 Bridge Cross-Section at Midspan



12.9.1.1 Plan Geometry

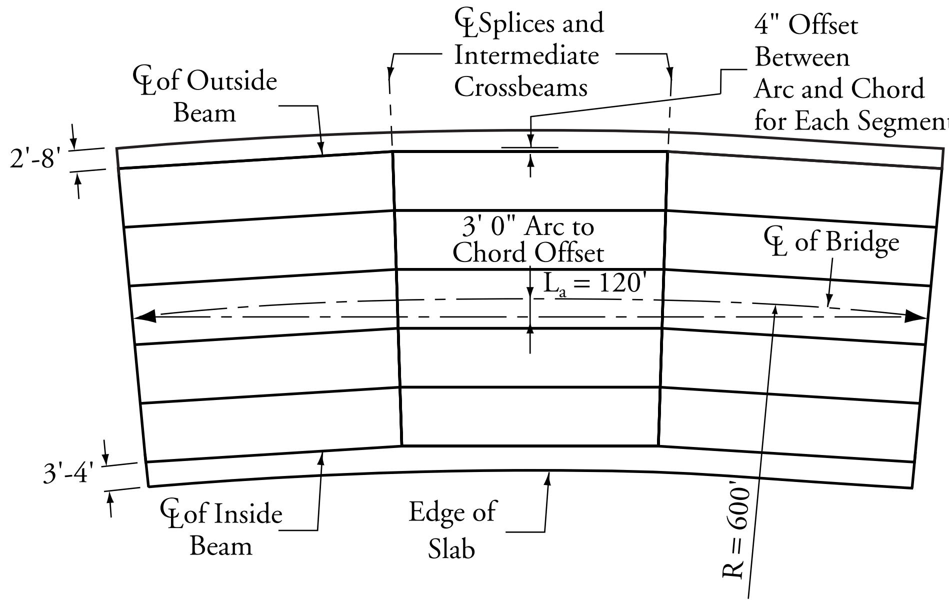
Check to see if straight beams might be used. The arc-to-chord offset is $L_a^2/8R = (120)^2/(8 \times 600) = 3$ ft. This exceeds the maximum recommended offset of 1.5 ft. If the beam is subdivided into three chords, the maximum offset will be reduced by a factor of $(3)^2$, producing an offset of 4 in. at the center of each chord. This will be barely detectable visually and will be acceptable. In order to minimize the overhang on the outside of the curve, the 3-ft overhang will be set at the middle of each chorded segment. At the ends of the chorded segments, the overhang from beam centerline on the outside will be 2'-8", and 3'-4" on the inside. **Figure 12.9.1.1-1** shows the plan geometry.

12.9.1.2 Construction

With a nominal chord offset of 3 ft for the span, the torsion will be too great for a plant post-tensioned, full-length beam. Therefore, segmental (spliced) construction will be used. Each 40-ft (nominal length) straight segment will be precast with enough pretension to compensate for self-weight on the 40-ft span. Shoring will be erected at the 1/3 points of the 120-ft span. The 40-ft segments will then be set on the shores and on the end bearings at the abutments. Crossbeams, 12-in.-thick by 66-in.-deep, will be cast at the ends. Crossbeams with the same cross-section will be cast at the 1/3 points (splice locations). The beams will then be post-tensioned and the shores removed.

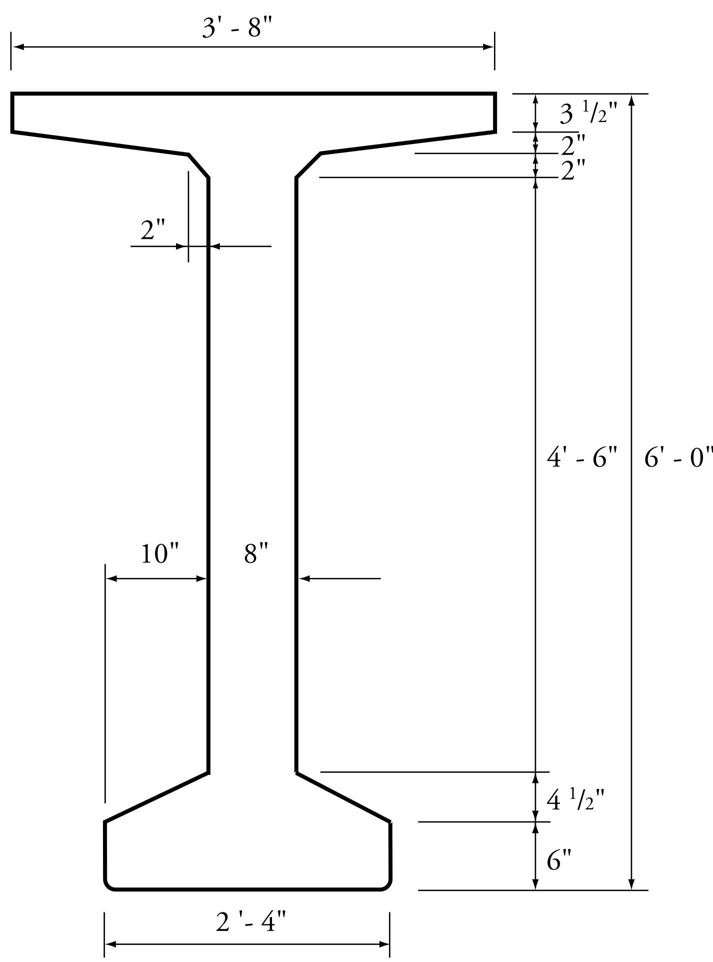
The deck is generally cast after post-tensioning. This procedure makes it feasible to replace the deck in the future, should that become necessary.

Figure 12.9.1.1-1
Beam Framing Plan Geometry



Because the girders are post-tensioned, a thicker web will be used to provide necessary cover over the ducts. This may be accomplished by spreading the side forms for an AASHTO-PCI BT-72 by 2 in., creating an 8-in.-thick web. See **Figure 12.9.1.2-1** for modified section dimensions.

Figure 12.9.1.2-1
AASHTO-PCI BT-72 Dimensions with 2 in. Added to Width



12.9.2 Materials

These are almost identical to those used in the Section 9.4 example.

Cast-in-place slab: Actual thickness, t_s = 8.0 in.

Structural thickness = 7.5 in.

Note that a 1/2-in. wearing surface is considered an integral part of the 8-in. deck.

Concrete strength at 28 days, f'_c = 4.0 ksi

Precast beams: AASHTO-PCI Bulb-tee with 2-in.-added width as shown in **Figure 12.9.1.2-1**

Concrete strength of beam at post-tensioning, f'_{ci} = 6.5 ksi

Concrete strength at 28 days, f'_c = 6.5 ksi

Concrete unit weight, w_c = 0.150 kcf

Design span = 120.0 ft (Arc length at centerline of bridge)

Post-tensioning strands: 0.6-in. dia, seven-wire, low-relaxation

Area of one strand = 0.217 in.^{2}

Ultimate strength, f_{pu} = 270.0 ksi

Yield strength, $f_{py} = 0.9f_{pu}$ = 243.0 ksi [LRFD Table 5.4.4.1-1]

Stress limits for post-tensioning strands: [LRFD Table 5.9.3-1]

at jacking: $f_{pj} = 0.80f_{pu}$ = 216.0 ksi

at service limit state (after all losses):
$f_{pe} < 0.80f_{py}$ = 194.4 ksi

Modulus of elasticity, E_p = 28,500 ksi [LRFD Art. 5.4.4.2]

Reinforcing bars: Yield strength, f_y = 60 ksi

Modulus of elasticity, E_s = 29,000 ksi [LRFD Art. 5.4.3.2]

Future wearing surface: 2 in. additional concrete, unit weight = 0.150 kcf

New Jersey-type barrier: Unit weight = 0.300 kip/ft/side

12.9.3 Cross-Section Properties

12.9.3.1 Non-Composite Sections

A = area of cross-section of precast beam = 911 in.^{2}

h = overall depth of beam = 72 in.

I = moment of inertia about the centroid of the precast beam = 608,109 in.^{4}

y_b = distance from centroid to extreme bottom fiber of the precast beam = 36.51 in.

y_t = distance from centroid to extreme top fiber of the precast beam = 35.49 in.

S_b = section modulus for the extreme bottom fiber of the precast beam = 16,657 in.^{3}

S_t = section modulus for the extreme top fiber of the precast beam = 17,134 in.^{3}

I_{lat} = lateral moment of inertia of precast beam = 46,014 in.^{4}

w_g = beam self-weight per unit length = 0.949 kip/ft

E_c = modulus of elasticity, ksi = $33{,}000(w_c)^{1.5}\sqrt{f'_c}$ [LRFD Eq. 5.4.2.4-1]

where

w_c = unit weight of concrete = 0.150 kcf

The *LRFD Specifications*, Commentary C5.4.2.4, indicates that the unit weight of normal weight concrete is 0.145 kcf. However, precast concrete mixes typically have a relatively low water/cementitious materials ratio and high density. Therefore, a unit weight of 0.150 kcf is used in this example. For high strength concrete, this value may need to be increased based on test results.

f'_c = specified strength of concrete, ksi

Therefore, the modulus of elasticity for:

cast-in-place slab, $E_{cs} = 33{,}000(0.150)^{1.5}\sqrt{4.0} = 3{,}834$ ksi

precast beam at transfer of post-tensioning (at 28 days minimum)

$$E_{ci} = 33{,}000(0.150)^{1.5}\sqrt{6.50} = 4{,}888 \text{ ksi}$$

precast beam at service loads, $E_c = 33{,}000(0.150)^{1.5}\sqrt{6.50} = 4{,}888$ ksi

The torsional constant, J, is estimated in accordance with *LRFD Specifications*, and Section 7.6.5.

$$J \approx \frac{A^4}{40.0 I_p}$$

The polar moment of inertia I_p is equal to the sum of I and I_{lat}. $I_p = 654{,}123$ in.^{4}

$$J \approx \frac{911^4}{40(654{,}123)} = 26{,}324 \text{ in.}^4$$

Properties of the 12- by 66-in. crossbeam:

$A = 792$ in.^{2}

$I = 287{,}496$ in.^{4}

$I_{lat} = 9{,}504$ in.^{4} (for lateral bending)

$J = 33{,}120$ in.^{4}

$w_g = 0.825$ kip/ft

12.9.3.2 Composite Sections

12.9.3.2.1 Effective Flange Width

Because this is a preliminary design it is reasonable to assume the same properties for interior and exterior beams. Therefore, the properties for a typical interior beam are used. Final designs will require more thorough calculations.

Effective flange width for interior beams shall be the lesser of: [LRFD Art. 4.6.2.6.1]

- (1/4) span = (120 x 12)/4 = 360 in.
- $12t_s$ plus greater of web thickness or 1/2 beam top flange width
  = (12 x 7.5) + (0.5 x 44) = 112 in.
- average spacing between beams = (9 x 12) = 108 in.

Therefore, the effective flange width is 108 in. for the beam.

For the interior crossbeams, the effective flange width is (12 x 7.5) + 12 = 102 in.

Note that the crossbeam in a curved bridge is not an ordinary beam spanning between main beams (9 ft in this case). Rather, it transfers load all the way across the bridge from inside to outside beams.

12.9.3.2.2 Modular Ratio

Modular ratio between slab and beam materials, $n = \dfrac{E_{cs}}{E_c} = \dfrac{3{,}834}{4{,}888} = 0.7845$

12.9.3.2.3 Transformed Section Properties

Transformed flange width for interior beams = n (effective flange width)
= (0.7845)(108) = 84.73 in.

Transformed flange area for interior beams = n(effective flange width)(structural thickness)
= (0.7845)(108)(7.5) = 635.45 in.^{2}

Note: Only the structural thickness of the deck, 7.5 in., is considered.

A minimum haunch thickness of 1/2 in. at midspan is considered in the structural properties of the composite section. The superelevation will cause the average thickness of the haunch to be greater than 1/2 in. The extra weight will be accounted for, but the extra thickness caused by superelevation will conservatively be neglected in computing composite section properties. In addition, the width of haunch must be transformed.

Transformed width of haunch = (0.7845)(44) = 35.52 in.

Transformed area of haunch = (0.7845)(44)(0.5) = 17.26 in.^{2}

Note that the haunch should only be considered to contribute to section properties if it is required to be provided in the completed structure. Therefore, some designers neglect its contribution to the section properties.

A_c = total area of the composite section = 1,564 in.^{2}

h_c = overall depth of the composite section = 80 in.

I_c = moment of inertia of the composite section = 1,208,734 in.^{4}

y_{bc} = distance from the centroid of the composite section to the extreme bottom fiber of the precast beam = 53.05 in.

y_{tg} = distance from the centroid of the composite section to the extreme top fiber of the precast beam = 18.95 in.

y_{tc} = distance from the centroid of the composite section to the extreme top fiber of the deck = 26.95 in.

S_{bc} = composite section modulus for the extreme bottom fiber of the precast beam = 22,784 in.^{3}

S_{tg} = composite section modulus for the top fiber of the precast beam = 63,792 in.^{3}

S_{tc} = composite section modulus for the extreme top fiber of the deck slab = 57,176 in.^{3}

I_{clat} = moment of inertia of composite section for lateral bending = 666,423 in.^{4}

For computing J_c, the torsional constant for the composite beam, half the composite flange width is used to compute the area A_c and the polar moment of inertia I_{pc} for substitution in Eq. C4.6.2.2.1-2 in the *LRFD Specifications*. The transformed area A_c is 1,246 in.^{2} and I_{pc} is 1,118,680 in.^{4} This results in a value of J_c of 53,865 in.^{4}

Composite properties of interior crossbeams:

A_c = 1,397 in.^{2}
I_c = 765,432 in.^{4}
I_{clat} = 529,860 in.^{4} for lateral bending
J_c = 54,204 in.^{4}

12.9.4 Loads

For a first approximation, all loads except the truck load will be assumed to be distributed over the area of the deck. Later, after a beam gridwork model is created, the computer program will generate member self-weights.

12.9.4.1 Dead Loads

12.9.4.1.1 Dead Loads Acting on the Non-Composite Structure

Beam and crossbeam weight:
Beams = (6)(120 ft)(0.949 kip/ft) = 683 kips
Crossbeams = (4)(45 ft)(0.825 kip/ft) = 149 kips
Total weight of beams and crossbeams = 683 + 149 = 832 kips

Deck weight:
Gross area of deck = (120 ft)(51 ft) = 6,120 ft^2
Actual thickness = 8 in.
Deck weight = [8 in./(12 in./ft)](0.150 kcf)(6,120 ft^2) = 612 kips

For a minimum haunch thickness of 0.5 in., the superelevation of 0.06 will cause the average haunch thickness to be 0.5 in. + 0.06(22 in.) = 1.82 in., say 2 in. The haunch weight is 0.150 kcf (2 in.)(44 in.)/(144 in.^{2} /ft^2) = 0.092 kip/ft/beam

Haunch weight = (6)(120 ft)(0.092 kip/ft) = 66 kips
Weight of deck, including haunch = 678 kips

12.9.4.1.2 Dead Loads Acting on the Composite Structure

Barrier weight is given as 0.3 kip/ft/side
Barrier weight = (2)(120 ft)(0.3 kip/ft) = 72 kips
Future wearing surface is 0.025 ksf
(0.025 ksf)(120 ft)(48 ft) = 144 kips
Dead load on composite structure = 72 + 144 = 216 kips

12.9.4.1.3 Total Dead Load

Total dead load = 832 + 678 + 216 = 1,726 kips

12.9.4.2 Live Loads

Design live loading is HL-93 which consists of a combination of: [LRFD Art. 3.6.1.2.1]

1. Design truck or design tandem with dynamic allowance.
   - The design truck is the same as the HS20 design truck specified by the *Standard Specifications*. [LFRD Art. 3.6.1.2.2]
   - The design tandem consists of a pair of 25.0-kip axles spaced at 4.0 ft apart. [LFRD Art. 3.6.1.2.3]
2. Design lane load of 0.64 kip/ft without dynamic allowance. [LRFD Art. 3.6.1.2.4]

IM = 33% [LFRDTable 3.6.2.1-1]

where IM = dynamic load allowance, applied to design truck or design tandem only

The number of design lanes is computed as:

Number of design lanes = the integer part of the ratio of w/12, where w is the clear roadway width, ft, between the curbs: [LRFD Art. 3.6.1.1.1]

w = 48 ft

Number of design lanes = integer part of (48/12) = 4 lanes

Multiple presence factor, m: [LRFD Table 3.6.1.1.2-1]

For 4 lanes, m = 0.65.

Stresses from truck and lane loads obtained from refined analysis will be multiplied by 0.65.

12.9.4.2.1 Lane Loading

The lane load is positioned over a 10-ft width within the 12-ft design lane. [LRFD Art. 3.6.1.3.1]

To maximize the effect of the live load, the 10-ft loaded width is shifted to the left within each design lane. This causes the lane load to have an eccentricity of 1 ft relative to the lane centerline, and the four lane loads have an eccentricity of 1 ft relative to the bridge centerline. The average arc length increases by the ratio of 601-ft radius/600-ft radius, to 120.2 ft.

The total lane loading for the four design lanes is (4)(120.2 ft)(0.64 kip/ft)(0.65) = 200.0 kips.

The 0.65 factor above is the factor, m.

12.9.4.2.2 Truck Loading

The total weight of the design truck is 8 + 32 + 32 = 72 kips.

Including 33% impact, 1.33 x 72 = 95.76 kips.

For 4 trucks, including the multiple presence factor, m:

4(95.76)(0.65) = 249.0 kips

Note that because this is a preliminary design of the main members of a 120-ft span, the tandem load need not be considered at this time.

12.9.4.2.3
Total Live Load

Total live load = 200.0 + 249.0 = 449.0 kip

12.9.4.2.4
Centrifugal Force

[LRFD Art. 3.6.3]

The design speed is 40 mph. The centrifugal force coefficient is given by:

$$C = \left(\frac{4}{3}\right)\frac{v^2}{gR}$$ [LRFD Eq. 3.6.3-1]

where

C = coefficient to compute centrifugal force

v = highway design speed, ft/sec

g = gravitational acceleration, 32.2 ft/sec^2

R = radius of curvature of traffic lane, ft

The design speed in ft/sec = 40 mph/0.682 = 58.65 ft/sec

$$C = \left(\frac{4}{3}\right)\frac{(58.65)^2}{(32.2)(600)} = 0.2374$$

This is applied to the truck axle loads only, without the dynamic load allowance, and with the factor, m. The centrifugal force for four trucks is 4(72 kip)(0.2374)(0.65) = 44.4 kips.

12.9.5
Correction Factors

The bending moments in the exterior beam on the outside of the curve will be greater than in a straight bridge for three reasons:

1. The additional span length on the outside of the curve.
2. The center of gravity of the curved centerline lies outside of a line through the centerline of the supports.
3. The center of gravity of an area load is further shifted outward, because there is more area outside of the centerline than inward of the centerline.

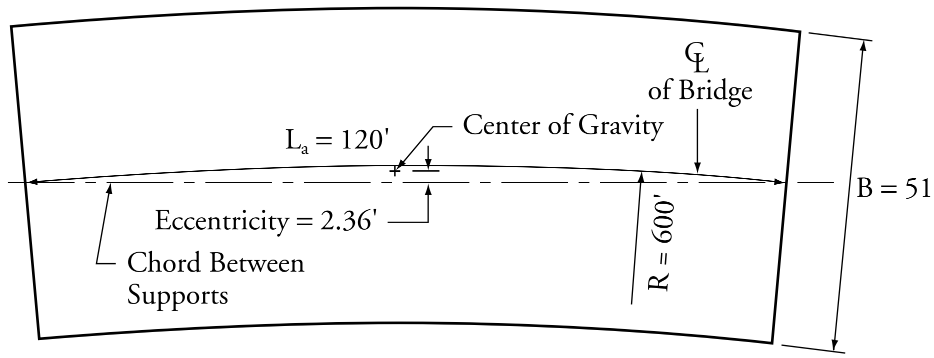
12.9.5.1
Additional Span Length Factor

The outside beam is on a radius of 622.5 ft. This increases the span length by a factor of 622.5/600 = 1.0375.

12.9.5.2 Shift in Center of Gravity

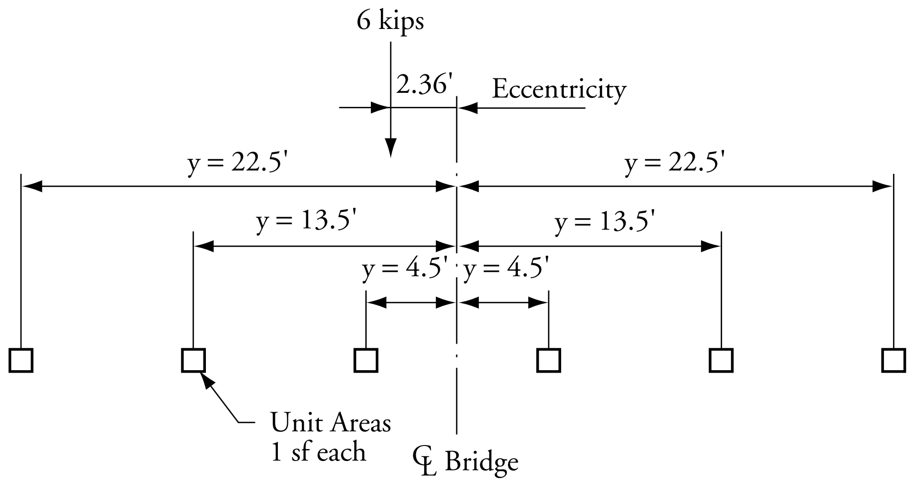
The center of gravity (in plan) of the centerline arc is offset from a line through the center of the bearings by an amount equal to 2/3 of the arc-to-chord offset (2/3)(3 ft) = 2 ft. The additional eccentricity caused by the extra area outside the centerline is equal to $B^2/12R = (51\ ft)^2/(12)(600) = 0.36$ ft, as shown in **Figure 12.9.5.2-1**. For the initial simplification that all dead load is an area load, the eccentricity of the dead load is 2.36 ft.

Figure 12.9.5.2-1
Center of Gravity of Curved Area



The next step is to find how much the load on the outside beam is increased because of this eccentricity. The procedure is analogous to one described in the LRFD Commentary [Article C4.6.2.2.2d] (see **Figure 12.9.5.2-2**). For six unit areas at 9-ft spacing, the moment of inertia is 1,417.5 ft^4 and the section modulus is 63 ft^3. For an arbitrary load of 1 kip per bearing, or 6 kips, at 2.36 ft eccentricity, P/A + Pe/S = 1 + 6(2.36)/63 = 1.2248. This is the increase in load on the outside exterior beam caused by the eccentricity of the load. The total correction factor for bending moment due to dead load is (1.0375)(1.2248) = 1.271.

Figure 12.9.5.2-2
Properties of Group of Beam Supports



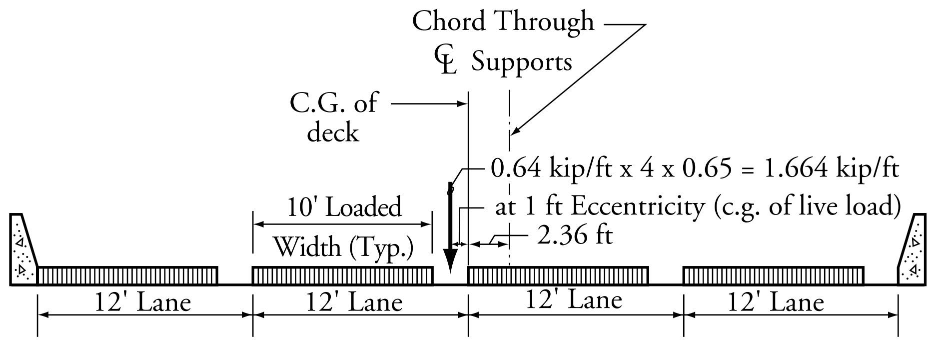
$$A = 6\ ft^2$$

$$I = \Sigma[(\text{Unit Area})(y^2)] = 1{,}417.5\ ft^4$$

$$S = I/y_{max} = 63\ ft^3$$

For the lane loading, the LRFD requirement to place the load off-center of the lane adds 1 ft to the eccentricity. See **Figure 12.9.5.2-3**. For a 6-kip load at 3.36-ft eccentricity, the load on the outside beam is 1 + 6(3.36)/63 = 1.32. The total correction factor for lane loading is (1.0375)(1.32) = 1.370.

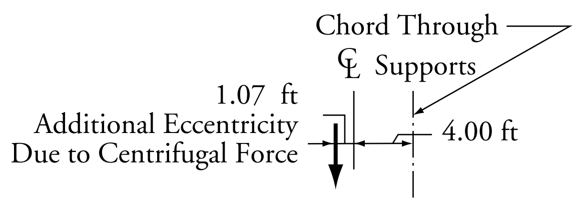
Figure 12.9.5.2-3
Lane Load Eccentricity



Lane Loading [LRFD Article 3.6.1.2.4]

For the truck loading, LRFD Article 3.6.1.3.1 specifies that the center of the wheel load be placed 2 ft from the curb. This causes the center of the vehicle to be 5 ft from the curb (also the lane edge), so the eccentricity from the centerline of the lane is 1 ft. The trucks are in the center of the bridge, which has a 3-ft eccentricity with respect to the supports. Thus, the vertical truck loading has an eccentricity of 4 ft as shown in **Figure 12.9.5.2-4**.

Figure 12.9.5.2-4
Truck Load Eccentricity



Truck Loading [LFRD Articles 3.6.1.3.1 and 3.6.3]

The effects of centrifugal force must also be taken into account. The total centrifugal force of 44.4 kips acts at a height of 6 ft [LRFD Art. 3.6.3]. The vertical truck loading is 249 kips. The horizontal force acting at 6 ft increases the eccentricity of the vertical load by (44.4/249)(6 ft) = 1.07 ft. The total eccentricity of the vertical truck load is 5.07 ft, and the correction is 1 + 6(5.07)/63 = 1.483, as shown in **Figure 12.9.5.2-4** The total correction factor due to centrifugal force and truck loading is (1.0375)(1.483) = 1.538.

12.9.6 Bending Moments – Outside Exterior Beam

The bending moments in the outside exterior beam may now be estimated. For all loads, the bending moment may be estimated as that for a 120-ft straight beam multiplied by the correction factor. For all loads except the truck loadings, the 120-ft straight beam bending moment is WL/8 divided by six beams in the bridge. For the truck loading, the bending moment is scaled from that for a standard truck on a 120-ft straight span. **Table 12.9.6-1** is a summary of the estimated midspan bending moments for the outside exterior beam.

Table 12.9.6-1
Estimated Bending Moments in Outside Beam

	Total Weight W, kips	Moment for 120-ft Straight Beam ft-kips	Correction Factor	Moment for Curved Beam ft-kips	Interior Beam, Straight Bridge, ft-kips*
Beam & Crossbeam	832	2,080	1.271	2,644	1,438
Deck & Haunch	678	1,695	1.271	2,154	1,660
Barrier	72	180	1.271	229	180
Wearing Surface	144	360	1.271	458	360
Truck Loading, w/impact	249	1,080	1.538	1,662	1,830
Lane Loading	200	500	1.370	685	843
Total	–	**5,895**	–	**7,830**	**6,311**

** Bending moments in the right column are taken from Table 9.4.4-1.*

Comparing these estimates to the values in the right column taken from Table 9.4.4-1, it may be seen that the dead load moments are substantially increased, compared to the interior beam of a straight bridge. However, the live loads are decreased somewhat, because of the factor, m [LRFD Art. 3.6.1.1.2], which is not used in the approximate distribution method. It should also be noted that the curved beam is almost 20% heavier than the straight beam.

12.9.7 Stresses – Outside Exterior Beam

The next step is to verify that the chosen beam section is adequate. It is assumed that the bottom fiber stress due to the weight of the beams and crossbeams can be compensated by the prestensioning.

Table 12.9.7-1
Estimate of Bottom Fiber Stress

LOAD	Bending Moment, ft-kips	S_b or S_{bc}, in.^{3}	Bottom Fiber Stress, ksi
1. Self Weight of Beams and Crossbeams (Compensated by pretensioning)	2,644	16,657	1.905
2. Deck and Haunch	2,154	16,657	1.552
3. Superimposed Dead Load	687	22,784	0.362
4. Live Load (0.8)(2,347) =	1,878	22,784	0.989
5. Sum of 2 + 3 + 4			**2.903**
Allowable stress at transfer of post-tensioning = $(0.60)f'_{ci}$ = (0.6)(6.5) [LRFD Art. 5.9.4.1.1]			3.900

Table 12.9.7-1 shows the bottom fiber stress caused by deck weight, superimposed dead load, and live load. For service load tensile stress checks, the live load may be taken as 80 percent of the computed live load [LRFD Art. 3.4.1, limit state Service III]. The bottom fiber stress for these loads applied to the beams and crossbeams in **Table 12.9.7-1** is 2.903 ksi. The allowable temporary stress after post-tensioning (before time-dependant losses) is 3.90 ksi. Therefore, because there is sufficient margin between the actual stress after losses and the allowable stress before losses, the beam section should be adequate and the computer model may be constructed using this beam.

12.9.8 Beam Gridwork Computer Models

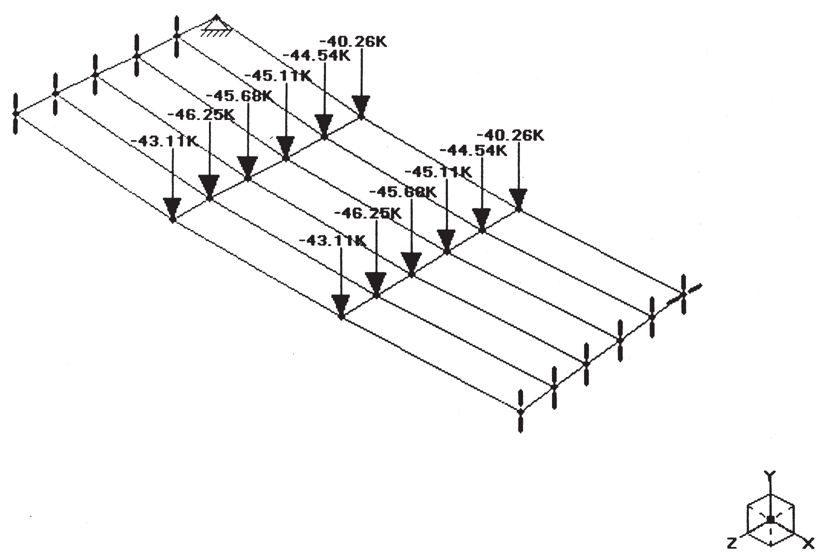
12.9.8.1 Model 1 – Beam Segments on Shores

This model (not shown) is a variation of Model 2 (see below). The ends of the members are released in bending to model the situation of simple span beam segments supported on shores. The simple span length is conservatively taken as the center-to-center distance of the crossbeams (40 ft). The reactions at each shoring location are computed. These loads are then applied to the Model 2 beam gridwork to represent removal of the shores. These loads are shown on **Figure 12.9.8.2-1**.

12.9.8.2 Model 2 – Shore Loads

Model 2 is the non-composite beam gridwork on the nominal 120-ft span. The loads applied to Model 2 are the loads that previously existed in the shores, as determined in Model 1. When the shores are removed, the loads previously existing in the shores are loads that are applied to Model 2. These loads are shown in **Figure 12.9.8.2-1**. The analysis done using Model 1 could be skipped, and the self-weight loads applied directly to Model 2. The difference in total self-weight bending moment in the outside exterior beam is less than 0.1 percent. However, it must be remembered that the moment consists of two parts, that applied to the 40-ft nominal span, and that applied to the 120-ft nominal span.

Figure 12.9.8.2-1
Non-Composite Model 2 – Shore Loads



12.9.8.3 Model 3 – Weight of Deck and Haunches

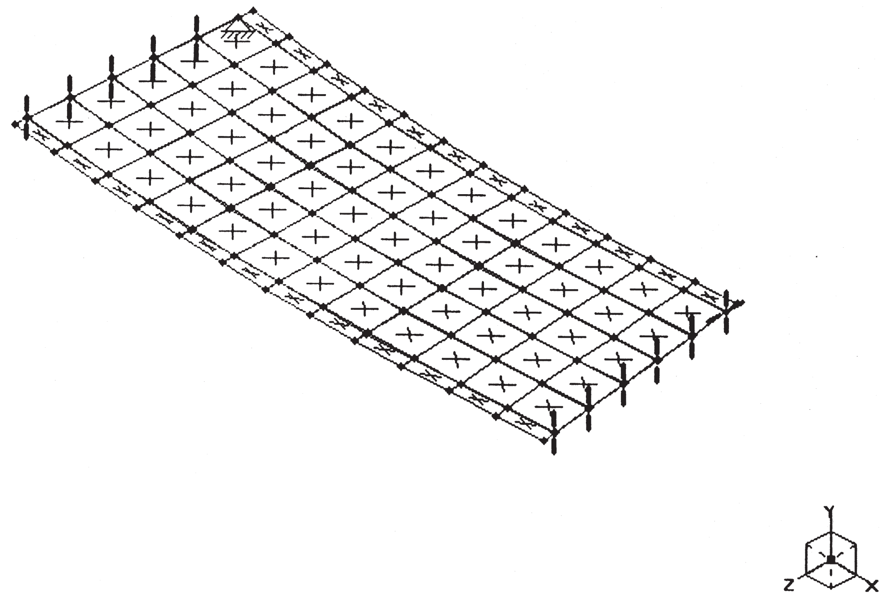
The total weight of the deck and the haunches between the deck and the top flanges of the beam was calculated to be 678 kips in Section 12.9.4.1.1. This load is assumed to be applied as a uniform load of 110.8 psf over the 6,120 sf gross area of the deck.

The model for deck weight is shown in **Figure 12.9.8.3-1**. The finite elements are only used as a means of applying a uniform load. The structural properties of the deck are zeroed out, because this is a non-composite model. The beam gridwork is the same as in Model 2.

12.9.8.4 Model 4 – Weight of Barriers and Future Wearing Surface

Model 4 represents the composite structure. Composite section properties are used in the beam gridwork. The general appearance of the model is the same as Model 3 (see **Figure 12.9.8.3-1**). The 0.025 ksf uniform load is applied over the entire 51-ft width of the deck. A net barrier load of 0.263 kip/ft (0.3 kip/ft less the 0.025 ksf acting over the 1.5-ft width occupied by the barrier) is applied as a line load along the longitudinal edges of the model.

Figure 12.9.8.3-1
Non-Composite Model 3 – Deck Weight

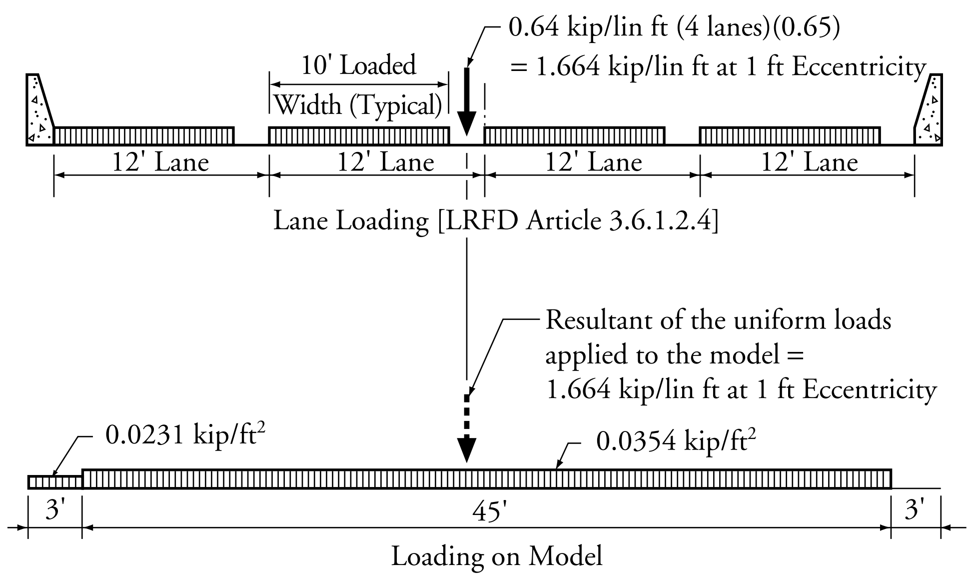


12.9.8.5
Model 5 – Lane Loading

As noted in Section 12.9.4.2.1, LRFD Article 3.6.1.3 specifies that the design lane load be applied over a 10-ft width within the design lane width (of 12 ft in this case). This causes the resultant of the lane loads to be shifted 1 ft towards the outside of the curve.

The upper part of **Figure 12.9.8.5-1** shows the specified location of the lane loads in a cross-section through the bridge. The lower part of **Figure 12.9.8.5-1** shows the actual loads applied to the model. The loads were chosen so that deck elements would be loaded uniformly and the total load would have the correct location of the resultant load.

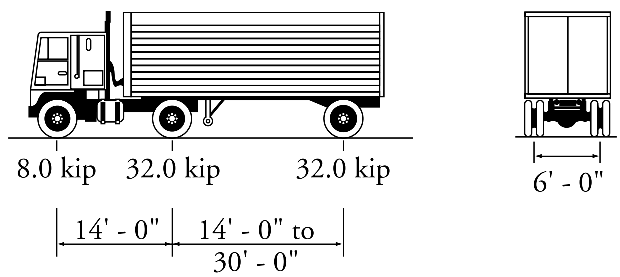
Figure 12.9.8.5-1
Lane Loading



12.9.8.6 Model 6 – Truck Loading with Centrifugal Force

The design truck is shown in **Figure 12.9.8.6-1**, which is Fig. 3.6.1.2.2-1 from the *LRFD Specifications*. For maximum positive moment, the minimum rear axle spacing of 14 ft controls. The maximum bending moment occurs with the middle axle load placed 2.33 ft from midspan.

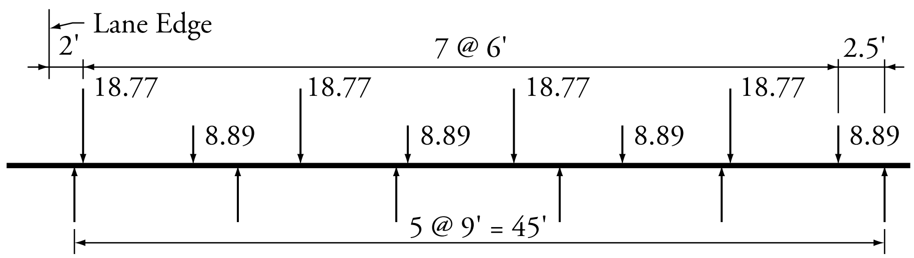
Figure 12.9.8.6-1 Characteristics of the Design Truck



The main axle wheel loads are 16 kips each, plus a 33 percent dynamic allowance, or 21.28 kips. For the design speed of 40 mph, the centrifugal force is 0.2374 of the truck weight (without dynamic allowance). This force acts 6 ft above the roadway. The overturning moment per main axle is 0.2374 times 32 kips times 6 ft, or 45.58 ft-kips. Dividing by the 6-ft wheel spacing, the wheel loads due to centrifugal force are ±7.6 kip. The total main axle wheel loads, including the 0.65 factor, m, are 0.65 (21.28 ±7.6) = 18.77 kips and 8.89 kips. The front axle wheel loads are 1/4 of this, or 4.69 kips and 2.22 kips.

The wheel loads are placed on fictitious, pin-ended members in order to transfer the loads to the main beams, as shown in **Figure 12.9.8.6-2** for the heavier axles.

Figure 12.9.8.6-2 Wheel Load Placement across Model for Heavy Axles



The added pin-ended members and loads that represent the truck loading for the condition producing maximum moment are shown in **Figure 12.9.8.6-3**.

12.9.8.7 Summary of Bending Moments

The bending moments for each of the six beams from the six loading models are summarized in **Table 12.9.8.7-1**. Pretensioning counteracts the moments from Model 1 while post-tensioning is used to counteract the moments from Models 2 through 6. The *LRFD Specifications* [Article 3.4.1 and Table 3.4.1.1], states that for checking tension in prestressed members at service load, the Service III load combination may be used. This combination is 1.00 (DC + DW) + 0.8(L + IM).

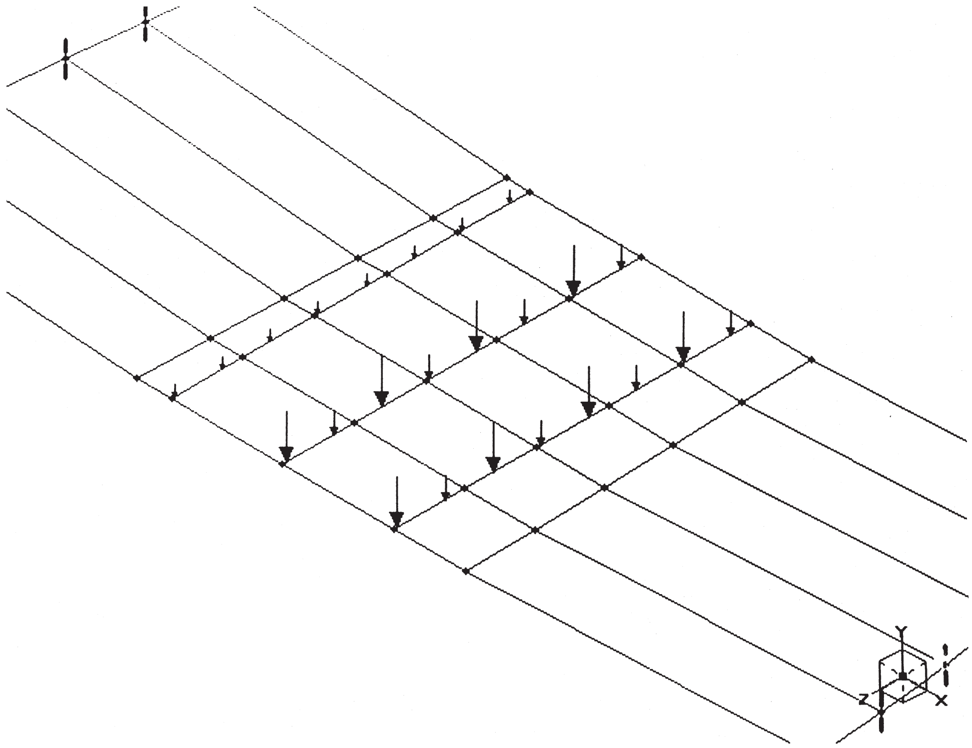


Figure 12.9.8.6-3
Truck Loading on Model 6

Table 12.9.8.7-1
Bending Moments in Each Beam

	Maximum Bending Moments, ft-kips					
	Beam Number					
	Outside					**Inside**
Load	**1**	**2**	**3**	**4**	**5**	**6**
Model 1 – Segments on Shores	204	199	193	187	181	176
Model 2 – Shore Loads	2,249	2,067	1,883	1,694	1,491	1,270
Model 3 – Deck & Haunch	2,119	1,973	1,823	1,662	1,479	1,286
Model 4 – Barrier & Surfacing	720	610	565	513	444	446
Model 5 – Lane Loading	649	605	551	491	420	341
Model 6 – Truck Loading	1,468	1,324	1,204	1,045	917	603
(0.8)Live Load = (0.8)(Models 5 + 6)	1,694	1,543	1,404	1,229	1,070	755
Models 2 + 3 + 4 + (0.8)(5 + 6)	6,782	6,193	5,675	5,098	4,484	3,757

12.9.9 Selection of Prestress Force

12.9.9.1 Pretensioning

The maximum self-weight bending moment for a beam segment is 204 ft-kips. The bottom fiber stress is M/S_b, or (204)(12)/(16,657) = 0.147 ksi. For y_b = 36.51 in., the eccentricity, e, is 34.51 in for strands centered at 2 in. from bottom of the beam. Try (4) 1/2 in.-dia strands with a force of 25 kips each.

Table 12.9.9.1-1 Stress Due to Pretensioning

Stress	Fiber Stress, ksi	
	Top	Bottom
P/A = 100/911	0.110	0.110
Pe/S = (100)(34.51)/S	-0.201	0.207
M/S = (204)(12)/S	0.143	-0.147
Pretension & Self-Weight	0.052	0.170

Because this is a temporary condition, a check for minimum reinforcement is not necessary.

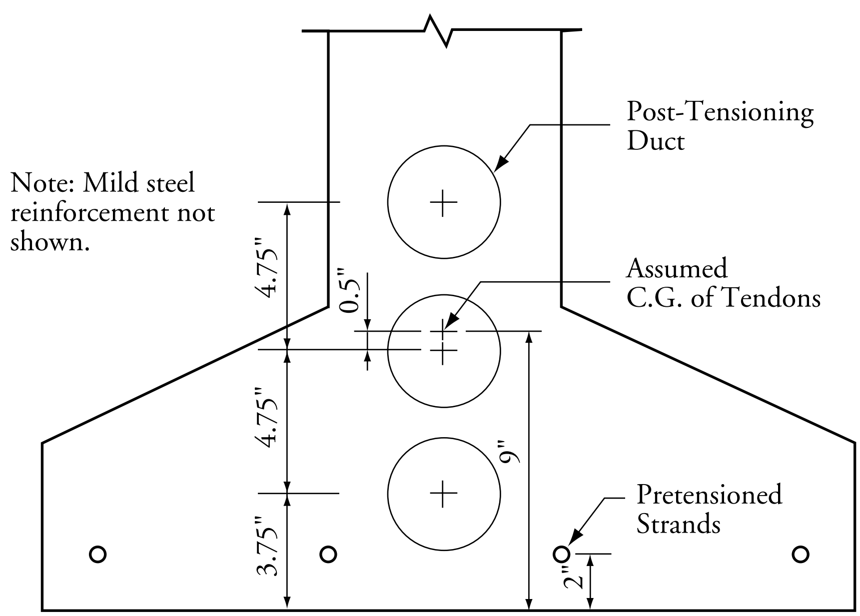
12.9.9.2 Post-Tensioning

Table 12.9.9.2-1 shows the stress to be resisted by post-tensioning. Assuming three tendons, the maximum eccentricity is estimated to be y_b less 9 in. (at the location of maximum moment) as seen in **Figure 12.9.9.2-1**.

Table 12.9.9.2-1 Bottom Fiber Stresses in Outside Beam at Location of Maximum Moment

Load	M/S	Bottom Stress, ksi
Shore Loads	(2,249)(12)/16,657 =	1.620
Deck & Haunch	(2,119)(12)/16,657 =	1.527
Barrier & Surfacing	(720)(12)/22,784 =	0.379
(0.8)Live	(1,694)(12)/22,784 =	0.892
Total Stress to be Compensated by Post-Tensioning, f_b =		**4.418**

Figure 12.9.9.2-1 Bottom Flange Detail at Maximum Moment Location



For preliminary design, assume zero tension in the bottom fiber. The required final force, P, is computed below, using non-composite section properties because the tensioning is assumed to be completed before casting the deck.

$P = f_b/(1/A + e/S_b)$

$P = 4.418/(1/911 + 27.51/16{,}657)$

$P = 1{,}607$ kips

Try (48) 0.6-in.-dia strands at 162 ksi ($0.6f_{pu}$)

$P = (48)(0.217)(162) = 1{,}687$ kips

A review of the total bending moments in **Table 12.9.8.7-1** indicates that the post-tensioning should be reduced in the other beams. Try 44 strands in Beam 2, 40 in Beam 3, 36 in Beam 4, 32 in Beam 5, and 28 in Beam 6.

12.9.9.3 Model 7 – Post-Tensioning

For the preliminary design, the post-tensioning trajectory is simplified to be three straight segments, with horizontal and vertical angle changes at the interior diaphragms. The tendons are modeled as bar elements, with a thermal coefficient equal to $1/E_p$. The tensioning of the model is done by applying a negative temperature change equal to effective prestress.

Figure 12.9.9.3-1 Model - 7 Post-Tensioning

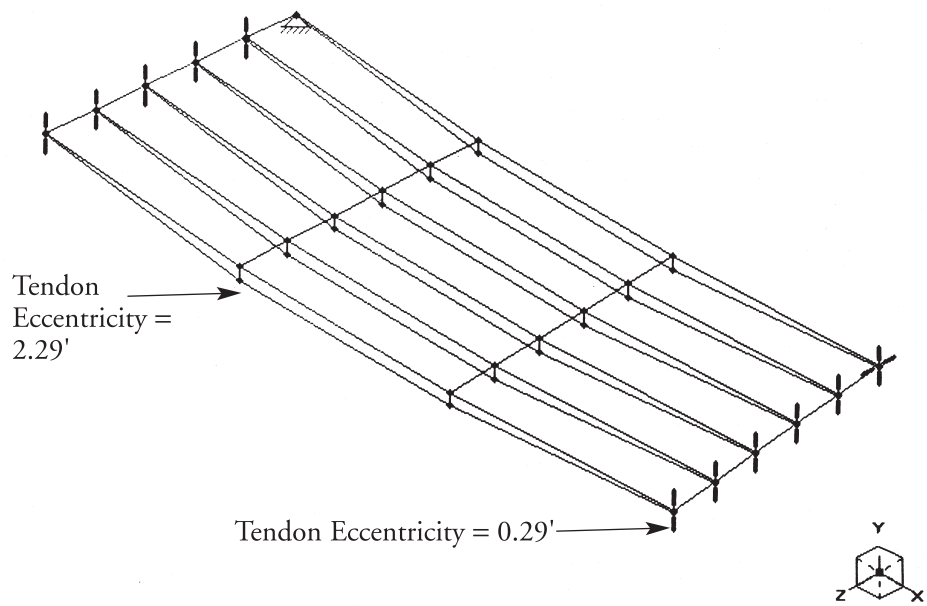


Figure 12.9.9.3-1 shows the post-tensioning model. Short, rigid stubs are used to connect the tendons to the beam gridwork. The length of these stubs is equal to the tendon eccentricity, 0.29 ft at the ends and 2.29 ft at the interior crossbeams. These stubs will also resist the transverse forces caused by the angle change of the tendons at the crossbeams.

For the middle third of the outer beam, the axial force is found to be 1,663 kips, and the bending moment due to post-tensioning is 3,650 ft-kips which agrees well with our assumptions. The tendon profiles will be held constant but the post-tensioning force changed for the remaining beams.

12.9.10 Results

12.9.10.1 Stresses in Outside Exterior Beam

Table 12.9.10.1-1 summarizes the stress history of the outside beam for the stages of service loads. The stresses are within the limits. As stated in LRFD Article 3.4.1, the Service I load combination is used to check compressive stresses, and the Service III load combination is used to check tensile stresses. Service I uses a load factor of 1.0 for live loads, whereas Service III uses a load factor of 0.8 for live loads.

Table 12.9.10.1-1 Stress Summary for Outside Beam

Load	Stresses, ksi		
	Top Slab	Top Beam	Bottom Beam
1. Pretensioning + Beam Segment Self Weight		0.152	0.170
2. Post-Tensioning: P/A = 1,663/911		1.825	1.825
3. Post-Tensioning: M/S = (3,650)(12)/S		-2.556	2.630
4. Shore Loads: M/S = (2,249)(12)/S		1,575	-1.620
5. Stress after Losses*		0.996	3.005
6. Deck & Haunch: M/S = (2,119)(12)/S		1.484	-1.527
7. Barrier & Surface: M/S = (720)(12)/S	0.151	0.135	-0.379
8. Dead Load**	0.151	2.615	1.099
9a. 0.8 Live Load: M/S = (1,694)(12)/S			-0.892
9b. 1.0 Live Load: M/S = (2,117)(12)/S	0.445	0.399	
	0.596	3.014	0.207
Stress Limits $0.60f'_c$ Compression & 0 Tension	2.400	3.900	0 Tension

* The stress before losses should also be checked. The allowable temporary compressive stress for this condition is $(0.60)f'_{ci} = (0.60)(6.5) = 3.90$ ksi. [LRFD Art. 5.9.4.2.1]

It appears, by inspection, that this stress should be OK.

** The allowable compressive stress in the beam under full dead load is $(0.45)f'_c = 2.925$ [LRFD Art. 5.9.4.2.1] OK.

12.9.10.2 Strength Limit State

The check for the strength limit state is done in the same manner as that presented in Section 9.4.9 for a straight beam. For the straight beam, the provided strength was 22 percent in excess of that required, and a similar amount of excess strength would be found for the beams in the curved bridge.

12.9.10.3 Crossbeams

The diaphragms function as crossbeams in the beam gridwork. They transfer load from the inside to the outside of the curve. This load transfer maintains equilibrium without the necessity of large torsional moments.

Figure 12.9.10.3-1 Model 2 - Crossbeam Shears and Moments

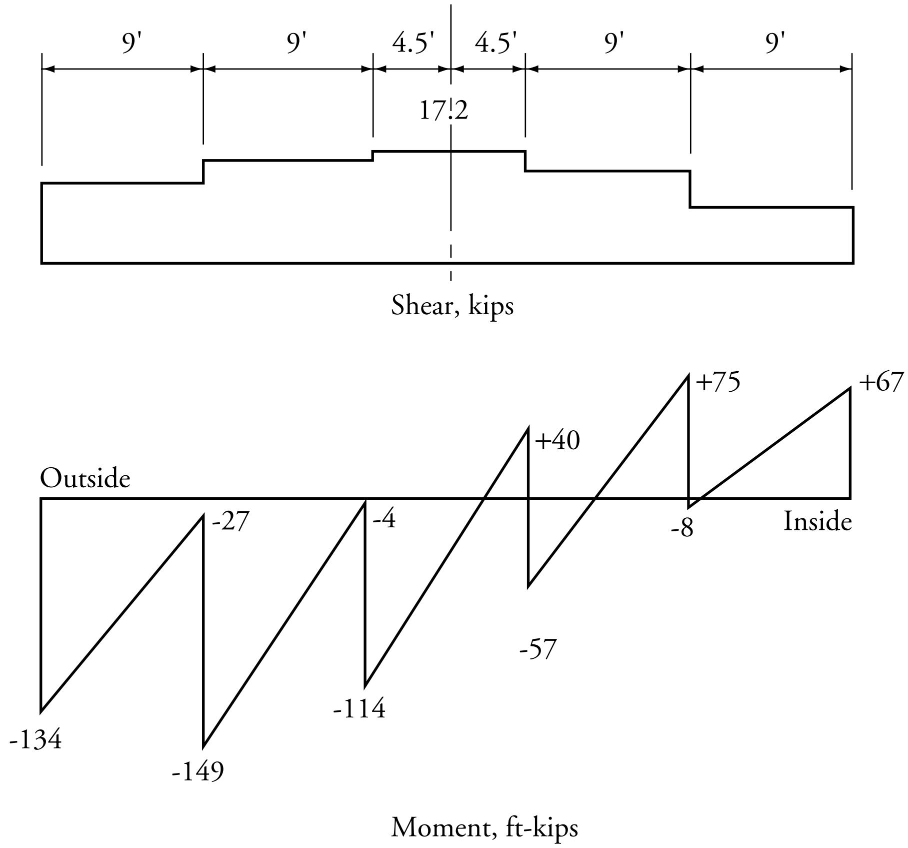


Figure 12.9.10.3-1 shows the shear and moment curves for an interior crossbeam for Model 2. The shear is relatively constant, transferring load to the outside. The crossbeam is also loaded by bending moments at each stringer. These moments equilibrate the primary bending moments in the stringers as they turn through an angle at the joint with the crossbeam.

Table 12.9.10.3-1 Factored Bending Moments in Crossbeam and at First Interior Beam

Load	M ft-kips	Load Factor	M_u ft-kips
Model 2 – Beams	-149	1.25	-186
Model 3 – Deck	-130	1.25	-163
Model 4 – Barrier & Surface	-138	1.50	-207
Model 5 + 6 – Live Loading	-110	1.75	-193
Model 7 – Prestress	-34	1.25	- 42
Total			-791

The maximum bending moment occurs at the first interior beam. **Table 12.9.10.3-1** shows the factored bending moments at this location. The *LRFD Specifications* do not give a load factor for prestressing. Because the bending from prestressing is additive to that from loads, a load factor of 1.25 (the same as for dead load) was conservatively used. The bending moments are well within the capacity of a non-prestressed beam. Although the crossbeam could be post-tensioned, the simple solution is to use a conventionally reinforced (non-prestressed) member.

12.9.10.4
Behavior Check

The behavior of the beam gridwork may be checked manually by observation of the bending moments applied to the crossbeam. The beams are bent through an angle ψ of 0.0667 radians at the crossbeam. The crossbeams must resist a moment of 0.0667 times the flexural bending moment in the beam.

Table 12.9.10.4-1
Beam Gridwork Behavior Check

Beam Number	Beam Bending Moment, ft-kips	M x ψ, ft-kips	Moment on Crossbeam, M_c ft-kips	Torque, M_t in Beam, ft-kips	$M_c + M_t$ ft-kips
1 (outside)	2,255	150	134	16	150
2	2,067	138	122	16	138
3	1,883	126	110	16	126
4	1,694	113	97	16	113
5	1,491	99	83	16	99
6 (inside)	1,264	84	67	17	84

Table 12.9.10.4-1 shows the flexural bending moment in each of the beams for Model 2, Shore Loads. The third column shows the bending moments multiplied by the angle ψ. The fourth column shows the moments in the crossbeams, from the gridwork analysis. The difference is resisted by torsion in the beams. This is compatibility torsion, caused by the fact that the bridge tilts slightly toward the outside of the curve.

12.9.10.5
Shear and Torsion

The beam gridwork is stable without torsional moments in its members. However, some torsion occurs due to the deformations of the gridwork. LRFD Article 5.8.2.1 requires torsion to be investigated when:

$$T_u > 0.25\phi T_{cr} \qquad \text{[LFRD Art. 5.8.2.1-3]}$$

where

T_u = factored torsional moment, in.-kips

ϕ = resistance factor [LRFD Article 5.5.4.2]

T_{cr} = torsional cracking moment, in.-kips, and where:

$$T_{cr} = 0.125\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_c}\right)\sqrt{1+\frac{f_{pc}}{0.125\sqrt{f_c'}}} \qquad \text{[LRFD Eq. 5.8.2.1-4]}$$

where

A_{cp} = total area enclosed by outside perimeter of concrete cross-section, in.^{2}

p_c = the length of the outside perimeter of the concrete section, in.

f_{pc} = compressive stress in concrete after prestress losses have occurred either at the centroid of the cross-section resisting transient loads or at the junction of the web and flange where the centroid lies in the flange, ksi

For consistency, the transformed section is used to compute A_{cp}, p_c, and the average f_{pc} on the transformed section.

$A_{cp} = A_c = 1{,}564$ in.^{2}

$p_c = 400$ in.

$$f_{pc} = P/A_c = 1{,}663/1{,}564 = 1.063 \text{ ksi}$$

$$T_{cr} = 0.125\sqrt{6.5}\left(\frac{(1{,}564)^2}{400}\right)\sqrt{1+\frac{1.063}{0.125\sqrt{6.5}}} = 4{,}058 \text{ in.-kips} = 338 \text{ ft-kips}$$

Check if $T_u < 0.25\phi T_{cr} = 0.25(0.9)(338) = 76.1$ ft-kips

Torsion may be neglected if the ultimate torque is less than 76.1 ft-kips. Examine torsion in the outside exterior beam.

Table 12.9.10.5-1 Torsional Moments in Outside Beam

Load	T ft-kips	Load Factor	T_u ft-kips
Model 2 – Beams	-16.1	1.25	-20.1
Model 3 – Deck	-15.9	1.25	-19.9
Model 4 – Barrier & Surface	-9.5	1.50	-14.3
Models 5 + 6: Live Loads	-15.8	1.75	-27.6
Model 7 – Prestress	+21.7	0.9*	+19.5
Total			62.4

*Because the prestress acts to oppose the other torsional moments, a load factor of 0.9 was conservatively assumed.

Table 12.9.10.5-1 shows that T_u is less than 76.1 ft-kips. Therefore, torsion may be neglected.

The shear design is performed in a manner similar to that shown in Section 9.4.11 for a straight beam. Note that for post-tensioned beams, LRFD Article 5.8.2.7 requires that the effective web width, b_v, be computed deducting one half of the diameter of grouted ducts. The actual web width is 8 in., but the effective width, b_v, will be approximately 6 in., similar to that for the straight beam.

12.9.11 Comparison To Straight Bridge

Compared to the straight bridge of Section 9.4, the additional cost items for this curved bridge are as follows:

1. Additional design cost.
2. The cost and inconvenience of shoring. This may be at least partially offset by the reduced shipping and erection costs for the beam segments, as compared to full-length beams.
3. The additional cost of concrete for 2 in.increase in width of beams (1 cf/lf of added concrete) due to addition of post-tensioning.
4. The cost of intermediate crossbeams (not required for straight bridge).
5. Additional cost of post-tensioning compared to pretensioning.
6. The cost of additional strand. Less strand is used in the other five beams, but the total strand area (including pretensioned strands) for the six beams is about 20 percent greater than for the straight bridge.
7. A wider cap beam may be necessary to allow clearance for the post-tensioning jacks between the ends of the beams.

12.10 DETAILED DESIGN

The detailed final design of the curved beam bridge will generally follow the design for a similar straight bridge, as described in Section 9.4. Some special points relating to the post-tensioned curved bridge are given below.

12.10.1 Loss of Prestress

The calculation of prestress losses for post-tensioned beams is somewhat different from that for pretensioned beams. Refer to *LRFD Specifications*, Article 5.9.5.

12.10.2 Computer Models

The computer models used in the preliminary design to analyze the effect of vertical loads are adequate for use in the detailed final design. The model for the post-tensioning (Model 7) should be refined, using more realistic tendon trajectories and accurate estimates of the initial and final prestress forces. In addition, the optimum prestress levels for all six beams needs to be investigated more thoroughly. This is a trial-and-error process.

12.10.3 Crossbeam Details

The detailing of crossbeams between the beam segments is similar to that described in Chapter 11 for spliced beams. Refer to Chapter 11.

Initial stresses in the beams at the crossbeam location need to be calculated in order to determine the required initial concrete strength, f_{ci}, for the crossbeam concrete at the time the beams are post-tensioned.

The post-tensioning tendons undergo an angle change at the crossbeams. This creates an inward radial force equal to the tension in the tendon multiplied by the angle change in radians. At the exterior beam on the inside of the curve, reinforcement must be provided to tie this force back into the crossbeam. See Podolny 1985 for a further discussion of this problem.

12.10.4 Post-Tensioning Anchorages

Post-tensioned beams will generally be detailed with end blocks to contain the tendon anchors. The design of post-tensioned anchorage zones is given in *LRFD Specifications*, Article 5.10.9. An alternate method is to place anchorages in the end walls to eliminate the need for end blocks on the beams.

12.11 REFERENCES

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